Risk Assessment for Banking Systems – Appendix

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A Distribution of value for residual loan portfolio positions

Percentiles	10	25	50	75	90	μ	σ
Small	56%	67%	79%	89%	100%	78%	16%
Medium	38%	49%	59%	68%	77%	58%	17%
Large	11%	17%	33%	43%	89%	35%	27%
All banks	37%	50%	62%	74%	88%	62%	20%

Table 1. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for September 2000 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.

Percentiles	10	25	50	75	90	μ	σ
Small	57%	67%	78%	88%	100%	76%	17%
Medium	37%	48%	58%	68%	77%	57%	18%
Large	10%	20%	32%	44%	92%	37%	26%
All banks	36%	49%	61%	74%	88%	61%	21%

Table 2. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for September 2001 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.

Percentiles	10	25	50	75	90	μ	σ
Small	55%	67%	77%	87%	100%	76%	17%
Medium	37%	47%	57%	66%	76%	56%	18%
Large	9%	22%	35%	46%	93%	39%	26%
All banks	36%	49%	61%	74%	87%	60%	21%

Table 3. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for September 2002 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.

Percentiles	10	25	50	75	90	μ	σ
Small	54%	66%	78%	88%	100%	76%	18%
Medium	35%	47%	57%	66%	77%	56%	18%
Large	6%	22%	34%	46%	93%	38%	27%
All banks	35%	49%	60%	73%	87%	60%	21%

Table 4. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for December 2002 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.

B Estimating the L matrix

Assume that we have, in total, K constraints that include all constraints on row and column sums as well as on the value of particular entries. Let us write these constraints as

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{kij} l_{ij} = b_k \tag{1}$$

for $k = 1, ..., K$ and $a_{kij} \in \{0, 1\}$.

We seek to find the matrix L that has the least discrepancy to some a priori matrix U with respect to the (generalized) cross entropy measure

$$\mathcal{C}(L, U) = \sum_{i=1}^{N}\sum_{j=1}^{N} l_{ij} \ln(\frac{l_{ij}}{u_{ij}}) \tag{2}$$

among all the matrices satisfying (1) with the convention that $l_{ij} = 0$ whenever $u_{ij} = 0$ and $0 \ln(\frac{0}{0})$ is defined to be 0.

The constraints for the estimations of the matrix L are not always consistent. For instance, the liabilities of all banks in sector k against all banks in sector l typically do not equal the claims of all banks in sector l against all banks in sector k. We deal with this problem by applying a two step procedure:

In a first step, we replace an a priori matrix U reflecting only possible links between banks by an a priori matrix V that takes actual exposure levels into account. As there are seven sectors, we partition V and U into 49 sub-matrices V^{kl} and U^{kl}, which describe the liabilities of the banks in sector k against the banks in sector l and our a priori knowledge. Given the bank balance sheet data, we define $u_{ij} = 1$ if bank i belonging to sector k might have liabilities against bank j belonging to sector l and $u_{ij} = 0$ otherwise. The (equality) constraints are that the liabilities of bank i against the sector l equal the row sum of the sub-matrix and that the claims of bank j against the sector k equal the column sum of the sub-matrix, i.e.,

$$\sum_{j \in l} v_{ij} = \text{liabilities of bank i against sector l} \tag{3}$$

$$\sum_{i \in k} v_{ij} = \text{claims of bank j against sector k} \tag{4}$$

For the matrices describing claims and liabilities within a sector (i.e. V^{kk}) that has a central institution, we get further constraints. Suppose that bank j^* is the central institution. Then

$$v_{ij^*} = \text{liabilities of bank i against central institution} \tag{5}$$

$$v_{j^*i} = \text{claims of bank i against central institution} \tag{6}$$

Though these constraints are inconsistent, given our data, we use the information to get a revised matrix V, which reflects our a priori knowledge better than the initial matrix U. Contrary to U, which consists only of zeroes and ones, the entries in V are adjusted to the actual exposure levels.[1]

