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ONLY AVAILABLE IN ELECTRONIC FORM

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Proof of Propositions and Lemmas

This document contains proofs for all Propositions and Lemmas.

EC.1. Proof of Proposition 1.

The first order condition for $\pi_s\,(N)$ is,

$$F'(N^*) = -\frac{c}{[R + P/\theta]\lambda}.$$

Under a PPC-only contract, the first order condition for $\pi_v\,(N)$ is,

$$F'(N_v^*) = -\frac{c}{r\lambda}.$$

Therefore, $N_v^* = N^*$ only if $r = R + P/\theta$. Given this condition, the client's profit is,

$$\pi_c(N^*) = R\lambda(1 - F(N^*)) - \frac{P\lambda F(N^*)}{\theta} - \left(R + \frac{P}{\theta}\right)\lambda(1 - F(N^*))$$

$$= -\frac{P\lambda}{\theta} < 0.$$

□.

EC.2. Proof of Proposition 2.

Conditions (i) and (ii) imply that the vendor will make a capacity choice N such that $N \geq N^*$. Assume that $N > N^*$.

$\pi_v(N) = \pi_s(N) - \pi_c(N) \leq \pi_s(N^*) - \pi_c(N) \leq \pi_s(N^*) - \pi_c(N^*) \leq \pi_v(N^*)$,

therefore $N = N^*$.

The vendor's profit

$\pi_v(N^*) = r\lambda(1\text{-}F(N^*)) - cN^* = V.$

Therefore the client's profit

$\pi_c(N^*) = \pi_s(N^*) - V$.

Thus, the client maximizes profit. □.

EC.3. Proof of Proposition 3.

The profit function of the vendor is

$$\pi_v(N) = R\lambda(1-F(N)) - P\frac{\lambda}{\theta}F(N) - cN - \pi_s^* + V$$
$$= \pi_s(N) - \pi_s^* + V \, .$$

Therefore, the vendor's profit maximizing capacity decision is the same as the service supply chain profit maximizing capacity. In addition, the vendor's profit $\pi_v(N^*) = V$. Therefore the client's profit $\pi_c(N^*) = \pi_s(N^*) - V$.

Thus the client maximizes profit. □.

EC.4. Proof of Proposition 4.

From Proposition 2 we know that a high productivity type vendor will earn a profit equal to V under contract CS-H. Assume the high type vendor accepts CS-L and chooses a staffing level of N_H.

According to the SLA constraint,

$$G_H(N_H,t) \geq \alpha_L = G_L(N_L^*,t) \, .$$

Now, $G_H(N_L^*,t) \geq G_L(N_L^*,t)$.

Therefore N_L^* is a feasible solution for the vendor's problem. This implies,

$$\pi_v(N_H) \geq \pi_v(N_L^*)$$
$$= r_L\lambda(1-F_H(N_L^*)) - cN_L^*$$
$$\geq r_L\lambda(1-F_L(N_L^*)) - cN_L^* = V \, .$$

Thus, the high productivity type vendor will accept CS-L. Therefore offering a choice between CS-H and CS-L will not screen between a high and a low productivity type vendor. Note that while the high productivity-type vendor will not accept the correct contract (CS-H), the low type will accept the correct contract (CS-L) – see Proposition 7. □.

EC.5. Proof of Proposition 5.

From Proposition 3 we know that a high productivity type vendor will earn a profit equal to V under contract CW-H. Assume the high productivity type vendor accepts CW-L. The vendor's profit function is

$$\pi_v(N) = R\lambda(1 - F_H(N)) - \frac{P}{\theta}\lambda F_H(N) - cN + V - \pi^*_{L,s}$$

This implies that the vendor's profit maximizing capacity decision under contract CW-L is equal to N^*_H. Therefore the vendor's profit is

$$\pi_v(N^*_H) = V - \pi^*_{L,s} + \pi^*_{H,s} \geq V$$

Therefore the high productivity-type vendor will accept contract CW-L and earn information rents on top of its reservation value. Hence offering a choice between CW-H and CW-L will not screen between a high and a low type vendor. □.

EC.6. Proof of Lemma 1.

