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Online Supplement for Pricing and Operational Recourse in Co-Production Systems.

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Appendix A: A Single-Class, Random Yield Model with Pricing (RYP1)

In this appendix, we consider the special case in which (i) production results in a random quantity of a single product and (ii) there is a single class of customers. We therefore set $a_{1L} = a_{2L} = a_{1H} = 0$. For notational simplicity we let $a = a_{2H}$ and remove the class subscript $i = 1, 2$ from all parameters. Downconversion is not relevant as there is no lower-quality product. Allocation is not relevant as there is only one customer class. The relevant decisions facing the firm are the price p and the production quantity Q. Recall that the distribution for the utility of customers' outside options is $G(\cdot)$, and so for a given price p, the fraction of customers who prefer the product to their outside options is $G(a-p)$.

We first consider the quantity-and-price setting problem under recourse-pricing and then consider the case of advance-pricing. Finally we contrast this random-yield model with the co-production model with a single class of customers, i.e., CPP1.

A.1. Recourse Pricing

In the case of recourse pricing, the firm sets prices after yield and market uncertainties are resolved. The firm chooses its prices to maximize its revenue, which is given by the equation

$$\mathbf{r}(q) = p \min\{xG(a-p), q\}.$$

THEOREM EC.1. *For any realization of product quantities q and market potential x, the optimal recourse price satisfies*

$$p^* = \frac{G(a-p^*)}{g(a-p^*)}, \quad x \le \frac{q}{\left(a - G^{-1}\left(\frac{q}{x}\right)\right) g\left(G^{-1}\left(\frac{q}{x}\right)\right)},$$

$$p^* = a - G^{-1}\left(\frac{q}{x}\right), \qquad \textit{otherwise.}$$

We note that the optimal prices are increasing in the market potential, reflecting the fact that the firm can charge higher prices when demand is high relative to supply.

Corollary EC.1. *Assume $G(\cdot) \sim U(0,1)$ and a is scaled between 0 and 1, the optimal recourse price $p^*(q,x)$ is given by*

$$p^*(q,x) = \frac{a}{2}, \qquad x \leq \frac{2q}{a},$$
$$p^*(q,x) = a - \frac{q}{x}, \qquad \textit{otherwise.}$$

For the special case of $G(\cdot) \sim U(0,1)$, we can use the optimal prices from Corollary EC.1 to develop expression for the optimal revenue $\mathbf{r}_r^*(q,x)$ as a function of the product quantity and market potential,

$$\mathbf{r}_r^*(Q,y,x) = \left(\frac{a^2}{4}\right)x, \qquad x \leq \frac{2yQ}{a},$$
$$\mathbf{r}_r^*(Q,y,x) = \left(a - \frac{yQ}{x}\right)yQ, \qquad \textit{otherwise.}$$

Note that the subscript r on the price vector is used to indicate that we are considering revenue with recourse pricing and not advance pricing. As one would expect, the optimal revenue is non decreasing in the market potential x, the production quantities Q and the customer valuation a.

We are now in a position to characterize the firm's optimal production-quantity Q^*. The firm chooses Q to maximize its expected profit,

$$\Pi_r(Q) = -c_P Q + E_{\tilde{Y},\tilde{X}}\left[\mathbf{r}_r^*(Q,y,x)\right].$$

The first term is the production cost and the second term is the expected revenue, where the expectation is taken over the yield and market-potential random variables. Using the above expressions for $\mathbf{r}_r^*(Q,y,x)$, we can write the expected profit function as

$$\Pi_r(Q) = -c_P Q + \int_0^1 \left(\int_0^{\frac{2yQ}{a}} \left(\frac{a^2}{4}\right) x dF_X(x) + \int_{\frac{2yQ}{a}}^{\infty} \left(a - \frac{yQ}{x}\right) yQ dF_X(x) \right) dF_Y(y).$$

It is relatively straight forward to show that $\Pi_r(Q)$ is concave in Q, and so the optimal production quantity Q^* is given by the first-order condition. Closed form solution to Q^* will not, however, typically exist.

A.2. Advance Pricing

In the case of advance pricing, the firm jointly sets the production quantity Q and the price p before yield and market uncertainties are resolved. We can then formulate the firm's joint quantity-and-price setting problem as

$$\max_{Q\geq 0, p\geq 0} \Pi_a (Q,p),$$

where

$$\Pi_a (Q,p) = -c_P Q + E_{\tilde{Y},\tilde{X}} [\mathbf{r}_a (Q,p,y,x)], \tag{EC.1}$$

$$\mathbf{r}_a (Q,p,y,x) = p \min \{xG(a-p), yQ\}, \tag{EC.2}$$

and the subscript a is used to indicate that we are considering advance pricing. Substituting equation (EC.2) into (EC.1), we then obtain,

$$\Pi_a (Q,p) = -c_P Q + \int_0^1 \left(\int_0^{\frac{yQ}{G(a-p)}} pG(a-p)\, x dF_X(x) + \int_{\frac{yQ}{G(a-p)}}^{\infty} pyQ dF_X(x) \right) dF_Y(y).$$

The above expected profit function is in general not concave in price p, and therefore is in general not jointly concave in Q and p. However, $\Pi_a (Q,p)$ is concave in Q for any given price p. The optimal Q^* is implicitly given by

$$\int_0^1 y F_X \left(\frac{yQ^*}{G(a-p)} \right) dF_Y(y) = E[y] - \frac{c_P}{p}.$$

In addition, note that $\frac{\partial^2 \Pi_a(Q,p)}{\partial p^2}$ is given by

$$\int_0^1 \left(\int_0^{\frac{yQ}{G(a-p)}} (-2g(a-p) + pg'(a-p)) dF_X(x) - \frac{py^2Q^2g^2(a-p)}{G^3(a-p)} f\left(\frac{yQ}{G(a-p)} \right) \right) dF_Y(y).$$

Therefore, if $\frac{pg'(a-p)}{g(a-p)} < 2$ (which is true for a wide class of distributions including the Uniform, Exponential and certain specifications of the Weibull, Gamma and truncated-Normal families), then $\Pi_a (Q,p)$ is concave in p for any given quantity Q. It is straightforward to show that $0 < p^* < a$ and $G(a-p)F_X^{-1}\left(1 - \frac{c_P}{pE[y]}\right) \leq Q^* \leq \nu G(a-\nu)\frac{E[X]}{c_P}$, where ν satisfies $\nu = \frac{G(a-\nu)}{g(a-\nu)}$. The optimal solution can therefore be efficiently computed.

A.3. Comparison with CPP1 Model

The RYP1 model is a special case of the CPP1 model in which $a_L = 0$. It is of interest to see how the simultaneous production of a valued "by-product", i.e., product L, influences the optimal price for product H and the expected profit. Because H is the only product in the RYP1 model, we do not subscript it in that model.

COROLLARY EC.2. *For any given realized product quantities and market potential, the optimal recourse price (for product H) under CPP1 is no higher than that under RYP1, i.e.,* $p_H|_{CPP1} \le p|_{RYP1}$.

Corollary EC.2 tells us that with a valued by-product the firm may charge a lower price for the high-quality product. When market potential is small relative to the supply of high-quality product, the price is identical. When market potential is large, however, the price is lower for CPP1 because there is more overall (i.e., including the by-product) supply to sell. The following theorem establishes that, as expected, the simultaneous production of a valued by-product gives the firm a higher expected profit, regardless of whether the firm adopts advance or recourse pricing.

THEOREM EC.2. *Regardless of recourse pricing or advance pricing, the optimal expected profit under CPP1 is at least as high as that under RYP1.*

Our numeric study[8] shows that the expected profit for CPP1 (averaged over $a_L = 0.2, 0.4$, and 0.6) can be significantly higher than that for RYP1. The average expected profit for CPP1 is 29.55% and 15.73% higher than that for RYP1 for advance pricing and recourse pricing, respectively. Note that the value of CPP1 over RYP1 is significantly reduced with recourse pricing. Consistent with Corollary EC.2, the average expected price for product H under CPP1 is 4.35% and 3.43% lower than that under RYP1.

Appendix B: A Two-Class, Random Yield Model with Pricing (RYP2)

In this appendix, we consider the special case in which (i) production results in a random quantity of a single product and (ii) there are two classes of customers. We therefore set $a_{1L} = a_{2L} = 0$. For notational simplicity we let $a_1 = a_{1H}$ and $a_2 = a_{2H}$. Without loss of generality, we assume $a_1 \ge a_2$. Downconversion is not relevant as there is no lower-quality product. We note that the firm is indifferent between allocation rules because both class of customers pay the same price and there is no spill over to another product. Therefore, the relevant decisions facing the firm are the price p, the production quantity Q, and the allocation policy. Recall that for a given price p, the fraction of customers who prefer the product to their outside options is $G_i(a_i - p)$, $i = 1, 2$. We consider the quantity-and-price setting problem under both recourse- and advance-pricing.

In the case of recourse pricing, the firm sets prices after yield and market uncertainties are resolved. The firm chooses its prices to maximize its revenue, which is given by the equation

$$\mathbf{r}(q) = p \min\left\{\sum_{i=1}^{2} x_i G_i(a_i - p), q\right\},$$

[8] Similar to that described in §4.3, except that uniform distributions were used for the outside utility, market potential and yield. Details available upon request.

where $q = Qy$. In what follows we assume that the distribution functions $G_i(\cdot)$ satisfy condition $T2$ (see Lemma A1 in Appendix A4.) We note that a sufficient condition for $T2$ to hold is that the $G_i(\cdot)$ be concave, and so $T2$ holds for a wide class of distribution functions including the Uniform, Exponential and certain specifications of the Weibull, Gamma and truncated-Normal families.

Theorem EC.3. *Assume (T2) holds. For any realization of product quantities q and market potentials $\mathbf{x} = (x_1, x_2)$, define $\bar{p} = 0$ if $\sum_{i=1}^{2} x_i G_i(a_i) \le q$. Otherwise define $\bar{p}$ as the unique solution to $\sum_{i=1}^{2} x_i G_i((a_i - \bar{p})^+) = q$. In addition, define ν_1 as the solution to $\nu = \frac{G_1(a_1-\nu)}{g_1(a_1-\nu)}$, and ν_{12} as the solution to $\nu = \frac{\sum_{i=1}^{2} x_i G_i(a_i-\nu)}{\sum_{i=1}^{2} x_i g_i(a_i-\nu)}$. The optimal recourse price is given by*

$$
\begin{aligned}
p^* &= \bar{\nu}_{12}, \quad \frac{G_1(a_1-a_2)}{g_1(a_1-a_2)} \le a_2, \\
p^* &= \bar{\nu}_{12}, \quad \frac{G_1(a_1-a_2)}{g_1(a_1-a_2)} > a_2, \ \frac{x_2}{x_1} > \frac{G_1(a_1-a_2) - a_2 g_1(a_1-a_2)}{a_2 g_2(0)}, \ r(q,\bar{\nu}_{12}) \ge r(q,\bar{\nu}_1), \\
p^* &= \bar{\nu}_1, \quad \frac{G_1(a_1-a_2)}{g_1(a_1-a_2)} > a_2, \ \frac{x_2}{x_1} > \frac{G_1(a_1-a_2) - a_2 g_1(a_1-a_2)}{a_2 g_2(0)}, \ r(q,\bar{\nu}_{12}) < r(q,\bar{\nu}_1), \\
p^* &= \bar{\nu}_1, \quad \frac{x_2}{x_1} \le \frac{G_1(a_1-a_2) - a_2 g_1(a_1-a_2)}{a_2 g_2(0)},
\end{aligned}
$$

where $\bar{\nu}_1 = \max\{\bar{p}, \nu_1\}$ and $\bar{\nu}_{12} = \max\{\bar{p}, \nu_{12}\}$.

We note that the condition $r(q,\bar{\nu}_{12}) \ge r(q,\bar{\nu}_1)$ can be specified by model primitives but is omitted here for the sake of brevity. Note that the above theorem collapses to the RYP1 model when the market potential x_2 and/or the valuation a_2 are set to zero. In fact, this theorem generalizes Theorem EC.1. Theorem EC.3 tells us that, depending on customer valuations $a_i, i = 1,2$, realized market potentials $x_i, i = 1,2$, and the realized production quantity q, the firm may not serve both class of customers. If the customer valuations are sufficiently close, then it is optimal to serve both customer classes unless the realized quantity q is too small. On the other hand, if realized market sizes are sufficiently different, then it is optimal to serve only the high-valuation class.

