MANAGEMENT SCIENCE
DOI 10.1287/mnsc.1070.0708ec
pp. ec1–ec7

e-companion

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Electronic Companion—"Supply Chain Relationships and Contracts: The Impact of Repeated Interaction on Capacity Investment and Procurement" by Terry A. Taylor and Erica L. Plambeck, *Management Science* 2007, DOI 10.1287/mnsc.1070.0708.

Electronic Companion

Proof of Proposition 3

If $c \le \sigma\lambda r$, then $\underline{K} = \overline{K} = H$ and the no-monitoring relational contract and the optimal relational contract yield identical expected profit $\Pi^n = \Pi^* = \underline{\Pi}_B + \underline{\Pi}_S = \overline{\Pi}$. Therefore, in what remains we restrict attention to

$$c \in (\sigma\lambda r, \lambda r), \tag{EC1}$$

so that $\underline{K} < \overline{K} = H$ and $\underline{\Pi}_B + \underline{\Pi}_S < \overline{\Pi}$. Trivially, we also restrict attention to relational contracts in which the buyer orders the full capacity in the high-demand state $q(H) = K$ and the supplier builds capacity $K \le H$. The proof proceeds in five steps. An optimal relational contract has $q(L) \ge \min(K, L)$, and therefore must satisfy one of the following three constraints:

$$q(L) = \min(K, L) \tag{EC2}$$

$$L < q(L) < K \tag{EC3}$$

$$q(L) = K > L. \tag{EC4}$$

In each of the first three steps, we add exactly one of the above three constraints to the fixed point problem (2)–(7) and solve for the constrained optimal relational contract. Of the resulting three solutions, the one which maximizes the supplier's capacity, and hence total system expected profit, is the (unconstrained) optimal relational contract. The three solutions have similar structure, and in Step 4, we use this structure to prove that the optimal relational contract yields strictly greater expected profit than the no-monitoring relational contract if and only if (21). Step 5 completes the proof by characterizing the parameter region in which $c_1 < c_2$ and the no-monitoring relational contract is worthless.

Step 1. *The* (EC2)-*constrained optimal relational contract*: The (EC2)-constrained optimal relational contract is the no-monitoring relational contract, because (EC2) is equivalent to the no-monitoring constraint (13). The optimal capacity K^n is the solution to problem (17)–(18), that with demand distribution (19) becomes problem

$$\begin{aligned} f^n(\pi) = \max_{0 \le K \le H} \quad & \{r[\lambda K + (1-\lambda)\min(K, L)] - cK\} \\ \text{subject to} \quad & -cK + \sigma r[\lambda K + (1-\lambda)\min(K, L)] + \delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B - \underline{\Pi}_S) \\ & \ge \max\big(0, (\sigma r - c)\min(K, L) + (1-\lambda)\delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B - \underline{\Pi}_S)\big). \end{aligned} \tag{EC5}$$

We will solve this problem for two cases, mutually exclusive and exhaustive capacity-cost-parameter regions. (We will consider these two cases again within Steps 2, 3, and 4.)

Case 1. $c \in (\sigma\lambda r, \min(\sigma, \lambda)r]$: Because $c \le \sigma r$, we can restrict attention to $K \ge L$. Substituting $\pi = r[\lambda K + (1-\lambda)L] - cK$ and $\underline{\Pi}_B + \underline{\Pi}_S = (r - c)L$ in constraint (EC5), the constraint simplifies to

$$[\delta(1-\delta)^{-1}\lambda(\lambda r - c) - (c - \sigma\lambda r)](K - L) \ge 0.$$

Because the objective value is increasing in K for $K \in [L, H]$, if

$$c \le \frac{\lambda[\delta(\lambda - \sigma) + \sigma]r}{1 - \delta(1-\lambda)}, \tag{EC6}$$

then the no-monitoring relational contract has $K=H$ and it achieves the first best; otherwise, the no-monitoring relational contract has $K=L$ and yields the same profit as the noncooperative outcome $\underline{\Pi}_B+\underline{\Pi}_S=(r-c)L$.

