A Theory of Banks' Industry Expertise, Market Power, and Credit Risk: Appendix

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Proof of Lemma 1: Given the realization of its signal, $\sigma_i \in \{0, 1\}$, bank i expects that a skilled entrepreneur realizes an NPV of $NPV_{\sigma_i} = k(\mathrm{E}[p|\sigma_i, , n_i] - c)$ by producing k units of output. It must be the case that $NPV_{\sigma_i} \geq kx_I$ for (at least) one of the two possible values of σ_i: otherwise, bank i (as a representative bank with industry expertise) could never expect to break even on the cost of using technology T_I.

I will next show that $NPV_{\sigma_i} < 0$ if and only if $\sigma_i = 0$, and hence $NPV_{\sigma_i} \geq kx_I$ for $\sigma_i = 1$. To this end, I will first show that there exists some value of $\sigma_i \in \{0, 1\}$ for which $NPV_{\sigma_i} < 0$; then, I will show that this is the case for $\sigma_i = 0$. The first result follows from the condition in the first sentence of Lemma 1 which can be stated more formally as in condition (10) (that must hold in any equilibrium with free entry):

$$x_0 = (\mathrm{E}[p] - c) = 0.5(\mathrm{E}[p|\sigma_i = 0, n_i] - c) + 0.5(\mathrm{E}[p|\sigma_i = 1, n_i] - c). \qquad (10')$$

In the above-stated equation, the two terms in brackets equal NPV_{σ_i}/k for $\sigma_i = 0$ and $\sigma_i = 1$, respectively. Since I have argued above that there must exist a value of $\sigma_i \in \{0, 1\}$ for which $NPV_{\sigma_i} \geq kx_I$, equation (10') and $x_I > x_0$ imply that $NPV_{\sigma_i} < 0$ for the other possible value of σ_i. Hence, there must exist a value of σ_i for which bank i cannot profit from refinancing borrowers if it observes this signal realization. Below, I will show that this the case for the signal realization which indicates low demand, $\sigma_i = 0$.

To obtain a contradiction, I assume that $NPV_0 > 0$ and $NPV_1 < 0$. This implies that bank i can only expect to profit from refinancing a borrower if it observes the low-demand signal, $\sigma_i = 0$. If $\sigma_i = 1$, the bank will reject any borrower's request for refinancing: $\lambda[\sigma_i = 1, n_i, r] = 0$, since $NPV_1 < 0$ implies that $\bar{\pi}[\sigma_i = 1, n_i, r] < 0$ for any r. If $\sigma_i = 0$, the bank refinances the borrower with probability $\lambda[\sigma_i = 0, n_i, r] \geq 0$ (with a strict inequality for high enough values of r, or else bank i could never break even on the cost of its industry expertise). As a consequence, the expected marketable output of the industry increases if the bank observes the low-demand signal, since this makes it more likely that the borrower obtains refinancing, allowing her to produce output. Since the same must be true also for any other bank with industry expertise, the expected aggregate marketable output of the industry will be higher in the low-demand state than in the high-demand state. By the downward-sloping demand function (1), this implies that the expected price of such output is strictly smaller in the low-demand state than in the high-demand state, $\mathrm{E}[p|\alpha = -a] < \mathrm{E}[p|\alpha = +a]$. (This inequality is strict since the intercept of the inverse demand function (1) is lower if $\alpha = -a$ than if $\alpha = +a$.) This inequality contradicts the assumption that $NPV_0 > 0 > NPV_1$ since it implies that $\mathrm{E}[p|\sigma_i = 0, n_i] < \mathrm{E}[p|\sigma_i = 1, n_i]$, and hence $NPV_0 = k(\mathrm{E}[p|\sigma_i = 0, n_i] - c) \leq NPV_1 = k(\mathrm{E}[p|\sigma_i = 1, n_i] - c)$.

