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Electronic Companion—"Buyer-Initiated vs. Seller-Initiated Information Revelation" by Pradeep Bhardwaj, Yuxin Chen, and David Godes, *Management Science*, DOI 10.1287/mnsc.1070.0840.

Proposition 1 (with Detailed Conditions): *If* $c \leq Min\left\{\frac{1}{3}, \frac{1-\beta\phi}{2-\beta\phi}, \frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}\right\}$ *or* $c \leq Min\left\{\frac{1}{3}, \frac{1-\beta\phi}{2-\beta\phi}, \frac{1-\phi(\beta\phi+1-\beta)}{1+\beta(1-\phi)}\right\}$ *and* $c > \frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$, *there exists an equilibrium in which* $P^* = 1$, $A^* = 11$ *and* $F^* = B$. *In this equilibrium the* $1-\beta$ *customers select a* $q^* \in [\frac{(1-\beta)(1-c)-(1-2c)+\beta(1-\phi)(1-c)}{(1-\beta)(1-c)}, \frac{1-2c-\beta\phi(1-c)}{(1-\beta)(1-c)}]$ *and the* β *customers inspect their preferred feature. Off-path customer actions are such that for* $P' \leq \phi$, $q' = 1$. *For* $P' \in (\phi, 1]$ *if* $P' \in \Delta(\beta, \phi, c)$ *and* $F' = B$, *then* q' *is chosen from* $\chi(\beta, \phi, c, P')$. *If* $P' \notin \Delta(\beta, \phi, c)$, *then the* $1-\beta$ *customers choose* $q' = 1$. *Pre-inspection off-path beliefs are such that when* $E[\Pi'|q, A' = 11] > E[\Pi'|q, A' = 10]$, $\mu_{11} = 1$ *and otherwise* $\mu_{10} = 1$. *Post-inspection off-path beliefs are such that the probability of the existence of the non-inspected feature is unaffected by the realization of the inspection of the other feature.*

Proof of Proposition 1: We prove this by showing existence directly. Let $A^* = 11$ and $P^* = 1$. The firm's equilibrium profits are $\pi^* = 1 - 2c$ since everyone buys in equilibrium. Under $F^* = B$, the customer chooses $\widehat{i}$. The $\beta\phi$ customer chooses $\widehat{i} = 1$ and the $\beta(1-\phi)$ customer chooses $\widehat{i} = 2$. The $1-\beta$ customer is uncertain about his preferences. We consider first hidden deviation on product features. Let q^* represent the (possibly degenerate) distribution over the uncertain customer's choice of which feature to view. In equilibrium, q^* satisfies the following conditions:

$$1 - 2c \geq [\beta\phi + (1-\beta) q^*](1-c) \quad (1)$$

$$1 - 2c \geq [\beta(1-\phi) + (1-\beta)(1-q^*)](1-c) \quad (2)$$

Equation (1) ensures that q^* is chosen such that the firm isn't strictly better off dropping feature 2. Equation (2) implements the analogous condition with respect to feature 1. Note that implicit in these conditions is that the customer will not buy upon seeing $a_{\widehat{i}} = 0$ given that $P = 1$. This holds for any $\mu_{11} < 1$ following the observation of either $a_1 = 0$ or $a_2 = 0$.

To satisfy both (1) and (2), there must exist a $q^* \in [0, 1]$ such that

$$\frac{(1-\beta)(1-c) - (1-2c) + \beta(1-\phi)(1-c)}{(1-\beta)(1-c)} \leq q^* \leq \frac{1-2c-\beta\phi(1-c)}{(1-\beta)(1-c)} \quad (3)$$

which exists as long as (a) the RHS exceeds the LHS: $1 - 2c - \beta\phi(1-c) \geq (1-\beta)(1-c) - (1-2c) + \beta(1-\phi)(1-c)$, (b) the LHS is less than 1: $(1-\beta)(1-c) - (1-2c) + \beta(1-\phi)(1-c) \leq$

$(1-\beta)(1-c)$ and (c) the RHS is greater than 0: $1-2c-\beta\phi(1-c)\geq 0$. Clearly, (a) holds for all $c\leq\frac{1}{3}$, (b) always holds when $c\leq\frac{1}{3}$ since $\beta<1$ and $1-\phi<\frac{1}{2}$. Finally, (c) holds if $c\leq\frac{1-\beta\phi}{2-\beta\phi}$.