Case 2. $c\in(\sigma r,\lambda r)$: Substituting $\pi=r[\lambda K+(1-\lambda)\min(K,L)]-cK$ and $\underline{\Pi}_B+\underline{\Pi}_S=0$ in constraint (EC5), the constraint simplifies to

$$\begin{aligned}&\delta(1-\delta)^{-1}\big(r[\lambda K+(1-\lambda)\min(K,L)]-cK\big)\\&\quad\geq\max\big(cK-\sigma r[\lambda K+(1-\lambda)\min(K,L)],\,(c-\sigma\lambda r)[K-\min(K,L)]/\lambda\big).\end{aligned}\tag{EC7}$$

Define

$$c_A=\frac{[\delta(1-\sigma)+\sigma][\lambda H+(1-\lambda)L]r}{H}$$
$$c_B=\frac{\lambda(\delta[\lambda H+(1-\lambda)L]+(1-\delta)\sigma(H-L))r}{\delta\lambda H+(1-\delta)(H-L)}$$
$$\tilde{c}=\Big[\delta(1-\lambda+\lambda^2)(1-\sigma)+(1+\lambda)\sigma+\sqrt{(\delta(1-\lambda+\lambda^2)(1-\sigma)+(1+\lambda)\sigma)^2-4\lambda\sigma(\delta(1-\sigma)+\sigma)}\Big]r\Big/2.$$

Because the objective value is increasing in K for $K\in[0,H]$, the optimal solution is the largest $K\in[0,H]$ satisfying (EC7). With some algebra, one can show, if

$$c\leq\min(c_A,c_B),$$

then the no-monitoring relational contract has $K=H$ and it achieves the first best; if

$$c>[\delta(1-\sigma)+\sigma]r,$$

then the no-monitoring contract has $K=0$ and yields the same profit as the noncooperative outcome $\underline{\Pi}_B+\underline{\Pi}_S=0$; otherwise, the no-monitoring relational contract has

$$K=\begin{cases}K_B & \text{if } c\leq\tilde{c}\\ K_A & \text{otherwise,}\end{cases}$$

where

$$K_A=\frac{(1-\lambda)[\delta(1-\sigma)+\sigma]rL}{c-r\lambda(\delta(1-\sigma)+\sigma)}$$
$$K_B=\frac{\{(1-\delta)c+\lambda[\delta(1-\lambda)-(1-\delta)\sigma]r\}L}{[1-\delta(1-\lambda)]c-\lambda(\delta(\lambda-\sigma)+\sigma)r},$$

and yields profit which is greater than the noncooperative profit but less than the first best.

Step 2. *The* (EC3)-*constrained optimal relational contract*: Problem (2)–(7) with (EC3) simplifies to

$$f(\pi)=\max_{d(H),\,d(L),\,q(L),\,K}\ \{r[\lambda K+(1-\lambda)L]-cK\}$$

$$\text{subject to}\quad d(L)\leq\min(d(H),\sigma rL)\tag{EC8}$$
$$rK-d(H)\geq\max(rq(L)-d(L),(1-\sigma)rK)\tag{EC9}$$
$$L<q(L)<K\leq H\tag{EC10}$$
$$d(H)+\delta(1-\delta)^{-1}(\pi-\underline{\Pi}_B-\underline{\Pi}_S)\geq\sigma rK\tag{EC11}$$
$$d(L)+\delta(1-\delta)^{-1}(\pi-\underline{\Pi}_B-\underline{\Pi}_S)\geq\sigma rL\tag{EC12}$$
$$\begin{aligned}&-cK+\lambda d(H)+(1-\lambda)d(L)+\delta(1-\delta)^{-1}(\pi-\underline{\Pi}_B-\underline{\Pi}_S)\\&\quad\geq\max((\sigma r-c)L,0)\end{aligned}\tag{EC13}$$
$$\begin{aligned}&-cK+\lambda d(H)+(1-\lambda)d(L)+\delta(1-\delta)^{-1}(\pi-\underline{\Pi}_B-\underline{\Pi}_S)\\&\quad\geq-cq(L)+\lambda\sigma rq(L)+(1-\lambda)[d(L)+\delta(1-\delta)^{-1}(\pi-\underline{\Pi}_B-\underline{\Pi}_S)],\end{aligned}\tag{EC14}$$

where $D = cK - \lambda d(H) - (1-\lambda)d(L) + \pi - \underline{\Pi}_B$, and for brevity $d(M)$ denotes $d(q(M))$ for $M \in \{H, L\}$. Because all the constraints except (EC8) are (weakly) relaxed as $d(L)$ increases and $d(L)$ is absent from the objective, an optimal solution has $d(L) = \min(d(H), \sigma r L)$. Similarly, all constraints except (EC9) are (weakly) relaxed as $d(H)$ increases and $d(H)$ is absent from the objective, so an optimal solution has $d(H) = \min(r(K + \sigma L - q(L)), \sigma r K)$ and $d(L) = \sigma r L$. Thus, the problem simplifies to a maximization in $\{q(L), K\}$ subject to constraints (EC10)–(EC14), which simplify to