In a second step, we recombine the results of the 49 approximations V^{kl} to get an entire $N \times N$ improved a priori matrix V of inter-bank claims and liabilities. Now we replace the original constraints by just requiring that the sum of *all* (interbank) liabilities of each bank equals the row sum of L and the sum of *all* claims of each bank equals the column sum of L.

$$\sum_{j=1}^{N} l_{ij} = \text{liabilities of bank i against all other banks} \tag{7}$$

$$\sum_{i=1}^{N} l_{ij} = \text{claims of bank j against all other banks} \tag{8}$$

Again, we face the problem that the sum of all liabilities does not equal the sum of all claims but corresponds to only 96 of them. By scaling the claims of each bank by 0.96 we enforce

[1]Note that the algorithm that calculates the minimum entropy entries does not converge to a solution if data are inconsistent. Thus to arrive at the approximation V, we terminate after ten iterations immediately after all row constraints are fulfilled.

consistency.[2] Given these constraints and the prior matrix V we estimate the matrix L.

Finally, we can use the information on claims and liabilities with the central bank and with banks abroad. By adding two further nodes and by appending the rows and columns for these nodes to the L matrix, we get a closed (consistent) system of the interbank network.

C Robustness Checks

We want to check our results for robustness along several lines. In Section C.1 we extend the model for possible correlation between market and credit losses. We extend the model to include operational risk in Section C.2 and provide an overview of results from different samples in Section C.3. We examine the role of the clearing date in Section C.4, look at the role of the network structure in Section C.5, and examine the effect of netting in Section C.6.

C.1 Correlation between Market and Credit Risks

In our main analysis we follow the approach of the Basel Committee and assume that losses from market and credit risks are independent. This might actually not be the case. Up to now we are not aware of any model suitable for our analysis which considers correlation between these two risk categories. We therefore decide to implement a simple heuristic approach, which assumes perfect correlation between system wide losses as a robustness check.

We compute for all market risk scenarios the system wide losses as the sum of all banks' individual market risk losses and sort them according to size. Following the same procedure for credit losses, we subsequently re-combine the sorted scenarios, thus creating cases where large losses from credit risk are paired with large losses from market risk. This way we ensure that system wide losses from market and credit risk are perfectly correlated. Note, however, that this does not necessarily imply that market and credit risk losses are perfectly correlated at the level of the individual bank.[3] Figure 1 illustrates the results of our approach to modeling inter-risk correlations on the level of individual banks. Compared to the base case the distribution of individual correlation coefficients is now rather dispersed and rises from a mean value of zero to 0.2.

[2] The remaining four percent of the claims are added to the vector e. Hence they are assumed to be fulfilled exactly.

[3] Requiring perfect correlation at the level of the individual bank would require us to consider inconsistent scenarios.

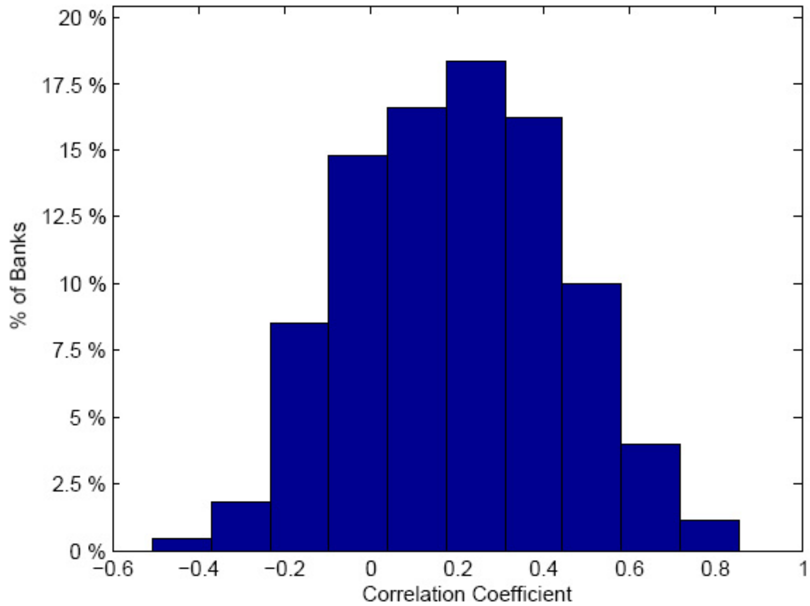

Figure 1. Histogramm of correlation coefficients between credit and market losses (gains) on the level of individual banks.