For the special case of $G(\cdot) \sim U(0,1)$ (and, without loss of generality, valuations scaled between 0 and 1.), we can derive explicit expressions for the optimal recourse price.

Corollary EC.3. *Assume $G_i(\cdot) \sim U(0,1)$ $i = 1,2$. For any realization of product quantities $\mathbf{q} = (q_H, q_L)$ and market potential $\mathbf{x} = (x_1, x_2)$, (a) If $\frac{a_1}{a_2} \le 2$, then the optimal recourse price is given by*

$$
\begin{aligned}
p^*(q,\mathbf{x}) &= a_1 - \frac{q}{x_1}, && q \le x_1(a_1 - a_2), \\
p^*(q,\mathbf{x}) &= \frac{\sum_{i=1}^{2} x_i a_i - q}{\sum_{i=1}^{2} x_i}, && x_1(a_1 - a_2) < q < \frac{\sum_{i=1}^{2} x_i a_i}{2},
\end{aligned}
$$

$$p^*(q,\mathbf{x}) = \frac{\sum_{i=1}^2 x_i a_i}{2\sum_{i=1}^2 x_i}, \qquad q \geq \frac{\sum_{i=1}^2 x_i a_i}{2}.$$

(b) If $\frac{a_1}{a_2} > 2$*, then the optimal recourse price is given by*

$$\begin{aligned}
p^*(q,\mathbf{x}) &= a_1 - \frac{q}{x_1}, && q \leq \frac{x_1 a_1}{2}, && \\
p^*(q,\mathbf{x}) &= \frac{a_1}{2}, && \frac{x_1 a_1}{2} < q, && \frac{x_2}{x_1} \leq \frac{a_1}{a_2} - 2, \\
p^*(q,\mathbf{x}) &= \frac{a_1}{2}, && \frac{x_1 a_1}{2} < q < \frac{M}{2}, && \frac{a_1}{a_2} - 2 < \frac{x_2}{x_1}, \\
p^*(q,\mathbf{x}) &= \frac{\sum_{i=1}^2 x_i a_i - q}{\sum_{i=1}^2 x_i}, && \frac{M}{2} < q < \frac{\sum_{i=1}^2 x_i a_i}{2}, && \frac{a_1}{a_2} - 2 < \frac{x_2}{x_1}, \\
p^*(q,\mathbf{x}) &= \frac{a_1}{2}, && \frac{\sum_{i=1}^2 x_i a_i}{2} \leq q, && \frac{a_1}{a_2} - 2 < \frac{x_2}{x_1} < \frac{a_1}{a_2}\left(\frac{a_1}{a_2} - 2\right), \\
p^*(q,\mathbf{x}) &= \frac{\sum_{i=1}^2 x_i a_i}{2\sum_{i=1}^2 x_i}, && \frac{\sum_{i=1}^2 x_i a_i}{2} \leq q, && \frac{a_1}{a_2}\left(\frac{a_1}{a_2} - 2\right) \leq \frac{x_2}{x_1},
\end{aligned}$$

where $M = \sum_{i=1}^2 x_i a_i - \sqrt{(\sum_{i=1}^2 x_i a_i)^2 - a_1^2 x_1 \sum_{i=1}^2 x_i}$.

We note that if $a_1 \leq 2a_2$, the optimal recourse price p^* is non-increasing in the realized production quantity q for any realized market potentials $x_i, i = 1, 2$, that is, the larger the supply the lower the price. However, if $a_1 > 2a_2$, that is, the classes differ greatly in their valuation, then the optimal recourse price p^* is not necessarily monotonic in q. However, one can show that the optimal revenue is non decreasing in the market potentials $x_i, i = 1, 2$, the launched quantity Q and the customer valuations $a_i, i = 1, 2$.

Using the optimal prices from Corollary EC.3 , we can develop an expression for the optimal revenue $\mathbf{r}_r^*(q,\mathbf{x})$ as a function of the product quantity and market potential. For $a_1 \leq 2a_2$,

$$\begin{aligned}
\mathbf{r}_r^*(Q,y,\mathbf{x}) &= \left(a_1 - \frac{yQ}{x_1}\right) yQ, && yQ \leq x_1(a_1 - a_2), \\
\mathbf{r}_r^*(Q,y,\mathbf{x}) &= \left(\frac{\sum_{i=1}^2 x_i a_i - yQ}{\sum_{i=1}^2 x_i}\right) yQ, && x_1(a_1 - a_2) < yQ < \frac{\sum_{i=1}^2 x_i a_i}{2}, \\
\mathbf{r}_r^*(Q,y,\mathbf{x}) &= \frac{\left(\sum_{i=1}^2 x_i a_i\right)^2}{4\sum_{i=1}^2 x_i}, && yQ \geq \frac{\sum_{i=1}^2 x_i a_i}{2}.
\end{aligned}$$

For $a_1 > 2a_2$,

$$\begin{aligned}
\mathbf{r}_r^*(Q,y,\mathbf{x}) &= \left(a_1 - \frac{yQ}{x_1}\right) yQ, && (Q,y,\mathbf{x}) \in \Lambda_0, \\
\mathbf{r}_r^*(Q,y,\mathbf{x}) &= \frac{a_1^2 x_1}{4}, && (Q,y,\mathbf{x}) \in \Lambda_1, \\
\mathbf{r}_r^*(Q,y,\mathbf{x}) &= \left(\frac{\sum_{i=1}^2 x_i a_i - yQ}{\sum_{i=1}^2 x_i}\right) yQ, && (Q,y,\mathbf{x}) \in \Lambda_2,
\end{aligned}$$

$$\mathbf{r}_r^*(Q,y,\mathbf{x}) = \frac{\left(\sum_{i=1}^2 x_i a_i\right)^2}{4\sum_{i=1}^2 x_i}, \qquad (Q,y,\mathbf{x}) \in \Lambda_3.$$

As one would expect, the optimal revenue is non decreasing in the market potentials $x_i, i=1,2$, the production quantity Q and the customer valuations $a_i, i=1,2$.

In the case of advance pricing, the firm jointly sets the production quantity Q and the price p before yield and market uncertainties are resolved. We can then formulate the firm's joint quantity-and-price setting problem as

$$\max_{Q\geq 0, p\geq 0} \Pi_a(Q,p),$$

where

$$\Pi_a(Q,p) = -c_P Q + E_{\tilde{Y},\tilde{\mathbf{X}}}\left[\mathbf{r}_a(Q,p,y,\mathbf{x})\right], \tag{EC.3}$$

$$\mathbf{r}_a(Q,p,y,\mathbf{x}) = p\min\left\{\sum_{i=1}^2 x_i G_i(a_i-p), yQ\right\}, \tag{EC.4}$$

and the subscript a is used to indicate that we are considering advance pricing. Substituting equation (EC.4) into (EC.3), we then obtain,

$$\begin{aligned}\Pi_a(Q,p) = &-c_P Q + \int_0^1 \Bigg\{ \int_0^{\frac{yQ}{G_1(a_1-p)}} \int_0^{\frac{yQ-x_1G_1(a_1-p)}{G_2(a_2-p)}} p\sum_{i=1}^2 x_i G_i(a_i-p)\, dF_{X_2}(x_2)\, dF_{X_1}(x_1) \\ &+ \int_0^{\frac{yQ}{G_1(a_1-p)}} \int_{\frac{yQ-x_1G_1(a_1-p)}{G_2(a_2-p)}}^{\infty} pyQ dF_{X_2}(x_2)\, dF_{X_1}(x_1) \\ &+ \int_{\frac{yQ}{G_1(a_1-p)}}^{\infty} pyQ dF_{X_1}(x_1) \Bigg\} dF_Y(y).\end{aligned}$$

The above expected profit function is in general not concave in price p, and therefore is in general not jointly concave in Q and p. However, $\Pi_a(Q,p)$ is concave in Q for any given price p. The optimal Q^* is implicitly given by

$$\int_0^1 y \left(\int_0^{\frac{yQ^*}{G_1(a_1-p)}} \overline{F}_{X_2}\left(\frac{yQ^* - x_1G_1(a_1-p)}{G_2(a_2-p)}\right) dF_{X_1}(x_1) + \overline{F}_{X_1}\left(\frac{yQ^*}{G_1(a_1-p)}\right)\right) dF_Y(y) = \frac{c_P}{p}.$$

In addition, note that $\partial^2\Pi_a(Q,p)/\partial p^2$ is $\int_0^1 V'' dF_Y(y)$, where

$$V'' = \int_0^{\frac{yQ}{G_1(a_1-p)}} \int_0^{\frac{yQ-x_1G_1(a_1-p)}{G_2(a_2-p)}} \left(\sum_{i=1}^2(-2x_i g_i(a_i-p) + p x_i g_i'(a_i-p))\right) dF_{X_2}(x_2) dF_{X_1}(x_1)$$

$$-\frac{p}{G_2(a_2-p)}\int_0^{\frac{yQ}{G_1(a_1-p)}}\left(x_1 g_1(a_1-p)+\frac{yQ-x_1G_1(a_1-p)}{G_2(a_2-p)}g_2(a_2-p)\right)^2\cdot$$
$$f_{X_2}\left(\frac{yQ-x_1G_1(a_1-p)}{G_2(a_2-p)}\right)dF_{X_1}(x_1).$$

Therefore, if $\frac{pg'(a-p)}{g(a-p)}<2$ (which is true for a wide class of distributions including the Uniform, Exponential and certain specifications of the Weibull, Gamma and truncated-Normal families), then $\Pi_a(Q,p)$ is concave in p for any given quantity Q. Because both p and Q are bounded, the optimal solution can be efficiently computed.

B.1. Comparison with RYP1 Model

We now investigate how the optimal recourse price in RYP2 compares to that in RYP1, i.e., how is the price influenced by the presence of a second class of customers. As the following corollary demonstrates, the optimal recourse price under RYP2 can be higher or lower than that under RYP1.

Corollary EC.4. *Define p_1^* and p_2^* as the optimal recourse prices in the RYP1 and RYP2 models, respectively. In addition, define x as the realized market potential in RYP1. Let j denote the additional customer class in RYP2 and $\bar{j}$ the original class. Let $x_{\bar{j}}=x$, i.e., the original class has the same realized potential in RYP2. Let ν_j be the unique solution to $\nu=\frac{G_j(a_j-\nu)}{g_j(a_j-\nu)}$. Then, for any realized product quantities, $p_1^*\le p_2^*$ if (a) $\nu_j>\nu_{\bar{j}}$, or (b) $x>\frac{q}{a-G^{-1}\left(\frac{q}{x}\right)}g\left(G^{-1}\left(\frac{q}{x}\right)\right)$, or (c) $\bar{p}\ge\nu_{\bar{j}}$, where $\bar{p}$ is defined in Theorem EC.3. Otherwise, $p_1^*>p_2^*$ if $p_2^*=\bar{\nu}_{12}$ and $p_1^*\le p_2^*$ if $p_2^*=\bar{\nu}_1$, where $\bar{\nu}_1$ and $\bar{\nu}_{12}$ are defined in Theorem EC.3 (note $\bar{\nu}_1$ and the optimality condition in Theorem EC.3 can be conversely defined if $a_j>a_{\bar{j}}$.)*

Corollary EC.4 tells us that the optimal price in RYP2 is at least as high as that in RYP1 if, in absence of the existing class of customers, it is optimal to induce the additional class of customers to pay a higher price; or if the realized market potential is relatively large. Otherwise, if it is optimal to serve both classes under RYP2, then the optimal price in RYP2 can be less than that in RYP1. However, as expected, the optimal expected profit under RYP2 is always higher than or equals that under RYP1.

Theorem EC.4. *Regardless of recourse pricing or advance pricing, the optimal expected profit under RYP2 is at least as high as that under RYP1.*

Our numeric study[9] shows that the expected profit for RYP2 (averaged over $a_H=0.3,0.4,0.5,$ and 0.6) is 8.62% and 10.47% higher than that for RYP1 for advance pricing and recourse pricing, respectively. In

[9] Similar to that described in §5.4, except that uniform distributions were used for the outside utility, market potentials and yield, and the market correlation was fixed at -0.5. Details available upon request.

contrast to CPP1, whose value over RYP1 is dampened by recourse pricing, the value of RYP2 over RYP1 is strengthened by recourse pricing. In another words, the firm accrues more benefits by adopting recourse pricing if there are two class of customers, one possible reason being that there is more uncertainty in the two class case. As expected, the expected profit under RYP2 is increasing in a_{2H}, i.e., as product H becomes more valuable to the second class of customers.