$$L < q(L) < K \le H \tag{EC15}$$

$$\min(r(K + \sigma L - q(L)), \sigma r K) + \delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B - \underline{\Pi}_S) \ge \sigma r K \tag{EC16}$$

$$\delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B - \underline{\Pi}_S) \ge 0 \tag{EC17}$$

$$\begin{aligned} &-cK + \lambda \min(r(K + \sigma L - q(L)), \sigma r K) + (1-\lambda)\sigma r L \\ &\quad + \delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B - \underline{\Pi}_S) \ge \max((\sigma r - c)L, 0) \end{aligned} \tag{EC18}$$

$$-cK + \lambda \min(r(K + \sigma L - q(L)), \sigma r K) + \delta(1-\delta)^{-1}\lambda(\pi - \underline{\Pi}_B - \underline{\Pi}_S) \ge -(c - \lambda\sigma r)q(L). \tag{EC19}$$

Because $q(L)$ is absent from the objective and because as $q(L)$ increases on $(L, \sigma L + (1-\sigma)K]$ or decreases on $[\sigma L + (1-\sigma)K, K)$, constraints (EC15)–(EC19) are (weakly) relaxed, an optimal solution has

$$q(L) = \sigma L + (1-\sigma)K, \tag{EC20}$$

$d(H) = \sigma r K$ and $d(L) = \sigma r L$.

Case 1. $c \in (\sigma\lambda r, \min(\sigma, \lambda)r]$: Substituting (EC20), $\pi = r[\lambda K + (1-\lambda)L] - cK$ and $\underline{\Pi}_B + \underline{\Pi}_S = (r-c)L$, constraints (EC15)–(EC19) simplify to

$$\delta(1-\delta)^{-1}\lambda(\lambda r - c) - \max(\sigma, \lambda)(c - \lambda\sigma r) \ge 0 \qquad \text{and} \qquad K \in (L, H].$$

Because the objective value is increasing in K for $K \in (L, H]$, if

$$c \le \frac{\lambda[\delta\lambda + (1-\delta)\sigma \max(\sigma, \lambda)]r}{\delta\lambda + (1-\delta)\max(\sigma, \lambda)}, \tag{EC21}$$

then a (EC3)-constrained optimal relational contract has $K = H$; otherwise, no self-enforcing relational contract satisfying (EC3) exists.

Case 2. $c \in (\sigma r, \lambda r)$: Substituting (EC20), $\pi = r[\lambda K + (1-\lambda)L] - cK$ and $\underline{\Pi}_B + \underline{\Pi}_S = 0$, constraints (EC15)–(EC19) simplify to

$$\delta(1-\delta)^{-1}(r[\lambda K + (1-\lambda)L] - cK) - cK + \sigma r[\lambda K + (1-\lambda)L] \ge 0 \qquad \text{and} \qquad K \in (L, H]. \tag{EC22}$$

Because the objective value is increasing in K for $K \in (L, H]$, the optimal capacity is the largest K satisfying (EC22). If

$$c \le c_A, \tag{EC23}$$

then the (EC3)-constrained optimal relational contract has $K = H$ and it achieves the first best; if

$$c > [\delta(1-\sigma) + \sigma]r,$$

then no self-enforcing relational contract satisfying (EC3) exists; otherwise, a (EC3)-constrained optimal relational contract has

$$K = K_A.$$

Step 3. *The* (EC4)*-constrained optimal relational contract*: Problem (2)–(7) with (EC4) simplifies to

$$f(\pi) = \max_{d, K} \; \{r[\lambda K + (1-\lambda)L] - cK\}$$

$$\text{subject to} \quad \sigma r L \ge d \tag{EC24}$$

$$L < K \le H$$

$$d + \delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B - \underline{\Pi}_S) \ge \sigma r[\lambda K + (1-\lambda)L] \tag{EC25}$$

$$-cK + d + \delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B - \underline{\Pi}_S) \ge \max((\sigma r - c)L, 0), \tag{EC26}$$

where $D = cK - d + \pi - \underline{\Pi}_B$. Constraint (EC26) implies (EC25). Because as d increases, constraint (EC26) is relaxed, in an optimal solution, (EC24) binds: $d = \sigma r L$.

