MANAGEMENT SCIENCE
DOI 10.1287/mnsc.1070.0828ec
pp. ec1–ec9

e-companion
ONLY AVAILABLE IN ELECTRONIC FORM

Electronic Companion—"A Theoretical Framework for Managing the New Product Development Portfolio: When and How to Use Strategic Buckets" by Raul O. Chao and Stylianos Kavadias, *Management Science*, DOI 10.1287/mnsc.1070.0828.

Technical Appendix

In EC.1 we provide proofs for Propositions 1, 2, and 3. In EC.2 and EC.3 we provide extensions of the analytic model based on a search for the best possible performance and a classic two-armed bandit. In EC.4 we provide details of the experimental design for the simulation and in EC.5 we provide an analysis of cost that is relevant to the evolutionary analysis presented in the manuscript. We restate equations from the manuscript when necessary to ease exposition.

EC.1. Proofs

Recall that the expected performance for an m period commitment (considered at $t=0$) to an NPD initiative of type d was derived as:

$$J_0^d = \left[-c(d) + rp(d)\hat{V}(d)\right] \frac{1-r^m[1-p(d)]^m}{1-r[1-p(d)]} \tag{EC.1}$$

Proposition 1. *Behavior of NPD Program Return Curves.* J_0^d is increasing and concave in m. Furthermore, for $d_1 < d_2$, $J_0^{d_1} > J_0^{d_2}$ for $m=1$ provided that $p(d_1)\hat{V}(d_1) > p(d_2)\hat{V}(d_2)$ and $c(d_1) < c(d_2)$. Additionally, there exist threshold values $\bar{p}$ and $\underline{p}$ such that $p(d_1) > \bar{p} > \underline{p} > p(d_2) \Rightarrow J_0^{d_1} < J_0^{d_2}$ as $m \to \infty$.

Proof of Proposition 1. We assume that J_0^d is continuous in m and $r=1$ to ease exposition. To see that J_0^d is increasing and concave in m note that the only term in Equation EC.1 that is a function of m is $a(m) = 1-[1-p(k)]^m$, so it suffices to analyze this term. Differentiating with respect to m gives $da/dm = -[1-p(k)]^m ln[1-p(k)] > 0$ because $p(k) \in (0,1)$. Similarly, $d^2a/dm^2 = -\big(ln[1-p(k)]\big)^2[1-p(k)]^m < 0$. Since $da/dm > 0$ and $d^2a/dm^2 < 0$, $a(m)$ is increasing and concave in m and so is J_0^d.

$J_0^{d_1} > J_0^{d_2}$ for $m=1$ follows directly by letting $m=1$ in Equation 5. To see that $J_0^{d_1} < J_0^{d_2}$ as $m \to \infty$ first note that $m \to \infty$ implies that $[1-p(k)]^m \to 0$. From Equation 5 we are left to show that $\big[-c(d_1)+p(d_1)\hat{V}(d_1)\big]\frac{1}{p(d_1)} < \big[-c(d_2)+p(d_2)\hat{V}(d_2)\big]\frac{1}{p(d_2)}$. The threshold values $\bar{p}$ and $\underline{p}$ imply that for sufficiently high $p(d_1)$ and sufficiently low $p(d_2)$ this inequality holds. The conditions on

$p(d_1)$ and $p(d_2)$ make sense because d_1 represents incremental innovation and d_2 represents radical innovation. QED.

Having described the structure of the NPD program return curves, we now turn our attention towards a comparative statics analysis for the crossing time. We define the function $f(m)$ as the difference between $J_0^{d_1}$ and $J_0^{d_2}$:

$$f(m) = \Big[-c(d_1) + rp(d_1)\hat{V}(d_1)\Big]\frac{1-r^m[1-p(d_1)]^m}{1-r[1-p(d_1)]} - \Big[-c(d_2) + rp(d_2)\hat{V}(d_2)\Big]\frac{1-r^m[1-p(d_2)]^m}{1-r[1-p(d_2)]} \tag{EC.2}$$

We note the following properties without providing a formal proof. Provided a crossing time exists, at $\bar{m}$ we have $J_0^{d_1} = J_0^{d_2}$. For $m < \bar{m}$ we have $J_0^{d_1} > J_0^{d_2} \Rightarrow f(m) > 0$ and for $m > \bar{m}$ we have $J_0^{d_1} < J_0^{d_2} \Rightarrow f(m) < 0$. Proposition 2 is a comparative statics analysis of $\bar{m}$ in order to understand the factors that make incremental or radical innovation more favorable.

