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**e-companion**

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# Electronic Companion—"Implications of Renegotiation for Optimal Contract Flexibility and Investment" by Erica L. Plambeck and Terry A. Taylor, *Management Science*, DOI 10.1287/mnsc.1070.0731.

Lemmas 1 and 2 are useful in the proofs of Theorems 1 and 3.

LEMMA 1. *The buyers' innovation investments are substitutes*:

$$\frac{\partial^2}{\partial e_1 \partial e_2} E_{e_1 e_2}\left[\sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c))\right] \le 0. \tag{EC1}$$

*Capacity and innovation are complements*: (3), (4), *and* (5).

PROOF OF LEMMA 1. Denote $r^*(m_1, m_2) = \sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c))$. By the Envelope Theorem, $(\partial/\partial m_1) r^*(m_1, m_2) = (\partial/\partial m_1) R_1(m_1, q_1^*(m_1, m_2, c))$. Assumption (2), specifically that $(\partial^2/\partial m_2 \partial q) R_2(m_2, q) \ge 0$, implies $(\partial/\partial m_2) q_1^*(M_1, M_2, c) \le 0$. Again using Assumption (2), specifically $(\partial^2/\partial m_1 \partial q) R_1(m_1, q) \ge 0$, we conclude that

$$\frac{\partial^2}{\partial m_1 \partial m_2} r^*(m_1, m_2) = \frac{\partial}{\partial m_1 \partial q} R_1(m_1, q_1^*(m_1, m_2, c)) \frac{\partial}{\partial m_2} q_1^*(m_1, m_2, c) \le 0. \tag{EC2}$$

Applying Stieltje's integration by parts with $F_i(-\infty, e_i) = 0$ and $F_i(\infty, e_i) = 1$, and then applying Leibnitz's Rule,

$$\begin{aligned}
&\frac{\partial^2}{\partial e_1 \partial e_2} E_{e_1 e_2}\left[\sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c))\right] \\
&\quad = \frac{\partial^2}{\partial e_1 \partial e_2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} r^*(m_1, m_2)\, dF_1(m_1, e_1)\, dF_2(m_2, e_2) \\
&\quad = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\partial}{\partial e_1} F_1(m_1, e_1) \frac{\partial}{\partial e_2} F_2(m_2, e_2) \frac{\partial^2}{\partial m_1 \partial m_2} r^*(m_1, m_2)\, dm_1\, dm_2 \\
&\quad \le 0.
\end{aligned}$$

The inequality follows from (EC2) and Assumption (1), specifically that $(\partial/\partial e_i) F_i(m_i, e_i) \le 0$. By similar analysis,

$$\frac{\partial^2}{\partial e_i \partial q} E_{e_i}[R_i(M_i, q)] = -\int_{-\infty}^{+\infty} \frac{\partial}{\partial e_i} F_i(m_i, e_i) \frac{\partial^2}{\partial m_i \partial q} R_i(m_i, q)\, dm_1 \ge 0$$

$$\begin{aligned}
&\frac{\partial^2}{\partial e_i \partial c} E_{e_1 e_2}\left[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))\right] \\
&\quad = -\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\partial}{\partial e_i} F_i(m_i, e_i) \frac{\partial^2}{\partial m_i \partial q} R_i(m_i, q_i^*(m_1, m_2, c) \frac{\partial}{\partial c} q_i^*(m_1, m_2, c)\, dm_i\, dF_j(m_j, e_j) \ge 0,
\end{aligned}$$

where the inequalities follow from (1) and (2). Finally, (5) follows from analogous arguments and $q_i^*(M_1, M_2, c) \le c$. □

We generalize the definition of $\hat{Q}_i$ in (19) to allow dependency on the capacity $c$: For $c \ge 0$, let $\hat{Q}_i(c)$ denote the unique value of $Q_i \in [0, \bar{Q}_i]$ that satisfies

$$(\partial/\partial e_i) E_{e_i^*(c)}[R_i(M_i, Q_i)] = (\partial/\partial e_i) E_{e_1^*(c) e_2^*(c)}\left[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))\right]. \tag{EC3}$$

Thus, the definition in (19) corresponds to $\hat{Q}_i(c^*)$.

Lemma 2. *Suppose that buyer i has a quantity flexibility contract with* $\Delta_i = Q_i = Q$ (*i.e.*, $p_i(q) = wq$ *for* $q \in [0, Q]$, *for some constant* $w \geq 0$). *Then there exists a unique solution* $e_i^R$ *to problem* (10), *buyer i's best response;* $e_i^R$ *is a continuous function of the exercise price w. If buyer i instead has a fixed-quantity contract* (*i.e.*, $\Delta_i = 0$), *then his unique best response* $e_i^R$ *is a continuous, increasing function of his fixed quantity Q;* $e_i^R$ *is a continuous, decreasing function of the other buyer's innovation* $e_j$; *and* $e_i^R$ *is a continuous function of the buyer's bargaining confidence* $\alpha_i$ *with*

$$(\partial/\partial\alpha_i)e_i^R \geq 0 \quad \text{if and only if} \quad e_i^R \leq e_i^*(c, e_j),$$

*where the inequality on the left hand side is strict if and only if the inequality on the right hand side is strict. Finally,* $\hat{Q}_i(c)$ *is a well-defined, continuous function of c that satisfies* $\hat{Q}_i(c) < \bar{Q}_i$ *for* $0 \leq c < \sum_{i=1,2} \bar{Q}_i$ *and* $\hat{Q}_i(c) = \bar{Q}_i$ *for* $c \geq \sum_{i=1,2} \bar{Q}_i$.