Case 1. $c \in (\sigma\lambda r, \min(\sigma, \lambda)r]$: Substituting $d = \sigma r L$, $\pi = r[\lambda K + (1-\lambda)L] - cK$ and $\underline{\Pi}_B + \underline{\Pi}_S = (r-c)L$, constraint (EC26) simplifies to

$$[\delta(1-\delta)^{-1}(\lambda r - c) - c](K - L) \geq 0.$$

Because the objective value increases as K increases for $K \in (L, H]$, we conclude that if

$$c \leq \delta\lambda r, \tag{EC27}$$

then a (EC4)-constrained optimal relational contract has $K = H$ and it achieves the first best. If (EC27) does not hold, then no self-enforcing relational contract satisfying (EC4) exists.

Case 2. $c \in (\sigma r, \lambda r)$: Substituting $d = \sigma r L$, $\pi = r[\lambda K + (1-\lambda)L] - cK$ and $\underline{\Pi}_B + \underline{\Pi}_S = (r-c)L$, constraint (EC26) simplifies to

$$\delta(1-\delta)^{-1}(r[\lambda K + (1-\lambda)L] - cK) - cK + \sigma r L \geq 0.$$

Because the objective value increases as K increases or $K \in (L, H]$, we conclude that if

$$c \leq \frac{(\delta[\lambda H + (1-\lambda)L] + (1-\delta)\sigma L)r}{H},$$

then a (EC4)-constrained optimal relational contract has $K = H$ and it achieves the first best; if

$$c > [\delta(1-\sigma) + \sigma]r,$$

then no self-enforcing relational contract satisfying (EC4) exists; otherwise, a (EC4)-constrained optimal relational contract has

$$K = \frac{[\delta(1-\lambda) + (1-\delta)\sigma]rL}{c - \delta\lambda r}.$$

To reiterate, of the three solutions identified above satisfying (EC2), (EC3), and (EC4), respectively, the solution with the largest capacity investment is the optimal relational contract.

Step 4. *Characterizing when the optimal relational contract yields strictly greater expected profit than the no-monitoring relational contract*: Now, we establish that the optimal relational contract yields strictly greater expected profit than the no-monitoring relational contract if and only if (21), where

$$c_1 = \min\left(\frac{\lambda[\delta(\lambda-\sigma) + \sigma]r}{1 - \delta(1-\lambda)}, \sigma r\right)$$

$$c_2 = \min\left(\max\left(\delta\lambda r, \frac{\lambda[\delta\lambda + (1-\delta)\sigma\max(\sigma, \lambda)]r}{\delta\lambda + (1-\delta)\max(\sigma, \lambda)}\right), \sigma r, \lambda r\right)$$