Let $f(a,x) = ah(x)\text{-}h(ax)$. We know that,

$$\frac{\partial f(a,x)}{\partial a} = h(x) - xh'(ax).$$

Therefore we need to show that $h(x) - xh'(ax) \geq 0 \; \forall \; a, x$. We know that the hazard function for the standard normal distribution is an increasing function. This implies

$\frac{\partial h(x)}{\partial x} = h(x)(h(x) - x) \geq 0, \forall \; x$. Therefore $h(x) \geq x$ for all x.

Thus it is sufficient to show that $h'(ax) \le 1 \; \forall \; a, x$ or $h'(y) \le 1 \; \forall \; y$ to complete the proof.

Using L'Hopital's rule it can be shown that, $\lim_{x \to \infty} h(x)(h(x) - x) = 1$.

Now to complete the proof it is sufficient to show that $\nexists x < \infty$ such that $h(x)(h(x)\text{-}x) > 1$.

Assume $\exists x < \infty$ such that $h(x)(h(x)\text{-}x) > 1$. We know $\lim_{x \to \infty} h(x)(h(x) - x) = 1$, this implies that $\exists y < \infty$ such that $h(y)(h(y)\text{-}y) > 1$ and $\frac{\partial [h(y)(h(y) - y)]}{\partial y} < 0$.

Now, $\frac{\partial [h(y)(h(y) - y)]}{\partial y} = h(y)[(h(y) - y)(2h(y) - y) - 1]$.

This implies,

$$(h(y) - y)(2h(y) - y) < 1. \tag{10}$$

But we know $h(y)(h(y)\text{-}y) > 1$. This implies $h(y)\text{-}y > 0$. Therefore the inequality (10) can be written as,

$$(h(y) - y)(2h(y) - y) < h(y)(h(y) - y).$$

By re-arranging terms,

$$(h(y) - y)(h(y) - y) < 0. \tag{11}$$

The inequality (11) is not true. This leads to a contradiction and hence completes our proof.

□.

EC.7. Proof of Proposition 6.

First we prove that **Property 1** holds. Assume $\frac{\mu_H}{\mu_L} N_H \ge N_L^*$ is not true. This implies $N_H < \frac{\mu_L}{\mu_H} N_L^*$. By the SLA constraint for the high productivity type vendor under contract TS-L,

$$G_H(\frac{\mu_L}{\mu_H}N_L^*,t) \geq G_H(N_H,t) \geq \alpha_L = G_L(N_L^*,t)\,. \tag{12}$$

We now use the diffusion approximations,

$$N_L^* = \frac{\lambda}{\mu_L} + \beta_L{}^* \sqrt{\frac{\lambda}{\mu_L}}$$
$$\frac{\mu_L}{\mu_H}N_L^* = \frac{\lambda}{\mu_H} + \beta_L{}^* \sqrt{\frac{\mu_L}{\mu_H}}\sqrt{\frac{\lambda}{\mu_H}}.$$

$$G_H(\frac{\mu_L}{\mu_H}N_L^*,t) = 1 - \left(\frac{h(\beta_L{}^*\sqrt{\frac{\mu_L}{\mu_H}}\sqrt{\mu_H/\theta})}{\Psi(\beta_L{}^*\sqrt{\frac{\mu_L}{\mu_H}}\sqrt{\mu_H/\theta},\sqrt{N_L^*\mu_L\theta t})}\right)\left[1 + \frac{h(\beta_L{}^*\sqrt{\frac{\mu_L}{\mu_H}}\sqrt{\frac{\mu_H}{\theta}})}{\sqrt{\frac{\mu_H}{\theta}}h(-\beta_L{}^*\sqrt{\frac{\mu_L}{\mu_H}})}\right]^{-1} e^{-\theta t}$$

$$= 1 - \left(\frac{h(\beta_L{}^*\sqrt{\mu_L/\theta})}{\Psi(\beta_L{}^*\sqrt{\mu_L/\theta},\sqrt{N_L^*\mu_L\theta t})}\right)\left[1 + \frac{h(\beta_L{}^*\sqrt{\frac{\mu_L}{\theta}})}{\sqrt{\frac{\mu_H}{\theta}}h(-\beta_L{}^*\sqrt{\frac{\mu_L}{\mu_H}})}\right]^{-1} e^{-\theta t}$$