Proposition 2. *Comparative Statics Analysis for $\bar{m}$.* The threshold time, $\bar{m}$, is higher when: (i) $\hat{V}(d_1)$ is higher, (ii) $p(d_1)$ is higher (iii) $c(d_1)$ is lower, (iv) $\hat{V}(d_2)$ is lower, (v) $p(d_2)$ is lower, (vi) $c(d_2)$ is higher.

Proof of Proposition 2. First, note that if an $\bar{m}$ exists it is unique since the individual NPD program return curves are increasing and concave in m, $J_0^{d_1} > J_0^{d_2}$ for $m = 1$, and $J_0^{d_1} < J_0^{d_2}$ as $m \to \infty$. The differential of $\bar{m}$ with respect to any parameter x is $d\bar{m}/dx = -(\partial f/\partial x)/(\partial f/\partial m)$. Furthermore, we know that $\partial f/\partial m < 0$ at $\bar{m}$. For notational convenience let $\hat{V}_i = \hat{V}(d_i)$, $p_i = p(d_i)$, and $c_i = c(d_i)$ for $i = 1, 2$ and assume that $r = 1$.

We provide a proof for each part of Proposition 2 in succession. (i) $\partial f/\partial \hat{V}_1 = 1 - [1 - p(d_1)]^m > 0 \Rightarrow d\bar{m}/d\hat{V}_1 > 0$. (ii) $\partial f/\partial p_1 = (-c_1 + p_1\hat{V}_1)\partial/\partial p_1\{[1-(1-p_1)^m]/p_1\} + [1-(1-p_1)^m]\hat{V}_1/p_1 > 0 \Rightarrow d\bar{m}/dp_1 > 0$. (iii) $\partial f/\partial c_1 = \big(-1 + [1-p(d_1)]^m\big)/p(d_1) < 0 \Rightarrow d\bar{m}/dc_1 < 0$. (iv) $\partial f/\partial \hat{V}_2 = -1 + [1-p(d_2)]^m < 0 \Rightarrow d\bar{m}/d\hat{V}_2 < 0$. (v) $\partial f/\partial p_2 = -(-c_2 + p_2\hat{V}_2)\partial/\partial p_2\{[1-(1-p_2)^m]/p_2\} - [1-(1-p_2)^m]\hat{V}_2/p_2 < 0 \Rightarrow d\bar{m}/dp_2 < 0$. (vi) $\partial f/\partial c_2 = \big(1 - [1-p(d_2)]^m\big)/p(d_2) > 0 \Rightarrow d\bar{m}/dc_2 > 0$. QED.

Recall that the difference between incremental and radical NPD program performance for the renewal process defined by technological and market disruptions is given by

$$\Delta J = \frac{q}{1-qr^t} f(t) + \frac{1-q}{1-qr^t} f(m) \tag{EC.3}$$

Proposition 3. *Technological and Market Disruptions.* For $t < \bar{m} < m$, ΔJ is a increasing function of q. Furthermore, there exists a $\bar{q} \in (0,1)$ such that $q < \bar{q} \Rightarrow \Delta J < 0$ and $q > \bar{q} \Rightarrow \Delta J > 0$.

Proof of Proposition 3. We focus on the case of $t < \bar{m} < m$ because any case in which $m < t$ is trivial. Also, the cases in which $\bar{m} < t < m$ and $t < m < \bar{m}$ result in a straightforward choice between incremental and radical innovation regardless of the disruption probability.

For $t < \bar{m} < m$, $f(t) > 0$ and $f(m) < 0$. From Equation EC.3 we can write $\lim_{q \to 0} \Delta J = f(m) < 0$ and $\lim_{q \to 1} \Delta J = \frac{1}{1-r^t} f(t) > 0$. To prove that there exists a unique threshold probability $\bar{q}$, we are left to show that $\partial \Delta J / \partial q > 0$. After some algebraic manipulation we get $\partial \Delta J / \partial q = \frac{f(t) + f(m)[r^t - 1]}{(1-qr^t)^2} > 0$. The final inequality follows because $f(t) > 0$, $f(m) < 0$, and $r^t < 1$. QED.

