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e-companion

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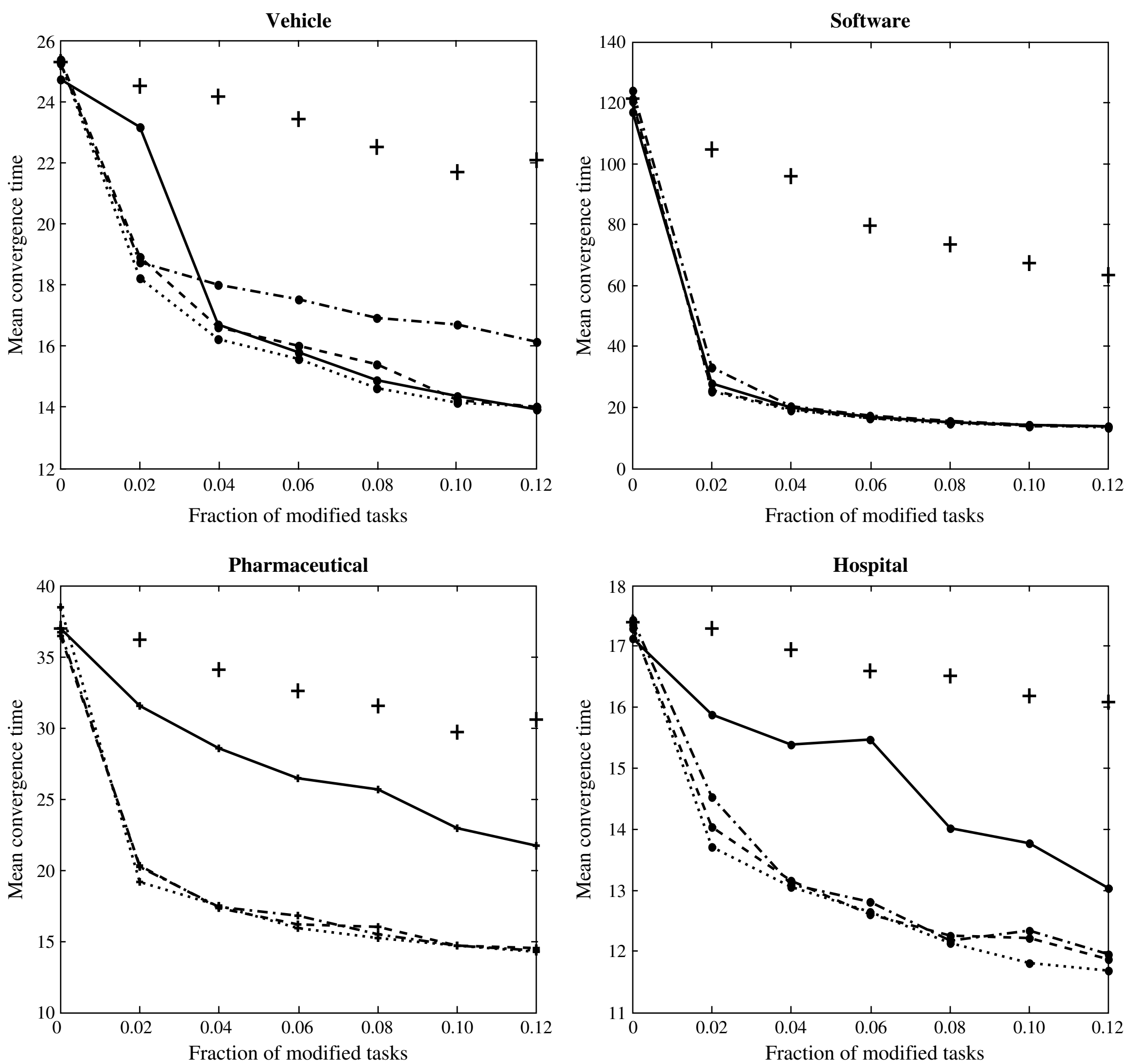
Electronic Companion—"The Statistical Mechanics of Complex Product Development: Empirical and Analytical Results" by Dan Braha and Yaneer Bar-Yam, *Management Science* 2007, 53(7) 1127–1145.