Proof of Lemma 2. Let $\pi_1(m_1, w)$ denote the expected profit for buyer 1 conditioned on his market size $M_1 = m_1$:

$$\pi_1(m_1, w) = \max_{q \in [0, Q]} \left\{ (1-\alpha)R_1(m_1, q) - wq + \alpha E_{e_2}\left[ \sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c)) - R_2(M_2, c-q) \right] \right\}.$$

Clearly, $\pi_1(m_1, w)$ decreases with $w$. Let $q^*(m_1, w)$ denote an optimal solution, and suppose that the unit price $w$ increases to $w + \delta$; $q^*(m_1, w) \leq Q$ remains feasible so $\pi_1(m_1, w+\delta) \in [\pi_1(m_1, w) - \delta Q, \pi_1(m_1, w)]$. We conclude that $\pi_1(m_1, w)$ is a decreasing, $Q$-Lipschitz continuous function of $w$ for all $(m_1, e_2) \in \mathscr{R} \times [0, 1]$.

Buyer 1's optimization problem (10) can be written as

$$\max_{e_1 \in [0,1]} \left\{ \int_{-\infty}^{+\infty} \pi_1(m_1, w)\, dF_1(m_1, e_1) - g_1(e_1) \right\}, \tag{EC4}$$

and the objective is a $Q$-Lipschitz continuous, decreasing function of $w$ (properties inherited from $\pi_1$). Applying Stieltje's integration by parts with $F_1(-\infty, e_1) = 0$ and $F_1(\infty, e_1) = 1$, and then applying Leibnitz's Rule

$$\frac{\partial^2}{\partial e_1^2} \int_{-\infty}^{+\infty} \pi_1(m_1, w)\, dF_1(m_1, e_1) = -\int_{-\infty}^{+\infty} \frac{\partial^2}{\partial e_1^2} F_1(m_1, e_1) \frac{\partial}{\partial m_1} \pi_1(m_1, w)\, dm_1 \leq 0.$$

The inequality follows from Assumption (1), specifically $(\partial^2/\partial e_1^2)F_1(m_1, e_1) \geq 0$, and Assumption (2), specifically $(\partial/\partial m_1)R_1(m_1, q) \geq 0$, which implies $(\partial/\partial m_1)\pi_1(m_1, w) \geq 0$. We have assumed that for all $e_1 \in [0, 1]$, $g_1''(e_1) \geq \underline{g} > 0$. Therefore buyer 1's optimization problem (EC4) has a strictly concave objective function and hence a unique optimal solution $e_1^R(w)$. In particular, $e_1^R(w)$ is the unique value of $e_1$ that satisfies the first-order condition

$$\int_{-\infty}^{+\infty} \frac{\partial}{\partial e_1} F_1(m_1, e_1) \frac{\partial}{\partial m_1} \pi_1(m_1, w)\, dm_1 + g_1'(e_1) = 0 \tag{EC5}$$

(our assumptions $g_1'(0) = 0$ and (1)–(2) imply $\int_{-\infty}^{+\infty} (\partial/\partial e_1)F_1(m_1, 0)(\partial/\partial m_1)\pi_1(m_1, w)\, dm_1 + g_1'(0) \leq 0$ and with our assumption that $g_1'(1) = \infty$ guarantee that (EC5) is satisfied for some $e_1 \in [0, 1)$).

Now let us establish that $e_1^R(w)$ is continuous in $w$. Let $H(e_1, w)$ denote the objective function in buyer 1's optimization problem (EC4). Consider a sequence $\{w^n\}_{n=1..\infty}$ with $\lim_{n\to\infty} w^n = w$. Because $(\partial^2/\partial e_1^2)H(e_1, w) \leq -\underline{g} < 0$,

$$H(e, w^n) \leq H(e_1^R(w^n), w^n) - (e_1^R(w^n) - e)^2 \underline{g}/2 \quad \text{for all } w^n \geq 0 \text{ and } e \in [0, 1]. \tag{EC6}$$

Fix $\epsilon > 0$, $\delta \in (0, \epsilon^2 \underline{g}/4Q)$ and finite $N$ with $|w^n - w| < \delta$ for $n \geq N$. Because $H(e_1, w)$ is $Q$-Lipschitz in $w$ for all $e_1 \in [0, 1]$,

$$H(e, w) \leq H(e, w^n) + \delta Q \quad \text{for all } n \geq N \text{ and } e \in [0, 1]. \tag{EC7}$$

Applying (EC6) and (EC7), we find that for all $n \geq N$ and $e$ satisfying $|e - e_1^R(w^n)| > \epsilon$

$$H(e, w) \leq H(e_1^R(w^n), w) + 2\delta Q - \epsilon^2 \underline{g}/2 < H(e_1^R(w^n), w)$$

and therefore

$$|e_1^R(w) - e_1^R(w^n)| \le \epsilon \quad \text{for } n \ge N.$$

We conclude that $e_1^R(w)$ is continuous in $w$.

Now let us focus on fixed-quantity contracts ($\Delta = 0$) and explicitly represent the dependence of $e_1^R$ and $\pi_1(m_1)$ on $(Q, e_2, \alpha_1)$. From (EC5), the buyer's best response $e_1^R(Q, e_2, \alpha_1)$ is the unique solution to

$$\int_{-\infty}^{+\infty} \frac{\partial}{\partial e_1} F_1(m_1, e_1) \frac{\partial}{\partial m_1} \pi_1(m_1, Q, e_2, \alpha_1)\, dm_1 + g_1'(e_1) = 0.$$