Appendix C: A Single-Class, Co-Production Model with Advance Pricing

In the case of advance pricing, the firm jointly sets the production quantity Q and the price vector $\mathbf{p} = (p_H, p_L)$ before yield and market uncertainties are resolved, but downconversion occurs after uncertainties are resolved. For a given price vector, market-size and yield realization, the optimal downconversion quantity is given by Theorem 1. Let $\pi(Q, \mathbf{p}, y, x)$ denote the resulting revenue less the downconversion cost. We can then formulate the firm's joint quantity-and-price setting problem as

$$\Pi_a(Q, \mathbf{p}) = \max_{Q \geq 0, \mathbf{p} \geq \mathbf{0}} \{-c_P Q + E_{\widetilde{Y}, \widetilde{X}}[\pi^*(Q, \mathbf{p}, y, x)]\}, \tag{EC.5}$$

where using Theorem 1, if $a_H - p_H \geq a_L - p_L$

$$\begin{aligned}\pi^*(Q, \mathbf{p}, y, x) = &\, p_H \min\{xG(a_H - p_H), yQ\} \\ &+ p_L \min\left\{[xG(a_H - p_H) - yQ]^+ \left(\frac{G(a_L - p_L)}{G(a_H - p_H)}\right), (1-y)Q\right\},\end{aligned}$$

and if $a_H - p_H < a_L - p_L$

$$\begin{aligned}\pi^*(Q, \mathbf{p}, y, x) = &\, p_L \min\{xG(a_L - p_L), (1-y)Q\} \\ &+ p_H \min\left\{[xG(a_L - p_L) - (1-y)Q]^+ \left(\frac{G(a_H - p_H)}{G(a_L - p_L)}\right), yQ\right\}, \\ &\qquad \text{if } c_D \geq p_L - p_H \frac{G(a_H - p_H)}{G(a_L - p_L)}, \\ \pi^*(Q, \mathbf{p}, y, x) = &\, -c_D q_D^* + p_L \min\{xG(a_L - p_L), (1-y)Q + q_D^*\} \\ &+ p_H \min\left\{[xG(a_L - p_L) - (1-y)Q - q_D^*]^+ \left(\frac{G(a_H - p_H)}{G(a_L - p_L)}\right), yQ - q_D^*\right\}, \\ &\qquad \text{if } c_D < p_L - p_H \frac{G(a_H - p_H)}{G(a_L - p_L)},\end{aligned}$$

where $q_D^* = \min\left\{z, \left(yQ - z\frac{G(a_H - p_H)}{G(a_L - p_L)}\right) / \left(1 - \frac{G(a_H - p_H)}{G(a_L - p_L)}\right)\right\}$ and $z = (xG(a_L - p_L) - (1-y)Q)^+$. Substituting above equations into (EC.5), we then obtain, if $a_H - p_H \geq a_L - p_L$,

$$\Pi_a(Q, \mathbf{p}) = -c_P Q + \int_0^1 \int_0^{\frac{yQ}{G(a_H - p_H)}} p_H G(a_H - p_H)\, x dF_X(x)\, dF_Y(y)$$

$$+\int_0^1 \int_{\frac{yQ}{G(a_H-p_H)}}^{\frac{yQ}{G(a_H-p_H)}+\frac{(1-y)Q}{G(a_L-p_L)}} \left(p_H yQ + p_L \left(xG(a_H-p_H) - yQ\right)\frac{G(a_L-p_L)}{G(a_H-p_H)}\right) dF_X(x)\, dF_Y(y)$$

$$+\int_0^1 \int_{\frac{yQ}{G(a_H-p_H)}+\frac{(1-y)Q}{G(a_L-p_L)}}^{\infty} \left(p_H y + p_L(1-y)\right) Q\, dF_X(x)\, dF_Y(y),$$

and if $a_H - p_H < a_L - p_L$,

$$\Pi_a(Q,\mathbf{p}) = -c_P Q + \int_0^1 \int_0^{\frac{(1-y)Q}{G(a_L-p_L)}} p_L G(a_L-p_L)\, x\, dF_X(x)\, dF_Y(y)$$

$$+\int_0^1 \int_{\frac{(1-y)Q}{G(a_L-p_L)}}^{\frac{yQ}{G(a_H-p_H)}+\frac{(1-y)Q}{G(a_L-p_L)}} \left(p_L(1-y)Q + p_H\left(xG(a_L-p_L) - (1-y)Q\right)\frac{G(a_H-p_H)}{G(a_L-p_L)}\right) \cdot$$

$$dF_X(x)\, dF_Y(y)$$

$$+\int_0^1 \int_{\frac{yQ}{G(a_H-p_H)}+\frac{(1-y)Q}{G(a_L-p_L)}}^{\infty} \left(p_H y + p_L(1-y)\right) Q\, dF_X(x)\, dF_Y(y), \text{ if } c_D \geq p_L - p_H \frac{G(a_H-p_H)}{G(a_L-p_L)},$$

and

$$\Pi_a(Q,\mathbf{p}) = -c_P Q + \int_0^1 \int_0^{\frac{Q}{G(a_L-p_L)}} p_L G(a_L-p_L)\, x\, dF_X(x)\, dF_Y(y)$$

$$+\int_0^1 \int_{\frac{Q}{G(a_L-p_L)}}^{\frac{yQ}{G(a_H-p_H)}+\frac{(1-y)Q}{G(a_L-p_L)}} \left(p_L \frac{Q - xG(a_H-p_H)}{G(a_L-p_L) - G(a_H-p_H)} G(a_L-p_L)\right.$$

$$\left. + p_H \frac{xG(a_L-p_L) - Q}{G(a_L-p_L) - G(a_H-p_H)} G(a_H-p_H)\right) dF_X(x)\, dF_Y(y)$$

$$+\int_0^1 \int_{\frac{yQ}{G(a_H-p_H)}+\frac{(1-y)Q}{G(a_L-p_L)}}^{\infty} \left(p_H y + p_L(1-y)\right) Q\, dF_X(x)\, dF_Y(y)$$

$$-c_D \int_0^1 \int_{\frac{(1-y)Q}{G(a_L-p_L)}}^{\frac{Q}{G(a_L-p_L)}} \left(xG(a_L-p_L) - (1-y)Q\right) dF_X(x)\, dF_Y(y)$$

$$-c_D \int_0^1 \int_{\frac{Q}{G(a_L-p_L)}}^{\frac{yQ}{G(a_H-p_H)}+\frac{(1-y)Q}{G(a_L-p_L)}} \frac{yQ - \left(xG(a_L-p_L) - (1-y)Q\right)\frac{G(a_H-p_H)}{G(a_L-p_L)}}{1 - \frac{G(a_H-p_H)}{G(a_L-p_L)}} dF_X(x)\, dF_Y(y),$$

$$\text{if } c_D < p_L - p_H \frac{G(a_H-p_H)}{G(a_L-p_L)}.$$

For the recourse-pricing case, we were able to obtain implicit solutions for the optimal price vector (and closed form solutions in the case of a uniform utility distribution), and show that a first-order condition was sufficient for optimality for the production quantity Q. Not surprisingly, closed form solutions to the optimal price vector and optimal production quantity do not exist in the advance-pricing case. The function is in general not jointly concave in Q and $\mathbf{p}$. However, the revenue function is concave in Q for any given price vector $\mathbf{p}$ and the price vector is bounded. Therefore, an optimal solution to the joint quantity-and-price problem can be found efficiently.

Appendix D: Two Customer Classes (CPP2) with Randomized Allocation Policy

In this appendix, we analyze the CPP2 model under a randomized allocation policy. For a price vector $\mathbf{p}=(p_H,p_L)$, realized quantities $\mathbf{q}=(q_H,q_L)$ and market-potential realizations $\mathbf{x}=(x_1,x_2)$, the firm's revenue as a function of the downconversion quantity q_D is

$$r(q_D) = p_H \min\left\{\sum_{i=1}^{2} d_{iH}, (q_H-q_D)\right\} + p_L \min\left\{\left(\sum_{i=1}^{2} d_{iH}-(q_H-q_D)\right)^{+}\frac{\sum_{i=1}^{2} d_{iL}}{\sum_{i=1}^{2} d_{iH}}, (q_L+q_D)\right\}, \quad \mathbf{p}\in\Gamma_1,$$

$$r(q_D) = p_H \min\left\{d_{jH}+(d_{\bar{j}L}-(q_L+q_D))^{+}s_{\bar{j}H}, (q_H-q_D)\right\} + p_L \min\left\{d_{\bar{j}L}+(d_{jH}-(q_H-q_D))^{+}s_{jL}, (q_L+q_D)\right\}, \quad \mathbf{p}\in\Gamma_2,$$

$$r(q_D) = p_H \min\left\{d_{\bar{j}H}+(d_{jL}-(q_L+q_D))^{+}s_{jH}, (q_H-q_D)\right\} + p_L \min\left\{d_{jL}+(d_{\bar{j}H}-(q_H-q_D))^{+}s_{\bar{j}L}, (q_L+q_D)\right\}, \quad \mathbf{p}\in\Gamma_3,$$

$$r(q_D) = p_H \min\left\{\left(\sum_{i=1}^{2} d_{iL}-(q_L+q_D)\right)^{+}\frac{\sum_{i=1}^{2} d_{iH}}{\sum_{i=1}^{2} d_{iL}}, (q_H-q_D)\right\} + p_L \min\left\{\sum_{i=1}^{2} d_{iL}, (q_L+q_D)\right\}, \quad \mathbf{p}\in\Gamma_4,$$

where Γ_1, Γ_2, Γ_3, and Γ_4 partition the pricing space and are given by

$$\begin{aligned}
\Gamma_1 &: \quad p_H - p_L \le \min_i\{a_{iH}-a_{iL}\},\\
\Gamma_2 &: a_{\bar{j}H}-a_{\bar{j}L} < p_H - p_L \le a_{jH}-a_{jL},\\
\Gamma_3 &: a_{jH}-a_{jL} < p_H - p_L \le a_{\bar{j}H}-a_{\bar{j}L},\\
\Gamma_4 &: \quad p_H - p_L > \max_i\{a_{iH}-a_{iL}\}.
\end{aligned}$$

Note that Γ_2 and Γ_3 cannot exist simultaneously.

THEOREM EC.5. *For a price vector $\mathbf{p}=(p_H,p_L)$, realized quantities $\mathbf{q}=(q_H,q_L)$ and market-potential realizations $\mathbf{x}=(x_1,x_2)$, the optimal downconversion quantity $q_D^*=0$ if (a) both class of customers prefer product H to L; or (b) class j customers prefer product L and $c_D \ge p_L - p_H s_{jH}$; or (c) both class customers* *prefer product L and* $c_D \geq p_L - p_H \frac{\sum_{i=1}^2 d_{iH}}{\sum_{i=1}^2 d_{iL}}$*. Otherwise, the optimal downconversion quantity is given by* $q_D^* = \min\{z, \hat{q}_D\}$*, where*

$$
\begin{aligned}
\hat{q}_D &= \frac{(q_H - d_{jH}) - (d_{\bar{j}L} - q_L)s_{\bar{j}H}}{1 - s_{\bar{j}H}}, && \mathbf{p} \in \Gamma_2,\ d_{\bar{j}L} \geq q_L \ \cap\ (d_{\bar{j}L} - q_L)s_{\bar{j}H} < q_H - d_{jH},\\
\hat{q}_D &= \frac{(q_H - d_{\bar{j}H}) - (d_{jL} - q_L)s_{jH}}{1 - s_{jH}}, && \mathbf{p} \in \Gamma_3,\ d_{jL} \geq q_L \ \cap\ (d_{jL} - q_L)s_{jH} < q_H - d_{\bar{j}H},\\
\hat{q}_D &= \frac{q_H - \left(\sum_{i=1}^2 d_{iL} - q_L\right)\frac{\sum_{i=1}^2 d_{iH}}{\sum_{i=1}^2 d_{iL}}}{1 - \frac{\sum_{i=1}^2 d_{iH}}{\sum_{i=1}^2 d_{iL}}}, && \mathbf{p} \in \Gamma_4,\ \sum_{i=1}^2 d_{iL} \geq q_L \ \cap\ \left(\sum_{i=1}^2 d_{iL} - q_L\right)\frac{\sum_{i=1}^2 d_{iH}}{\sum_{i=1}^2 d_{iL}} < q_H,\\
\hat{q}_D &= 0, && \textit{otherwise},
\end{aligned}
$$

and

$$
\begin{aligned}
z &= \max\{0, d_{\bar{j}L} - q_L\}, && \mathbf{p} \in \Gamma_2,\\
z &= \max\{0, d_{jL} - q_L\}, && \mathbf{p} \in \Gamma_3,\\
z &= \max\left\{0, \sum_{i=1}^2 d_{iL} - q_L\right\}, && \mathbf{p} \in \Gamma_4.
\end{aligned}
$$

We conducted a numeric investigation[10] to compare the expected profits under randomized and prioritized allocation. Our numeric investigation shows that, relative to recourse prioritization, randomized allocation results in a decrease in the expected profit of 0.58% on average under advance pricing, and 1.21% on average under recourse pricing. The maximum decrease in profit is even more significant: 8.76% under advance pricing and 7.21% under recourse pricing. This indicates that knowledge of customers' identity can be of significant value to the firm.

