Value of Flexibility in Managing R&D Projects Revisited*

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Abstract

In this paper we consider the question of whether an increase in uncertainty increases the value of an R&D project. We also consider the related question of the impact of increased project uncertainty on the value of management flexibility, defined as the difference in value when the project is managed "actively" versus when it is under "passive" management. These questions have already been formulated in an insightful paper in the literature where different sources of variability and uncertainty in R&D projects are identified, and abandonment and improvement at interim stages are considered as options that provide management flexibility. We follow the same formulation. We derive a set of negative results that are contrary to the results of the above-mentioned paper and a set of positive results that are different from those presented. Our negative results indicate that when the source of variability is development uncertainty or market requirement uncertainty, one cannot make a general statement about the impact of increased uncertainty. In some cases the value of flexibility (and project value) increases and in others it decreases. On the other hand, if the source of variability is market payoff, we show that increased variability increases either the overall project value or the project option value. If the increased variability of market payoff increases the "passive" value of the project, the overall project value also increases, and if it decreases the "passive" value, the value of flexibility, i.e., the project option value increases.

7 Electronic supplement pages

We include supplementary material in this section. The material is generally organized according to the sections of the paper.

7.1 Stochastic order

Most of the proofs in the paper rely on the notion of stochastic order. We briefly review this concept and results related to it (see, e.g., Ross 1996 Chapter 9, for more details).

Definition. *A random variable X is said to be stochastically greater than or equal to random variable Y if $P(X > x) \geq P(Y > x)$, or equivalently, $F_X(x) \leq F_Y(x)$, for any real number x (F_X and F_Y are cumulative distributions of X and Y respectively). We denote this relation by $X \geq^{st} Y$.*

The following results will be used in the proofs in this section.

- *$X \geq^{st} Y$ if and only if there exists a coupling of X and Y such that $X \geq Y$.* In other words, it is possible to pair up realizations of X and Y such that each sample of X is greater than or equal to a sample from Y [12].
- *$X \geq^{st} Y$ if and only if for any non-decreasing function f, $E[f(X)] \geq E[f(Y)]$.*

7.2 Supplements to Section 3

Proposition 3.1. *If the final payoff function $\Pi(.)$ is monotone non-decreasing, the project value evaluated at any stage before launch, i.e., $V_t(.)$, is also monotone non-decreasing.*

Proof. The proof is by backward induction. $V_T(x) = \Pi(x)$ is monotone non-decreasing in x by assumption. Now assume that $V_{t+1}(x)$ is monotone non-decreasing in x. Let x' and x be two states at stage t and $x' > x$. We need to show that $V_t(x') \geq V_t(x)$. Let the optimal action at state x be u^*. Assume that the same action is applied at state x' (after this stage optimal decisions are made). Let $V_t^c(x')$ be the project value at state x' in stage t under

[12] Using a single uniform random number and the inverse transform method for generating samples of X and Y gives one such coupling. See, e.g., Ross 1996 Proposition 9.2.2.

this assumption, then:

$$V_t^c(x') - V_t(x) = \begin{cases} & \text{if } u^* = \text{ continue} \\ \frac{1}{1+r}E\left[V_{t+1}(x' + k(u^*) + \omega_t) - V_{t+1}(x + k(u^*) + \omega_t)\right] & \\ & \text{or improve;} \\ 0 & \text{if } u^* = \text{ abandon.} \end{cases}$$

$x' \geq x$ implies $x' + k(u^*) + \omega_t \geq x + k(u^*) + \omega_t$. Moreover, monotonicity of $V_{t+1}(.)$ implies

$$V_{t+1}(x' + k(u^*) + \omega_t) - V_{t+1}(x + k(u^*) + \omega_t) \geq 0.$$

Given that the expected value of a non-negative random variable is non-negative, we have

$$E[V_{t+1}(x' + k(u^*) + \omega_t) - V_{t+1}(x + k(u^*) + \omega_t)] \geq 0.$$

Hence, $V_t^c(x') - V_t(x) \geq 0$. Note that $V_t(x') \geq V_t^c(x')$ because $V_t(x')$ is the project value under optimal action at state x'. Therefore,$V_t(x') - V_t(x) \geq V_t^c(x') - V_t(x) \geq 0$, and the proof by induction is complete. □