We present the results of our simulations in Table 5. Both the fraction of crisis scenarios as well as the number of contagious defaults increases. The latter increases from 0.86 percent to 1.31 percent in the short run case. As in the base case there are hardly any contagious defaults in the long run scenario for moderate levels of joint fundamental defaults. Contagion is a problem if there are many simultaneous defaults or if crisis resolution is poor.

The assumed correlation between market and credit risk changes the individual default probabilities in the long run scenario only a little as can be seen in Table 6. But in the short run default probabilities are approximately twice as high as in the base case. This is caused by an increased probability of contagious defaults. Thus, the consequences of an inefficient bankruptcy procedure are strengthened in the case of high inter–risk correlation.

C.2 Operational Risk

To detect whether the inclusion of operational risk alters our results critically we perform a robustness check by including an analysis of operational risk based on the following heuristic approximation: Building on the *Basic Indicator Approach* described in (Basel Committee on

Fundamental Defaults	Total	Short-Run		Long-Run	
		no contagion	contagion	no contagion	contagion
0-5	97.436%	97.344%	0.092%	97.435%	0.001%
6-10	1.064%	0.901%	0.163%	1.060%	0.004%
11-20	0.782%	0.370%	0.412%	0.768%	0.014%
21-50	0.513%	0.080%	0.433%	0.501%	0.012%
more	0.205%	0%	0.205%	0.125%	0.080%
Total	100.00%	98.695%	1.305%	99.889%	0.111%
Base case	100.00%	99.137%	0.863%	99.955%	0.045%

Table 5. Probabilities of fundamental and contagious defaults in the short-run and in the long-run under the assumption of inter-risk correlations. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the inter-bank market. The short run analysis is under the assumption that insolvent banks pay nothing in the interbank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

	Panel A: Short-Run			Panel B: Long-Run		
	10%	Median	90%	10%	Median	90%
Correlated Case						
Probability of Default	0.17%	0.98%	1.06%	0.00%	0.00%	0.25%
Probability of Contagious Default	0.17%	0.97%	0.98%	0.00%	0.00%	0.00%
Base Case						
Probability of Default	0.08%	0.54%	0.61%	0.00%	0.00%	0.18%
Probability of Contagious Default	0.08%	0.54%	0.54%	0.00%	0.00%	0.00%

Table 6. Total and contagious default probabilities of individual banks in the short-run and the long-run. A bank defaults contagiously because other banks do not fully honor their promises. The short-run analysis (Panel A) is under the assumption that insolvent banks pay nothing in the interbank market; in the long-run (Panel B) the residual value of the bank is proportionally shared among claimants assuming zero bankruptcy costs.

Banking System	Mean	Std.-Dev.	Quantiles			
			10%	5%	1%	0.5%
Mill Euro	-883	30	-921	-932	-953	-961
% of Total Assets	-0.15%	0.01%	-0.16%	-0.16%	-0.17%	-0.17%

Table 7. Aggregate losses due to operational risk.

Banking Supervision (2004)) we assume that banks must hold capital for operational risk equal to the average over the previous three years of 15 percent of positive annual gross income. Figures for any year in which annual gross income is negative or zero are excluded from both the numerator and denominator when calculating the average. We use gross annual income data from the OeNB Quartalsbericht for this calculation. We assume that the capital requirement is equal to the 99 percent quantile of a Poisson distribution, which we fit to this quantile for each bank. We then add operational risk to the simulation by drawing for each bank independently from its operational loss distribution in addition to the market and credit risk distribution.