$$c_3 = \min(\max(c_B, \sigma r), \lambda r)$$

$$c_4 = \min(\tilde{c}, \lambda r).$$

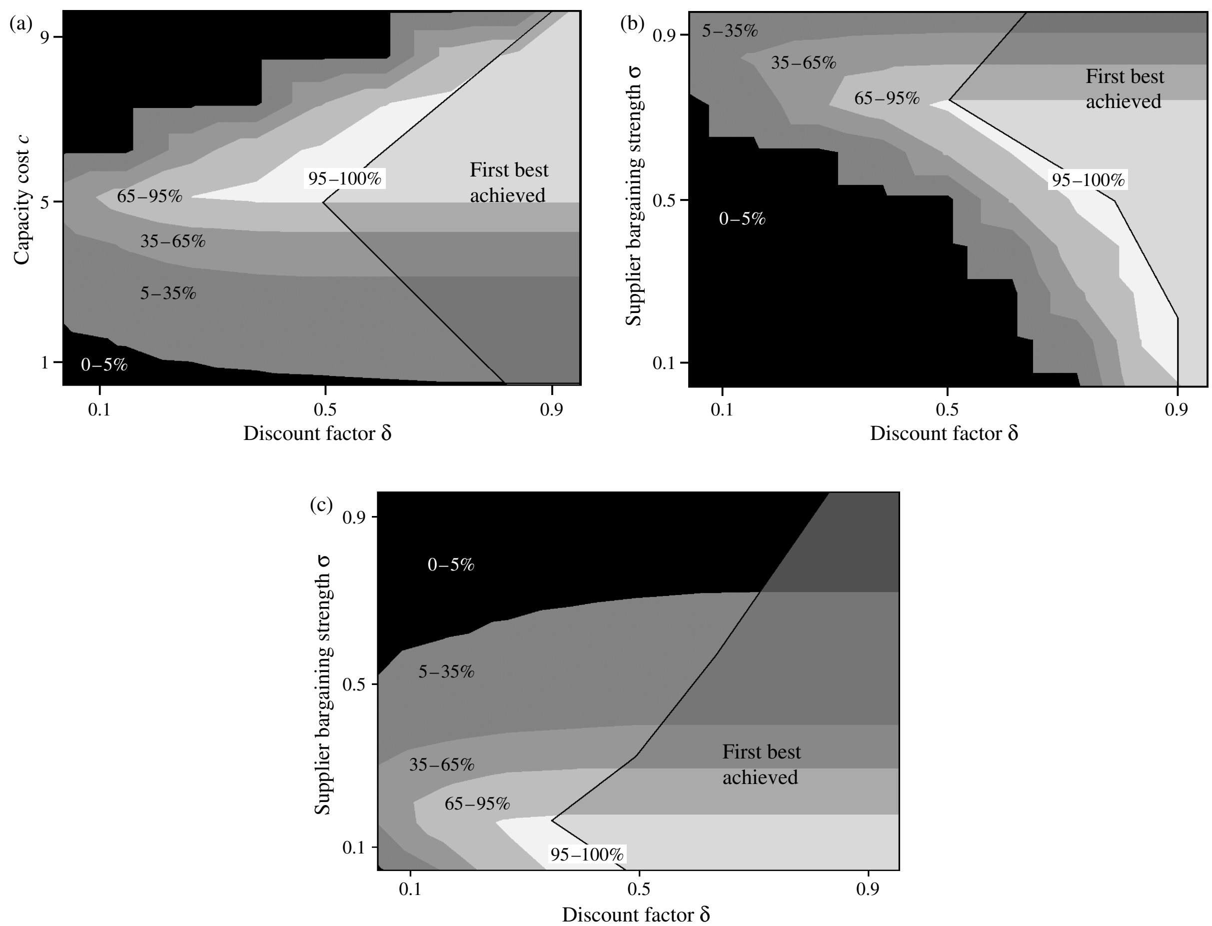
Note that $\{c_1, c_2\} \in (\sigma\lambda r, \min(\sigma, \lambda)r]$, the capacity cost range in Case 1, and $\{c_3, c_4\} \in [\sigma r, \lambda r]$, the capacity cost range in Case 2.

Case 1. $c \in (\sigma\lambda r, \min(\sigma, \lambda)r]$: We next establish that for this range of capacity costs, the optimal relational contract yields strictly larger profit than the no-monitoring relational contract if and only if $c \in (c_1, c_2]$. We established in Step 1 that if (EC6) holds, $\Pi^n = \overline{\Pi}$; otherwise, $\Pi^n = \underline{\Pi}_B + \underline{\Pi}_S$. We established in Steps 1, 2, and 3, that if (EC6), (EC21), or (EC27) holds, then $\Pi^* = \overline{\Pi}$; otherwise, $\Pi^* = \underline{\Pi}_B + \underline{\Pi}_S$. Further, $c_1 \leq c_2$. Therefore, if $c \in (\sigma\lambda r, c_1]$, $\Pi^n = \Pi^* = \overline{\Pi}$; if $c \in (c_2, \min(\sigma, \lambda)r]$, then $\Pi^n = \Pi^* = \underline{\Pi}_B + \underline{\Pi}_S$; and if $c \in (c_1, c_2]$ then

$$\Pi^n = \underline{\Pi}_B + \underline{\Pi}_S < \Pi^* = \overline{\Pi}. \tag{EC28}$$

Case 2. $c \in (\sigma r, \lambda r)$: We next establish that for this range of capacity costs, the optimal relational contract yields strictly larger profit than the no-monitoring relational contract if and only if $c \in (c_3, c_4]$. To see this, first observe that the (EC3)-constrained optimal relational contract yields weakly larger

Figure EC.1 Gain from Optimal Relational Contract: Exponential Demand



Notes. The regions indicate where the optimal relational contract achieves the first best and the gain from using an optimal relational contract as a percentage of the first best. Parameters are $r = 10$ and ξ is an exponential(1) random variable. In panel (a) $\sigma = 0.5$; in panel (b) $c = 7.5$; in panel (c) $c = 2.5$.