Online Supplements

Supplement 1. Figure EC.1

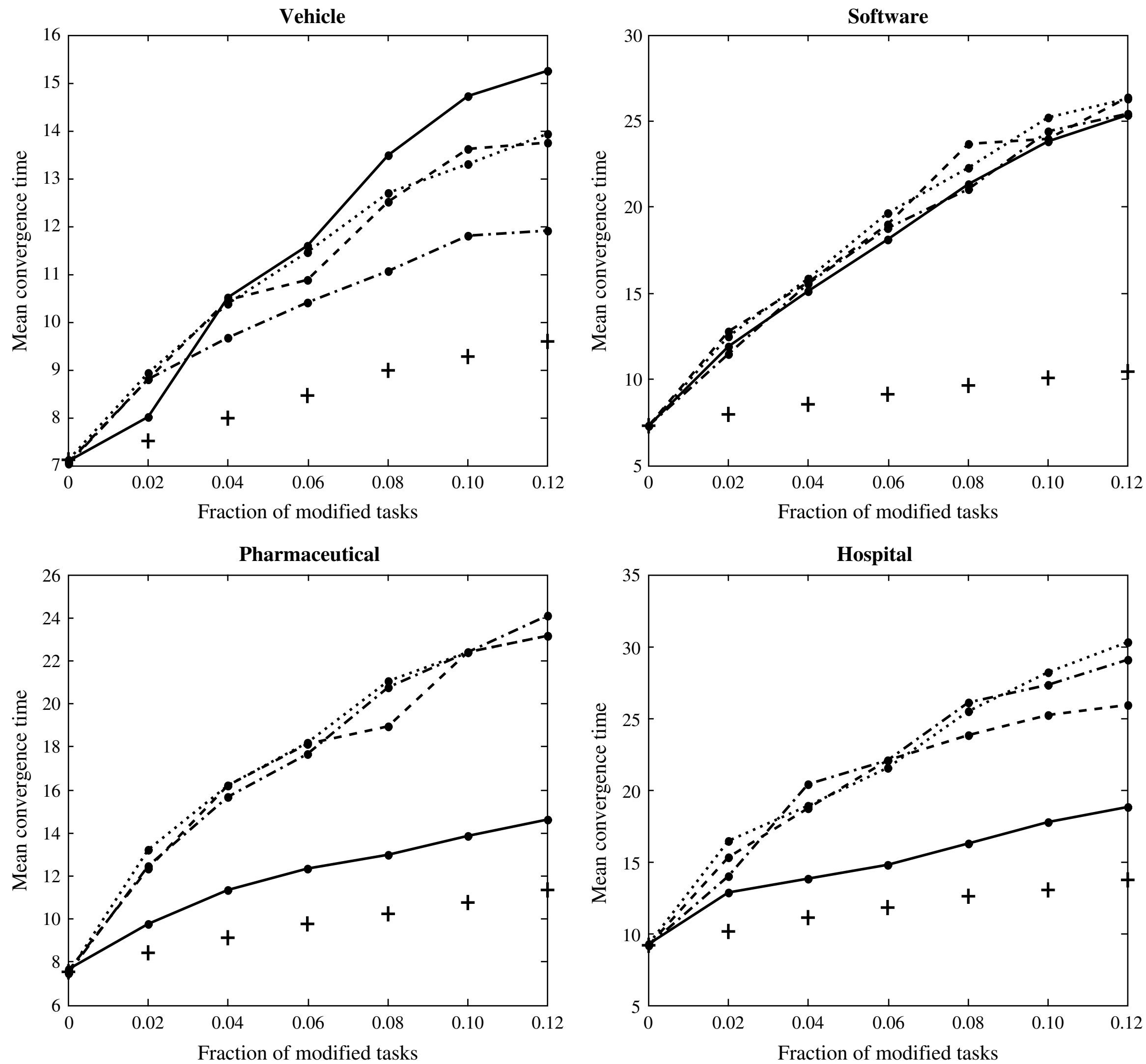
Figure EC.1 Product Development Performance vs. Fraction of Modified Tasks for Which Sensitivity Rates Are Improved