From the Implicit Function Theorem, $e_1^R(Q, e_2, \alpha_1)$ is continuous in $Q$, $e_2$, and $\alpha_1$ and has partial derivatives

$$\frac{\partial}{\partial Q} e_1^R(Q, e_2, \alpha_1) = \frac{-\int_{-\infty}^{+\infty} (\partial/\partial e_1) F_1(m_1, e_1)(\partial^2/\partial m_1 \partial Q)\pi_1(m_1, Q, e_2, \alpha_1)\, dm_1}{\int_{-\infty}^{+\infty} (\partial^2/\partial e_1^2) F_1(m_1, e_1)(\partial/\partial m_1)\pi_1(m_1, Q, e_2, \alpha_1)\, dm_1 + g_1''(e_1)} \bigg|_{e_1 = e_1^R(Q, e_2, \alpha_1)} \tag{EC8}$$

$$\frac{\partial}{\partial e_2} e_1^R(Q, e_2, \alpha_1) = \frac{-\int_{-\infty}^{+\infty} (\partial/\partial e_1) F_1(m_1, e_1)(\partial^2/\partial m_1 \partial e_2)\pi_1(m_1, Q, e_2, \alpha_1)\, dm_1}{\int_{-\infty}^{+\infty} (\partial^2/\partial e_1^2) F_1(m_1, e_1)(\partial/\partial m_1)\pi_1(m_1, Q, e_2, \alpha_1)\, dm_1 + g_1''(e_1)} \bigg|_{e_1 = e_1^R(Q, e_2, \alpha_1)} \tag{EC9}$$

$$\frac{\partial}{\partial \alpha_1} e_1^R(Q, e_2, \alpha_1) = \frac{-\int_{-\infty}^{+\infty} (\partial/\partial e_1) F_1(m_1, e_1)(\partial^2/\partial m_1 \partial \alpha_1)\pi_1(m_1, Q, e_2, \alpha_1)\, dm_1}{\int_{-\infty}^{+\infty} (\partial^2/\partial e_1^2) F_1(m_1, e_1)(\partial/\partial m_1)\pi_1(m_1, Q, e_2, \alpha_1)\, dm_1 + g_1''(e_1)} \bigg|_{e_1 = e_1^R(Q, e_2, \alpha_1)}. \tag{EC10}$$

The denominator (identical in (EC8), (EC9), and (EC10)) is strictly positive because $g_1''(e_1) > 0$, $(\partial^2/\partial e_1^2)F_1(m_1, e_1) \ge 0$ and $(\partial/\partial m_1)\pi_1(m_1, Q, e_2, \alpha_1) \ge 0$. The numerator in (EC8) is negative because $(\partial/\partial e_1)F_1(m_1, e_1) \le 0$ and $(\partial^2/\partial m_1 \partial Q)\pi_1(m_1, Q, e_2, \alpha_1) \ge 0$. The numerator in (EC9) is positive because $(\partial^2/\partial m_1 \partial e_2)\pi_1(m_1, Q, e_2, \alpha_1) = -\alpha_1 \int_{-\infty}^{+\infty} (\partial/\partial e_2) F_2(m_2, e_2)(\partial^2/\partial m_1 \partial q) R_1(m_1, q_1^*(m_1, m_2, c)) \cdot (\partial/\partial m_2) q_1^*(m_1, m_2)\, dm_2 \le 0$. The numerator in (EC10) can be rewritten as

$$\left\{ (\partial/\partial e_1) E_{e_1}[R_1(M_1, Q_1)] - (\partial/\partial e_1) E_{e_1 e_2}\left[ \sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c)) \right] \right\} \bigg|_{e_1 = e_1^R},$$

which is negative if and only if $e_1^R \le e_1^*(c, e_2)$ and is zero if and only if $e_1^R = e_1^*(c, e_2)$.

Existence and uniqueness of $\hat{Q}_i(c)$ follows from (5), $(\partial/\partial e_i) E_{e_i^*(c)}[R_i(M_i, 0)] = 0$, and our assumption that for $i = 1, 2$, there exists $\bar{Q}_i$ such that $(\partial/\partial q) E_{e_i}[R_i(M_i, q)] = 0$ for $q \ge \bar{Q}_i$ and $(\partial^2/\partial e_i \partial q) E_{e_i}[R_i(M_i, q)] > 0$ for $q < \bar{Q}_i$. Strict joint concavity of the integrated expected profit function guarantees that $e_i^*(c)$ is continuous in $c$ for $i = 1, 2$, and, with Assumptions (1)–(2), guarantees that the left hand side and the right hand side of (EC3) are continuous in $c$. Then continuity of $\hat{Q}_i(c)$ in $c$ follows from existence of $(\partial^2/\partial e_i \partial q) E_{e_i}[R_i(M_i, q)]$. Assumptions (1), (2) and $(\partial/\partial q) E_{e_i}[R_i(M_i, q)] = 0$ for $q \ge \bar{Q}_i$ imply that $(\partial/\partial e_i) E_{e_1 e_2}[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))] = (\partial/\partial e_i) E_{e_i}[R_i(M_i, \bar{Q}_i)]$ for $c \ge \sum_{i=1,2} \bar{Q}_i$ and $(e_1, e_2) \in [0, 1]^2$, which establishes $\hat{Q}_i(c) = \bar{Q}_i$ for $c \ge \sum_{i=1,2} \bar{Q}_i$. Then, $\hat{Q}_i(c) < \bar{Q}_i$ for $c < \sum_{i=1,2} \bar{Q}_i$ follows immediately from our assumption that the inequalities (3) and (4) are strict for $q < \bar{Q}_i$ and $c < \sum_{i=1,2} \bar{Q}_i$. □

Proof of Theorem 1. It is straightforward to show that (1) and (2) imply that

$$(\partial^2/\partial e_i^2) E_{e_i}[R_i(M_i, Q_i)] \le 0 \quad \text{and} \quad (\partial^2/\partial e_i^2) E_{e_1 e_2}\left[ \sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c)) \right] \le 0,$$

so $g_i''(e_i) > 0$ implies that buyer $i$'s objective function under fixed-quantity contracts is strictly concave in the settings with and without renegotiation. This, together with the facts that each buyer's objective function is continuous in his innovation and each buyer's strategy space is compact and convex, implies that there exists at least one Nash equilibrium in each setting. Under fixed-quantity contracts, in the case without renegotiation, buyer $i$'s optimal innovation $e_i^n$ is given by the unique solution to the first-order condition

$$(\partial/\partial e_i) E_{e_i}[R_i(M_i, Q_i)] - g_i'(e_i) = 0.$$

Therefore $(e_1^n, e_2^n)$ is unique. In the case with renegotiation, buyer 1's best response $e_i^R(e_j)$ to innovation level $e_j$ is the unique solution to

$$(1-\alpha_i)(\partial/\partial e_i)E_{e_i}[R_i(M_i, Q_i)] + \alpha_i(\partial/\partial e_i)E_{e_1e_2}\left[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))\right] - g_i'(e_i) = 0.$$