Appendix E: Downgrading versus Downconversion

As discussed in the main paper, the firm could downgrade rather than downconvert when filling a low-quality demand from its high-quality inventory. However, downgrading has some tactical and strategic disadvantages. Tactically, the practice of downgrading cannibalizes high-quality demand as it does not extract the high-quality price from those customers willing to spill up to the high-quality product. Strategically, the practice may promote undesirable reselling on the part of customers who receive a high-quality product for the price of a low-quality one. Downconversion is therefore a common practice in the semiconductor industry. Downgrading, however, has the advantage of being free whereas downconversion incurs a cost. Therefore, downgrading may, in theory, be preferable to downconversion. In this section, we investigate the tradeoff

[10] Comprising 1920 problem instances. Details available upon request.

between the cost disadvantage of downconversion and the cannibalization disadvantage of downgrading. Strategic disadvantages of downgrading are not considered but these would only serve to make downgrading less desirable.

The practice of downgrading is captured by a slight modification to the model presented in §3: if low-quality demand, i.e., demand for product L, exceeds the low-quality inventory, q_L, then the firm fulfills as much as it can of this unsatisfied demand (at price p_L) using any excess inventory of product H, i.e., any inventory of H left over after satisfying first-choice demand for H.

THEOREM EC.6. *(a) Downgrading will not occur in the single customer-class case if prices are set optimally. (b) In the two customer-class case, (i) downconversion (weakly) dominates downgrading if the downconversion cost, c_D, is zero, (ii) downgrading can be preferred to downconversion if $c_D > 0$.*

The single-class result echoes the earlier result for downconversion. The two-class results suggest that, as one might expect, downgrading is more likely to be preferred as the downconversion cost increases. We investigate this using the same numeric study as described in §5.4 (restricting attention to advanced pricing and recourse allocation) but using a wider range of relative downconversion costs, from 0% of the production cost up to 7.5%. Table EC.1 presents, as a function of the downconversion cost and class 2's valuation of product H, the percentage of cases in which the firm 1) prefers downconversion, 2) prefers downgrading, or 3) is indifferent between the two practices. As can be seen, downconversion becomes less attractive relative

		downconversion cost*				
a_{2H}	preference	0%	0.5%	2.5%	5.0%	7.5%
0.6	DC	43.08	42.04	38.33	35.21	32.08
	ID	56.92	49.92	50.08	50.00	50.25
	DG	0	8.04	11.58	14.79	17.67
0.5	DC	2.67	2.39	1.83	1.50	1.44
	ID	97.33	16.11	14.72	14.00	13.83
	DG	0	81.50	83.44	84.50	84.72
0.4	DC	1.67	1.58	1.58	1.58	1.58
	ID	98.33	18.58	16.58	15.67	15.42
	DG	0	79.83	81.83	82.75	83.00
0.3	DC	3.33	3.17	3.17	3.17	3.17
	ID	96.67	30.50	27.17	25.83	25.67
	DG	0	66.33	69.67	71.00	71.17

Table EC.1 Percentage (%) of cases where downconversion (DC) or downgrading (DG) is preferred. (ID denotes indifference). *Relative to production cost.

to downgrading as the cost of downconversion increases. While downgrading was quite often preferred to downconversion, it is important to note that when downgrading was preferred, the average improvement over downconversion was 0.09%, whereas, when downconversion was preferred, the average improvement over downgrading was 0.67%. Furthermore, the maximum improvement of downgrading over downconversion was 1.52% while the maximum improvement of downconversion over downgrading was 6.50%.

Finally we note that downconversion is increasingly preferred as a_{2H}, class-2's valuation of product H, increases. The reason is as follows. In situations in which the optimal prices induce class 2 to prefer product L, an increase in a_{2H} makes class-2 customers more willing to spill up to product H if there is insufficient inventory of product L. Downgrading does not take advantage of spill up and, therefore, the cannibalization disadvantage of downgrading is more significant as a_{2H} increases.

Appendix F: Residual Market Uncertainty

In the main paper, we have assumed that the firm has perfect market information when making the downconversion and (recourse) pricing decisions. That is, market uncertainty is completely resolved by the end of the production lead time. We note that single-period models in which resource allocation and/or pricing decisions can be postponed until after the market size is perfectly observed are common in both the operations and marketing literatures. Ex-post resource allocation is assumed in the flexibility literature (e.g., Fine and Freund (1990), Van Mieghem (1998, 2004)), in the delayed-differentiation literature (e.g., Swaminathan and Tayur (1998), Anand and Girotra (2006)), in the transshipment literature (e.g., Rudi et al. (2001), Dong and Rudi (2004)), and in the subcontracting literature (e.g., Van Mieghem (1999)). As discussed in §2, ex-post pricing is assumed in Bish and Wang (2004) and Chod and Rudi (2005), and is one of the cases considered in Van Mieghem and Dada (1999). It is also assumed in Desai et al. (2007) who note that "given the lead time needed for manufacturing, the production decision may be based on the firm's expectations about the market demand [and] thus, subsequent marketing decisions, such as price and advertising, are conditional not only on the realized demand, but also on the production decision made earlier, when the firm did not have complete information about market demand."

Market-size uncertainty may be reduced over the production lead time as the firm obtains new information from its interactions with potential customers and/or from observations of economic indicators that influence market size. When prices are set in advance, customers might pre order based on advanced showings and prototype demonstrations. If prices are not set in advance then customer will not pre order; they may, however, signal their interest (with the actual purchase decision dependent on the eventual price.) Economic indicators, such as interest rates, macro-level industrial demand, etc., may also contain useful signals of market sizes. While market uncertainty may not be fully resolved based on customer interactions and economic indicators, the common assumption that postponed decisions can be made with perfect market-size information is an acceptable approximation if market uncertainty reduces significantly over the production lead time.

There may, however, be situations in which market uncertainty is not even close to being fully resolved after production. Yield uncertainty is, of course, resolved after production. In our context, downconversion and (recourse) pricing decisions would then be made with perfect yield information but imperfect market information. Would this residual market uncertainty effect our earlier findings, and, if so, how? To address this question, we focus on the single-class case. Let $F_R(\cdot)$ denote the distribution function for the market potential x at the point in time at which the firm chooses the downconversion quantity (and prices in the recourse case.) In other words, $F_R(\cdot)$ represents the firm's market forecast after production. If $F_R(\cdot)=F_X(\cdot)$, the original forecast, then no uncertainty is resolved over the lead time.

Recall that the realized inventories of product H and L are given by $q_H = yQ$ and $q_H = (1-y)Q$ respectively. For any price vector (p_H, p_L), Theorem 1 specified the optimal downconversion quantity assuming the firm had perfect market information. The following theorem extends that earlier result to the case in which there is residual market uncertainty when downconversion occurs.

THEOREM EC.7. *For any realized inventory vector (q_H, q_L) (i) if $a_H - p_H \geq a_L - p_L$ then $q_D^* = 0$, (ii) if $a_H - p_H < a_L - p_L$ then (a) the optimal downconversion quantity $q_D^* = 0$ if*

$$c_D \geq p_L \overline{F}_R\left(\frac{q_L}{G(a_L-p_L)}\right) - p_H \frac{G(a_H-p_H)}{G(a_L-p_L)}\left(F_R\left(\frac{q_L}{G(a_L-p_L)} + \frac{q_H}{G(a_H-p_H)}\right) - F_R\left(\frac{q_L}{G(a_L-p_L)}\right)\right) - p_H \overline{F}_R\left(\frac{q_L}{G(a_L-p_L)} + \frac{q_H}{G(a_H-p_H)}\right),$$

and, otherwise, q_D^* is the unique solution to

$$c_D = p_L \overline{F}_R\left(\frac{q_L+q_D}{G(a_L-p_L)}\right) - p_H \frac{G(a_H-p_H)}{G(a_L-p_L)}\left(F_R\left(\frac{q_L+q_D}{G(a_L-p_L)} + \frac{q_H-q_D}{G(a_H-p_H)}\right) - F_R\left(\frac{q_L+q_D}{G(a_L-p_L)}\right)\right) - p_H \overline{F}_R\left(\frac{q_L+q_D}{G(a_L-p_L)} + \frac{q_H-q_D}{G(a_H-p_H)}\right).$$

(b) $0 \leq q_D^* < q_H$, i.e., the firm never downconverts all product H to L.

For the case of perfect market information (and a single customer class), we established that the firm does not downconvert if prices are set optimally. While we have not been able to analytically establish the same result when there is residual market uncertainty at the time of downconversion and pricing, extensive numeric investigations did not uncover a single instance when downconversion occurred. This suggests that our earlier finding is robust.

Clearly, recourse pricing will be less beneficial if market uncertainty is not fully resolved over the production lead time. To investigate how residual market uncertainty affects the firm's expected profit, we model market uncertainty as follows. Let $X = X_L + X_R$, where X_L is realized over the lead time and X_R is a zero-mean random variable representing the residual uncertainty. In particular, let X_L and X_R be normally distributed with $X_L \sim N(\mu, \lambda\sigma)$ and $X_R \sim N(0, (1-\lambda)\sigma)$. Then, $X \sim N(\mu, \sigma)$. The parameter λ represents the fraction of market size uncertainty that is resolved over the production lead time. At $\lambda = 0$, no uncertainty is resolved. The $\lambda = 1$ case corresponds to our original model in which all market uncertainty is resolved.

For the base case scenario described in §4.3, Figure EC.1 plots the increase in expected profit (relative to the advanced pricing case) as a function of λ. We see from Figure EC.1 that the relative benefit of recourse pricing is convex increasing in λ. However, there is substantial benefit even for low values of λ, suggesting that recourse pricing is of significant value even with imperfect market information. In fact, at $\lambda = 0$, recourse pricing has no additional market information over advanced pricing, yet there is a 1% increase in profit over advanced pricing. This indicates that recourse pricing is quite beneficial even if only the yield has been observed.

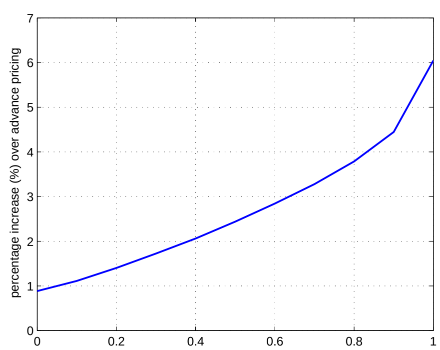

Figure EC.1 **Percent increase in the optimal expected profit as residual market uncertainty decreases.**

Appendix G: Proofs

We first present a useful technical lemma which is used in some of the subsequent proofs.