Notes. Comparison between five-task modification policies: Multiplicative (dotted line), additive (dashed line), information-generating (dashed-dotted line), information-consuming (solid line), and random (+). The average in-degree, internal completion rate, sensitivity rate *prior to modification*, and modified sensitivity rate are, respectively, as follows: Vehicle: $\langle k_{in} \rangle = 3.475$, $r = 0.75$, $\beta = 0.2$, $\beta^- = 0.1$; Software: $\langle k_{in} \rangle = 3.163$, $r = 0.75$, $\beta = 0.1$, $\beta^- = 0.05$; Pharmaceutical: $\langle k_{in} \rangle = 6.371$, $r = 0.75$, $\beta = 0.1$, $\beta^- = 0.05$; Hospital: $\langle k_{in} \rangle = 9.741$, $r = 0.75$, $\beta = 0.05$, $\beta^- = 0.025$.

Supplement 2. Figure EC.2

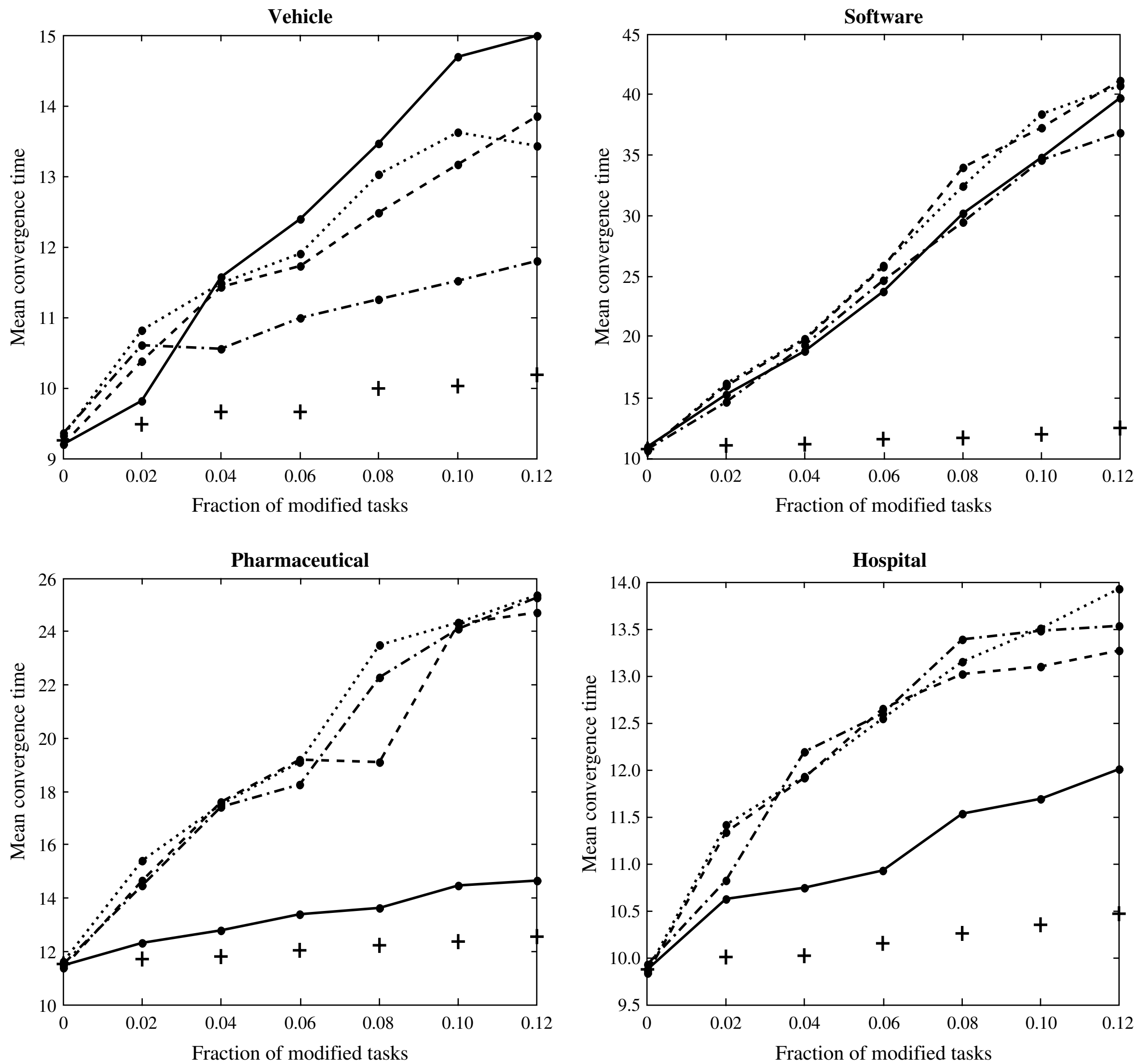
Figure EC.2 Product Development Performance vs. Fraction of Modified Tasks for Which Completion Rates Are Impaired