Therefore

$$G_L(N_L^*,t) = 1 - \left(\frac{h(\beta_L{}^*\sqrt{\mu_L/\theta})}{\Psi(\beta_L{}^*\sqrt{\mu_L/\theta},\sqrt{N_L^*\mu_L\theta t})}\right)\left[1 + \frac{h(\beta_L{}^*\sqrt{\frac{\mu_L}{\theta}})}{\sqrt{\frac{\mu_L}{\theta}}h(-\beta_L{}^*)}\right]^{-1} e^{-\theta t}\,.$$

Therefore the inequality (12) can be written as

$$h\left(-\beta_L{}^*\sqrt{\frac{\mu_L}{\mu_H}}\right) \leq \sqrt{\frac{\mu_L}{\mu_H}}h(-\beta_L{}^*)\,.$$

From Lemma 1 we know that $ah(x)-h(ax)$ is increasing in a for all x. Also $ah(x)-h(ax)=0$ for $a=1$. This implies that $ah(x)-h(ax) \leq 0$ for all a ≤ 1. Therefore

$$h\left(-\beta_L^{*}\sqrt{\frac{\mu_L}{\mu_H}}\right) \geq \sqrt{\frac{\mu_L}{\mu_H}}h(-\beta_L^{*}) .$$

This leads to a contradiction and hence our assumption that $N_H < \frac{\mu_L}{\mu_H}N_L^*$ is not true.

Now we prove that **Property 2** holds. Let us assume

$$1 - F_L(\frac{\mu_H}{\mu_L}N) < 1 - F_H(N) . \tag{13}$$

From the diffusion approximations,

$$N = \frac{\lambda}{\mu_H} + \beta\sqrt{\frac{\lambda}{\mu_H}}$$
$$\frac{\mu_H}{\mu_L}N = \frac{\lambda}{\mu_L} + \beta\sqrt{\frac{\mu_H}{\mu_L}}\sqrt{\frac{\lambda}{\mu_L}}$$

$$F_H(N) = \left(1 - \frac{h(\beta\sqrt{\mu_H/\theta})}{h(\beta\sqrt{\mu_H/\theta} + \sqrt{\theta/(N\mu_H)})}\right)\left[1 + \frac{h(\beta\sqrt{\frac{\mu_H}{\theta}})}{\sqrt{\frac{\mu_H}{\theta}}h(-\beta)}\right]^{-1}$$

$$F_L(\frac{\mu_H}{\mu_L}N) = \left(1 - \frac{h(\beta\sqrt{\frac{\mu_H}{\mu_L}}\sqrt{\mu_L/\theta})}{h(\beta\sqrt{\frac{\mu_H}{\mu_L}}\sqrt{\mu_L/\theta} + \sqrt{\theta/(N\mu_H)})}\right)\left[1 + \frac{h(\beta\sqrt{\frac{\mu_H}{\mu_L}}\sqrt{\frac{\mu_L}{\theta}})}{\sqrt{\frac{\mu_L}{\theta}}h(-\beta\sqrt{\frac{\mu_H}{\mu_L}})}\right]^{-1} .$$

The inequality (13) can be written as

$$\sqrt{\frac{\mu_H}{\mu_L}}h(-\beta) < h(-\beta\sqrt{\frac{\mu_H}{\mu_L}}) .$$

From Lemma 1 we know that $ah(x)$-$h(ax)$ is increasing in a for all x. Also $ah(x)$-$h(ax)$ =0 for a=1. This implies that $ah(x)$-$h(ax) \geq 0$ for all a ≥ 1. Therefore

$$\sqrt{\frac{\mu_H}{\mu_L}} h(-\beta) \geq h(-\beta \sqrt{\frac{\mu_H}{\mu_L}}) .$$

This leads to a contradiction and hence completes our proof. □.

EC.8. Proof of Lemma 2.

Assume that the vendor has low productivity and accepts the contract TS-L. By the definition of N_L^*, the vendor will maximize its profit at N_L^*, and we know that

$$\pi_v(N_L^*) = V .$$

For all $N \geq N_L^*$, the SLA constraint is satisfied i.e., all $N \geq N_L^*$ are a feasible solution to the vendor's problem. This implies

$$r_L \frac{\lambda}{\mu_L}(1 - F_L(N)) - cN \leq V .$$

Therefore, $\dfrac{\mu_L(V + cN)}{\lambda(1 - F_L(N))} \geq r_L \ \forall \ N \geq N_L^*$. □.