Table 7 shows statistics of the simulated operational losses for the entire banking system. The average aggregate loss, which amounts to 883 mill. euros (0.15 percent of total assets) is of approximately the same size as the average aggregate loss due to credit risk. As operational losses are simulated independently across banks, the dispersion of these losses is low compared to the dispersion of credit risk losses.

Comparing Table 8, which summarizes the probabilities of fundamental and contagious defaults with the results for the case without operational losses (Table 4 of the paper) shows that the inclusion of operational risk does not change the main findings, which is also consistent with the findings of Kuritzkes, Schuermann, and Weiner (2003). The increase of default probabilities is very moderate.

C.3 Systemic Risk Across Time

In our analysis, we have based all calculations on a dataset collected for September 2002. To check how our results change between different observation periods, we have also done all calculations for September 2000, September 2001, and December 2002. The basic findings remain robust across these different data sets as can be seen in Table 9. In the years 2000 and 2001, the fraction of contagious defaults was slightly higher than in the other observation periods. All numbers are calculated under the assumptions of the base case.

Our results on bankruptcy costs and contagion as well as on the value at risk for a lender of last resort also remain robust across the observation periods. A complete set of tables is

Fundamental Defaults	Total	Short-Run		Long-Run	
		no contagion	contagion	no contagion	contagion
0-5	98.130%	98.075%	0.055%	98.130%	0%
6-10	0.775%	0.590%	0.185%	0.770%	0.005%
11-20	0.570%	0.365%	0.205%	0.570%	0%
21-50	0.405%	0.065%	0.340%	0.395%	0.001%
more	0.120%	0%	0.120%	0.070%	0.050%
Total	100.00%	99.095%	0.905%	99.935%	0.065%
Base case	100.00%	99.137%	0.863%	99.955%	0.045%

Table 8. Probabilities of fundamental and contagious defaults in the short-run and in the long-run including operational risk. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the interbank market. The short run analysis is under the assumption that insolvent banks pay nothing in the interbank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

Observation Period	Short-Run		Long-Run	
	no contagion	contagion	no contagion	contagion
2000-09	98.682%	1.318%	99.839%	0.161%
2001-09	98.960%	1.040%	99.766%	0.234%
2002-09	99.137%	0.863%	99.955%	0.045%
2002-12	99.241%	0.759%	99.979%	0.021%

Table 9. Comparison of fractions of fundamental and contagious defaults in the short-run and in the long-run across four different observation periods. The basic features found in the base case 2002-09 remain the same.

available from the authors upon request.

C.4 Role of the Holding Period

The simultaneous consideration of market and credit risk forced us to make a decision on holding time horizons for non interbank market and credit portfolios. For all positions exposed to market risk we chose a holding period of ten trading days and a quarter for the loan portfolio. The choice of the loan portfolio horizon was determined by data availability. For the market portfolio, we took daily data to have a sufficient amount of data for historic simulation. We scaled the daily returns to ten days following the common standard of the Basel Committee. We therefore had to check whether different assumptions about holding periods lead to different empirical conclusions. Instead of assuming a holding period for the market portfolio of ten

Holding Period	Short-Run		Long-Run	
	no contagion	contagion	no contagion	contagion
4 Weeks – 1 Quarter	99.548%	0.452%	99.997%	0.003%
3 Months – 1 Quarter	96.821%	3.179%	99.378%	0.622%
63 Days – 1 Year	99.240%	0.760%	99.982%	0.018%
Base Case	99.137%	0.863%	99.955%	0.045%

Table 10. Comparison of fractions of fundamental and contagious defaults in the short-run and in the long-run for different holding periods. The basic features found in the base case 2002-09 remain the same.

days we tried all our simulations with the assumption of a holding period of four weeks using non overlapping rescaled weekly data, of three month using non overlapping rescaled monthly data. Moreover, we repeated our analysis with a holding period of 63 trading days for all positions exposed to market risk and one year for the loan portfolio. The return over the 63 days was calculated as the sum of 63 independently drawn daily returns.