Proof of Proposition 4: A bank with industry expertise refinances its borrowers if its experts predict high demand. Since this happens with probability 0.5, the value of the bank's expertise equals 0.5 times its expected profit from refinancing the borrowers, $\Omega^*[n^*]$. To derive the expression for $\Omega^*[n^*]$, I use condition (9). This condition implies that, per borrower, a bank earns the following expected profit in refinancing its borrowers if its industry experts predict high demand:

$$\bar{\pi}[n^*, r^*] = \delta[n^*]o[n^*]k. \tag{27}$$

Per unit of capacity, this implies an expected profit of $\bar{\pi}[n^*, r^*]/k = \delta[n^*]o[n^*]$, bringing the bank's overall expected profit to $\Omega^*[n^*] = n^*\pi[n^*]/k = \delta[n^*]o[n^*]n^*$.

Proof of Lemma 2: Given the result in Proposition 4, a bank with industry expertise can break even if the following condition holds:

$$0.5\Omega^*[n^*] = 0.5\delta[n^*]o[n^*]n^* = 0.5(1 - \rho[n^*])(n^*)^2 \geq n^*x_I, \tag{28}$$

where the first equation follows from expression (18), and the second equation is obtained since $\delta[n^*] = \mathrm{Prob}[p < c(1 + r^*)|\sigma_i = 1, n_i = n^*] = \mathrm{Prob}[\alpha = -a|\sigma_i = 1, n_i = n^*] = (1 - \rho[n^*])$ (by $\alpha = -a \Leftrightarrow p < c(1 + r^*)$), and $o[n^*] = n^* - k \rightarrow n^*$ for $k \rightarrow 0$. If $\rho[\cdot]$ satisfies the condition stated in Lemma 2, the above-stated inequality is satisfied for $n^* \geq n_0$ since the left-hand side increases fast enough to eventually exceed the right-hand side, where n_0 satisfies the equation:

$$0.5(1 - \rho[n_0])(n_0)^2 = n_0x_I. \tag{29}$$

Proof of Proposition 5: As stated in Section 3.2.3, the equilibrium is determined by condition (10') (as equivalent to the condition stated in Proposition 2), condition (21) (as equivalent to condition (14') for $k \to 0$), and condition (16) (as equation). Of relevance for the analysis below, condition (10') determines the expected price of marketable output, $\mathrm{E}[p] = (c + x_0)$, and hence the conditional expected price of such output if demand is low, $\alpha = -a$ and bank i finances its borrowers' output production since $\sigma_i = 1$:

$$\begin{aligned}\mathrm{E}[p|\alpha = -a] &= \quad \mathrm{E}[p] \quad - a \quad +(\mathrm{E}[Q_b^{-i}] - \mathrm{E}[Q_b^{-i}|\alpha = -a]) \quad -0.5n^* \\ &= c + x_0 \quad - a \quad +(b^* - 1)(\rho[n^*] - 0.5)n^* \qquad -0.5n^*,\end{aligned} \tag{30}$$

where I have used that $\mathrm{E}[Q_b^{-i}|\alpha = -a] = (b^* - 1)(1 - \rho[n^*])n^*$ and $\mathrm{E}[Q_b^{-i}] = (b^* - 1)0.5n^*$ since n^* is the aggregate marketable output of the borrowers of one bank with industry expertise, if the bank refinances the borrowers.

Before deriving closed form solutions, I will rearrange the equilibrium conditions in order to state them more conveniently. Condition (21) determines the optimal level at which each of the banks with industry expertise finances industrial capacity formation. Upon using expression (22) to substitute for $\Delta_\Omega[n^*, r^*]$, I obtain the condition:

$$0.5\rho'[n^*]\gamma[r^*]n^* - x_I = 0, \tag{31}$$

where $\gamma[r^*] = c(1+r^*) - \mathrm{E}[p|p < c(1+r^*)]$ (by expression (13)) for $\mathrm{E}[p|p < c(1+r^*)] = \mathrm{E}[p|\alpha = -a]$ given by the expression stated above. Under the assumption that $\rho[\cdot]$ satisfies equation (23), $\rho'[n^*]$ is given by $\rho'[n^*] = \nu(1 - \rho[n^*])/n^*$. By substituting for $\rho'[n^*]$ and $\gamma[r^*]$ and rearranging, I obtain the following condition:

$$(1 - \rho[n^*])(c(1 + r^*) - \mathrm{E}[p|\alpha = -a])n^* - \frac{2x_I n^*}{\nu} = 0. \tag{32}$$