Cutting price alone can never be profitable. A deviation to $P'=\phi$ would be accompanied by $A'=11$ iff $c\leq\frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$. In this case, the firm is strictly worse off. If $c>\frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$, the firm deviates to $A'=10$ when $P'=\phi$. This is unprofitable iff $c\leq\frac{1-\phi(\beta\phi+1-\beta)}{1+\beta(1-\phi)}$ which is guaranteed by the condition stated.

Consider a deviation to $F'=S$ and $\widehat{i}=1$. In this case, $\mu_{10}=1$ implying that $\beta\phi$ of the customers will pay 1, $\beta(1-\phi)$ customers will pay 0 and $1-\beta$ customers will pay ϕ. If $P'=1$, then the deviation is strictly profitable if $\beta\phi(1-c)>1-2c$ which is not possible for $c\leq\frac{1-\beta\phi}{2-\beta\phi}$. If $P'=\phi$, it is strictly profitable if $[\beta\phi+(1-\beta)](\phi-c)>1-2c$ which is not possible for $c\leq\frac{1-\phi(\beta\phi+1-\beta)}{1+\beta(1-\phi)}$.

It is useful to note the role played by the off-path beliefs and actions as specified in Section 2.1. Recall that, conditional on an observed deviation, the firm chooses the product features optimally and that the customer's beliefs are consistent with this. If, alternatively, we considered a context in which the firm made suboptimal decisions such as a deviation to $P'=\phi$ and $A'=10$ when $c\leq\frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$, then the firm would only be worse off. Thus, our assumptions about the firm's off-path actions have no material impact on the equilibrium presented here. Off-path beliefs play a stronger role, however. Most crucial is the implication of our off-path belief structure that $\mu_{10}=1$ when (a) the customer sees $P'=\phi$ when $c>\frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$ or (b) the customer sees $F'=S$. First, recall that these represent the rational inferences of the customer in that they are associated with what is "best" for the firm, conditional on the observables. If, alternatively, we allowed for $\mu_{11}>0$ in case (a), our equilibrium would not be affected since the customer's likelihood of purchase would not change (if she sees $a_1=1$ she buys, otherwise she doesn't) and the price is the same. In the latter case, if the customer's beliefs were such that $\mu_{11}\neq 0$ when $F'=S$, then this would potentially impact the equilibrium. This can be seen simply by considering the limiting case in which $\mu_{11}=1$ when $F'=S$. In this case, the firm could choose $P'=P^*=1$, $\widehat{i}=1$ and $A'=10$ which would clearly be a profitable deviation for low values of β and high values of ϕ. We don't consider this for the reasons laid out in Section 2.1: it seems unreasonable to assume that one player (a customer) would place positive probability on another player (the firm) taking an action in a one-shot game that was strictly detrimental to its profit (producing a costly feature that doesn't affect the likelihood of a sale). It is relatively straightforward to show that there exists a $\underline{\mu}_{11}$ such that for $0 < \mu_{11} < \underline{\mu}_{11}$ the equilibrium stated in the Proposition will persist.

As noted in the Proposition, we assume that off-path beliefs about the existence of a non-inspected feature are not impacted by the outcome of the inspection of the other feature. Note that this represents a conservative assumption in that relaxing it would simply make it easier to establish the existence of the high-quality equilibrium. To see why, consider for example a case in which the customer inspects feature 1 and finds $a_1 = 0$. Currently, we assume that the off-path belief is given by $\Pr[a_2 = 1] = 1$ which maximizes the expected value associated with a purchase of the product, conditional on the deviation. For any $\Pr[a_2 = 1] < 1$, this expected utility would be lower, decreasing the attractiveness of a deviation.