EC.9. Proof of Proposition 7.

A low productivity-type vendor will earn a profit equal to V under contract TS-L. Assume that a low productivity type vendor accepts CS-H. Let N_L be the capacity decision that the low productivity type vendor makes under contract CS-H. Now,

$$\pi_{H,s}^* \geq R\lambda(1 - F_L(N_L)) - \frac{P\lambda}{\theta} F_L(N_L) - cN_L .$$

This implies

$$\frac{\pi^*_{H,s} + cN_L + \frac{P\lambda}{\theta}}{R\lambda + \frac{P\lambda}{\theta}} \geq (1 - F_L(N_L)). \tag{14}$$

Because we assume that the low-productivity vendor accepts CS-H,

$$G_L(N_L, t) \geq \alpha_H = G_H(N^*_H, t) .$$

We know that $G_L(N,t) \leq G_H(N,t)$ and $G(N_1,t) \leq G(N_2,t)$ for $N_1 \leq N_2$. Therefore, $N_L \geq N^*_H$. This implies that,

$$\begin{aligned}
\pi_v &= \left(\frac{V + cN^*_H}{(1 - F_H(N^*_H))}\right)(1 - F_L(N_L)) - cN_L \\
&\leq \left(\frac{V + cN^*_H}{(1 - F_H(N^*_H))}\right)\left(\frac{\pi^*_{H,s} + cN_L + \frac{P\lambda}{\theta}}{R\lambda + \frac{P\lambda}{\theta}}\right) - cN_L \\
&= \left(\frac{V + cN^*_H}{(1 - F_H(N^*_H))}\right)\left(1 - F_H(N^*_H) + \frac{c(N_L - N^*_H)}{R\lambda + \frac{P\lambda}{\theta}}\right) - cN_L \\
&= V - c(N_L - N^*_H)\left(\frac{\left(R\lambda + \frac{P\lambda}{\theta}\right)(1 - F_H(N^*_H)) - V - cN^*_H}{\left(R\lambda + \frac{P\lambda}{\theta}\right)(1 - F_H(N^*_H))}\right) \\
&\leq V - c(N_L - N^*_H)\left(\frac{\pi^*_{H,s} - V}{\left(R\lambda + \frac{P\lambda}{\theta}\right)(1 - F_H(N^*_H))}\right) \leq V.
\end{aligned}$$

Therefore a low productivity type vendor will not accept contract CS-H. Let us assume that a high productivity-type vendor accepts the contract TS-L. The profit earned by the vendor is

$$\pi_v = \frac{\mu_L(V + cN^*_L)}{\mu_H(1 - F_L(N^*_L))}(1 - F_H(N_H)) - cN_H$$

$$\pi_v \le \frac{\mu_L (V + c\frac{\mu_H}{\mu_L} N_H)}{\mu_H (1 - F_L(\frac{\mu_H}{\mu_L} N_H))}(1 - F_H(N_H)) - cN_H$$
$$\le \left(\frac{\mu_L}{\mu_H}\right) V \le V.$$

Therefore the high productivity-type vendor will not accept contract TS-L. □.

EC.10. Proof of Proposition 8.

A low productivity vendor will earn a profit equal to V under contract TW-L. Assume that a low productivity vendor accepts CW-H. The vendor's profit function is

$$\pi_v(N) = R\lambda(1 - F_L(N)) - \frac{P}{\theta}\lambda F_L(N) - cN + V - \pi^*_{H,s}.$$

This implies that the vendor's profit maximizing capacity decision under contract CW-H is equal to N^*_L.

Therefore the vendor's profit is

$$\pi_v(N^*_L) = V + \pi^*_{L,s} - \pi^*_{H,s} \le V.$$

Therefore the low productivity vendor will not accept contract CW-H.