Note that $e_i^R(e_j) > e_i^n$ if and only if

$$(\partial/\partial e_i)E_{e_1e_2}\left[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))\right]\Bigg|_{e_i=e_i^n} > (\partial/\partial e_i)E_{e_i}[R_i(M_i, Q_i)]|_{e_i=e_i^n}. \qquad \text{(EC11)}$$

Because by Lemma 1 innovation investments are substitutes (EC1), (EC11) holds if and only if $e_j < \bar{e}_j$. This implies (12).

Suppose the buyers and contracts are symmetric. In the setting with renegotiation suppose that $e_1^r = e_2^r = e_h^r$ and $e_1^r = e_2^r = e_l^r$ are symmetric equilibria with $e_h^r > e_l^r$. Then

$$\begin{aligned} g'(e_h^r) &= \left\{(1-\alpha)(\partial/\partial e_i)E_{e_i}[R(M_i, Q)] + \alpha(\partial/\partial e_i)E_{e_1e_2}\left[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))\right]\right\}\Bigg|_{e_1=e_2=e_h^r} \\ &\le \left\{(1-\alpha)(\partial/\partial e_i)E_{e_i}[R(M_i, Q)] + \alpha(\partial/\partial e_i)E_{e_1e_2}\left[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))\right]\right\}\Bigg|_{e_1=e_2=e_l^r} \\ &= g'(e_l^r), \end{aligned}$$

where the inequality follows because by Lemma 1 innovation investments are substitutes (EC1) and because $(\partial^2/\partial e_i^2)E_{e_i}[R_i(M_i, Q_i)] \le 0$ and $(\partial^2/\partial e_i^2)E_{e_1e_2}[\sum_{l=1,2} R_l(M_l, q_l^*(M_1, M_2, c))] \le 0$. Because $g(\cdot)$ is strictly convex, $g'(e_h^r) > g'(e_l^r)$; this contradiction establishes that the symmetric equilibrium in unique. (12) implies $e^r > e^n$ if and only if $e^r < \bar{e}_1$, which implies (13). □

Proof of Theorem 3. We will characterize an optimal quantity flexibility contract and associated Nash equilibrium in investment for three parameter regions.

Region 1: $\sum_{i=1,2} \hat{Q}_i(c^*) = c^*$. If the manufacturer chooses capacity $c^*$, then under fixed-quantity contracts $(Q_1, Q_2) = (\hat{Q}_1(c^*), \hat{Q}_2(c^*))$, $(e_1, e_2) = (e_1^*, e_2^*)$ is an equilibrium in innovation. Because $(e_1^*, e_2^*, c^*)$ is optimal for the integrated system, by the envelope theorem

$$(\partial/\partial c)E_{e_1^*e_2^*}\left[\sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c))\right]\Bigg|_{c=c^*} = k.$$

Because $E_{e_1e_2}[\sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c))]$ is concave in $c$ and $\alpha_m \in [0, 1)$, for $c \ge c^*$,

$$\alpha_m(\partial/\partial c)E_{e_1^*e_2^*}\left[\sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c))\right] < k.$$

Therefore, under fixed-quantity contracts $(\hat{Q}_1(c^*), \hat{Q}_2(c^*))$ and innovation levels $(e_1^*, e_2^*)$, the manufacturer's best response is $c^R = c^*$.

Region 2: $\sum_{i=1,2} \hat{Q}_i(c^*) < c^*$. We will construct quantity flexibility contracts that implement the first-best actions $(e_1^*, e_2^*, c^*)$. Buyer 1 has a fixed-quantity contract with $Q_1 = \hat{Q}_1(c^*)$, and buyer 2 has a quantity flexibility contract with $Q_2 = \Delta_2 = c^* - \hat{Q}_1(c^*)$. Assuming capacity investment $c = c^*$ and buyer 2 innovation $e_2 = e_2^*$, buyer 1 chooses innovation according to

$$\max_{e_1}\left\{E_{e_1}\left[R_1(M_1, \hat{Q}_1(c^*)) + \alpha_1 E_{e_2^*}\left[\sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c^*)) - R_1(M_1, \hat{Q}_1(c^*)) - R_2(M_2, c^* - \hat{Q}_1(c^*))\right]\right] - g_1(e_1)\right\}.$$

Because buyer 1's best response is unique, by construction of $\hat{Q}_1(c^*)$, buyer 1's best response is $e_1^R = e_1^*$. Assuming capacity investment $c = c^*$ and buyer 1 innovation $e_1 = e_1^*$, buyer 2 chooses innovation according to

$$\max_{e_2}\left\{E_{e_2}\left[\max_{q_2(M_2)\in[0,\,c^*-\hat{Q}_1(c^*)]}\left\{R_2(M_2, q_2(M_2)) + \alpha_2 E_{e_1^*}\left[\sum_{i=1,2} R_i(M_i, q_i^*(M_1, M_2, c^*)) - R_1(M_1, c^* - q_2(M_2)) - R_2(M_2, q_2(M_2))\right] - w_2 q_2(M_2)\right\}\right] - g_2(e_2)\right\}.$$

Observe that for $w = 0$, $q_2(M_2) = c^* - \hat{Q}_1(c^*) > \hat{Q}_2(c^*)$ for all $M_2$ and thus buyer 2's best response $e_2^R > e_2^*$; as $w \to \infty$, $q_2(M_2) \to 0$ for all $M_2$ and thus buyer 2's best response $e_2^R < e_2^*$. Because buyer 2's best response function is continuous in $w_2$ (by Lemma 2), there exists $\tilde{w}_2$ such that at $\tilde{w}_2$ buyer 2's best response $e_2^R = e_2^*$. Under the contracts $(Q_1, \Delta_1, w_1) = (\hat{Q}_1(c^*), 0, w_1)$ and $(Q_2, \Delta_2, w_2) = (c^* - \hat{Q}_1(c^*), c^* - \hat{Q}_1(c^*), \tilde{w}_2)$ and innovation levels $(e_1^*, e_2^*)$, the manufacturer's best response to build capacity $c^R = c^*$ (by the same argument as for Region 1).