Finally, we consider the role of the off-path customer shopping policy stated in the equilibrium. Were the customer to, instead, continue to randomize her feature inspection upon viewing $P' = \phi$, the firm would have more profitable deviation opportunities since it would decrease the likelihood of being "caught." Specifically, conditional on observing an off-path price $P' = \phi$, were the customer to inspect a_1 with probability $q' < 1$ and a_2 with probability $1-q'$, the firm would have a profitable deviation to $A' = 00$ if $(1-\beta)(1-q')\phi > 1-2c$ since the β type customers would never buy. In the extreme case, where we let $q' = 0$, this becomes $c > \frac{1-(1-\beta)\phi}{2}$ meaning we need $c \leq \frac{1-(1-\beta)\phi}{2}$ to ensure such a deviation does not take place. To see that such a constraint cannot hold in our equilibrium, recall that another condition for this equilibrium to exist is that $c > \frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$. Thus, one necessary condition – were we to allow $q'(F' = S) = 0$ – would be that $\frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)} > \frac{1-(1-\beta)\phi}{2}$ or $\beta\phi(1-\phi)(3-\beta) - (1-\phi) - \beta > 0$. Since the LHS of this expression is decreasing in β, we notice that $\beta\phi(1-\phi)(3-\beta) - (1-\phi) - \beta < -(1-\phi) < 0$ which is a contradiction. Thus, it must be the case that the customer places "enough" inspection probability on a_1 upon viewing $F' = S$. Again, our specification of off-path actions is such that $q' = 1$ since it seems unreasonable to assume that a customer would inspect an feature that is irrelevant to her purchase decision (and, therefore, not inspect one that is relevant).□

Proposition 2 (with Detailed Conditions): *There exist four types of equilibria in which* $A^* = 10$. *(i)* $\Sigma^* = \{10, S, \phi, 1\}$ *when* $\frac{\phi[1-\beta(2-\phi)]}{1-\beta} \geq c \geq \frac{1-\beta\phi}{2-\beta\phi}$; *(ii)* $\Sigma^* = \{10, B, \phi, 1\}$ *when* $\frac{\phi[1-\beta(2-\phi)]}{1-\beta} \geq c \geq \frac{1-\beta\phi}{2-\beta\phi}$; *(iii)* $\Sigma^* = \{10, S, 1, 1\}$ *when* $c \geq Max\left\{\frac{1-\beta\phi}{2-\beta\phi}, \frac{\phi[1-\beta(2-\phi)]}{1-\beta}\right\}$; *and (iv)* $\Sigma^* = \{10, B, 1, 1\}$ *when* $c \geq Max\left\{\frac{1-\phi}{2-\phi}, \frac{\phi[1-\beta(2-\phi)]}{1-\beta}\right\}$. *In each of these equilibria, all customers select a* $q^* = 1$. *The off-path customer shopping policy for the* $1-\beta$ *customers is such that if* $P' \leq \phi$, *then* $q\prime = 1$. *For* $P' \in (\phi, 1]$ *if* $P' \in \Delta(\beta, \phi, c)$ *and* $F' = B$, *then* q' *is chosen from* $\chi(\beta, \phi, c, P')$. *If* $P' \notin \Delta(\beta, \phi, c)$, *then the* $1-\beta$ *customers choose* $q' = 1$. *The* β *customers always inspect their preferred feature. Pre-inspection off-path beliefs are such that when* $E[\Pi'|A' = 11] > E[\Pi'|A' = 10]$, $\mu_{11} = 1$ *and otherwise* $\mu_{10} = 1$. *Post-inspection off-path beliefs are such that the probability of the existence of the non-inspected feature is unaffected by the realization of the inspection of the other feature.*

Proof of Proposition 2: Case (i), $\Sigma^* = \{10, S, \phi, 1\}$, demand is $\beta\phi + (1-\beta)$ and equilibrium profit is $[\beta\phi + (1-\beta)](\phi - c)$. As long as $F = S$, our off-path belief principle precludes any weight being placed on the state in which $A' = 11$. A deviation to $P' = 1$ with $\widehat{i} = 1$ yields demand of $\beta\phi$ and profit of $\beta\phi(1-c)$ and is thus unprofitable if $c \leq \frac{\phi[1-\beta(2-\phi)]}{1-\beta}$. A deviation to $\{00, S, \phi, 1\}$ or $\{01, S, \phi, 1\}$ is never profitable since the customer will never buy. Deviations to $\{00, S, \phi, 0\}$ and $\{00, S, 1-\phi, 1\}$ yield no sales.