profit than the (EC4)-constrained optimal relational contract. We proceed by establishing some preliminary results. Observe that $c_B - c_A$ is convex in L and has two roots, one of which is

$$\tilde{L} = \frac{H}{2(1-\lambda)[\delta(1-\sigma)+\sigma]}\Big\{\delta(1-3\lambda+\lambda^2)(1-\sigma)+(1-\lambda)\sigma + \sqrt{[\delta(1-3\lambda+\lambda^2)(1-\sigma)+(1-\lambda)\sigma]^2+4\delta\lambda(1-\lambda)^2[\delta(1-\sigma)+\sigma](1-\sigma)}\Big\},$$

and one of which is negative. Further, c_A and c_B are increasing in L, and

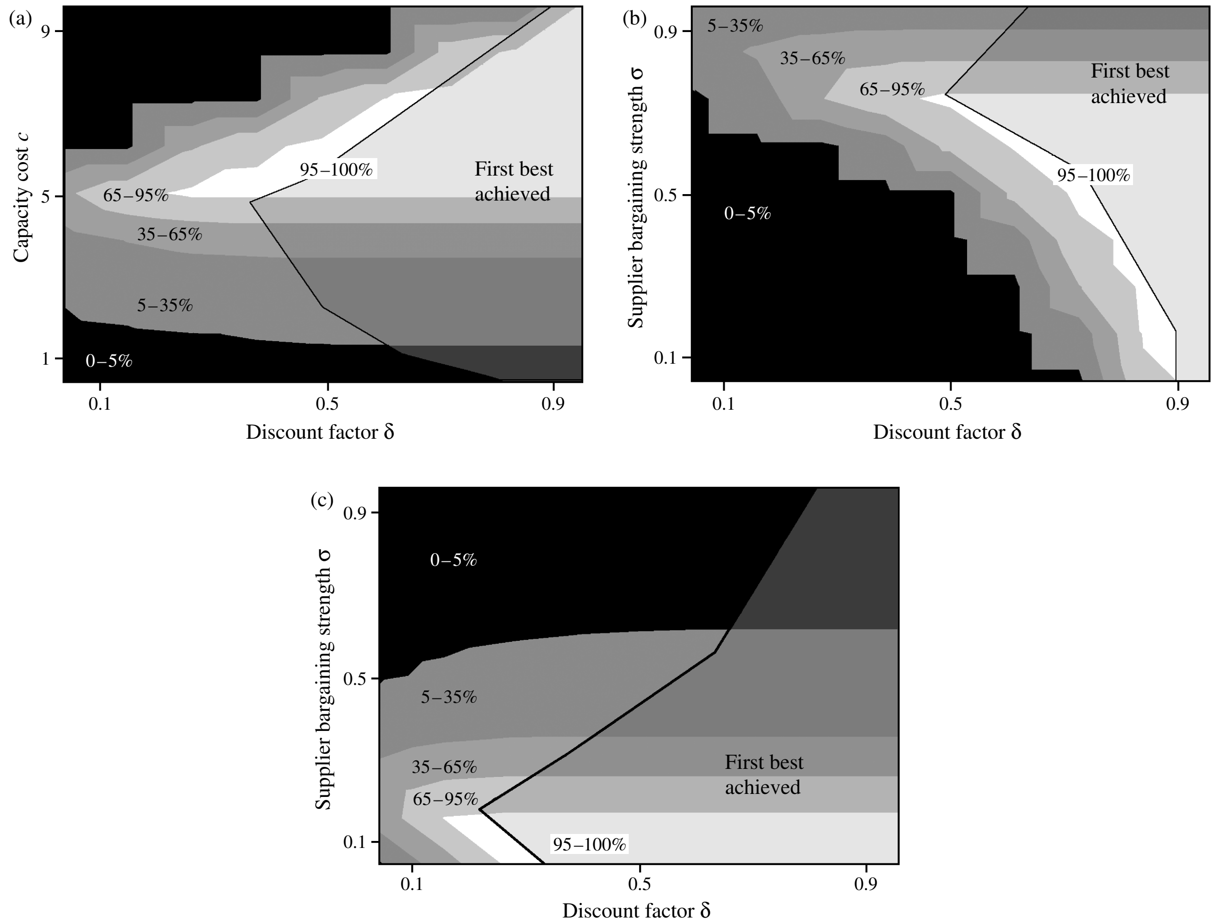
$$c_B = c_A = \tilde{c} \quad \text{if } L = \tilde{L}.$$

This implies that

$$c_B < c_A < \tilde{c} \quad \text{if } L < \tilde{L}$$
$$c_B > c_A > \tilde{c} \quad \text{if } L > \tilde{L}.$$