Corollary 3.1. *If it is optimal to abandon a project at state x at stage t, then it is optimal to abandon the project at any state smaller than x at that stage.*

Proof. Let $x' < x$. Since it is optimal to abandon the project at state x and stage t, $V_t(x) = 0$. Monotonicity of $V_t(.)$ implies that $V_t(x') \leq V_t(x) = 0$. On the other hand, $V_t(x') \geq 0$ since abandoning the project at x' gives a value of 0. Therefore, $V_t(x') = 0$ and it is optimal to abandon the project at x' in stage t.□

Optimal policy and optimal value function for Example 3.2

Table 3 gives the optimal value function for the last stage before launch of the product, $t = 5$ (not all nodes are shown).

7.3 Supplement to Section 4

Examples for Proposition 4.2.

Examples 4.3-1 Consider two projects with the following data. Project 1:($M = 250$, $m = 50$); ($\mu = -3$, $\sigma = 3$); ($p = 0.5$, $N = 2$); ($c_5 = 100$, $\alpha_5 = 10$); ($c_4 = 10$, $\alpha_4 = 15$);

State i	$V_5(i)$	Optimal Policy	$(V_5(i) - V_5(i-0.5))$
4.0	457.32	Continue	16.08
3.5	441.24	Continue	19.28
3.0	421.96	Continue	**22.51**
2.5	399.45	Continue	**25.62**
2.0	373.84	Continue	**24.38**
1.5	349.45	Improve	25.62
1.0	323.84	Improve	28.40
0.5	295.43	Improve	30.69
0.0	264.74	Improve	**32.32**
-0.5	232.42	Improve	**33.16**
-1.0	199.26	Improve	**33.16**
-1.5	166.10	Improve	**32.32**
-2.0	133.78	Improve	30.69
-2.5	103.08	Improve	26.53
-3.0	76.55	Continue	19.28
-3.5	57.28	Continue	16.08
-4.0	41.19	Continue	13.08

Table 3: Optimal project values and policy at $T - 1 = 5$.

$(c_3 = 4,\ \alpha_3 = 10)$; $(c_2 = 2,\ \alpha_2 = 5)$; $(c_1 = 1,\ \alpha_1 = 10)$; $(c_0 = 1,\ \alpha_0 = 6)$; $(I = 2,\ r = 0.08)$; T = 6. Project 2 has the same data as project 1, except $(\overline{M} = 300,\ \overline{m} = 0)$. For project 1: NPV= 47.99, option value = 6.26, and project value = 54.25. For project 2: NPV = 68.83, option value = 14.81, and project value = 81.65. Note that the option value for project 2 is higher then that of project 1. On the other hand, in the following example the option value of project 2 is smaller than that of project 1. We keep the same data as above and only change the continuation cost at the stage $T - 1$. Let $c(5) = 200$; then for project 1: NPV = -20.07, option value = 18.07, and project value = -2.00. For project 2: NPV = -1.22, option value = 14.81, and project value = 13.59.

Example 4.3-2 Consider two projects with the following data. Project 1:$(M = 350,\ m = 150)$; $(\mu = 2,\ \sigma = 3)$; $(p = 0.5,\ N = 2)$; $(c_5 = 100,\ \alpha_5 = 45)$; $(c_4 = 50,\ \alpha_4 = 35)$; $(c_3 = 8,\ \alpha_3 = 30)$; $(c_2 = 4,\ \alpha_2 = 25)$; $(c_1 = 2,\ \alpha_1 = 20)$; $(c_0 = 1,\ \alpha_0 = 6)$; $(I = 2,\ r = 0.08)$; T = 6. Project 2 has the same data as project 1, except $(\overline{M} = 450,\ \overline{m} = 50)$. For project 1: NPV = 11.41, option value = 6.87, and project value = 18.28. For project 2: NPV = -15.28, option value = 40.05, and project value = 24.77. Note that the project value for project 2 is higher than that of project 1. On the other hand, in the following example the project value of project 2 is smaller than that of project 1. We keep the same data and only change the market requirement mean, $\mu = 3$. For project 1: NPV = 0.41, option value = 5.61, and project value = 6.02. For project 2: NPV = −37.28, option value = 43.13, and project value = 5.85.