To check deviations to $F' = B$, note that when $P' = \phi$, we have $q' = 1$. If $c \geq \frac{\beta\phi(1-\beta)}{1+\beta(1-\phi)}$, $\mu_{10} = 1$ and $\Pi' = [\beta\phi + 1 - \beta](\phi - c) = \Pi^*$ so is never strictly profitable. When $c < \frac{\beta\phi(1-\beta)}{1+\beta(1-\phi)}$, $\mu_{11} = 1$ and $\Pi' = \phi - 2c$ which is always profitable in this region of the parameter space. Thus, we require that $c \geq \frac{\beta\phi(1-\beta)}{1+\beta(1-\phi)}$ which always holds when $c \geq \frac{1-\beta\phi}{2-\beta\phi}$. When $P' = 1$, $\mu_{11} = 1$ when $1 \in \Delta$ which occurs when $c \leq \frac{1-\beta\phi}{2-\beta\phi}$. In this region of the parameter space, $\Pi' = 1 - 2c$, and the deviation is profitable if $c < \frac{1-\phi[\beta\phi+1-\beta]}{2-[\beta\phi+1-\beta]}$ which always holds in the region of the parameter space in which $c \leq \frac{1-\beta\phi}{2-\beta\phi}$ so we require that $c > \frac{1-\beta\phi}{2-\beta\phi}$. When $c > \frac{1-\beta\phi}{2-\beta\phi}$, we have $\mu_{10} = 1$ when $P' = 1$ and $\Pi' = \beta\phi(1-c)$ which is unprofitable if $c \leq \frac{\phi[1+\beta(\phi-2)]}{1-\beta}$.

Case (ii) $\Sigma^* = \{10, B, \phi, 1\}$ and $\Pi^* = [\beta\phi + 1 - \beta](\phi - c)$. We first consider deviations that maintain $F = B$. A deviation to $A' = 11$ triggers no off-path actions/beliefs since there are no observable deviations from equilibrium. This yields $\Pi' = \phi - 2c$ and is unprofitable for $c \geq \frac{\beta\phi(1-\beta)}{1+\beta(1-\phi)}$

which always holds when $c \geq \frac{1-\beta\phi}{2-\beta\phi}$. A deviation to $A' = 00$ yields no sales. Consider now deviations to $P' = 1$. When $c \geq \frac{1-\beta\phi}{2-\beta\phi}$, $\mu_{10} = 1$ and $\widehat{q} = 1$. This yields $\Pi' = \beta\phi(1-c)$ which is unprofitable if $c \leq \frac{\phi[1+\phi\beta-2\beta]}{1-\beta}$. When $c \leq \frac{1-\beta\phi}{2-\beta\phi}$, $\mu_{11} = 1$ yielding $\Pi' = 1-2c$ which is unprofitable if $c \geq \frac{1-\phi[\beta\phi+1-\beta]}{2-[\beta\phi+1-\beta]}$ which is not possible when $c \leq \frac{1-\beta\phi}{2-\beta\phi}$ since $\frac{1-\beta\phi}{2-\beta\phi} \leq \frac{1-\phi[\beta\phi+1-\beta]}{2-[\beta\phi+1-\beta]}$. Thus, we require that $c \geq \frac{1-\beta\phi}{2-\beta\phi}$. Deviation to $F' = S$ implies that $\mu_{10} = 1$ and $\widehat{q}(F' = S) = 1$. If $P' = 1$ the deviation is unprofitable if $c \leq \frac{\phi[1+\phi\beta-2\beta]}{1-\beta}$. If $P' = \phi$, off-path profits are identical to those in the equilibrium.

In Case (iii), $\Sigma^* = \{10, S, 1, 1\}$ and profit is $\Pi^* = \beta\phi(1-c)$. Again, when $F = S$, off-path beliefs preclude any weight being placed on the state in which $A' = 11$. So, $\mu_{10} = 1$ and $\widehat{q}(F' = S) = 1$. Deviation to $P' = \phi$ is not profitable if $c \geq \frac{\phi[1+\beta(\phi-2)]}{1-\beta}$. Deviations to $F' = B$ at $P' = 1$ trigger q' when $c \leq \frac{1-\beta\phi}{2-\beta\phi}$ since $1 \in \Delta$. This deviation is always profitable in this region so we require that $c \geq \frac{1-\beta\phi}{2-\beta\phi}$. In this region, $\mu_{10} = 1$ and $\Pi' = \beta\phi(1-c)$ and so are never strictly profitable. A deviation to $P' = \phi$ triggers $q' = 1$. Here, $\mu_{11} = 1$ iff $\phi - 2c \geq (\beta\phi + 1 - \beta)(\phi - c)$ which requires that $\phi - 2c \geq \beta\phi(\phi - c)$ which requires that $c \leq \phi\frac{1-\beta\phi}{2-\beta\phi}$ contradicting the condition that $c \geq \frac{1-\beta\phi}{2-\beta\phi}$. So, we have $\mu_{10} = 1$. This deviation is not strictly profitable if $\beta\phi(1-c) \geq (\beta\phi + 1 - \beta)(\phi - c)$ or $c \geq \frac{\phi[1-\beta(2-\phi)]}{1-\beta}$.