Condition (16) holds as equation in an equilibrium with free entry. By the expression for $\Omega^*[n^*]$ stated in Proposition 4, this equation can be stated as follows:

$$0.5\delta[n^*]o[n^*]n^* - n^* x_I = 0, \tag{33}$$

where $\delta[n^*] = \mathrm{Prob}[p < c(1 + r^*)|\sigma_i = 1, n_i = n^*] = \mathrm{Prob}[\alpha = -a|\sigma_i = 1, n_i = n^*] = (1 - \rho[n^*])$ (by $\alpha = -a \Leftrightarrow p < c(1 + r^*)$), and $o[n^*] = n^* - k \to n^*$ for $k \to 0$.

In equilibrium, banks with industry expertise finance industrial capacity formation at the level of $n^* = n_0$ units, defined in the proof of Lemma 2 as the solution to condition (33). This implies that the banks' skilled borrowers request refinancing at an interest rate of $r^* = \underline{r}[n^*]$, given by condition (27) for $\bar{\pi}[n^*, r^*] = k(\rho[n^*]c(1 + r^*) + (1 - \rho[n^*])\mathrm{E}[p|\alpha = -a]) - kc$, $\delta[n^*] = (1 - \rho[n^*])$, and $o[n^*] = n^*$:

$$\rho[n^*]c(1 + r^*) + (1 - \rho[n^*])(\mathrm{E}[p|\alpha = -a]) - c = (1 - \rho[n^*])n^*. \tag{27'}$$

To summarize, I must solve the following system of equations for n^*, r^* and b^*:

$$\begin{aligned}
&\text{(i) } 0 = && \rho[n^*]c(1 + r^*) + (1 - \rho[n^*])(\mathrm{E}[p|\alpha = -a] - n^*) && -c, && (27')\\
&\text{(ii) } 0 = && ((1 - \rho[n^*])c(1 + r^*) - (1 - \rho[n^*])\mathrm{E}[p|\alpha = -a])n^* && -\frac{2x_I n^*}{\nu}, && (32')\\
&\text{(iii) } 0 = && (1 - \rho[n^*])(n^*)^2 && -2x_I n^*, && (33')
\end{aligned}$$

where b^* enters the first two equations as a determinant of the expected price $\mathrm{E}[p|\alpha = -a]$. Multiplying condition (i) by n^* and adding the result to condition (ii) yields the following equation:

$$c(1 + r^*)n^* - (1 - \rho[n^*])(n^*)^2 - cn^* - \frac{2x_I n^*}{\nu} = 0. \tag{34}$$

Condition (iii) implies that the second term of the above-stated condition equals $2x_I n^*$; upon substituting for this term, dividing the resulting expression by n^* and rearranging, I obtain result (26).

Condition (iii) can be written as follows:

$$(1 - \rho[n^*])n^* - 2x_I = 0. \tag{35}$$

Under the assumption that $\rho[\cdot]$ satisfies equation (23), the first term of the above-stated equation equals $\epsilon(n^*)^{1-\nu}$. Hence, solving this equation for n^* yields result (25).

Finally, I derive the expression for b^*. To do this, I use results (26) and (25) to substitute for r^* and n^* in condition (i) and solve the resulting equation for b^* to obtain result (24).

It remains to show that condition (21) identifies a global maximum if $\nu < 1$. This result is derived in a note that is available from the author upon request.