Notes. Comparison between five-task modification policies: Multiplicative (dotted line), additive (dashed line), information-generating (dashed-dotted line), information-consuming (solid line), and random (+). The average in-degree, sensitivity rate, internal completion rate *prior to modification*, and modified internal completion rate are, respectively, as follows: Vehicle: $\langle k_{in} \rangle = 3.475$, $\beta = 0.135$, $r = 1$, $r^- = 0.5$; Software: $\langle k_{in} \rangle = 3.163$, $\beta = 0.06$, $r = 1$, $r^- = 0.5$; Pharmaceutical: $\langle k_{in} \rangle = 6.371$, $\beta = 0.06$, $r = 1$, $r^- = 0.5$; Hospital: $\langle k_{in} \rangle = 9.741$, $\beta = 0.05$, $r = 1$, $r^- = 0.5$.

Supplement 3. Figure EC.3

Figure EC.3 Product Development Performance vs. Fraction of Modified Tasks for Which Sensitivity Rates Are Impaired



Notes. Comparison between five-task modification policies: Multiplicative (dotted line), additive (dashed line), information-generating (dashed-dotted line), information-consuming (solid line), and random (+). The average in-degree, internal completion rate, sensitivity rate *prior to modification*, and modified sensitivity rate are, respectively, as follows: Vehicle: $\langle k_{in} \rangle = 3.475$, $r = 0.75$, $\beta = 0.1$, $\beta^+ = 0.2$; Software: $\langle k_{in} \rangle = 3.163$, $r = 0.75$, $\beta = 0.05$, $\beta^+ = 0.1$; Pharmaceutical: $\langle k_{in} \rangle = 6.371$, $r = 0.75$, $\beta = 0.05$, $\beta^+ = 0.1$; Hospital: $\langle k_{in} \rangle = 9.741$, $r = 0.75$, $\beta = 0.025$, $\beta^+ = 0.05$.

Supplement 4. Appendix

The analysis presented in §5.3 neglected the correlations *among* different tasks' connectivities. For completeness, we present in this appendix a brief analysis of the case where there are explicit correlations between *pairs* of neighboring tasks. First, we modify the dynamical mean-field rate equations as follows:

$$\frac{d\alpha_k(t)}{dt} = (1-\alpha_k(t))\tanh(\beta k\theta_k(t)) - \alpha_k(t) r(1-\tanh(\beta k\theta_k(t))) \quad \forall k, \tag{EC1}$$

where $\theta_k(t)$ is the probability that any given incoming link (arc) to a task with in-degree k originates from an unresolved task.

It is easy to see that θ_k is given by

$$\theta_k(t) = \sum_j P(\tilde{k}_{\text{in}} = j \mid k_{\text{in}} = k)\alpha_j(t), \tag{EC2}$$

where $P(\tilde{k}_{\text{in}} = j \mid k_{\text{in}} = k)$ is the probability that an incoming link to a task with in-degree $k_{\text{in}} = k$ originates from another (neighboring) task with in-degree $\tilde{k}_{\text{in}} = j$. By plugging the expressions for $\theta_k(t)$ in (EC1), we obtain a first-order nonlinear system of differential equations,