Case (iv), $\Sigma^* = \{10, B, 1, 1\}$ and $\Pi^* = \beta\phi(1-c)$. A deviation to $A' = 11$ triggers no off-path actions/beliefs since there are no observable deviations from equilibrium. This yields $\Pi' = \beta(1-2c)$ and is unprofitable if $c \geq \frac{1-\phi}{2-\phi}$. A deviation to $P' = \phi$ triggers $q' = 1$. Given this, the firm prefers $A' = 10$ to $A' = 11$ if $c \geq \frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$ which always holds given the condition in the equilibrium since $\frac{1-\phi}{2-\phi} \geq \frac{\beta\phi(1-\phi)}{1+\beta(1-\phi)}$. So, $\Pi' = [\beta\phi + 1 - \beta](\phi - c)$ which is unprofitable if $c \geq \frac{\phi[1-\beta(2-\phi)]}{1-\beta}$. Deviations to $F' = S$ always have $\mu_{10} = 1$ and $\widehat{q}(F' = S) = 1$. If $P' = 1$ profits are the same as in the equilibrium. If $P' = \phi$ it's again unprofitable if $c \geq \frac{\phi[1-\beta(2-\phi)]}{1-\beta}$.

As with the high-quality equilibrium in Proposition **1**, were off-path beliefs adjusted to allow for $\mu_{11} > 0$ when $F = S$, there would exist profitable deviations for the firm, particularly at higher levels of μ_{11} since the $\beta(1-\phi)$ customers never buy in equilibrium but would buy were $\mu_{11} \geq P'$. Moreover, were we to allow the customer to randomize her shopping policy even when $P' = \phi$ then opportunities would exist, as in the high-quality equilibrium, for the firm to cut price to $\phi - \varepsilon$

and produce a product $A' = 00$. If q' is low enough (the customer inspects a_2 with high-enough frequency), then when the customer sees $a_2 = 0$, she still assigns the on-path belief that $a_1 = 1$. Again, it is our belief that both assumptions – that a customer would believe that a firm would produce a feature it could never communicate and that a customer would inspect a feature that is irrelevant to her purchase decision – are unreasonable.□

Proof of Proposition 3: It's clear that neither firm can offer $A = 11$. Thus, we only consider equilibria in which the firms offer $A = 01$ or $A = 10$. Let the subscript $j = 1, 2$ denote the firms. Clearly, any equilibrium in which $A_1 = A_2 = 10$ for both firms must be characterized by $P_1^* = P_2^* = c$ and zero profits. In fact, there is always a profitable deviation to $A_1 = 01, a = a_1$ and $P_1 = 1$. Now, consider the asymmetric equilibrium $A_1 = 10$ and $A_2 = 01$. Profits from the "captive" customers are $\beta\phi (P_1 - c)$ and $\beta (1 - \phi) (P_2 - c)$ for firms 1 and 2, respectively. With respect to the $1-\beta$ customer, their expected surplus from firms 1 and 2 are $\phi - P_1$ and $1 - \phi - P_2$, respectively. Any equal-price equilibrium at positive profit would be eliminated by price cuts.□

Proposition 4 (with Detailed Conditions): *If* $c \leq \frac{2(1-\phi)(\beta\phi+1-\beta)}{4(\beta\phi+1-\beta)-1}$ *and* $\beta\phi \leq \frac{1}{4}$, *then there exists a high-quality pure-strategy equilibrium characterized by both firms choosing* $A^* = 11$, $F^* = B$ *and setting prices* $P^* = 3c$ *and earning profits* $\Pi^* = \frac{c}{2} > 0$. *In equilibrium, the* $1 - \beta$ *customers choose* $q^* = \frac{1-2\beta\phi}{2(1-\beta)}$. *The off-path customer shopping policy for the* $1 - \beta$ *customers is such that if* $P' \leq \phi$, *then* $q\prime = 1$. *For* $P' \in (\phi, 1]$ *if* $P' \in \Delta(\cdot)$ *and* $F' = B$, *then* $q'(P')$ *is chosen from* $\chi(\cdot)$. *If* $P' \notin \Delta(\cdot)$, *then the* $1 - \beta$ *customers choose* $q' = 1$. *The* β *customers always inspect their preferred feature. Pre-inspection off-path beliefs are chosen such that, given the off-path shopping policy and the observed firm actions, the customer assumes that the firm has selected* A' *to maximize* $E[\Pi|P']$. *Post-inspection off-path beliefs are such that the probability of the existence of the non-inspected feature is unaffected by the realization of the inspection.*