Table 10 summarizes our simulation results.[4] The main finding that contagion is only a problem in case of inefficient bankruptcy resolution remains valid for two out of the three robustness checks. Only the case where we model a holding period of three months using rescaled monthly data yields different results. This is due to the fact that the distribution of the calculated returns is much more dispersed and has a lot more mass in the tails than quarterly return data. Yet, even under this extreme distributional assumption the main driving forces for simultaneous default are correlated exposures and poor bankruptcy resolution.

C.5 Role of the Network Structure

Since our interbank matrix L is only partially observed and has to be estimated, it is interesting to check our results for robustness with respect to variations in the estimates of L. We compare our initial analysis for the base case with a hypothetical alternative case where we ignore all the structural information about L that we know from the data and assume instead perfect diversification of interbank loans. This assumption can be modeled by estimating all matrix entries by maximum entropy using only row and column sums as constraints. In the terminology of Allen and Gale (2000), this amounts to a complete market structure. The results are presented in Table 11. In terms of the relative importance of contagious defaults, we see that the basic pattern is roughly the same but that the importance of contagion under the perfect diversification assumption increases especially under the long-run assumption.

[4] More detailed results are available from the authors upon request.

Fundamental Defaults	Total	Short-Run		Long-Run	
		no contagion	contagion	no contagion	contagion
0-5	97.350%	96.825%	0.525%	96.995%	0.355%
6-10	1.210%	0.945%	0.265%	1.155%	0.055%
11-20	0.830%	0.210%	0.620%	0.730%	0.100%
21-50	0.425%	0%	0.425%	0.145%	0.280%
more	0.185%	0%	0.185%	0.055%	0.130%
Total	100.00%	97.980%	2.020%	99.080%	0.920%
Base case	100.00%	99.137%	0.863%	99.955%	0.045%

Table 11. Probabilities of fundamental and contagious defaults in the short-run and in the long-run assuming perfect diversification of interbank loans. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the interbank market. The short run analysis is under the assumption that insolvent banks pay nothing in the inter-bank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

C.6 Netting

We perform a final robustness check with respect to simple netting arrangements. If we assume that all exposures in L are bilaterally netted before clearing, we get an almost identical picture to the base case. It is usually assumed that with respect to contagion, netting arrangements would perform a stabilizing task. Note that this is, however, not true in general. Elsinger, Lehar, and Summer (2005) show that the effect of netting depends on the precise structure of liabilities, or the specific form of the matrix L. Depending on this structure, contagion may increase or decrease. In our case, the basic empirical finding remain the same whether or not we bilaterally net all claims before clearing.

Fundamental Defaults	Total	Short-Run		Long-Run	
		no contagion	contagion	no contagion	contagion
0-5	98.300%	98.240%	0.060%	98.300%	0%
6-10	0.685%	0.520%	0.165%	0.680%	0.005%
11-20	0.495%	0.300%	0.195%	0.495%	0%
21-50	0.400%	0.115%	0.285%	0.390%	0.010%
more	0.120%	0%	0.120%	0.050%	0.070%
Total	100.00%	99.175%	0.825%	99.915%	0.085%
Base case	100.00%	99.137%	0.863%	99.955%	0.045%

Table 12. Probabilities of fundamental and contagious defaults in the short-run and in the long-run under the assumption of netting. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the inter-bank market. The short run analysis is under the assumption that insolvent banks pay nothing in the interbank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

References

Allen, Franklin, and Douglas Gale, 2000, Financial Contagion, *Journal of Political Economy* 108, 1–34.

Basel Committee on Banking Supervision, 2004, *International Convergence of Capital Measurement and Capital Standards, A Revised Framework.* (BIS).

Elsinger, Helmut, Alfred Lehar, and Martin Summer, 2005, Using Market Information for Banking System Risk Assessment, http://ssrn.com/abstract=787929.

Kuritzkes, Andrew, Til Schuermann, and Scott Weiner, 2003, Risk Measurement, Risk Management and Capital Adequacy of Financial Conglomerates, *Brookings-Wharton Papers in Financial Services* pp. 141–194.