Lemma EC.1. *(a) Define $h_i(u)=uG_i(a_i-u)$ in the domain $0\le u\le a_i$. A sufficient condition for $h_i(u)$ to be unimodal in u is*

$$\text{(T1) } G_i(a_i-u)g_i'(a_i-u)-2g_i^2(a_i-u)<0$$

where $R_i=\frac{g_i(a_i-u)}{G_i(a_i-u)}$. Note that (T1) $\Leftrightarrow R_i^2>R_i'$. (b) Define $h(u)$ as a convex combination of $h_i(u)$, i.e., $h(u)=\lambda h_1(u)+(1-\lambda)h_2(u)$, $0\le\lambda\le 1$. A sufficient condition for $h(u)$ to be unimodal in u over $0\le u\le \min\{a_1,a_2\}$ is

$$\text{(T2)}\quad \lambda^2\left(G_1(u_1)g_1'(u_1)-2g_1^2(u_1)\right)+(1-\lambda^2)\left(G_2(u_2)g_2'(u_2)-2g_2^2(u_2)\right)$$
$$+\lambda(1-\lambda)\left(G_1(u_1)g_2'(u_2)+G_2(u_2)g_1'(u_1)-4g_1(u_1)g_2(u_2)\right)<0,$$

where $u_i=a_i-u, i=1,2$. (c) If $\lambda=0$ or 1, then (T1) implies (T2).

Proof of Lemma EC.1 (a) Note $\frac{\partial h_i(u)}{\partial u}=G_i(a_i-u)-ug_i(a_i-u)$ and $\frac{\partial^2 h_i(u)}{\partial u^2}=-2g_i(a_i-u)-ug_i'(a_i-u)$. Because $\frac{\partial h_i(u)}{\partial u}|_{u=0}>0$ and $\frac{\partial h_i(u)}{\partial u}|_{u=a_i}\le 0$, a sufficient condition for $h_i(u)$ to be unimodal in u is $\frac{\partial^2 h_i(u)}{\partial u^2}<0$ whenever $\frac{\partial h_i(u)}{\partial u}=0$. Note $\frac{\partial h_i(u)}{\partial u}=0\Rightarrow u^*=\frac{G_i(a_i-u)}{g_i(a_i-u)}\Rightarrow \frac{\partial^2 h_i(u)}{\partial u^2}|_{u=u^*}=(-2g_i^2(a_i-u)+G_i(a_i-u)g_i'(a_i-u))/g_i(a_i-u)<0$ by (T1). (b) Follows analogously as (a).

Proof of Theorem 1 Define $\pi(q_D)=r(q_D)-c_Dq_D$. Also define $\alpha=\frac{G(a_H-p_H)}{G(a_L-p_L)}$.

(i) In this case, $a_H-p_H\ge a_L-p_L$. We will show that $\pi(q_D)$ is decreasing in q_D. Recall $r(q_D)=p_H\min\{xG(a_H-p_H),q_H-q_D\}+p_L\min\{[xG(a_H-p_H)-(q_H-q_D)]^+/\alpha,q_L+q_D\}$. For $q_D\le[q_H-xG(a_H-p_H)]^+$, $\pi(q_D)=p_HxG(a_H-p_H)-c_Dq_D$, which is decreasing in q_D. For $q_D>[q_H-xG(a_H-p_H)]^+$, $\pi(q_D)=p_H(q_H-q_D)+p_L\min\{(xG(a_H-p_H)-(q_H-q_D))/\alpha,q_L+q_D\}-c_Dq_D$. If $q_L+q_D>xG(a_H-p_H)-(q_H-q_D))/\alpha$, then $\pi(q_D)=p_H(q_H-q_D)+p_L(xG(a_H-p_H)-(q_H-q_D))/\alpha-c_Dq_D$. Note $\pi(q_D)$ is decreasing in q_D because $p_H>p_L>p_L/\alpha-c_D$. If $q_L+q_D\le xG(a_H-p_H)-(q_H-q_D))/\alpha$, then $\pi(q_D)=p_H(q_H-q_D)+p_L(q_L+q_D)-c_Dq_D$, which is again decreasing in q_D. Since $\pi(q_D)$ is always decreasing in q_D, it then follows that $q_D^*=0$.

(ii) In this case $a_H-p_H<a_L-p_L$. Therefore $\pi(q_D)=p_L\min\{xG(a_L-p_L),q_L+q_D\}+p_H\min\{[xG(a_L-p_L)-(q_L+q_D)]^+\alpha,q_H-q_D\}-c_Dq_D$. Because $p_L-c_D\le p_H\alpha$, $\pi(q_D)$ is decreasing in q_D. Therefore $q_D^*=0$.

(iii) Define $z=[xG(a_L-p_L)-q_L]^+$. If $q_D>z$, then $\pi(q_D)=p_LxG(a_L-p_L)-c_Dq_D$, which is decreasing in q_D. If $q_D\le z$, then $\pi(q_D)=p_L(q_L+q_D)+p_H\min\{(z-q_D)\alpha,q_H-q_D\}-c_Dq_D$. For $q_D\le\frac{[q_H-z\alpha]^+}{1-\alpha}$, $\pi(q_D)=p_L(q_L+q_D)+p_H(z-q_D)\alpha-c_Dq_D$, and $\pi(q_D)$ is increasing in q_D because $c_D<p_L-p_H\alpha$. For $q_D>\frac{[q_H-z\alpha]^+}{1-\alpha}$, $\pi(q_D)=p_L(q_L+q_D)+p_H(q_H-q_D)$, which is decreasing in q_D because $p_H>p_L+(a_H-a_L)>p_L$. Combining these results, we then have $q_D^*=\min\{z,\frac{[q_H-z\alpha]^+}{1-\alpha}\}$.

Proof of Theorem 2 We prove (a) by contradiction. Consider any arbitrary price vector p'_H and p'_L, such that $a_L - p'_L > a_H - p'_H$. Define $q'_H = q_H - q_D$ and $q'_L = q_L + q_D$.

If $xG(a_L - p'_L) \le q'_L$, then $r(p'_H, p'_L) = p'_L xG(a_L - p'_L)$. Define $\hat{p}_H = a_H - a_L + p'_L$. Then $r(\hat{p}_H, p'_L) = \hat{p}_H \min\{xG(a_H - \hat{p}_H), q'_H\} + p'_L \min\{[xG(a_H - \hat{p}_H) - q'_H]^+, q'_L\}$. Note $r(p'_H, p'_L) \le r(\hat{p}_H, p'_L)$ for any value of q'_H because $\hat{p}_H \ge p'_L$ and $xG(a_H - \hat{p}_H) = xG(a_L - p'_L)$. It then follows that $a_L - p'_L > a_H - p'_H$ cannot be optimal.

If $xG(a_L - p'_L) > q'_L$, define $\beta' = \frac{G(a_H - p'_H)}{G(a_L - p'_L)}$, then $r(p'_H, p'_L) = p'_L q'_L + p'_H \min\{(xG(a_L - p'_L) - q'_L)\beta', q'_H\}$. For $q'_H < (xG(a_L - p'_L) - q'_L)\beta'$, $r(p'_H, p'_L) = p'_L q'_L + p'_H q'_H \Rightarrow r(p'_H, p'_L)$ is increasing in p'_L. For $q'_H \ge (xG(a_L - p'_L) - q'_L)\beta'$, $r(p'_H, p'_L) = p'_L q'_L + p'_H (xG(a_L - p'_L) - q'_L)\beta'$, which is increasing in p'_L up to $p'_H - (a_H - a_L)$. We have now proven that $r(p'_H, p'_L)$ is increasing in p'_L for $p'_L < p'_H - (a_H - a_L)$. Therefore $a_H - p'_H < a_L - p'_L$ cannot hold.

(b)(i) By (a), for any post downconversion quantity, the optimal price vector satisfies $a_H - p_H \ge a_L - p_L$. For any given price vector that satisfies $a_H - p_H \ge a_L - p_L$, the total revenue is non-decreasing in q_H. Therefore, at the optimal price vector, downconversion will not occur, i.e., $q_D^* = 0$.

(b)(ii). From (a) and (b)(i), $a_H - p_H \ge a_L - p_L$ and $q_D^* = 0$. Therefore, the revenue as a function of price is given by $r(p_H, p_L) = p_H \min\{xG(a_H - p_H), q_H\} + p_L \min\{[xG(a_H - p_H) - q_H]^+ \frac{G(a_L - p_L)}{G(a_H - p_H)}, q_L\}$.

In the following, we prove theorem statements by establishing an upper bound on $r(p_H, p_L)$ and then considering $r(p_H, p_L)$ by different regions of p_H.

Let $\bar{r}(p_H, p_L) = \lim_{q_H \to \infty, q_L \to \infty} r(p_H, p_L) = p_H xG(a_H - p_H)$, then $\bar{r}(p_H^*, p_L^*) = p_H^* xG(a_H - p_H^*)$ is the upper bound on $r(p_H, p_L)$. By Lemma EC.1, $\bar{r}(p_H, p_L)$ is unimodal in p_H and by first order condition, $p_H^* = \frac{G(a_H - p_H^*)}{g(a_H - p_H^*)}$.

To fully characterize $r(p_H, p_L)$, we consider the revenue function by different regions of p_H. Define $\overline{p_H} = \max\{0, a_H - G^{-1}\left(\frac{q_H}{x}\right)\}$ and $\underline{p_H} = \max\left\{0, a_H - G^{-1}\left(\frac{q_H G(a_L - \nu_L)}{xG(a_L - \nu_L) - q_L}\right)\right\}$, where ν_L is the unique solution to $\nu = \frac{G(a_L - \nu)}{g(a_L - \nu)}$ (note $\nu G(a_L - \nu)$ is unimodal in ν for $\nu \in [0, a_L]$). Note that $0 \le \underline{p_H} \le \overline{p_H} \le a_H$. Define $\Gamma_0 : [\overline{p_H}, a_H]$, $\Gamma_1 : [\underline{p_H}, \overline{p_H})$, and $\Gamma_2 : [0, \underline{p_H})$ to partition p_H into three regions. We next show that $r(p_H, p_L)$ is unimodal in p_H in all three regions.

For $p_H \in \Gamma_0$, $r(p_H, p_L) = p_H xG(a_H - p_H)$, which is unimodal in p_H by above analysis. For $p_H \in \Gamma_1$, $r(p_H, p_L) = p_H q_H + p_L (xG(a_H - p_H) - q_H) \frac{G(a_L - p_L)}{G(a_H - p_H)} = p_H q_H + p_L G(a_L - p_L)\left(x - \frac{q_H}{G(a_H - p_H)}\right)$. Note that the optimal p_L is independent of p_H. Because $p_L G(a_L - p_L)$ is unimodal in p_L, we have $p_L^* = \frac{G(a_L - p_L^*)}{g(a_L - p_L^*}$. Substitute p_L^* into the revenue function, we have $r(p_H, p_L^*) = p_H q_H + p_L^* G(a_L - p_L^*)\left(x - \frac{q_H}{G(a_H - p_H)}\right)$. Note that $\frac{\partial r(p_H, p_L^*)}{\partial p_H} = q_H \left(1 - p_L^* G(a_L - p_L^*) \frac{g(a_H - p_H)}{G^2(a_H - p_H)}\right)$, and $\frac{\partial^2 r(p_H, p_L^*)}{\partial p_H^2} = -\frac{p_L^* G(a_L - p_L^*) q_H}{G^3(a_H - p_H)} (-g'(a_H - p_H)G(a_H - p_H) + 2g^2(a_H - p_H)) < 0$ by $R^2 > R'$ assumption. Therefore, $r(p_H, p_L)$ is concave in p_H for $p_H \in \Gamma_1$.

For $p_H \in \Gamma_2$, $r(p_H, p_L) = p_H q_H + p_L q_L$, where p_L satisfies the condition $\left(x - \frac{q_H}{G(a_H - p_H)}\right) G(a_L - p_L) = q_L$. Therefore, given p_H, $p_L(p_H) = a_L - G^{-1}\left(\frac{q_L}{x - \frac{q_H}{G(a_H - p_H)}}\right)$. Substitute $p_L(p_H)$ into the revenue function, we have $r(p_H, p_L(p_H)) = p_H q_H + \left(a_L - G^{-1}\left(\frac{q_L}{x - \frac{q_H}{G(a_H - p_H)}}\right)\right) q_L$. Note that $\frac{\partial r(p_H, p_L(p_H))}{\partial p_H} = q_H - q_L \frac{\partial G^{-1}(\kappa)}{\partial p_H}$, and $\frac{\partial^2 r(p_H, p_L(p_H))}{\partial p_H^2} = -q_L \frac{\partial^2 G^{-1}(\kappa)}{\partial p_H^2}$, where $\kappa = \frac{q_L}{x - \frac{q_H}{G(a_H - p_H)}}$.