Finally, we will show that fixed-quantity contracts cannot implement the first-best actions $(e_1^*, e_2^*, c^*)$. By extension of the argument above, assuming capacity investment $c = c^*$ and buyer 2 innovation $e_2 = e_2^*$, buyer 1 chooses innovation $e_1^R = e_1^*$ under a fixed-quantity contract if and only if $Q_1 = \hat{Q}_1(c^*)$. By extension of the argument for Region 1, assuming innovation $(e_1^*, e_2^*)$, the manufacturer will build the first-best capacity if and only if

$$\sum_{i=1,2} Q_i = c^*. \tag{EC12}$$

However, $\sum_{i=1,2} \hat{Q}_i(c^*) < c^*$ implies that the fixed-quantity contracts required to induce the first-best innovation investments assuming capacity investment $c = c^*$ violate (EC12), so fixed-quantity contracts cannot implement the first-best actions.

Region 3: $\sum_{i=1,2} \hat{Q}_i(c^*) > c^*$. Our analysis will proceed in three steps. Consider the relaxed contract design problem in which the constraint that the manufacturer's capacity is a best response is replaced with the weaker constraint that $c \geq \sum_{i=1,2} Q_i$; call this problem (R). First, we will construct a solution to (R) with fixed-quantity contracts $(\tilde{Q}_1, \tilde{Q}_2)$ capacity $\tilde{c} = \sum_{i=1,2} \tilde{Q}_i$ and associated innovation equilibrium $(\tilde{e}_1, \tilde{e}_2)$ that satisfies

$$\tilde{e}_1 \leq e_1^*(\tilde{c}, \tilde{e}_2) \qquad \text{and} \qquad \tilde{e}_2 \leq e_2^*(\tilde{c}, \tilde{e}_1). \tag{EC13}$$

Second, we will prove that $(\tilde{Q}_1, \tilde{Q}_2, \tilde{c}, \tilde{e}_1, \tilde{e}_2)$ is a solution to the original contract design problem (given fixed-quantity contracts $(\tilde{Q}_1, \tilde{Q}_2)$ and innovation $(\tilde{e}_1, \tilde{e}_2)$, the manufacturer optimally chooses capacity $\tilde{c}$). Third, we will prove that $(\tilde{c}, \tilde{e}_1, \tilde{e}_2) \neq (c^*, e_1^*, e_2^*)$.

In constructing a solution to (R), we can restrict attention to fixed-quantity contracts. To see this, consider any capacity $\check{c}$, general contracts $\{\check{Q}_i, \check{p}_i(q)\colon 0 \leq q \leq \check{Q}_i\}_{i=1,2}$ satisfying $\check{c} \geq \sum_{i=1,2} \check{Q}_i$ and associated innovation equilibrium $(\check{e}_1, \check{e}_2)$. Because capacity and innovation are complements (3), $\check{e}_1 \in [e_1^R(e_2, c, 0, 0, 0), e_1^R(e_2, c, \check{Q}_1, 0, 0)]$ and $e_1^R(e_2, c, \cdot, 0, 0)$ is increasing (by Lemma 2). Because $e_1^R(e_2, c, \cdot, 0, 0)$ is continuous (by Lemma 2), there exists $\check{Q}_1 \leq \check{Q}_1$ such that $e_1^R(e_2, c, \check{Q}_1, 0, 0) = \check{e}_1$. Thus, under fixed-quantity contracts $(\check{Q}_1, \check{Q}_2)$, where $\check{Q}_2$ is defined analogously, $(\check{e}_1, \check{e}_2)$ is an innovation equilibrium and $\check{c} \geq \sum_{i=1,2} \check{Q}_i$. Recall that expected profit strictly decreases with $c$ for $c \geq \bar{c}$ where $\bar{c} < \sum_{i=1,2} \bar{Q}_i$. Also by Lemma 2, $\hat{Q}_i(c) < \bar{Q}_i$ for $0 \leq c < \sum_{i=1,2} \bar{Q}_i$ and $\hat{Q}_i(c) = \bar{Q}_i$ for $c \geq \sum_{i=1,2} \bar{Q}_i$. Therefore, in solving (R), we can restrict attention to $c \leq \sum_{i=1,2} \bar{Q}_i$ and $Q_i \leq \bar{Q}_i$ for $i = 1, 2$. A solution $(\tilde{Q}_1, \tilde{Q}_2, \tilde{c}, \tilde{e}_1, \tilde{e}_2)$ exists because the contract parameters and innovation levels are restricted to compact intervals, and the integrated expected profit function and buyers' best response functions are continuous.