Let us assume that a high productivity vendor accepts contract TW-L and chooses a capacity equal to N_H. The profit earned by the vendor

$$\pi_v = \frac{\mu_L(V + R\lambda - \pi^*_{L,s})}{\mu_H}(1 - F_H(N_N)) - \left(\frac{P\lambda}{\theta} - V + \pi^*_{L,s}\right) F_H(N_H) - cN_H.$$

By re-arranging terms,

$$\pi_v = \frac{\mu_L}{\mu_H}\left((R\lambda + \frac{P\lambda}{\theta})(1 - F_H(N_H))\right) + \left(1 - \frac{\mu_L}{\mu_H}\right)\left(\frac{p_L\lambda}{\theta}\right)(1 - F_H(N_H)) - cN_H - \frac{P\lambda}{\theta} - \pi^*_{L,s} + V. \quad (15)$$

By the definition of $\pi^*_{L,s}$,

$$\pi^*_{L,s} \geq R\lambda(1-F_L(\frac{\mu_H}{\mu_L}N_H)) - \frac{P\lambda}{\theta}F_L(\frac{\mu_H}{\mu_L}N_H) - c\frac{\mu_H}{\mu_L}N_H .$$

Using Property 2,

$$\pi^*_{L,s} \geq R\lambda(1-F_H(N_H)) - \frac{P\lambda}{\theta}F_H(N_H) - c\frac{\mu_H}{\mu_L}N_H \quad . \tag{16}$$

Equations 15 and 16 imply

$$\pi_v \leq \left(\frac{\mu_L}{\mu_H}-1\right)\left((R\lambda+\frac{P\lambda}{\theta})(1-F_H(N_H))\right) + \left(1-\frac{\mu_L}{\mu_H}\right)\left(\frac{p_L\lambda}{\theta}\right)(1-F_H(N_H)) - cN_H\left(1-\frac{\mu_H}{\mu_L}\right) + V ,$$

and therefore,

$$\pi_v \leq \left(1-\frac{\mu_H}{\mu_L}\right)\left(\frac{r_L\lambda}{\mu_H}(1-F_H(N_H)) - cN_H\right) + V . \tag{17}$$

We have assumed that the high productivity-type vendor accepts contract TW-L. Therefore we must have

$$\pi_v = \frac{r_L\lambda}{\mu_H}(1-F_H(N_H)) - \frac{p_L\lambda}{\theta}F_H(N_H) - cN_H \geq 0 .$$

We can show that $p_L \geq 0$, and this implies that

$$\left(\frac{r_L\lambda}{\mu_H}(1-F_H(N_H)) - cN_H\right) \geq 0 .$$

Therefore from inequality 17 we find that $\pi_v \leq V$. Thus the high productivity-type vendor will not accept contract TW-L. □.

EC.11. Proof of Proposition 9.

A low productivity-type vendor will earn a profit equal to V under contract TS-L. Assume that a low productivity-type vendor accepts contract TSA-H. Let N_L be the capacity decision that the low productivity type vendor makes under contract TS-L. The profit function for the vendor is,

$$\pi_v = r_H \frac{\lambda}{\mu_L}(1 - F_L(N_L)) - cN_L - P_{AHT}.$$

Therefore,

$$\pi_v \le r_H \frac{\lambda}{\mu_L}(1 - F_L(N_L)) - cN_L - r_H \frac{\lambda}{\mu_L} \le 0 \le V.$$

Therefore a low type of vendor will not accept contract TSA-H.

A high-type vendor will earn a profit equal to V under contract TSA-H and not accept contract TS-L as it would earn a profit lower than V under contract TS-L (see the proof of Proposition 7). □.

EC.12. Proof of Proposition 10.

Assume two vendors, 1 and 2, with $\mu_1 > \mu_2$. We first consider a PPC+SLA contract. Let N_1 and N_2 be the profit maximizing capacity decisions and let π_1 and π_2 be the corresponding profits for vendors with service rates μ_1 and μ_2, respectively. Therefore the vendor profits are,

$\pi_1 = r\lambda(1 - F_1(N_1)) - cN_1$, where $G_1(N_1, t) \ge \alpha$ and,

$\pi_2 = r\lambda(1 - F_2(N_2)) - cN_2$, where $G_2(N_2, t) \ge \alpha$.

N_2 is a feasible solution for vendor 1. Therefore,

$\pi_1 \ge r\lambda(1 - F_1(N_2)) - cN_2$.

By definition, $F_1(N_2) \le F_2(N_2)$ and therefore $\pi_1 \ge r\lambda(1 - F_2(N_2)) - cN_2 = \pi_2$.