Example for Theorem 4.2.

Example 4.3.1-1 Consider two projects with the following data. Project 1:($M = 250$, $m = 50$); ($\mu = -3$, $\sigma = 3$); ($p = 0.5$, $N = 2$); ($c_5 = 100$, $\alpha_5 = 10$); ($c_4 = 10$, $\alpha_4 = 15$); ($c_3 = 4$, $\alpha_3 = 10$); ($c_2 = 2$, $\alpha_2 = 5$); ($c_1 = 1$, $\alpha_1 = 10$); ($c_0 = 1$, $\alpha_0 = 6$); ($I = 2$, $r = 0.08$); T = 6. Project 2 has the same data as project 1, except ($\overline{M} = 300$, $\overline{m} = 0$). For project 1: NPV= 47.99, option value = 6.26, and project value = 54.25. For project 2: NPV = 68.83, option value = 14.81, and project value = 81.65. Note that the option value for project 2 is higher then that of project 1. On the other hand, in the following example the option value of project 2 is smaller than that of project 1. We keep the same data as above and only change the continuation cost at the stage $T - 1$. Let $c(5) = 200$; then for project 1: NPV = −20.07, option value = 18.07, and project value = −2.00. For project 2: NPV = −1.22, option value = 14.81, and project value = 13.59.

Example 4.3.1-2 Consider two projects with the following data. Project 1:($M = 350$, $m = 150$); ($\mu = 2$, $\sigma = 3$); ($p = 0.5$, $N = 2$); ($c_5 = 100$, $\alpha_5 = 45$); ($c_4 = 50$, $\alpha_4 = 35$); ($c_3 = 8$, $\alpha_3 = 30$); ($c_2 = 4$, $\alpha_2 = 25$); ($c_1 = 2$, $\alpha_1 = 20$); ($c_0 = 1$, $\alpha_0 = 6$); ($I = 2$, $r = 0.08$); T = 6. Project 2 has the same data as project 1, except ($\overline{M} = 450$, $\overline{m} = 50$). For project 1: NPV = 11.41, option value = 6.87, and project value = 18.28. For project 2: NPV = −15.28, option value = 40.05, and project value = 24.77. Note that the project value for project 2 is higher than that of project 1. On the other hand, in the following example the project value of project 2 is smaller than that of project 1. We keep the same data and only change the market requirement mean, $\mu = 3$. For project 1: NPV = 0.41, option value = 5.61, and project value = 6.02. For project 2: NPV = −37.28, option value = 43.13, and project value = 5.85.

Example for Theorem 4.3.

Example 4.4-1 Consider three projects with the following data. Project 1:($M = 400$,

$m = 100$); ($\mu = 7$, $\sigma = 0.01$); ($p = 0.5$, $N = 2$); ($c_1 = 60$, $\alpha_1 = 3$); ($c_0 = 35$, $\alpha_0 = 2$); ($I = 2$, $r = 0.08$); T = 2. Project 2 has the same data as project 1, except ($\overline{\sigma} = 2$). Project 3 has also the same data, except ($\overline{\sigma}' = 4$). For project 1: NPV = -6.82, option value = 4.82, and project value = -2.00. For project 2: NPV = -6.57, option value = 4.57, and project value = -2.00. For project 3: NPV = 7.00, option value = 12.82, and project value = 19.82. Note that the option value for project 1 is greater than the one for project 2, however it is smaller than the one for project 3.