We now show that our solution $(\tilde{Q}_1, \tilde{Q}_2, \tilde{c}, \tilde{e}_1, \tilde{e}_2)$ to (R) satisfies $\sum_{i=1,2} \hat{Q}_i(\tilde{c}) \geq \tilde{c}$. Because $\sum_{i=1,2} \hat{Q}_i(c^*) > c^*$, if $\sum_{i=1,2} \hat{Q}_i(\tilde{c}) < \tilde{c}$ then $\tilde{c} \neq c^*$. Suppose $\sum_{i=1,2} \hat{Q}_i(\tilde{c}) < \tilde{c} < c^*$. As in Region 2, if $\sum_{i=1,2} \hat{Q}_i(c) < c$ there exist contracts that induce $(e_1^*(c), e_2^*(c))$ and satisfy $c \geq \sum_{i=1,2} Q_i$. Because $\hat{Q}_i(c)$ is continuous (by Lemma 2), there exists $\delta \in (0, c^* - \tilde{c})$ such that $\sum_{i=1,2} \hat{Q}_i(\tilde{c} + \delta) < \tilde{c} + \delta$. Therefore, there exist contracts that induce $(e_1^*(\tilde{c} + \delta), e_2^*(\tilde{c} + \delta))$ and satisfy $\tilde{c} + \delta \geq \sum_{i=1,2} Q_i$. Because $\Pi(e_1^*(c), e_2^*(c), c)$ is strictly concave in $c$, these contracts result in strictly greater expected profit, so our solution to (R) cannot have $\sum_{i=1,2} \hat{Q}_i(\tilde{c}) < \tilde{c} < c^*$. By similar argument, our solution to (R) cannot have $\sum_{i=1,2} \hat{Q}_i(\tilde{c}) < \tilde{c}$ and $\tilde{c} > c^*$.

If $\sum_{i=1,2}\hat{Q}_i(\tilde{c})=\tilde{c}$, then as in Region 1, under fixed-quantity contracts $(\tilde{Q}_1,\tilde{Q}_2)=(\hat{Q}_1(c),\hat{Q}_2(c))$, $(\tilde{e}_1,\tilde{e}_2)=(e_1^*(\tilde{c}),e_2^*(\tilde{c}))$ is an equilibrium and $\sum_{i=1,2}\tilde{Q}_i=\tilde{c}$.

For the case $\sum_{i=1,2}\hat{Q}_i(\tilde{c})>\tilde{c}$, we will argue by contradiction to establish (EC13) and then $\sum_{i=1,2}\tilde{Q}_i=\tilde{c}$. Suppose

$$\tilde{e}_1>e_1^*(\tilde{c},\tilde{e}_2)\qquad\text{and}\qquad\tilde{e}_2<e_2^*(\tilde{c},\tilde{e}_2).\tag{EC14}$$

Consider the alternate contract $(Q_1',Q_2')=(\tilde{Q}_1-\delta,\tilde{Q}_2+\delta)$ for $\delta>0$. For $\delta$ sufficiently small, there exists an equilibrium $(e_1',e_2')$ with $e_1'\in(e_1^*(c,e_2'),e_1)$ and $e_2'\in[e_2,e_2^*(c,e_1))$. This follows from the following: $(\partial^2/\partial q\partial e_2)E_{e_2}[R_2(M_2,q)]\geq 0$ (by Lemma 1); $(\partial^2/\partial q\partial e_1)E_{e_1}[R_1(M_1,q)]>0$ for $q\in[\tilde{Q}_1-\delta,\tilde{Q}_1]$ (because $\tilde{Q}_i-\delta<\tilde{Q}_i\leq\bar{Q}_i$); $e_1^*(c,e_2)$ is continuous in $e_2$ (which is implied by strict joint concavity of the integrated system expected profit); and $e_i^R$ is continuous and decreasing in $e_j$ (by Lemma 2). By strict joint concavity of the integrated expected profit function,

$$\Pi(e_1,e_2,c)\leq\Pi(e_1,e_2',c)<\Pi(e_1',e_2',c),$$

so the equilibrium $(e_1',e_2')$ results in strictly greater profit, contradicting the optimality of $(\tilde{e}_1,\tilde{e}_2)$. Therefore, (EC14) cannot hold. Suppose

$$\tilde{e}_1>e_1^*(\tilde{c},\tilde{e}_2)\qquad\text{and}\qquad\tilde{e}_2\geq e_2^*(\tilde{c},\tilde{e}_2).\tag{EC15}$$

One can decrease $\tilde{Q}_1$ and $\tilde{Q}_2$ by small amounts so that there exists an equilibrium with $(e_1',e_2')$ with $e_1'\in[e_1^*(\tilde{c},e_2),\tilde{e}_1)$ and $e_2'=\tilde{e}_2$. To see this, note that with $\tilde{e}_2$ fixed, one can decrease buyer 1's optimal innovation slightly by decreasing $\tilde{Q}_1$. With $e_1'$ fixed, one can reduce $\tilde{Q}_2$ by a small amount so that the resulting best response $e_2'=\tilde{e}_2$. The equilibrium $(e_1',e_2')$ yields strictly greater expected profit, so (EC15) cannot hold. The same arguments that rule out (EC14) and (EC15) hold when the subscript indices are reversed. Thus, optimal contracts and associated equilibrium must satisfy (EC13). If (EC13) holds with equality for $i=1,2$, then by strict joint concavity of the integrated expected profit function, $(\tilde{e}_1,\tilde{e}_2)=(e_1^*(\tilde{c}),e_2^*(\tilde{c}))$. This holds if and only if the contracts are $(\tilde{Q}_1,\tilde{Q}_2)=(\hat{Q}_1(\tilde{c}),\hat{Q}_2(\tilde{c}))$, which because $\sum_{i=1,2}\hat{Q}_i(\tilde{c})>\tilde{c}$ implies $\sum_{i=1,2}\tilde{Q}_i>\tilde{c}$, a contradiction. Therefore, without loss of generality, we can suppose $\tilde{e}_1<e_1^*(\tilde{c},\tilde{e}_2)$. It remains to show that $\tilde{c}=\sum_{i=1,2}\tilde{Q}_i$. We noted above that $\tilde{c}\leq\sum_{i=1,2}\bar{Q}_i$ and $\tilde{Q}_i\leq\bar{Q}_i$ for $i=1,2$. If $\tilde{c}=\sum_{i=1,2}\bar{Q}_i$, then $\tilde{Q}_i=\bar{Q}_i$ for $i=1,2$, which implies $\tilde{c}=\sum_{i=1,2}\tilde{Q}_i$. If $\tilde{c}<\sum_{i=1,2}\bar{Q}_i$, then (5) and $(\partial^2/\partial e_i\partial q)E_{e_i}[R_i(M_i,q)]=0$ for $q\geq\bar{Q}_i$ imply that $(\partial/\partial e_i)E_{e_i}[R_i(M_i,\bar{Q}_i)]>(\partial/\partial e_i)E_{e_1e_2}[\sum_{l=1,2}R_l(M_l,q_l^*(M_1,M_2,\tilde{c}))]$ and $e_i^R(\tilde{e}_j,\tilde{c},\bar{Q}_i,0,0)>e_i^*(\tilde{c},\tilde{e}_j)$. Therefore, $\tilde{Q}_i<\bar{Q}_i$ for $i=1,2$ (or we would have a contradiction of (EC13)). If $\tilde{c}>\sum_{i=1,2}\tilde{Q}_i$, then one can increase $\tilde{Q}_1$ and $\tilde{Q}_2$ by small amounts so that there exists an equilibrium $(e_1',e_2')$ with $e_1'\in(\tilde{e}_1,e_1^*(c,\tilde{e}_2)]$ and $e_2'=\tilde{e}_2$. The equilibrium $(e_1',e_2')$ yields strictly greater expected profit, so it must be that $\tilde{c}=\sum_{i=1,2}\tilde{Q}_i$.