Next we consider a PPC+W contract. Again let N_1 and N_2 be the profit maximizing capacity decisions and let π_1 and π_2 be the corresponding profits for vendors with service rates μ_1 and μ_2, respectively. Therefore the vendor profits are:

$$\pi_1 = r\lambda(1 - F_1(N_1)) - p\frac{\lambda}{\theta}F_1(N_1) - cN_1, \text{ and,}$$

$$\pi_2 = r\lambda(1 - F_2(N_2)) - p\frac{\lambda}{\theta}F_2(N_2) - cN_2.$$

By definition,

$$\pi_1 \geq r\lambda(1 - F_1(N_2)) - p\frac{\lambda}{\theta}F_1(N_2) - cN_2$$

$$\geq r\lambda(1 - F_2(N_2)) - p\frac{\lambda}{\theta}F_2(N_2) - cN_2 = \pi_2. \qquad \square.$$

EC.13. Proof of Proposition 11.

Assume two vendors, 1 and 2, with $\mu_1 > \mu_2$. We first consider a PPT+SLA contract. Let N_1 and N_2 be the profit maximizing capacity decisions and let π_1 and π_2 be the corresponding profits for vendors with service rates μ_1 and μ_2, respectively. Therefore the vendor profits are:

$$\pi_1 = r\frac{\lambda}{\mu_1}(1 - F_1(N_1)) - cN_1, \text{ where } G_1(N_1, t) \geq \alpha$$

and,

$$\pi_2 = r\frac{\lambda}{\mu_2}(1 - F_2(N_2)) - cN_2, \text{ where } G_2(N_2, t) \geq \alpha.$$

Assume, $G_2\left(\frac{\mu_1 N_1}{\mu_2}, t\right) \geq G_1(N_1, t)$. If this assumption is not true then,

$$G_2\left(\frac{\mu_1 N_1}{\mu_2},t\right) < G_1(N_1,t).$$

Using the diffusion approximations, the above inequality can be rewritten as,

$$\sqrt{\frac{\mu_1}{\mu_2}}h(-\beta_1) - h\left(-\beta_1\sqrt{\frac{\mu_1}{\mu_2}}\right) \le 0.$$

Lemma 1 and our assumption that $\frac{\mu_1}{\mu_2} > 1$ imply that the above inequality does not hold. Therefore $G_2\left(\frac{\mu_1 N_1}{\mu_2},t\right) \ge G_1(N_1,t)$ is true. By definition $G_1(N_1,t) \ge \alpha$. This implies that $\frac{\mu_1 N_1}{\mu_2}$ is a feasible staffing level for vendor 2. This implies,

$$\pi_2 \ge r\frac{\lambda}{\mu_2}(1 - F_2(\frac{\mu_1 N_1}{\mu_2})) - c\frac{\mu_1 N_1}{\mu_2}.$$

Using Property 2, $\pi_2 \ge \frac{\mu_1}{\mu_2}\left[r\frac{\lambda}{\mu_1}(1 - F_1(N_1)) - cN_1\right] \ge \pi_1$.

Next we consider a PPC+W contract. Again let N_1 and N_2 be the profit maximizing capacity decisions and let π_1 and π_2 be the corresponding profits for vendors with service rates μ_1 and μ_2, respectively. Therefore the vendor profits are:

$$\pi_1 = r\frac{\lambda}{\mu_1}(1 - F_1(N_1)) - p\frac{\lambda}{\theta}F_1(N_1) - cN_1, \text{ and,}$$

$$\pi_2 = r\frac{\lambda}{\mu_2}(1 - F_2(N_2)) - p\frac{\lambda}{\theta}F_2(N_2) - cN_2.$$

By definition N_2 is optimal for vendor 2,

$$\pi_2 \ge r\frac{\lambda}{\mu_2}(1 - F_2(\frac{\mu_1 N_1}{\mu_2})) - p\frac{\lambda}{\theta}F_2(\frac{\mu_1 N_1}{\mu_2}) - c\frac{\mu_1 N_1}{\mu_2}.$$

Using Proposition 6,

$$\pi_2 \geq r\frac{\lambda}{\mu_2}(1-F_1(N_1)) - p\frac{\lambda}{\theta}F_1(N_1) - c\frac{\mu_1 N_1}{\mu_2}$$
$$\geq r\frac{\lambda}{\mu_2}(1-F_1(N_1)) - p\frac{\lambda}{\theta}F_1(N_1)\frac{\mu_1}{\mu_2} - c\frac{\mu_1 N_1}{\mu_2} \geq \pi_1.$$

□.