Example 4.4-2 Consider three projects with the following data. Project 1:($M = 300$, $m = 0$); ($\mu = -3$, $\sigma = 0.01$); ($p = 0.5$, $N = 2$); ($c_1 = 10$, $\alpha_1 = 5$); ($c_0 = 10$, $\alpha_0 = 15$); ($I = 2$, $r = 0.08$); T = 2. Project 2 has the same data as project 1, except ($\overline{\sigma} = 3$). Project 3 has also the same data, except ($\overline{\sigma}' = 4$). For project 1: NPV = 235.94, option value = 0.00, and project value = 235.94. For project 2: NPV = 191.02, option value = 13.04, and project value = 204.07. For project 3: NPV = 177.46, option value = 12.68, and project value = 190.14. Note that the option value for project 1 is zero, which is smaller than the one for project 2. On the other hand, option value for project 2 is greater than the one for project 3.

Example for Theorem 4.4.

Example 4.4-3 Consider two projects with the following data. Project 1:($M = 250$, $m = 50$); ($\mu = -3$, $\sigma = 3$); ($p = 0.5$, $N = 2$); ($c_1 = 100$, $\alpha_1 = 10$); ($c_0 = 10$, $\alpha_0 = 15$); ($I = 2$, $r = 0.08$); T = 2. Project 2 has the same data as project 1, except ($\overline{M} = 300$, $\overline{m} = 0$). For project 1: NPV= 79.80, option value = 2.90, and project value = 82.70. For project 2: NPV = 107.69, option value = 8.41, and project value = 116.10. Note that the option value for project 2 is higher then that of project 1. On the other hand, in the following example the option value of project 2 is smaller than that of project 1. We keep the same data as above and only change the continuation cost at the stage $T - 1$. Let $c(1) = 200$; then for project 1: NPV = -12.80, option value = 10.80, and project value = -2.00. For project 2: NPV = 15.10, option value = 8.41, and project value = 23.51.

Proof of Theorem 4.2.

In order to prove Theorem 4.2, we need some preliminary results.

Lemma 6.4. *Consider a symmetric project and assume the expected market requirement is zero. Then, the Net Present Value (NPV) of the project (evaluated at $t = 0$ and state 0) is given by*

$$NPV_0(0) = \frac{a}{(1+r)^T} - (\sum_{t=0}^{T-1} \frac{c(t)}{(1+r)^t} + I).$$

Proof. In NPV calculation it is assumed that the only option available to management is continuation of the project. Therefore, the total cost of the project is the second term on the right hand side of the above equation. All we need to show is that the expected payoff of the project is the first term on the right hand side of the above equation. Under the symmetry assumptions, the state space at the terminal stage T can be represented by

$$S_T = \{0, \pm x_1, \ldots, \pm x_J\}$$

for some J. Moreover, due to symmetry we have $P(X_T = x_j) = P(X_T = -x_j)$ for all $j = 1, \ldots, J$ and $F(x_j) + F(-x_j) = 1$. Therefore, the undiscounted expected payoff is given by

$$\begin{aligned} E[\Pi(X_T)] &= \sum_{j=1}^{J} \{(m(1 - F(x_j)) + M(F(x_j) + (m(1 - F(-x_j)) + MF(-x_j)\}P(X_T = x_j) \\ &\quad + \frac{M+m}{2} P(X_T = 0) \\ &= \sum_{j=1}^{J} (M+m) \frac{P(X_T = x_j) + P(X_T = -x_j)}{2} + \frac{M+m}{2} P(X_T = 0) \\ &= \frac{M+m}{2} = a. \end{aligned}$$

Therefore, the discounted payoff is as shown in equation above and the proof of the lemma is complete.

Next we consider the impact of increased market payoff variability on the NPV of a symmetric project.

Proposition 7.1 *Consider a symmetric project. Then, when market payoff variability increases, (1) if $\mu < 0$, the NPV of the project increases, (2) if $\mu = 0$, the NPV of the project remains unchanged, and (3) if $\mu > 0$, the NPV of the project decreases.*

Proof. The proof for case 2 ($\mu = 0$) is an immediate corollary of Lemma 6.4. To prove the result for case 1 and 3 let $Y(T)$, as before, denote the terminal state of project 1 (and therefore project 2). Furthermore let Π^{μ} and $\overline{\Pi}^{\mu}$ denote the expected payoff functions for projects 1 and 2 when market requirement mean equal to μ. It can be easily verified that $E[\Pi^{\mu}(Y(T)] = E[\Pi^0(Y(T) - \mu)]$. Hence

$$\overline{NPV}_0^{\mu}(0) - NPV_0^{\mu}(0) = \frac{1}{(1+r)^T} E[(\overline{\Pi}^{\mu} - \Pi^{\mu})(Y(T))] = \frac{1}{(1+r)^T} E[(\overline{\Pi}^0 - \Pi^0)(Y(T) - \mu)].$$