The second step is to show that our optimal solution to problem (R) is an optimal solution to the original problem which includes the constraint that the capacity is a best response for the manufacturer. First, we consider the case where $\tilde{c}>0$, and we begin by establishing that $\tilde{Q}_i>0$ for $i=1,2$. We established above for the case $\tilde{c}=\sum_{i=1,2}\bar{Q}_i$, that $\tilde{Q}_i=\bar{Q}_i>0$ for $i=1,2$. Otherwise, $\tilde{c}<\sum_{i=1,2}\bar{Q}_i$ and $(\partial/\partial e_i)E_{e_i}[R_i(M_i,c)]>(\partial/\partial e_i)E_{e_1e_2}[\sum_{l=1,2}R_l(M_l,q_l^*(M_1,M_2,c))]$ and (EC13) imply that $\tilde{Q}_i\in(0,\tilde{c})$ for $i=1,2$. Suppose that $c=\tilde{c}$ is not a best response. Because in the optimal solution to (R) $\sum_{i=1,2}\tilde{Q}_i=\tilde{c}$, this implies

$$\alpha_m(\partial/\partial c)E_{\tilde{e}_1\tilde{e}_2}\left[\sum_{i=1,2}R_i(M_i,q_i^*(M_1,M_2,c))\right]\Bigg|_{c=\tilde{c}}>k.$$

This implies

$$(\partial/\partial c)E_{\tilde{e}_1\tilde{e}_2}\left[\sum_{i=1,2}R_i(M_i,q_i^*(M_1,M_2,c))\right]\Bigg|_{c=\tilde{c}}>k,$$

so total system expected profit is strictly greater at $(\tilde{e}_1,\tilde{e}_2,\tilde{c}+\delta)$ for $\delta$ sufficiently small and strictly positive than at $(\tilde{e}_1,\tilde{e}_2,\tilde{c})$. Let $e_1^R(e_2,c,Q_1,\Delta_1,w_1)$ denote buyer 1's best response to innovation $e_2$ and capacity $c$ under contract $(Q_1,\Delta_1,w_1)$. Note that

$$\tilde{e}_1=e_1^R(\tilde{e}_2,\tilde{c},\tilde{Q}_1,\tilde{Q}_1,0)\leq e_1^R(\tilde{e}_2,\tilde{c}+\delta,\tilde{Q}_1,\tilde{Q}_1,0),$$

where the inequality holds because capacity and innovation are complements (4). Then, because $\tilde{Q}_1 > 0$,

$$\lim_{w_1\to\infty} e_1^R(\tilde{e}_2, \tilde{c}+\delta, \tilde{Q}_1, \tilde{Q}_1, w_1) < e_1^R(\tilde{e}_2, \tilde{c}, \tilde{Q}_1, \tilde{Q}_1, 0)$$

for $\delta$ sufficiently small. Because $e_1^R(e_2, c, Q_1, Q_1, w_1)$ is continuous in $w_1$ (by Lemma 2), there exists $\tilde{w}_1$ such that $e_1^R(\tilde{e}_2, \tilde{c}+\delta, \tilde{Q}_1, \tilde{Q}_1, \tilde{w}_1) = \tilde{e}_1$ for $\delta$ sufficiently small; let $\tilde{w}_2$ denote the analogous quantity for buyer 2. Thus, under contracts $(Q_i, \Delta_i, w_i) = (\tilde{Q}_i, \tilde{Q}_i, \tilde{w}_i)$ for $i = 1, 2$ and under capacity $\tilde{c}+\delta$ for $\delta$ sufficiently small, $(\tilde{e}_1, \tilde{e}_2)$ is an equilibrium in innovation; because total system expected profit is strictly greater at $(\tilde{e}_1, \tilde{e}_2, \tilde{c}+\delta)$, $(\tilde{e}_1, \tilde{e}_2, \tilde{c})$ cannot be an optimal solution to (R), a contradiction. Second, consider the case where $\tilde{c} = 0$. This implies $\tilde{e}_1 = \tilde{e}_2 = \tilde{Q}_1 = \tilde{Q}_2 = \Pi(\tilde{e}_1, \tilde{e}_2, \tilde{c}) = 0$. Let $(\hat{e}_1(c), \hat{e}_2(c))$ denote the system-profit maximizing innovation equilibrium under capacity $c$ and fixed-quantity contracts $Q_i = 0$ for $i = 1, 2$. If $c = \tilde{c}$ is not a best response, then $(\partial/\partial c)\Pi(0, 0, c)|_{c=0} > 0$, which implies $(\partial/\partial c)\Pi(\hat{e}_1(c), \hat{e}_2(c), c)|_{c=0} > 0$, which contradicts that $(\tilde{e}_1, \tilde{e}_2, \tilde{c}) = (0, 0, 0)$. This establishes that the solution to (R) identified in step two is an optimal solution to the original problem.