EC.2. Maximum Value Extension

In this section we provide an alternative probabilistic structure of search to show that the result in the base analytic model is robust. Consider that the two alternative types of innovation can be viewed as draws from an extreme value distribution (Gumbel distribution). The reason for the specific distributional assumption stems from the need to obtain a closed form expression for the distribution of the maximum value (for a more thorough discussion of extreme value theory and the Gumbel distribution see Dahan and Mendelson 2001). Let the expected performance for an m period commitment (considered at $t = 0$) to an NPD program of type d be denoted by J_0^d. If the $F(\cdot)$ are distributed according to a Gumbel distribution, we can state the following:

$$J_0^d = E\big[\max\{F(\omega_1), \ldots, F(\omega_m)\}\big] = \mu_d + \beta_d \gamma + \beta_d ln(m) \tag{EC.4}$$

where μ_d represents the *location* parameter of the Gumbel distribution, β_d the *scale* parameter, and γ is the *Euler-Mascheroni* constant.

The NPD program achieves a higher value (on expectation) with each period that passes. It is a straightforward exercise to show that J_0^d is increasing and concave in m. Now, contemplate the equivalence between our theoretical framework and the current structure. For $m = 1$ the result is $J_0^d = \mu_d + \beta_d\gamma$. As before, we assume that incremental innovation delivers higher immediate benefit (similar to our assumption that $-c(d_1) + p(d_1)V(d_1) > -c(d_2) + p(d_2)V(d_2)$. This implies that for $d_1 < d_2$ and $m = 1$ we have $J_0^{d_1} > J_0^{d_2}$. Mathematically, this condition translates to $\mu_{d_1} + \beta_{d_1}\gamma > \mu_{d_2} + \beta_{d_2}\gamma$ for $m = 1$. We assume that variance is higher for the radical innovation efforts because of the widespread search. For the Gumbel distribution this implies that $\beta_{d_2} > \beta_{d_1}$. Under these circumstances we can state the following: *there exists a crossing time, $\bar{m}$, such that prior to $\bar{m}$ incremental innovation achieves higher expected performance and after $\bar{m}$ radical innovation achieves higher expected performance.*

When NPD program performance is the maximum possible value achieved over a finite horizon (rather than a search for a target value) the intuition is similar to Proposition 1. Note that an underlying assumption here is the independence of the draws, which is consistent with our belief that managers do not have explicit knowledge regarding how the different performance attributes map to the performance function. Still, managers must engage in innovation efforts to improve product performance.

EC.3. Multi-Armed Bandit Extension

In this section we provide an alternative formulation of the commitment time discussed in the base analytic model. This extension considers a period-by-period decision regarding the type of innovation effort (incremental or radical) to pursue in each period of a finite horizon problem. We will show that the length of the horizon impacts the tradeoff between incremental and radical innovation. Specifically, we will show that for short horizon problems, it does not make sense to pursue radical innovation. Conversely, as the length of the horizon increases, there exists a threshold policy under which radical innovation makes sense. This intuition is similar to the result established in the manuscript.

We build this model using the same definitions established in §3 of the manuscript. Let $\hat{V}_i = \hat{V}(d_i)$, $p_i = p(d_i)$, and $c_i = c(d_i)$ for $i = 1, 2$. For any $d_1 < d_2$, recall the following assumptions: $\hat{V}_1 < \hat{V}_2$, $p_1 >$

p_2, $c_1 < c_2$, and $p_1\hat{V}_1 > p_2\hat{V}_2$. These assumptions are equivalent to saying that incremental innovation has lower potential value, higher probability of success, and lower cost relative to radical innovation. Also, the expected value of incremental innovation is greater than the expected value of radical innovation ($p_1\hat{V}_1 > p_2\hat{V}_2$). Finally, let $\upsilon_1 = -c_1 + p_1\hat{V}_1$ and $\upsilon_2 = -c_2 + p_2\hat{V}_2$ be the single period payoff (net of the cost of innovation) for each type of innovative effort. Based on our assumptions, $\upsilon_1 > \upsilon_2$. Thus, from a standpoint of short term financial metrics, incremental innovation dominates. Consider a firm that attempts to improve product performance over a finite horizon $t = 0, 1, 2, \ldots, T$. We can write the firm's decision problem in any period t as:

$$J_t = max\{\upsilon_1 + (1-p_1)J_{t+1}, \upsilon_2 + (1-p_2)J_{t+1}\} \tag{EC.5}$$

with boundary condition $J_T = 0$. We define a policy as a choice of the type of innovative effort in each period: $\pi = \{k_0^*, k_1^*, k_2^*, \ldots, k_{T-1}^*\}$ where $k_t^* \in \{I, R\}$ is the optimal choice between incremental (I) and radical (R) innovation at each decision epoch t.