To prove the revenue function is unimodal, it is necessary and sufficient to show that $\frac{\partial^2 G^{-1}(\kappa)}{\partial p_H^2} > 0$ whenever $\frac{\partial r(p_H, p_L(p_H))}{\partial p_H} = 0$. Note that $\frac{\partial G^{-1}(\kappa)}{\partial p_H} = \frac{1}{g(a_L - p_L)} \frac{q_L}{(x - q_H/G(a_H - p_H))^2} \frac{q_H}{G^2(a_H - p_H)} g(a_H - p_H) = \frac{g(a_H - p_H)}{g(a_L - p_L)} \frac{q_H q_L}{(xG(a_H - p_H) - q_H)^2}$, and $\frac{\partial^2 G^{-1}(\kappa)}{\partial p_H^2} = \left(-\frac{g'(a_H - p_H)}{g(a_L - p_L)} + \frac{g(a_H - p_H)}{(g(a_L - p_L))^2} g'(a_L - p_L) \frac{\partial p_L}{\partial p_H}\right)$ $\frac{q_H q_L}{(xG(a_H - p_H) - q_H)^2} + \frac{g(a_H - p_H)}{g(a_L - p_L)} \frac{q_H q_L}{(xG(a_H - p_H) - q_H)^3} 2xg(a_H - p_H) = \frac{q_H q_L}{(xG(a_H - p_H) - q_H)^2 g(a_L - p_L)} S$, where $S = -g'(a_H - p_H) + \frac{g(a_H - p_H) g'(a_L - p_L)}{g(a_L - p_L)} \frac{\partial p_L}{\partial p_H} + \frac{2g^2(a_H - p_H)x}{xG(a_H - p_H) - q_H}$.

By definition $\frac{\partial p_L}{\partial p_H} = -\frac{\partial G^{-1}(\kappa)}{\partial p_H}$. Setting $\frac{\partial r(p_H, p_L(p_H))}{\partial p_H} = 0$, we have $\frac{\partial p_L}{\partial p_H} = -\frac{q_H}{q_L}$. Substitute $\frac{\partial p_L}{\partial p_H} = -\frac{q_H}{q_L}$ into expression S and recognizing that $xG(a_L - p_L) = q_L + q_H \frac{G(a_L - p_L)}{G(a_H - p_H)}$, we have $S = -g'(a_H - p_H) - \frac{g(a_H - p_H) g'(a_L - p_L)}{g(a_L - p_L)} \frac{q_H}{q_L} + \frac{2xg^2(a_H - p_H) G(a_L - p_L)}{q_L G(a_H - p_H)} = -g'(a_H - p_H) - \frac{g(a_H - p_H) g'(a_L - p_L)}{g(a_L - p_L)} \frac{q_H}{q_L} + \frac{2g^2(a_H - p_H)}{q_L G(a_H - p_H)} (q_L + q_H \frac{G(a_L - p_L)}{G(a_H - p_H)}) = \frac{-g'(a_H - p_H) G(a_H - p_H) + 2g^2(a_H - p_H)}{G(a_H - p_H)} + \frac{q_H}{q_L} g(a_H - p_H) (\frac{2g(a_H - p_H) G(a_L - p_L)}{G^2(a_H - p_H)} - \frac{g'(a_L - p_L)}{g(a_L - p_L)})$. Note that $-g'(a_H - p_H) G(a_H - p_H) + 2g^2(a_H - p_H) > 0$ by $R^2 > R'$ assumption. If we assume $g'(\cdot) \le 0$, then we have $S > 0 \Rightarrow \frac{\partial^2 G^{-1}(\kappa)}{\partial p_H^2} > 0 \Rightarrow \frac{\partial^2 r(p_H, p_L(p_H))}{\partial p_H^2} < 0$ whenever $\frac{\partial r(p_H, p_L(p_H))}{\partial p_H} = 0$. Without the $g'(\cdot) \le 0$ assumption, the expression S is not necessarily positive. However, we can say more about the property of S.

Next we show that if there exists an optimal solution in Γ_2, then there exists an equivalent optimal solution that satisfies $a_H - p_H^* = a_L - p_L^*$. We prove this by construction.

First note that $r(p_H, p_L(p_H))$ is a continuous function in p_H. Therefore the optimal solution of p_H^* and p_L^* must be local stationary points, which is given by $\frac{\partial r(p_H, p_L(p_H))}{\partial p_H} = 0 \Rightarrow \frac{\partial p_L}{\partial p_H} = -\frac{q_H}{q_L}$. Now, suppose there exists an optimal solution pair p_H^* and p_L^* which does not satisfy $a_H - p_H^* = a_L - p_L^*$. Construct a new solution pair where $p'_H = p_H^* + \epsilon$ and $p'_L = p_L - \epsilon \frac{q_H}{q_L}$. Because $r(p'_H, p'_L) = r(p_H^*, p_L^*)$ and p'_H and p'_L satisfy local stationary point condition, the solution (p'_H, p'_L) is at least as good as the existing (p_H^*, p_L^*) solution. If $a_H - p'_H = a_L - p'_L$ then we have found an equivalent optimal solution; if not we can continue increase p'_H by ϵ until either $p_H \ge \underline{p_H}$ or $a_H - p'_H = a_L - p'_L$ is satisfied. In the former case, an equivalent optimal solution exists in region Γ_1. This proves that in region Γ_2, one only needs to search solution pairs where $a_H - p_H = a_L - p_L$. Substitute this condition into expression S, we have $S = \frac{-g'(a_H - p_H) G(a_H - p_H) + 2g^2(a_H - p_H)}{G(a_H - p_H)} + \frac{q_H}{q_L} (\frac{2g^2(a_H - p_H)}{G(a_H - p_H)} - g'(a_H - p_H)) = \frac{-g'(a_H - p_H) G(a_H - p_H) + 2g^2(a_H - p_H)}{G(a_H - p_H)} (1 + \frac{q_H}{q_L}) > 0$. This proves that in region Γ_2, $\frac{\partial^2 r(p_H, p_L(p_H))}{\partial p_H^2} < 0$ whenever $\frac{\partial r(p_H, p_L(p_H))}{\partial p_H} = 0$.

Finally, we prove that $r(p_H, p_L)$ is continuous at $\underline{p_H}$ but is in general not continuous at $\overline{p_H}$, i.e., the lower and upper limits of $\frac{\partial r(p_H,p_L)}{\partial p_H}$ are identical at $\underline{p_H}$ but not so at $\overline{p_H}$. Note that $\frac{\partial r(p_H,p_L)}{\partial p_H}|p_H \uparrow \underline{p_H} = q_H - q_L \frac{\partial G^{-1}(\kappa)}{\partial p_H} = q_H - q_L \frac{g(a_H-p_H)}{g(a_L-p_L)} \frac{q_H q_L}{(xG(a_H-p_H)-q_H)^2} = q_H - q_L \frac{g(a_H-p_H)}{g(a_L-p_L)} \frac{q_H q_L}{\left(x\frac{q_H G(a_L-p_L^*)}{xG(a_L-p_L^*)-q_L}-q_H\right)^2} = q_H - \frac{g(a_H-p_H)}{g(a_L-p_L)q_H}(xG(a_L-p_L^*)-q_L)^2 = q_H - \frac{g(a_H-p_H)G^2(a_L-p_L^*)}{g(a_L-p_L)q_H}\left(x - \frac{q_L}{G(a_L-p_L^*)}\right)^2 = q_H - \frac{p_L^* g(a_H-p_H)G(a_L-p_L^*)}{q_H}$
$\left(x - \frac{q_L}{G(a_L-p_L^*)}\right)^2 = q_H - p_L^* G(a_L - p_L^*) q_H \frac{g(a_H-p_H)}{\left(q_H/\left(x-\frac{q_L}{G(a_L-p_L^*)}\right)\right)^2} = q_H - p_L^* G(a_L - p_L^*) q_H \frac{g(a_H-p_H)}{G^2(a_H-p_H)} = \frac{\partial r(p_H,p_L)}{\partial p_H}|p_H \downarrow \underline{p_H}$. This proves that $r(p_H, p_L)$ is continuous at $\underline{p_H}$.

The optimal p_H^* lies in Γ_0 if $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H \downarrow \overline{p_H}} \geq 0$; it equals to $\overline{p_H}$ if $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H \downarrow \overline{p_H}} < 0$ and $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H \uparrow \overline{p_H}} \geq 0$; it lies in Γ_1 if $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H \uparrow \overline{p_H}} < 0$ and $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H = \underline{p_H}} \geq 0$; it lies in Γ_2 otherwise.

Note that $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H \downarrow \overline{p_H}} \geq 0 \Leftrightarrow x \leq \frac{q_H}{\left(a_H - G^{-1}\left(\frac{q_H}{x}\right)\right) g\left(G^{-1}\left(\frac{q_H}{x}\right)\right)}$; $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H \uparrow \overline{p_H}} \geq 0 \Leftrightarrow x \leq \frac{q_H}{\sqrt{g\left(G^{-1}\left(\frac{q_H}{x}\right)\right)\nu_L G(a_L-\nu_L)}}$; and $\frac{\partial r(p_H,p_L)}{\partial p_H}|_{p_H = \underline{p_H}} \geq 0 \Leftrightarrow x \leq \left(q_L + \frac{q_H}{\sqrt{g(a_H-\underline{p_H})/g(a_L-\nu_L)}}\right)/G(a_L - \nu_L)$. The theorem statement follows after applying first order conditions.

Proof of Corollary 1 The corollary statements are trivially true for $x \in \Omega_0 \cup \Omega_1$ because there is no demand for product L in these two regions. For $x \in \Omega_3$, $a_H - p_H^* = a_L - p_L^*$. Therefore, the corollary statements follow if $a_H - p_H^* = a_L - p_L^*$ for $x \in \Omega_2$. Note for $x \in \Omega_2$, p_H^* satisfies $\frac{G(a_H-p_H^*)}{\sqrt{g(a_H-p_H^*)}} = \frac{G(a_L-p_L^*)}{\sqrt{g(a_L-p_L^*)}} \Rightarrow \frac{G(a_L-(p_H^*-(a_H-a_L)))}{\sqrt{g(a_L-(p_H^*-(a_H-a_L)))}} = \frac{G(a_L-p_L^*)}{\sqrt{g(a_L-p_L^*)}} \Rightarrow p_H^* = p_L^* + (a_H - a_L)$ is one potential solution. Next we prove that this potential solution is unique. Suppose there exists an alternative solution $\hat{p_H} = p_H^* + \epsilon, \epsilon \neq 0$. This, however, cannot happen because in region Ω_2 the revenue function $r(p_H, p_L^*)$ is concave in p_H. Therefore, there exists one and only one p_H^*, which means ϵ must equal to zero and $\hat{p_H}$ cannot be an optimal solution.

Proof of Theorem 3 (a) (i) Consider $a_{iH} - p_H \geq a_{iL} - p_L$ and $a_{\bar{i}H} - p_H < a_{\bar{i}L} - p_L$, i.e., class i prefers H to L and the complement class $\bar{i}$ prefers L to H, $i = 1, 2$. In this case, there is no competition for first choice demand because customers have separating preferences. An allocation policy is only relevant for rationing class i's spill-over demand and class $\bar{i}$'s first choice demand, $i = 1, 2$. Such instances arise only when class i's first choice product is sold out, and class i's spill-over demand plus class $\bar{i}$'s first choice demand exceeds the firm's inventory. In these instances, however, the firm is indifferent among all allocation policies because all products are sold regardless of the policy. Thus, when customers have separating preferences, any allocation policy is optimal and priority allocation is one such policy.

(ii) First consider $a_{iH} - p_H \geq a_{iL} - p_L$, $i = 1, 2$, i.e., both customer classes prefer H to L. If $\sum_{i=1}^{2} x_i G(a_{iH} - p_H) \leq q_H$, then any allocation is optimal because all first choice demand can be filled. If $\sum_{i=1}^{2} x_i G(a_{iH} - p_H) > q_H$, then all first choice demand cannot be filled, and a fraction of customer demand will spill over to product

L. The firm's revenue is maximized when the number of spill-over customers are maximized. The fraction of class i customers willing to spill over is s_{iL}, $i=1,2$. Therefore, the optimal allocation policy would spill down class 1 customers if $s_{1L}>s_{2L}$, spill down class 2 customers if $s_{1L}<s_{2L}$, and be indifferent otherwise. So, if $s_{1L}>s_{2L}$, then the firm's revenue is maximized by first filling demand from class 2 customers, and then filling demand from class 1 customers. Conversely, if $s_{1L}<s_{2L}$, then the firm's revenue is maximized by first filling demand from class 1 customers, and then filling demand from class 2 customers. In either case, a priority allocation rule is thus optimal. An analogous argument holds when $a_{iH}-p_H<a_{iL}-p_L$, $i=1,2$, i.e., both customer classes prefer L to H.