We have observed that $\tilde{c} \neq c^*$ or $\tilde{e}_i < e_i^*(\tilde{c}, \tilde{e}_2)$ for at least one buyer. We conclude that in Region 3, the firms cannot induce the first-best investments with contracts of the form $\{Q_i, p_i(q): 0 \le q \le Q_i\}_{i=1,2}$, fixed-quantity contracts are optimal, and the associated Nash equilibrium is characterized by underinvestment in innovation (EC13).

In Region 1: $\sum_{i=1,2} \hat{Q}_i(c^*) = c^*$, the firms induce the first-best investments with fixed-quantity contracts for any level of bargaining confidence $\alpha_i \in (0, 1/2]$ for $i = 1, 2$. It remains to show that in Region 3: $\sum_{i=1,2} \hat{Q}_i(c^*) > c^*$, total expected profit (with the optimal contracts and associated Nash equilibrium) weakly increases with buyer $i$'s bargaining confidence $\alpha_i$. If $\tilde{c} = 0$, the result is immediate; suppose $\tilde{c} > 0$. Recall that $(\tilde{e}_1, \tilde{e}_2, \tilde{c}, \tilde{Q}_1, \tilde{Q}_2)$ represents the optimal contracts (fixed-quantity contracts) and associated investment equilibrium for a fixed $(\alpha_1, \alpha_2)$. Increase buyer 1's bargaining confidence to $\alpha_1 + \delta$, and let $e_1' = e_1^R(\tilde{e}_2, \tilde{c}, \tilde{Q}_1, \tilde{Q}_1, 0, \alpha_1+\delta)$ denote the associated best response. From Lemma 2, $e_1' \ge e_1^R(\tilde{e}_2, \tilde{c}, \tilde{Q}_1, \tilde{Q}_1, 0, \alpha_1) = \tilde{e}_1$. Because $\tilde{Q}_1 > 0$, for sufficiently small $\delta > 0$ there exists finite $w'$ such that $e_1^R(\tilde{e}_2, \tilde{c}, \tilde{Q}_1, \tilde{Q}_1, w', \alpha_1+\delta) = \tilde{e}_1$, so the firms can induce the same capacity and innovation investments $(\tilde{c}, \tilde{e}_1, \tilde{e}_2)$ by giving buyer 1 a quantity flexibility contract with parameters $Q_1 = \Delta_1 = \tilde{Q}_1$, and $w_1 = w'$. We conclude that problem (R), and hence the original contract design problem, has a solution with weakly greater expected profit than $(\tilde{e}_1, \tilde{e}_2, \tilde{c}, \tilde{Q}_1, \tilde{Q}_2)$.

It is straightforward to verify that in the symmetric biopharmaceutical example $\sum_{i=1,2} \hat{Q}_i(c^*) \le c^*$ if and only if $e^* \ge 1/2$. Further, it is straightforward to show that there exists $\tilde{k} < H$ such that $e^* \ge 1/2$ if and only if $k \le \tilde{k}$, and that $\tilde{k}$ has the asserted properties. □

Proof of Theorem 4. We will show that expected profit is greater with tradable options than in our basic setting with renegotiation. From Theorem 3, $\sum_{i=1,2} \hat{Q}_i(c^*) > c^*$ implies fixed-quantity contracts are optimal in the setting with renegotiation. Let $(Q_1^r(\alpha_1, \alpha_2), Q_2^r(\alpha_1, \alpha_2))$ denote the optimal fixed-quantity quantities and $(e_1^r(\alpha_1, \alpha_2), e_2^r(\alpha_1, \alpha_2), c^r(\alpha_1, \alpha_2))$ the associated optimal Nash equilibrium investments, for each $(\alpha_1, \alpha_2) \in [0, \frac{1}{2}]^2$ . From Theorem 3, total expected profit with investments $(e_1^r(\alpha_1, \alpha_2), e_2^r(\alpha_1, \alpha_2), c^r(\alpha_1, \alpha_2))$ is greater at $\alpha_1 = \alpha_2 = \frac{1}{2}$ than at any other $(\alpha_1, \alpha_2) \in [0, \frac{1}{2}]^2$. If each buyer $i = 1, 2$ has $Q_i^r(\frac{1}{2}, \frac{1}{2})$ tradable options, then the investments $(e_1^r(\frac{1}{2}, \frac{1}{2}), e_2^r(\frac{1}{2}, \frac{1}{2}), c^r(\frac{1}{2}, \frac{1}{2}))$ constitute a Nash equilibrium. To see this, observe that if the manufacturer does not speculate: $c^r(\frac{1}{2}, \frac{1}{2}) = Q_1^r(\frac{1}{2}, \frac{1}{2}) + Q_2^r(\frac{1}{2}, \frac{1}{2})$, the buyers' objective functions are identical in the setting with tradable options and the setting with fixed-quantity contracts and $\alpha = \frac{1}{2}$. The manufacturer's added value if he builds capacity $c > Q_1^r(\frac{1}{2}, \frac{1}{2}) + Q_2^r(\frac{1}{2}, \frac{1}{2})$ is lower in the setting with tradable options than with fixed-quantity contracts. Therefore, the fact that the manufacturer does not speculate in the setting with fixed-quantity contracts implies that the manufacturer does not speculate in the setting with tradable options. We conclude that the firms achieve greater expected profit with tradable options than with simple fixed-quantity contracts, and hence with any contracts of the form $\{Q_i, p_i(q): 0 \le q \le Q_i\}_{i=1,2}$. □