

## Customer Intimacy and Cross-Selling Strategy

Akcura, Srinivasan

### Technical Appendix

**Solution of the model.** The profit function is  $p + \theta\alpha i$ . Because of the participation constraint,  $p = \theta - r(\alpha)i^2$ . By replacing  $p$  with  $\theta - r(\alpha)i^2$  in the profit function and taking the derivative with respect to  $i$ , we find the optimal values for an interior solution. (Note that second derivative is negative and there exists a unique solution.) The optimal profit equals  $\theta(1 + \alpha i^*) - r(\alpha)i^{*2}$ . When  $i^* < i_0$ , the firm cannot profit from the information and finds it beneficial to ask either for no information or  $i_0$ . The firm asks for  $i^* = i_0$  as long as  $\alpha\theta i_0 - r(\alpha)i_0^2 \geq 0$ ; otherwise the firm asks for no information. On the other hand, the firm asks for the maximum information when the interior (unconstrained) optimal calls for more than  $\bar{i}$ . The complete solution can be represented as  $\{(i^*, p^*) \mid [I, \theta - r(\alpha)I^2] \text{ for } I \geq i_0; [i_0, \theta - r(\alpha)i_0^2] \text{ for } 2I \geq i_0 > I; (0, \theta) \text{ for } i_0 > 2I\}$ , where  $I$  equals  $\frac{\alpha\theta}{2r(\alpha)}$  if  $\bar{i} > \frac{\alpha\theta}{2r(\alpha)}$ ,  $\bar{i}$  otherwise.

**Proof of Proposition 1.**  $p^*$  becomes negative if  $\alpha^2\theta > 4r(\alpha)$ . As  $r(\alpha)$  increases, since

$\frac{d}{dr(\alpha)}i^* < 0$ ,  $\frac{d}{dr(\alpha)}p^* > 0$ , information level decreases and price increases. In turn, the profit

decreases since  $\frac{d}{dr(\alpha)}\Pi^* = -\left(\frac{\alpha\theta}{2r(\alpha)}\right)^2 < 0$ .  $\frac{d}{d\theta}p^* < 0$  if  $\alpha^2\theta > 2r(\alpha)$  and  $\frac{d}{d\theta}p^* > 0$

otherwise. Then, as customer valuation increases, low-risk customers pay a lower price while high-risk customers pay a higher price.

**Proof of Proposition 2.** The derivative of the optimum information level is

$\frac{d}{d\alpha} i^* = \frac{\theta}{2r(\alpha)} \left[ 1 - \frac{\alpha}{r(\alpha)} r_\alpha(\alpha) \right]$  which is negative when  $r_\alpha(\alpha) \frac{\alpha}{r(\alpha)} > 1$ . Similarly, for price and

profit, we have  $\frac{d}{d\alpha} p^* = -\frac{\theta^2 \alpha}{4r(\alpha)} \left[ 2 - \frac{\alpha}{r(\alpha)} r_\alpha(\alpha) \right]$  and  $\frac{d}{d\alpha} \Pi^* = \frac{\alpha \theta^2}{4r(\alpha)} \left[ 2 - \frac{\alpha}{r(\alpha)} r_\alpha(\alpha) \right]$ , which

are positive and negative when  $r_\alpha(\alpha) \frac{\alpha}{r(\alpha)} > 2$ . Hence, the profit increases as cross-selling

decreases if  $r_\alpha(\alpha) \frac{\alpha}{r(\alpha)} > 2$ . Then, the firm finds it beneficial to limit the cross-selling. The

optimum cross-selling is set to a level such that it will collect the maximum information. Then,

$\frac{r(\alpha^*)}{\alpha^*} = \frac{\theta}{2\bar{i}}$ . Note that  $\frac{r(\alpha)}{\alpha}$  is an increasing function of  $\alpha$  given  $r_\alpha(\alpha) \frac{\alpha}{r(\alpha)} > 2$ . Hence, as

$\bar{i}$  increases,  $\alpha^*$  decreases. When a firm does not follow a commitment strategy and the

customers do not know the cross-selling activity, the firm always has an incentive to sell more

than what customers expect. For example, assume that customer expectation for cross-selling is

$\hat{\alpha}$ . Then, the firm can charge according to  $r(\hat{\alpha})$ , but profit by setting  $\alpha$  to its maximum level.

However, the customers, knowing the firm incentives, rightly expect that and set  $\hat{\alpha}$  to its

maximum level.