$$\frac{d\boldsymbol{\alpha}}{dt} = \mathbf{f}(\boldsymbol{\alpha}). \tag{EC3}$$

We will analyze the stability of the uniformly resolved state $\alpha_k = 0\ \forall k$ by using a linearization technique. More specifically, the uniformly resolved state $\boldsymbol{\alpha} = 0$ will be unstable if the Jacobian matrix of $\mathbf{f}(\boldsymbol{\alpha})$ at the fixed point $\boldsymbol{\alpha} = 0$ has positive eigenvalues. The Jacobian matrix of $\mathbf{f}(\boldsymbol{\alpha})$ at the fixed point $\alpha = 0$ can be shown to be

$$J(\boldsymbol{\alpha})_{\boldsymbol{\alpha}=0} = \begin{pmatrix} \frac{\partial f_1}{\partial \alpha_1} & \frac{\partial f_1}{\partial \alpha_2} & \cdots & \frac{\partial f_1}{\partial \alpha_{n-1}} & \frac{\partial f_1}{\partial \alpha_n} \\ \frac{\partial f_2}{\partial \alpha_1} & \frac{\partial f_2}{\partial \alpha_2} & \ddots & \frac{\partial f_2}{\partial \alpha_{n-1}} & \frac{\partial f_2}{\partial \alpha_n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \frac{\partial f_{n-1}}{\partial \alpha_1} & \frac{\partial f_{n-1}}{\partial \alpha_2} & \ddots & \frac{\partial f_{n-1}}{\partial \alpha_{n-1}} & \frac{\partial f_{n-1}}{\partial \alpha_n} \\ \frac{\partial f_n}{\partial \alpha_1} & \frac{\partial f_n}{\partial \alpha_2} & \cdots & \frac{\partial f_n}{\partial \alpha_{n-1}} & \frac{\partial f_n}{\partial \alpha_n} \end{pmatrix}_{\boldsymbol{\alpha}=0} = \beta A - rI,$$

where $A = (a_{kj})$ is a matrix for which $a_{kj} = kP(\tilde{k}_{\text{in}} = j \mid k_{\text{in}} = k)$, and I is the identity matrix. Let λ^* be the largest eigenvalue of the matrix A. Thus, the largest eigenvalue of the Jacobian $J(\boldsymbol{\alpha})_{\boldsymbol{\alpha}=0}$ is $\beta\lambda^* - r$, and we conclude that the uniformly resolved state $\boldsymbol{\alpha} = 0$ is unstable if $\beta > (r/\lambda^*)$.

Finally, it is shown that if the correlations among different tasks' connectivities are neglected we recover the conditions derived in §5.3. If neighboring tasks are uncorrelated—that is, $P(\tilde{k}_{\text{in}} = j \mid k_{\text{in}} = k) \approx P(\tilde{k}_{\text{in}} = j)$—we obtain

$$\begin{aligned} a_{kj} &= kP(\tilde{k}_{\text{in}} = j \mid k_{\text{in}} = k) = \sum_m kP(\tilde{k}_{\text{in}} = j \mid \tilde{k}_{\text{out}} = m, k_{\text{in}} = k) P(\tilde{k}_{\text{out}} = m \mid k_{\text{in}} = k) \\ &\approx \sum_m kP(\tilde{k}_{\text{in}} = j \mid \tilde{k}_{\text{out}} = m) \frac{mP(\tilde{k}_{\text{out}} = m)}{\langle \tilde{k}_{\text{out}} \rangle} = \sum_m mP(\tilde{k}_{\text{in}} = j, \tilde{k}_{\text{out}} = m) \frac{k}{\langle \tilde{k}_{\text{out}} \rangle}. \end{aligned}$$