(b) (i) Follows directly from part (a). (ii) From part (a), the optimal priority class depends on the spill over ratio s_{iL}, $i=1,2$. In advance pricing, because both a_{iK}, $i=1,2$, $k\in\{H,L\}$, and $\mathbf{p}$ are known in advance, the optimal priority class can be determined a priori and therefore the priority based allocation rule is optimal.

Proof of Theorem 4 First define $r(q)$ and $r(q,\epsilon)$ as the revenue function with zero and $\epsilon>0$ units of product H converted to product L, respectively. We prove theorem statement by considering different regions of price vector $\mathbf{p}$. In what follows, we consider an exhaustive and mutually exclusive list of problem regions.

First consider $\mathbf{p}\in\Gamma_1$.

- A: $\sum_{i=1}^2 d_{iH}<q_H$. Then $r(q)=p_H\sum_{i=1}^2 d_{iH}\Rightarrow r(q,\epsilon)=r(q)-\epsilon c_D<r(q)\Rightarrow \hat{q}_D=0$.
- B: $\sum_{i=1}^2 d_{iH}\geq q_H$ and $d_{jH}<q_H$, Then $r(q)=p_Hq_H+p_L\min\{(\sum_{i=1}^2 d_{iH}-q_H)s_{\bar{j}L},q_L\}$.

—B1: $q_L\geq(\sum_{i=1}^2 d_{iH}-q_H)s_{\bar{j}L}$. Then $r(q)=p_Hq_H+p_L(\sum_{i=1}^2 d_{iH}-q_H)s_{\bar{j}L}\Rightarrow r(q,\epsilon)=r(q)-\epsilon(p_H-p_Ls_{\bar{j}L})<r(q)\Rightarrow\hat{q}_D=0$ because $s_{\bar{j}L}<1$ for $\mathbf{p}\in\Gamma_2$.

—B2: $q_L<(\sum_{i=1}^2 d_{iH}-q_H)s_{\bar{j}L}$. Then $r(q)=p_Hq_H+p_Lq_L\Rightarrow r(q,\epsilon)=r(q)-\epsilon(p_H-p_L)<r(q)\Rightarrow\hat{q}_D=0$.

- C: $\sum_{i=1}^2 d_{iH}\geq q_H$ and $d_{jH}\geq q_H$. Then $r(q)=p_Hq_H+p_L\min\{(d_{jH}-q_H)s_{jL}+d_{\bar{j}H}s_{\bar{j}L},q_L\}$.

—C1: $q_L>(d_{jH}-q_H)s_{jL}+d_{\bar{j}H}s_{\bar{j}L}$. $r(q)=p_Hq_H+p_L((d_{jH}-q_H)s_{jL}+d_{\bar{j}H}s_{\bar{j}L})\Rightarrow r(q,\epsilon)=r(q)-\epsilon(p_H-p_Ls_{jL}+c_D)<r(q)\Rightarrow\hat{q}_D=0$.

—C2: $q_L\leq(d_{jH}-q_H)s_{jL}+d_{\bar{j}H}s_{\bar{j}L}$. $r(q)=p_Hq_H+p_Lq_L\Rightarrow r(q,\epsilon)=r(q)-\epsilon(p_H-p_L+c_D)<r(q)\Rightarrow\hat{q}_D=0$.

Next consider $\mathbf{p}\in\Gamma_2$.

- D: $d_{jH}\geq q_H$, $r(q)=p_Hq_H+p_L\min\{d_{\bar{j}L}+(d_{jH}-q_H)s_{jL},q_L\}$.

—D1: $q_L>d_{\bar{j}L}+(d_{jH}-q_H)s_{jL}$. $r(q)=p_Hq_H+p_L(d_{\bar{j}L}+(d_{jH}-q_H)s_{jL})\Rightarrow r(q,\epsilon)=r(q)-\epsilon(p_H-p_Ls_{jL}+c_D)<r(q)\Rightarrow\hat{q}_D=0$.

—D2: $q_L \le d_{\bar{j}L} + (d_{jH} - q_H)s_{jL}$. $r(q) = p_H q_H + p_L q_L \Rightarrow r(q,\epsilon) = r(q) - \epsilon(p_H - p_L + c_D) < r(q) \Rightarrow \hat{q}_D = 0$.

- E: $d_{jH} < q_H$ and $d_{\bar{j}L} < q_L$. $r(q) = p_H d_{jH} + p_L d_{\bar{j}L} \Rightarrow r(q,\epsilon) = r(q) - \epsilon c_D < r(q) \Rightarrow \hat{q}_D = 0$.
- F: $d_{jH} < q_H$ and $d_{\bar{j}L} \ge q_L$. $r(q) = p_H \min\{d_{jH} + (d_{\bar{j}L} - q_L)s_{\bar{j}H}, q_H\} + p_L q_L$.

—F1: $(d_{\bar{j}L} - q_L)s_{\bar{j}H} < q_H - d_{jH}$. $r(q) = p_H(d_{jH} + (d_{\bar{j}L} - q_L)s_{\bar{j}H}) + p_L q_L \Rightarrow r(q,\epsilon) = r(q) + \epsilon(p_L - c_D - p_H s_{\bar{j}H}) \Rightarrow \partial r(q,\epsilon)/\partial\epsilon > 0$ if $p_L - c_D > p_H s_{\bar{j}H}$. The optimal $\hat{q}_D$ therefore satisfies $d_{jH} + (d_{\bar{j}} - (q_L + \hat{q}_D))s_{\bar{j}H} = q_H - \hat{q}_D \Rightarrow \hat{q}_D = \frac{(q_H - d_{jH}) - (d_{\bar{j}L} - q_L)s_{\bar{j}H}}{1 - s_{\bar{j}H}}$.

—F2: $(d_{\bar{j}L} - q_L)s_{\bar{j}H} \ge q_H - d_{jH}$. $r(q) = p_H q_H + p_L q_L \Rightarrow r(q,\epsilon) = r(q) - \epsilon(p_H - p_L + c_D) < r(q) \Rightarrow \hat{q}_D = 0$.

The region $\mathbf{p} \in \Gamma_3$ can be symmetrically proved as $\mathbf{p} \in \Gamma_2$.

Finally, consider $\mathbf{p} \in \Gamma_4$.

- G: $\sum_{i=1}^2 d_{iL} < q_L$. $r(q) = p_L \sum_{i=1}^2 d_{iL} \Rightarrow r(q,\epsilon) = r(q) - \epsilon c_D < r(q) \Rightarrow \hat{q}_D = 0$.
- H: $d_{jL} \ge q_L$. $r(q) = p_H \min\{(d_{jL} - q_L)s_{jH} + d_{\bar{j}L}s_{\bar{j}H}, q_H\} + p_L q_L$.

—H1: $(d_{jL} - q_L)s_{jH} + d_{\bar{j}L}s_{\bar{j}H} \ge q_H$. $r(q) = p_H q_H + p_L q_L \Rightarrow r(q,\epsilon) = r(q) - \epsilon(p_H - p_L + c_D) < r(q) \Rightarrow \hat{q}_D = 0$.

—H2: $(d_{jL} - q_L)s_{jH} + d_{\bar{j}L}s_{\bar{j}H} < q_H$. $r(q) = p_H((d_{jL} - q_L)s_{jH} + d_{\bar{j}L}s_{\bar{j}H}) + p_L q_L \Rightarrow r(q,\epsilon) = r(q) + \epsilon(p_L - c_D - p_H s_{jH}) > r(q) \Rightarrow \partial r(q,\epsilon)/\partial\epsilon > 0$ if $p_L - c_D > p_H s_{jH}$. Therefore, the optimal $\hat{q}_{D1}$ satisfies $(d_{jL} - (q_L + \hat{q}_{D1}))s_{jH} + d_{\bar{j}L}s_{\bar{j}H} = q_H - \hat{q}_{D1} \Rightarrow \hat{q}_{D1} = \frac{q_H - (d_{jL} - q_L)s_{jH} - d_{\bar{j}L}s_{\bar{j}H}}{1 - s_{jH}}$. Now, $r(\hat{q}_{D1}) = p_H \min\{q_H - \hat{q}_{D1}, (d_{jL} - q_L - \hat{q}_{D1})s_{jH} + d_{\bar{j}L}s_{\bar{j}H}\} + p_L(q_L + \hat{q}_{D1})$. Thus, if $d_{\bar{j}L}s_{\bar{j}H} < q_H - (d_{jL} - q_L - \hat{q}_{D1})s_{jH} - \hat{q}_{D1}$, one can follow similar logic and show that the optimal $\hat{q}_{D2} = \frac{(q_H - (d_{jL} - q_L - \hat{q}_{D1})s_{jH} - \hat{q}_{D1} - d_{\bar{j}L}s_{\bar{j}H})^+}{1 - s_{\bar{j}H}}$.

- I: $d_{jL} < q_L$ and $\sum_{i=1}^2 d_{iL} \ge q_L$. $r(q) = p_H \min\{(\sum_{i=1}^2 d_{iL} - q_L)s_{\bar{j}H}, q_H\} + p_L q_L$.

—I1: $(\sum_{i=1}^2 d_{iL} - q_L)s_{\bar{j}H} \ge q_H$. $r(q) = p_H q_H + p_L q_L \Rightarrow r(q,\epsilon) = r(q) - \epsilon(p_H - p_L + c_D) < r(q) \Rightarrow \hat{q}_D = 0$.

—I2: $(\sum_{i=1}^2 d_{iL} - q_L)s_{\bar{j}H} \ge q_H$. $r(q) = p_H(\sum_{i=1}^2 d_{iL} - q_L)s_{\bar{j}H} + p_L q_L \Rightarrow r(q,\epsilon) = r(q) + \epsilon(p_L - c_D - p_H s_{\bar{j}H}) > r(q) \Rightarrow \partial r(q,\epsilon)/\partial\epsilon > 0$ if $p_L - c_D > p_H s_{\bar{j}H}$. Therefore, the optimal $\hat{q}_D$ satisfies $(\sum_{i=1}^2 d_{iL} - (q_L + \hat{q}_D))s_{\bar{j}H} = q_H - \hat{q}_D \Rightarrow \hat{q}_D = \frac{q_H - (\sum_{i=1}^2 d_{iL} - q_L)s_{\bar{j}H}}{1 - s_{\bar{j}H}}$.

The theorem statements follow directly by imposing the appropriate upper bound z on the maximum downconversion quantity $\hat{q}_D$.

Proof of Theorem 5 We prove that downconversion can be optimal by constructing a particular example. We use $U(0,1)$ distributions for utility of outside options (and therefore can scale a_i's to be 0 to 1). We use a deterministic demand and yield example, in which case there is no distinction between advance or recourse pricing. Define x_1 and x_2 as the deterministic market size for class 1 and class 2 customers, and y as the deterministic yield. Let the customer valuations be $a_{1L}=0$ and $a_{2H}=a_{2L}$. To simplify notation, let $a_1=a_{1H}$ and $a_2=a_{2H}=a_{2L}$. Let $\alpha=\frac{a_2-p_L}{a_2-p_H}$ and $\beta=\frac{a_2-p_H}{a_2-p_L}$.

The profit $\Pi(Q,p_H,p_L,q_D)=-c_PQ+p_H\min\{x_1(a_1-p_H)+x_2(a_2-p_H),yQ-q_D\}+p_L\min\{[x_2(a_2-p_H)-[yQ-q_D-x_1(a_1-p_H)]^+]^+\alpha,(1-y)Q+q_D\}-c_Dq_D$ if $p_H\le p_L$, and $\Pi(Q,p_H,p_L,q_D)=-c_PQ+p_H\min\{x_1(a_1-p_H)+[x_2(a_2-p_L)-(1-y)Q-q_D]^+\beta,yQ-q_D\}+p_L\min\{[x_2(a_2-p_L),(1-y)Q+q_D\}-c_Dq_D$ otherwise. We first note that $p_H\le p_L$, i.e., class 2 prefers H to L, cannot be optimal (the proof involves showing that $p_H\le p_L\Rightarrow q_D^*=0$, and $p_H>p_L$ dominates $p_H\le p_L$; details available upon request). We can therefore restrict attention to $p_H>p_L$.