When $\mu < 0$, $Y(T) - \mu >^{st} Y(T)$, therefore

$$E[(\overline{\Pi}^0 - \Pi^0)(Y(T) - \mu)] \geq E[(\overline{\Pi}^0 - \Pi^0)(Y(T))] = 0.$$

Therefore, in this case the NPV increases and case 1 is proved.

When $\mu > 0$, $Y(T) >^{st} Y(T) - \mu$, therefore

$$E[(\overline{\Pi}^0 - \Pi^0)(Y(T) - \mu)] \leq E[(\overline{\Pi}^0 - \Pi^0)(Y(T))] = 0,$$

and therefore in this case NPV decreases and case 3 is proved.

Proof of Theorem 4.2. The proof follows directly from Theorem 4.1 and Proposition 7.1.

7.4 Analysis of increased variability and the value of financial options in the setting of the paper

In this section we show that we can cast the problem of studying the impact of increased variability/volatility on the value some financial options in the setting of this paper and derive known results using the approach used in the paper. This discussion makes the relationship between financial and real options from the point of view of the impact of increased variability quite clear.

Consider a European option on a financial asset following the standard stochastic differential equation

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW.$$

Assume that the initial value of the asset is S_0 and the strike price is K. For simplicity of the presentation assume the maturity of the option is at $T = 1$

One approach to valuation of financial options is to price them in a risk-neutral setting, i.e., in a setting where all asset values "grow" at the risk-free rate, say r. In other words, in the risk-neutral setting the asset value follows the stochastic differential equation

$$\frac{dS_t}{S_t} = rdt + \sigma dW_t.$$

Similar to R&D projects, in a European option the payoff is obtained at the terminal time. The terminal value of the asset, S_1 has a lognormal distribution and $E[ln(S_1/S_0)] = r - \frac{1}{2}\sigma^2$, and $Var(ln(S_1/S_0) = \sigma^2$.

To cast this problem in the setting of this paper, note that all of uncertainty is captured by the Brownian motion $\{W_t; t \geq 0\}$ (more precisely for the European option we are considering by the standard normal random variable W_1). We take this variable to be the state of the project. The initial state of the project is assume to be $W_0 = 0$.

We assume that the project has two stages. "Development" cost in the first stage is zero. The project state, W_1, is observed at the end of the first stage at time $t = 1$. There is a "development" cost of K during the second stage. The duration of the second stage is zero and the state of the project does not change during the second stage of the project. There is a terminal payoff that depends on the value of the terminal state. The terminal payoff function is given by

$$J(w, \sigma) = S_0 \ Exp[r - \frac{1}{2} \ \sigma^2 + \sigma \ w].$$

The decision maker can only make a decision at the end of stage 1 and the decision is whether to continue the project or to abandon it. The default decision is to continue the project.

It is easy to see that the passive value of the project is $S_0 - Ke^{-r}$ which is independent of σ, the project value is the value of a European call, and the value of flexibility is the value of European put.

Therefore we can analyze the impact of increasing volatility/variability, i.e., σ using the methodology of this paper. Note that the project is symmetric, the payoff function is a convex function of the terminal state w, and σ is a parameter of the terminal payoff function, i.e., exactly the setup for which we have derived positive results in this paper.

In this particular case of European call and put, the terminal payoff function as a function of σ does not satisfy the conditions specified in Proposition 6.5 and as a result the increase in value of European call and put as a result of an increase in σ does not follow directly from this Proposition. However, we can directly show that if $\overline{\sigma} > \sigma$ then $E[J(S_1, \overline{\sigma}) - J(S_1, \sigma)|\mathcal{A}'] > 0$ and the increase in value of European call and put follows immediately.