In the final decision epoch we have $J_{T-1} = max\{\upsilon_1, \upsilon_2\}$ and the optimal choice is $k_{T-1}^* = I$. Working backwards, when two periods remain in the decision horizon we have $J_{T-2} = max\{\upsilon_1 + (1-p_1)\upsilon_1, \upsilon_2 + (1-p_2)\upsilon_2\}$ and the optimal choice is

$$k_{T-2}^* = \begin{cases} I \text{ if } \frac{\upsilon_1-\upsilon_2}{\upsilon_1} > p_1 - p_2 \\ R \text{ otherwise} \end{cases} \tag{EC.6}$$

When three periods remain in the decision horizon we have $J_{T-3} = max\{\upsilon_1 + (1-p_1)J_{T-2}^*, \upsilon_2 + (1-p_2)J_{T-2}^*\}$. Using the information for k_{T-2}^* and k_{T-1}^* the optimal choice at $t = T-3$ is

$$k_{T-3}^* = \begin{cases} I \text{ if } \frac{\upsilon_1-\upsilon_2}{J_{T-2}^*} > p_1 - p_2 \\ R \text{ otherwise} \end{cases} \tag{EC.7}$$

Of the potential sub-policies for the final three decision epochs, we will show that $\{I, R, I\}$ is not a feasible policy. The intuition behind our analysis is that there exists one and only one switch between radical and incremental innovation (once incremental innovation is chosen, it is chosen for the remainder of the horizon). The following condition is necessary for $\{I, R, I\}$ to be a feasible policy:

$$\frac{\upsilon_1-\upsilon_2}{\upsilon_2+(1-p_2)\upsilon_1} > p_1-p_2 > \frac{\upsilon_1-\upsilon_2}{\upsilon_1} \tag{EC.8}$$

The left inequality establishes the optimal choice of incremental innovation in period $T-3$ and the right inequality establishes the optimality condition for $\{k^*_{T-2}, k^*_{T-1}\} = \{R, I\}$. Based on this we can write:

$$\begin{aligned}\upsilon_1-\upsilon_2 &> (p_1-p_2)[\upsilon_2+(1-p_2)\upsilon_1]\\ &> (p_1-p_2)[\upsilon_1+(1-p_1)\upsilon_1]\\ &= \upsilon_1(p_1-p_2)[1+(1-p_1)]\\ &> (\upsilon_1-\upsilon_2)[1+(1-p_1)] \end{aligned}\tag{EC.9}$$

The final inequality in Equation EC.9 is a contradiction because $[1+(1-p_1)] > 1$. Thus, the sub-policy $\{k^*_{T-3}, k^*_{T-2}, k^*_{T-1}\} = \{I, R, I\}$ is not feasible. Therefore, the only sub-policies that are feasible for the final three decision epochs are $\{I, I, I\}$, $\{R, I, I\}$, and $\{R, R, I\}$. This logic can be repeated for each preceding decision epoch. We state the conditions for a threshold policy in the following Claim.

Claim. There exists a period m after which incremental innovation is always chosen. If $1+(1-p_1)+\ldots+(1-p_1)^{m-2} > (\upsilon_1-\upsilon_2)/(p_1-p_2)$, radical innovation is chosen in every period until period m.

Proof of Claim The proof is by induction on m. Assume that $k^*_{T-m} = I$ if $(\upsilon_1-\upsilon_2) > [1+(1-p_1)+\ldots+(1-p_1)^{m-2}](p_1-p_2)$ and $k^*_{T-m+1} = I$. We will show that $k^*_{T-m-1} = I$ if $(\upsilon_1-\upsilon_2) > [1+(1-p_1)+\ldots+(1-p_1)^{m-1}](p_1-p_2)$. The value function for $t = T-m-1$ is $J_{T-m-1} = max\{\upsilon_1+(1-p_1)J^*_{T-m}, \upsilon_2+(1-p-2)J^*_{T-m}\}$. This results in

$$k^*_{T-m-1} = \begin{cases} I \text{ if } \frac{\upsilon_1-\upsilon_2}{J^*_{T-m}} > p_1-p_2 \\ R \text{ otherwise}\end{cases} \tag{EC.10}$$

The proof that $k^*_{T-m-1} = I$ if $k^*_{T-m} = I$ and $1 + (1-p_1) + \ldots + (1-p_1)^{m-1} < (\upsilon_1 - \upsilon_2)/(p_1 - p_2)$ is straightforward. If instead $k^*_{T-m} = R$ then we have $\upsilon_1 + (1-p_1)J^*_{T-m} > \upsilon_2 + (1-p_2)J^*_{T-m}$. However, an alternate policy would imply $J^*_{T-m} > \upsilon_1[1 + (1-p_1) + \ldots + (1-p_1)^{m-2}]$ and we obtain a contradiction as in Equation EC.9. QED.