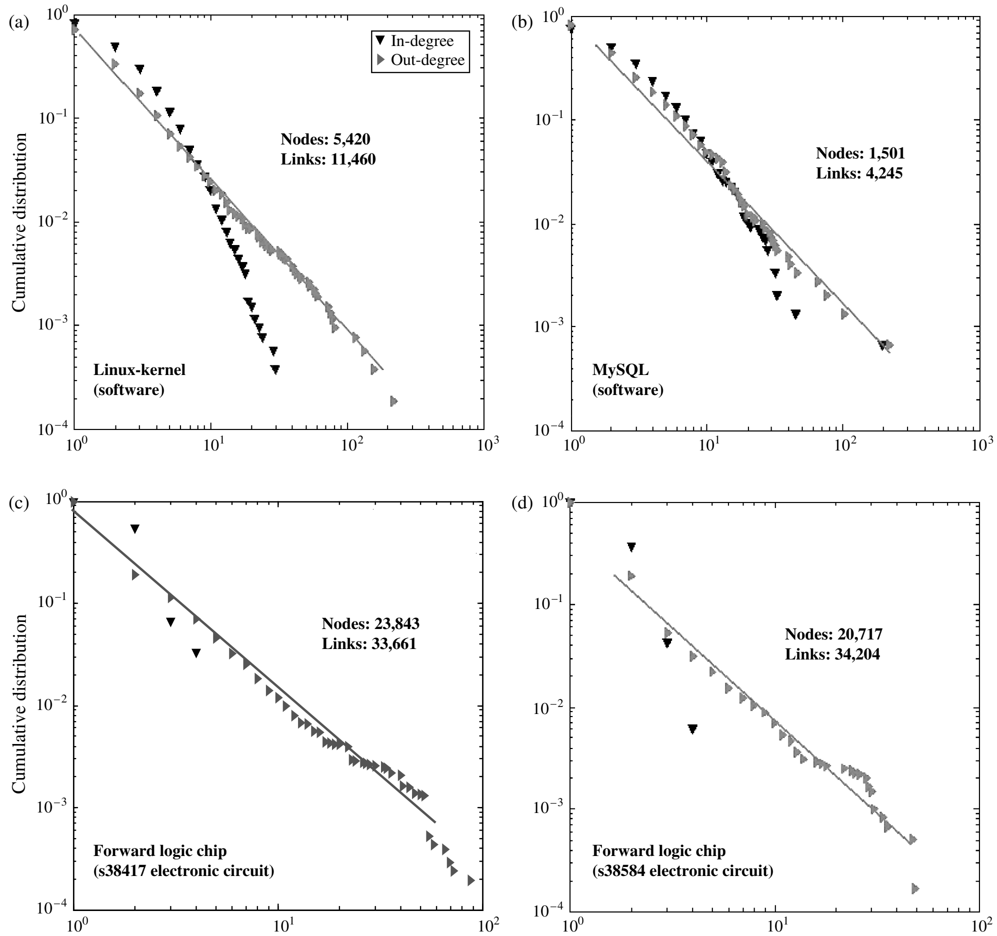
A direct calculation shows that the matrix A thus obtained has the eigenvalue $\lambda^* = \langle k_{\text{in}} k_{\text{out}} \rangle / \langle k_{\text{out}} \rangle$ with the corresponding eigenvector $\mathbf{v} = (1, 2, \ldots, n-1, n)$. Moreover, it is easy to check that λ^* is the unique and thus largest eigenvalue of A. Consequently, we conclude that the uniformly resolved state $\boldsymbol{\alpha} = 0$ is unstable if

$$\beta > \frac{r}{\lambda^*} = \frac{r\langle k_{\text{out}} \rangle}{\langle k_{\text{in}} k_{\text{out}} \rangle}$$

recovering the result established in §5.3.

Supplement 5. Figure EC.4

Figure EC.4 Degree Distributions for Open Source Software and Electronic Circuit Networks



Notes. (a and b) Open source software systems: The software system networks were generated from the call graphs of the Linux operating system kernel, and the MySQL relational database system (version 2.4.19 and version 3.23.32 respectively, data courtesy of Chris Myers, Cornell University). A call graph is a directed graph that represents calling relationship among subroutines. (c and d) Electronic circuits: The electronic circuit networks were generated from the ISCAS89 benchmark set of sequential logic electronic circuits. The nodes represent logic gates and flip-flops (data courtesy of Ron Milo, Weizmann Institute). The log-log plots of the cumulative distributions of incoming and outgoing links show a power law regime for the out-degree distributions with a fast decaying tail for the in-degree distributions. Similar to the product development networks (see Figure 3 in main text), the product design networks exhibit a noticeable asymmetry (related to the cut-offs) between the distributions of incoming and outgoing information flows, suggesting that the incoming capacities of "components" are much more limited than their counterpart outgoing capacities. These product design networks also exhibit a low correlation between the in-degrees and out-degrees of nodes as observed for the PD networks.