If $L \geq \tilde{L}$, then $c_3 \geq c_4$ and the capacity under the no-monitoring contract is the same as the capacity under the optimal relational contract (if $c \in (\sigma r, c_A)$, $K^n = K^* = \bar{K}$; if $c \in (c_A, \min([\delta(1-\sigma)+\sigma]r, \lambda r))$, $K^n = K^* = K_A$; and if $c \in ([\delta(1-\sigma)+\sigma]r, \lambda r)$, $K^n = K^* = 0$), so $\Pi^n = \Pi^*$. Suppose $L < \tilde{L}$. If $c \in (\sigma r, c_3]$, then the capacity under both the no-monitoring contract and the optimal relational contract is the first best capacity $K^n = K^* = \bar{K}$, so $\Pi^n = \Pi^* = \bar{\Pi}$. If $c \in (c_4, \lambda r)$, then the capacity under the no-monitoring

Figure EC.2 Gain from Optimal Relational Contract: Uniform Demand



Notes. The regions indicate where the optimal relational contract achieves the first best and the gain from using an optimal relational contract as a percentage of the first best. Parameters are $r = 10$ and ξ is an uniform(0, 1) random variable. In panel (a) $\sigma = 0.5$; in panel (b) $c = 7.5$; in panel (c) $c = 2.5$.

contract is the same as the capacity under the optimal relational contract (if $c \in (c_A, \min([\delta(1-\sigma)+\sigma]r, \lambda r))$, $K^n = K^* = K_A$; and if $c \in ([\delta(1-\sigma)+\sigma]r, \lambda r)$, $K^n = K^* = 0$), so $\Pi^n = \Pi^*$. If $c \in (c_3, c_4]$, then the capacity under the no-monitoring contract is strictly lower than the capacity under the optimal relational contract (if $c \in (c_3, c_A)$, $K^n = K_B < K^* = \bar{K}$; and if $c \in (c_A, c_4)$, $K^n = K_B < K_A = K^*$), so $\Pi^n < \Pi^*$.

Step 5. *Sufficient conditions so that* $c_1 < c_2$: We begin by showing that there exists $\bar{L} \in (0, H]$ such that if $\bar{L} < L$, then $c_1 < c_2$. Define

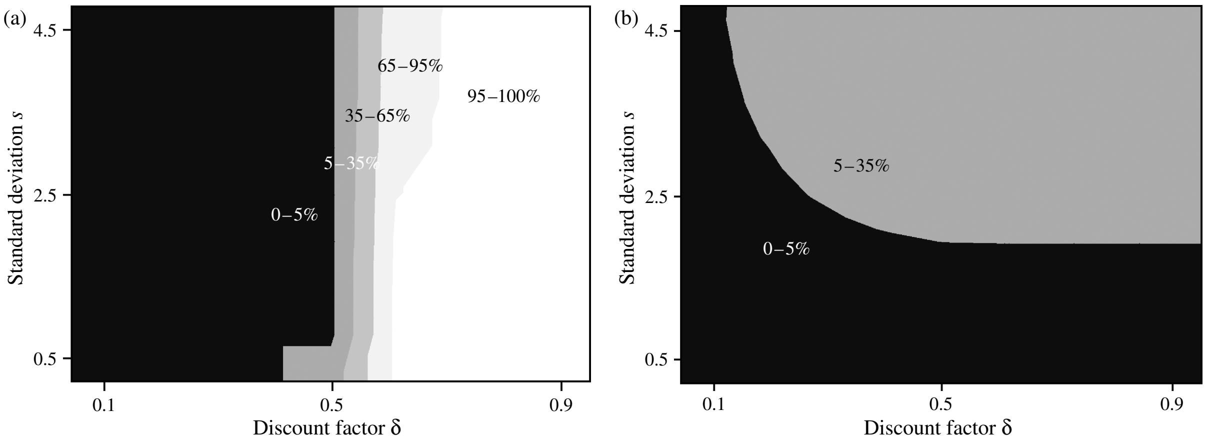
$$\bar{L} = \begin{cases} H & \text{if } \sigma \geq \lambda \\ \min\left(\dfrac{(1-\delta)(1-\sigma)H}{\delta(1-\lambda)+(1-\delta)(1-\sigma)}, \tilde{L}\right) & \text{otherwise.} \end{cases}$$