EC.4. Simulation Experimental Design

Table EC.1 shows the parameters and value used for the simulation experiments in §5. The choice of $N = 15$ and $S = 2$ define the size of the technology/market landscape. Because our interest lies in understanding complexity in terms of performance interactions rather than size, we have limited these variables to static values. This assumption is in line with other work in complex performance landscapes and it does not alter the qualitative results of our study. The contribution of each attribute to overall product performance is $U(0,1)$ to account for the fact that managers cannot infer the performance function. Research on complex performance landscapes has shown that the landscape structure is robust with respect to the choice of distribution for the f_j. In particular, the landscape structure is similar if $f_j \sim N(\mu, \sigma^2)$ or $f_j \sim exp(\lambda)$. Our full experimental design varies K, d, and s as shown in Table EC.1. This results in 15 x 15 x 27 = 6075 experiments. In order to conserve space in the manuscript we cannot show the results for every combination of K, d, and s. Instead, we show results that highlight robust phenomena. A complete data set is available from the authors by request.

Because our analysis is focused on dynamic phenomena, the standard error for NPD program performance is a function of time. Thus, we provide bounds for the standard error. In our case, $\hat{\sigma}^2 \in (0, 0.004)$ and $n = 500$. This gives a standard error of $SE \in (0, 0.00283)$. In the worst case situation, this results in a 99.0% confidence interval for NPD program performance. In many cases, the confidence interval approaches 99.9% and theoretically, the confidence inteval approaches 100% for a number of experiments.

Parameter and Value	Description
$N = 15$	Number of attributes per product.
$S = 2$	Number of states per attribute
$f_j \sim U(0,1)$	Contribution of each attribute to overall product performance
$K \in \{0, 1, 2, \ldots, N-1\}$	Number of interactions between product attributes
$d \in \{1, 2, \ldots, N\}$	Search distance for NPD programs.
$s \in \{1.000, 0.9990, 0.9980, \ldots, 0.9750\}$	Environmental stability (with probability $1-s$ a disruption will occur in each period).

Table EC.1 Parameters and values used for simulation experiments.

EC.5. Impact of Cost on Simulation

Note that our simulation analysis results in NPD program performance curves that are increasing and concave in time and we do not optimize with respect to commitment time. Assuming that the per-period cost of innovation is constant, such an optimization would result in an optimum commitment time for each type of innovative effort (increasing and concave performance and linear cumulative cost). If the crossing time between incremental and radical NPD program performance occurs before the minimum optimal commitment time, our insights are valid. If the crossing time occurs after the optimal commitment times the problem becomes very complex as a function of the costs. Figure EC.1 depicts NPD program performance net of per period costs in an environment where $K = 4$ (the dashed line begins at the optimal commitment time). Despite the fact that $K = 4$ in both graphs, the difference in costs results in a different crossing time. Still, we feel that our insights regarding performance (revenue) are robust and contribute to the understanding of how value is created by NPD programs.

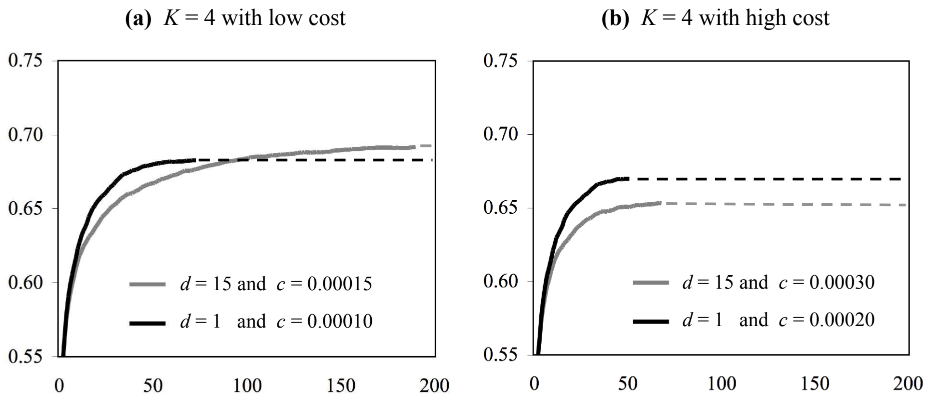

Figure EC.1 NPD program performance net of per period costs.