EC.14. Proof of Proposition 12.

The service supply chain profit for a given staffing level *N* is:

$$\pi_s(\mu, N) = R\lambda(1-F(\mu,N)) - P\frac{\lambda}{\theta}F(\mu,N) - cN\ .$$

With the PPC+W contract, the vendor's profit is:

$$\pi_v(\mu, N) = r\lambda(1-F(\mu,N)) - p\frac{\lambda}{\theta}F(\mu,N) - cN\ .$$

By definition, $r+\frac{p}{\theta} = R + \frac{P}{\theta}$.

Rewriting the vendor's profit,

$$\pi_v(\mu,N) = r\lambda - F(\mu,N)\left(r+\frac{p}{\theta}\right)\lambda - cN$$
$$= \pi_s(\mu,N) - (R-r)\lambda.$$

Therefore $r+\frac{p}{\theta} = R+\frac{P}{\theta}$ ensures that the PPC+W contract is coordinating, as long as $\pi_v(\mu,N) \geq V$.

□.

EC. 15. Proof of Proposition 13.

Let us assume that (r,p) defines a coordinating pay per call contract with a linear penalty for waiting. Let μ_l be such that $\pi_v^*(\mu_l) = V$. The client's expected profit under this contract is

$$\pi_c = M\,\Pr(\mu < \mu_l) + (\pi_s^*(\mu_l) - V)\Pr(\mu \geq \mu_l)\,. \tag{18}$$

Now assume that the vendor is offered a choice between a PPT+W+AHT (or PPT+SLA+AHT) contract that maximizes the client's profit if the vendor's service rate is μ_l and a coordinating PPC+W contract that maximizes the client's profit if the vendor's service rate is $\mu_h \in (\mu_l, \infty)$. Note that if the vendor's service rate $\mu \in [\mu_l, \mu_h)$ then the vendor will accept the pay per time contract and choose to work at $\mu_v = \mu_l$ (see Propositions 8 and 11). If the vendor's service rate $\mu \geq \mu_h$ then the vendor will accept the pay per call contract and choose to work at $\mu_v = \mu$ (see Propositions 8 and 10). In this case the service supply chain will be coordinated (see Proposition 12). It can be shown that for this case, the supply chain profit increases as $\mu \geq \mu_h$ and the vendor earns an information rent of $\pi_s^*(\mu) - \pi_s^*(\mu_h)$ above its reservation value. The client's expected profit is,

$$\pi_c = M\,\Pr(\mu < \mu_l) + (\pi_s^*(\mu_l) - V)\Pr(\mu_h > \mu \geq \mu_l) + (\pi_s^*(\mu_h) - V)\Pr(\mu_h \leq \mu)\,. \tag{19}$$

The proof of the theorem follows by comparing equations 18 and 19. □.

EC.16. Proof of Proposition 14.

Assume that the client offers the vendor a choice between a PPT+W+AHT (or PPT+SLA+AHT) that maximizes the client's profit if the vendor's service rate is μ_l (we will call this the lower-PPT-contract) and a PPT+W+AHT (or PPT+SLA+AHT) contract that maximizes the client's profit if the vendor's service rate is μ_h (we will call this the higher-PPT-contract). Note that if the vendor's service rate $\mu \in [\mu_l, \mu_h)$ then the vendor will accept the lower-PPT-contract and choose to work at $\mu_v = \mu_l$ (see Propositions 9 and 11). If the vendor's service rate $\mu \geq \mu_h$ then the vendor will accept the higher-PPT-contract and choose to work at $\mu_v = \mu_h$ (see Propositions 9 and 10). In this case the service supply chain will not be coordinated and therefore the supply chain earns less profit than under the contract described in Proposition 13. The client's expected profit is the same as under the screening contract described in Proposition 13,

$$\pi_c = M\,\Pr(\mu < \mu_l) + (\pi_s^*(\mu_l) - V)\Pr(\mu_h > \mu \geq \mu_l) + (\pi_s^*(\mu_h) - V)\Pr(\mu_h \leq \mu)\,. \qquad \square.$$