Proof of Proposition 6: Differentiating result (24) yields the expressions,

$$\begin{aligned}
\frac{\partial b^*}{\partial x_I} &= -\frac{\epsilon((a-x_0)((n^*)^\nu - 2\epsilon(1-\nu)) + 2x_I(n^*)^\nu(2\nu-1))}{((n^*)^\nu - 2\epsilon)^2 x_I^2(1-\nu)}, \\
\frac{\partial b^*}{\partial \epsilon} &= \frac{((a-x_0)\nu + 2x_I(2\nu-1))(n^*)^\nu}{((n^*)^\nu - 2\epsilon)^2 x_I \nu(1-\nu)}.
\end{aligned} \tag{36}$$

Since banks with industry expertise observe informative signals (i.e. $\rho[n^*] > 0.5$, and, hence, $n^* > 2\epsilon$), the first of these expressions decreases in the size of the demand shock, a, while the second increases in a. Since the number of banks also increases in a, $b^* \geq 1$ is a sufficient condition for $\partial b^*/\partial x_I < 0$ ($\partial b^*/\partial \epsilon > 0$), if the first (second) derivative is negative (positive) for that critical value of a that implies $b^* = 1$. Indeed, solving this equation for this critical value of a and using the result to substitute for a in the above-stated derivatives yields,

$$\begin{aligned} \tfrac{\partial b^*}{\partial x_I}|_{b^*=1} &= -\tfrac{(n^*)^\nu(2-\nu)-2\epsilon(1-\nu^2)}{((n^*)^\nu-2\epsilon)x_I\nu(1-\nu)} < 0, \\ \tfrac{\partial b^*}{\partial \epsilon}|_{b^*=1} &= \tfrac{(n^*)^\nu(2-\nu)}{((n^*)^\nu-2\epsilon)\epsilon\nu(1-\nu)} > 0. \end{aligned} \tag{37}$$

Proof of Proposition 7: As discussed above Proposition 7, a bank with industry expertise incurs an expected loss of $\gamma[r^*]n^*$ if it refinances its skilled borrowers in the low-demand state. The recovery rate is given by $\gamma[r^*]n^*/(c(1+r^*)n^*) = \gamma[r^*]/(c(1+r^*))$. By expression (13) and $\mathrm{E}[p|p < c(1+r^*)] = \mathrm{E}[p|\alpha = -a]$, $\gamma[r^*]/(c(1+r^*))$ is given by,

$$\frac{\gamma[r^*]}{c(1+r^*)} = 1 - \frac{\mathrm{E}[p|\alpha = -a]}{c(1+r^*)} = \frac{cr^*}{c(1+r^*)}\frac{1}{1-\rho[n^*]} - \frac{n^*}{c(1+r^*)}, \tag{38}$$

where the second equation follows from condition (27') (in the proof of Proposition 5) which implies the following expression for the expected price $\mathrm{E}[p|\alpha = -a]$:

$$\mathrm{E}[p|\alpha = -a] = c(1+r^*) - cr^*\frac{1}{1-\rho[n^*]} + n^*. \tag{39}$$

Notice that $\gamma[r^*]$ does not depend on the size of the demand shock, a. To obtain the results in Proposition 7, I use results (25) and (26) to substitute for n^* and r^* in expression (38), and differentiate the resulting expression with respect to x_I and ϵ:

$$\begin{aligned} \tfrac{\partial}{\partial x_I}\tfrac{\gamma[r^*]}{c(1+r^*)} &= \tfrac{1}{\nu}\tfrac{(2c/\epsilon)(2x_I/\epsilon)^{\nu/(1-\nu)}}{(c+2x_I(1+\nu)/\nu)^2} > 0, \\ \tfrac{\partial}{\partial \epsilon}\tfrac{\gamma[r^*]}{c(1+r^*)} &= -\tfrac{1}{\nu}\tfrac{(2x_I/\epsilon^2)(2x_I/\epsilon)^{\nu/(1-\nu)}}{c+2x_I(1+\nu)/\nu} < 0. \end{aligned} \tag{40}$$