If $\sigma \geq \lambda$, then (EC1) reduces to Case 1 and $c_1 < c_2$. If $\sigma < \lambda$, then Case 2 is relevant as well. If $c_1 < c_2$, the the result is immediate. Suppose that $c_1 = c_2$; with some algebra, one can show that $L < \bar{L}$ implies $c_3 < c_4$. Because the numbering of the indices was arbitrary, we can relabel c_3 as c_1 and c_4 as c_2, which establishes the result. Finally, we observe that with $\bar{\delta} = \min(\sigma(1-\lambda)/(\lambda^2+\sigma-2\lambda\sigma), 1)$, if $\delta < \bar{\delta}$, then $c_1 < c_2$ and for $c \in (c_1, c_2]$, the optimal relational contract achieves the first best ($\Pi^* = \bar{\Pi}$) but the no-monitoring relational contract is worthless ($\Pi^n = \underline{\Pi}_B + \underline{\Pi}_S$) (see (EC28)). □

Numerical Study

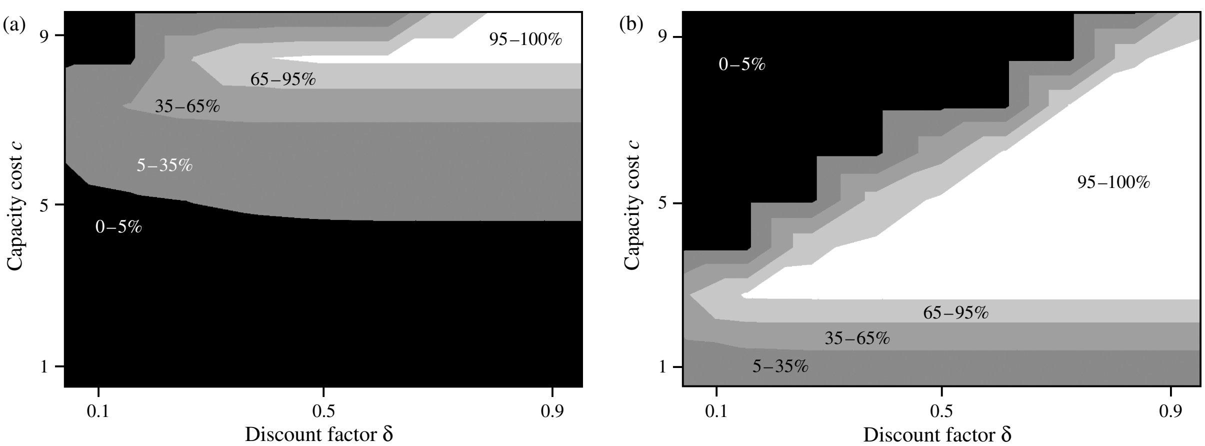
The body of the paper points out that Figure 2 is representative of a larger numerical study. The remainder of this electronic companion provides details of this study.

Figure 3 Gain from Optimal Relational Contract: Impact of Standard Deviation Under Normal Demand



Note. The regions indicate the gain from using an optimal relational contract as a percentage of the first best. Parameters are $r = 10$, $\sigma = 0.5$, and ξ is an normal random variable with mean 5 and standard deviation s, truncated such that its probability mass is distributed over $\xi \geq 0$. In panel (a) $c = 7.5$; in panel (b) $c = 2.5$.

Figure 4 Gain from Optimal Relational Contract: Normal Demand



Note. The regions indicate the gain from using an optimal relational contract as a percentage of the first best. Parameters are $r = 10$, $c = 5$, and ξ is an normal random variable with mean 5 and standard deviation 3, truncated such that its probability mass is distributed over $\xi \geq 0$. In panel (a) $\sigma = 0.75$; in panel (b) $\sigma = 0.25$.

Figures EC.1 and EC.2 show that the form of Figure 2 is structurally similar with demand is an exponential(1) or uniform$(0, 1)$ random variable instead of a truncated normal random variable as depicted in the text.

Figure EC.3 shows the impact of the standard deviation of demand when demand is a truncated normal random variable.

Figure EC.4 substantiates the statement that in Figure 2(a) in the main paper, when σ is larger (smaller), the region with gains shrinks and shifts up (expands and shifts down). This can be seen by comparing the panels in Figure EC.4 where $\sigma = 0.75$ and $\sigma = 0.25$ with Figure 2(a) where $\sigma = 0.5$.

The boundaries that mark the regions in Figure 2 are much smoother because they are based on a study with 99×99 data points. The boundaries that mark the regions in Figures EC.1–EC.4 are much rougher because they are based on a study with less granularity: 9×9 data points.