If $q_D\le x_2(a_2-p_L)-(1-y)Q$, then $\Pi(Q,p_H,p_L,q_D)=-c_PQ+p_H\min\{x_1(a_1-p_H)+(x_2(a_2-p_L)-(1-y)Q-q_D)\beta,yQ-q_D\}+p_L((1-y)Q+q_D)-c_Dq_D$. In this case one can show that Q^* satisfies $yQ-q_D\ge x_1(a_1-p_H)+(x_2(a_2-p_L)-(1-y)Q-q_D)\beta$. We therefore restrict attention to profit function $\Pi(Q,p_H,p_L,q_D)=-c_PQ+p_H(x_1(a_1-p_H)+(x_2(a_2-p_L)-(1-y)Q-q_D)\beta)+p_L((1-y)Q+q_D)-c_Dq_D$. Note $\Pi(Q,p_H,p_L,q_D)$ is increasing in q_D if $c_D<p_L-p_H\beta$ and decreasing otherwise.

If $x_2(a_2-p_L)<(1-y)Q+q_D$, then $\Pi(Q,p_H,p_L,q_D)=-c_PQ+p_H\min\{x_1(a_1-p_H),yQ-q_D\}+p_Lx_2(a_2-p_L)-c_Dq_D\Rightarrow\Pi(Q,p_H,p_L,q_D)$ is strictly decreasing in q_D. Combining the above, we have $q_D^*(Q,p_H,p_L)=[x_2(a_2-p_L)-(1-y)Q]^+$ if $c_D<p_L-p_H\beta$ and $q_D^*(Q,p_H,p_L)=0$ otherwise.

Using this $q_D^*(Q,p_H,p_L)$, we can then solve for the optimum Q, p_H, and p_L. One can show that if $c_D\le\frac{(a_2-c_P)x_2y-(a_1-c_P)x_1(1-y)}{x_1(1-y)^2+x_2y^2}$, then $Q^*=\frac{1}{2}(a_1x_1+a_2x_2+c_Dx_1-(c_P+yc_D)(x_1+x_2))$, $p_H^*=\frac{1}{2}(a_1+c_P-(1-y)c_D)$, $p_L^*=\frac{1}{2}(a_2+c_P+yc_D)$, and therefore $q_D^*=\frac{1}{2}((a_2-c_P)x_2y-(a_1-c_P)x_1(1-y)-c_D(x_1(1-y)^2+x_2y^2))>0$. $q_D^*>0$ proves that downconversion can be optimal. We also note that if $\frac{a_1-c_P}{a_2-c_P}>\frac{x_2y}{x_1(1-y)}$ then downconversion is never optimal.

Proof of Theorem EC.6 (a) By Theorem 2, the optimal price vector always induces customers to prefer product H over L. Downgrading can never be optimal because either all demands are satisfied by product H or there is not leftover of product H. (b) Follows from Theorem 5. (c) (i) and (ii) follows from the fact that, at zero downconversion cost, downgrading is a special case of downconversion.

Proof of Theorem EC.7 (i) The case of $a_H-p_H\ge a_L-p_L$ follows from part (i) of Theorem 1. (ii) We prove that the optimal expected revenue is a unimodal function in q_D. Note that if $a_H-p_H<a_L-p_L$, then the firm's expected revenue as a function of downconversion quantity, q_D, is given by

$$E_X[r(q_D)]=p_L\left(\int_0^{\frac{q_L+q_D}{G(a_L-p_L)}}xG(a_L-p_L)dF_R(x)+\int_{\frac{q_L+q_D}{G(a_L-p_L)}}^{\infty}(q_L+q_D)dF_R(x)\right)$$

$$+p_H \int_{\frac{q_L+q_D}{G(a_L-p_L)}}^{\frac{q_L+q_D}{G(a_L-p_L)}+\frac{q_H-q_D}{G(a_H-p_H)}} \left(xG(a_L-p_L)-(q_L+q_D)\right)\frac{G(a_H-p_H)}{G(a_L-p_L)}dF_R(x)$$
$$+p_H \int_{\frac{q_L+q_D}{G(a_L-p_L)}+\frac{q_H-q_D}{G(a_H-p_H)}}^{\infty} (q_H-q_D)dF_R(x)-c_Dq_D. \quad \text{(EC.6)}$$

By (EC.6), we have

$$\frac{\partial E_X[r(q_D)]}{\partial q_D}=-c_D+p_L\overline{F}_R\left(\frac{q_L+q_D}{G(a_L-p_L)}\right)-p_H\frac{G(a_H-p_H)}{G(a_L-p_L)}\int_{\frac{q_L+q_D}{G(a_L-p_L)}}^{\frac{q_L+q_D}{G(a_L-p_L)}+\frac{q_H-q_D}{G(a_H-p_H)}} dF_R(x)$$
$$-p_H\int_{\frac{q_L+q_D}{G(a_L-p_L)}+\frac{q_H-q_D}{G(a_H-p_H)}}^{\infty} dF_R(x). \quad \text{(EC.7)}$$

Note that

$$\frac{\partial^2 E_X[r(q_D)]}{\partial q_D^2}=-\frac{p_L}{G(a_L-p_L)}f_R\left(\frac{q_L+q_D}{G(a_L-p_L)}\right)+p_H\frac{G(a_H-p_H)}{G^2(a_L-p_L)}f_R\left(\frac{q_L+q_D}{G(a_L-p_L)}\right)$$
$$-p_H\frac{\left(G(a_H-p_H)-G(a_L-p_L)\right)^2}{G^2(a_L-p_L)G(a_H-p_H)}f_R\left(\frac{q_L+q_D}{G(a_L-p_L)}+\frac{q_H-q_D}{G(a_H-p_H)}\right). \quad \text{(EC.8)}$$

By (EC.7), we have

$$\left(p_L-p_H\frac{G(a_H-p_H)}{G(a_L-p_L)}\right)\overline{F}_R\left(\frac{q_L+q_D}{G(a_L-p_L)}\right)>c_D\Rightarrow p_L>p_H\frac{G(a_H-p_H)}{G(a_L-p_L)}. \quad \text{(EC.9)}$$

Substitute (EC.9) into (EC.8), we have $\frac{\partial^2 E_X[r(q_D)]}{\partial q_D^2}<0$ whenever $\frac{\partial E_X[r(q_D)]}{\partial q_D}=0$. Part (ii)(a) then follows directly from (EC.7). (ii)(b) The theorem statement follows by substituting $q_D=q_H$ into (EC.7).

Proof of Theorem EC.1 See the proof of Theorem 2, which proves a more general case.

Proof of Corollary EC.2 By Theorem 2 and EC.1, the optimal recourse prices $p_H^*|_{RYP1}=p^*|_{CPP1}$ for $x\in\Omega_0\cup\Omega_1$. For $x\in\Omega_2$, by definition, $\frac{q_H}{x}<\sqrt{g\left(G^{-1}\left(\frac{q_H}{x}\right)\right)\nu_L G(a_L-\nu_L)}\Rightarrow\frac{q_H}{x}<\sqrt{g(a_H-p_H^*)p_L^*G(a_L-p_L^*)}=G(a_L-p_L^*)\sqrt{\frac{g(a_H-p_H^*)}{g(a_L-p_L^*)}}\Rightarrow p^*|_{RYP1}=a-G^{-1}\left(\frac{q}{x}\right)=a_H-G^{-1}\left(\frac{q_H}{x}\right)>a_H-G^{-1}\left(G(a_L-p_L^*)\sqrt{\frac{g(a_H-p_H^*)}{g(a_L-p_L^*)}}\right)=p_H^*|_{CPP1}$. For $x\in\Omega_3$, $p^*|_{RYP1}=a-G^{-1}\left(\frac{q}{x}\right)\geq a_H-G^{-1}\left(\frac{q_H+q_L}{x}\right)=p_H^*|_{CPP1}$. Thus, $p^*|_{RYP1}\geq p_H^*|_{CPP1}$ for any realization of market size x and yield y.

Proof of Theorem EC.2 The theorem statement follows by recognizing that the CPP1 model is a relaxation of RYP1 model by allowing a_L to take on positive values.

Proof of Theorem EC.3 We first characterize the revenue function $r(\infty)=\lim_{q\to\infty}r(q)=p\sum_{i=1}^2 x_iG_i(a_i-p)$. Partition p into two regions: $\Gamma_0=\{p:0\leq p\leq a_2\}$ and $\Gamma_1=\{p:a_2<p\leq a_1\}$. We next prove $r(\infty)$ is unimodal in p in Γ_0 and Γ_1.

For $p\in\Gamma_0$, $r(\infty,p)=p\sum_{i=1}^2 x_iG_i(a_i-p)$, which is unimodal in p by assumption (T2) in Lemma EC.1.

For $p\in\Gamma_1$, $r(\infty,p)=px_1G_1(a_1-p)$, which is also unimodal in p by assumption (T2) in Lemma EC.1. Note that $r(\infty,p)$ is in general not continuous at the $p=a_2$. Because $\frac{\partial r(\infty,p)}{\partial p}|_{p\downarrow a_2}=x_1G_1(a_1-a_2)-a_2x_1g_1(a_1-$

a_2) and $\frac{\partial r(\infty,p)}{\partial p}|_{p\uparrow a_2} = x_1G_1(a_1-a_2) - a_2(x_1g_1(a_1-a_2)+x_2g_2(0))$, we have $\frac{\partial r(\infty,p)}{\partial p}|_{p\downarrow a_2} \geq \frac{\partial r(\infty,p)}{\partial p}|_{p\uparrow a_2}$. If $\frac{\partial r(\infty,p)}{\partial p}|_{p\downarrow a_2} \leq 0$, then $p^* \in \Gamma_0$ and is given by $p^* = \frac{\sum_{i=1}^2 x_iG_i(a_i-p^*)}{\sum_{i=1}^2 x_ig_i(a_i-p^*)}$, which is the definition of ν_{12}. If $\frac{\partial r(\infty,p)}{\partial p}|_{p\uparrow a_2} \geq 0$, then $p^* \in \Gamma_1$ and is given by $p^* = \frac{G_1(a_1-p^*)}{g_1(a_1-p^*)}$, which is the definition of ν_1. Finally, if $\frac{\partial r(\infty,p)}{\partial p}|_{p\downarrow a_2} > 0$ and $\frac{\partial r(\infty,p)}{\partial p}|_{p\uparrow a_2} < 0$, then $r(\infty,p)$ is separately concave in Γ_0 and Γ_1. In this case, the optimal p^* is given by $\arg\max r(\infty,p), p \in \{p_m, p_s\}$.

For a finite realized q, p^* is lower bounded by $\bar{p}$, since $r(q,\bar{p}-\epsilon) < r(q,\bar{p}), \forall\ \epsilon > 0$. Theorem statements then follow directly.

Proof of Corollary EC.4 (a) By Theorem EC.3, $\nu_j > \nu_{\bar{j}} \Rightarrow p_2^* > p_1^*$ for any realization of x_i and q. (b) $x > \frac{q}{a - G^{-1}\left(\frac{q}{x}\right)} g\left(G^{-1}\left(\frac{q}{x}\right)\right) \Rightarrow \bar{p} > p_1^* \Rightarrow p_2^* > p_1^*$. (c) Follows from (b). Otherwise, $p_2^* = \nu_{12} \Rightarrow \min\{\bar{p}, \nu_j\} \leq p_2^* < \nu_{\bar{j}} \Rightarrow p_2^* \leq p_1^*$. The case of $a_1 \leq a_2$ can be analogously proved.

Proof of Theorem EC.4 The theorem statement follows by recognizing that the RYP2 model is a relaxation of RYP1 model by allowing a_2 and x_2 to take on positive values.

Proof of Theorem EC.5 This theorem can be analogously proved as Theorem 4.

Proof of Corollary 2, EC.3, and EC.1 Follow directly from Theorems 2, EC.3, and EC.1 respectively by setting $G_i(\cdot)$ to $U(0,1)$ and (without loss of generality) scaling the valuations to between 0 and 1.

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