Proof of Proposition 4: Note first that equilibrium profits are always non-negative. Now, for each firm to not deviate from $A^* = 11$, we need a customer inspection policy $q \in [0, 1]$ that will ensure that the firm will not find it profitable to deviate to a low-quality product. To ensure this, q must satisfy the following conditions at the equilibrium price P:

$$\frac{1}{2}\left(\beta\phi+q\left(1-\beta\right)\right)\left(P-c\right) \leq \frac{1}{2}\left(P-2c\right) \tag{4}$$

$$\frac{1}{2}\left(\beta\left(1-\phi\right)+\left(1-q\right)\left(1-\beta\right)\right)\left(P-c\right) \leq \frac{1}{2}\left(P-2c\right) \tag{5}$$

which imply the following joint condition for the existence of a proper q;

$$\frac{-P\beta\phi+c\left(1+\beta\phi\right)}{\left(1-\beta\right)\left(P-c\right)} \leq q \leq \frac{P\left(1-\beta\phi\right)-c\left(2-\beta\phi\right)}{\left(1-\beta\right)\left(P-c\right)} \tag{6}$$

This condition ensures that the customer checks feature 2 often enough (Equation (4)) to prevent the firm from deviating to $A'=10$ and that she checks feature 1 often enough (Equation (5)) to prevent the firm from deviating to $A'=01$. To ensure that such a $q\in[0,1]$ exists that satisfies the condition in (10), we need P to satisfy three conditions:

$$P \geq 3c \tag{7}$$

$$P \geq c\frac{(2-\beta\phi)}{(1-\beta\phi)} \tag{8}$$

$$P \geq c\frac{2-\beta\left(1-\phi\right)}{1-\beta\left(1-\phi\right)} \tag{9}$$

Condition (7) ensures that $\frac{-P\beta\phi+c(1+\beta\phi)}{(1-\beta)(P-c)} \leq q \leq \frac{P(1-\beta\phi)-c(2-\beta\phi)}{(1-\beta)(P-c)}$. Condition (8) ensures that $\frac{P(1-\beta\phi)-c(2-\beta\phi)}{(1-\beta)(P-c)} \geq 0$ and condition (9) ensures that $\frac{-P\beta\phi+c(1+\beta\phi)}{(1-\beta)(P-c)} \leq 1$. Note that, since $\frac{(2-\beta\phi)}{(1-\beta\phi)}$ is increasing in $\beta\phi$, (8) is implied by (7) for $\beta\phi \leq \frac{1}{4}$. Again, since $\frac{2-x}{1-x}$ is increasing in x, we know that (9) is implied by (8) since $\phi > \frac{1}{2}$. So, the only candidate for a binding constraint in a high-quality equilibrium is (7). Note that, if this does bind, profits are strictly positive.

In the proposed equilibrium, $P^*=3c$ and $\Pi^*=\frac{c}{2}$. Note that when $P^*=3c$ the condition (6) reduces to

$$q=\frac{1-2\beta\phi}{2\left(1-\beta\right)} \tag{10}$$

A deviation to $P' > P^*$ yields no sales and is thus strictly worse. A deviation to $P' < 3c$ implies that $\chi=\varnothing$ so $q'=1$ and $\mu_{10}=1$. Given this, such a price deviation has the potential to capture the whole market but only if the surplus offered is higher than the other (on-path) firm:

$$\begin{aligned} 1 - P^* &= 1 - 3c < \phi - P' \\ &\Leftrightarrow P' < 3c - (1 - \phi) \end{aligned} \tag{11}$$

That is, the firm can't simply cut the price by ε and get the market. The customer's inference process introduces a discontinuity into the problem implying that the firm must cut by a first-order amount in order to capture the market. This deviation is profitable only if:

$$\frac{c}{2} < (\beta\phi + 1 - \beta)(3c - (1 - \phi) - c) \tag{12}$$

where we've substituted in $P' = 3c - (1 - \phi)$, the highest price at which the deviating firm would capture the market. If $2c < (1 - \phi)$ – which occurs at low values of c and ϕ – then this never holds and the deviation is always prevented. Assuming $2c > (1 - \phi)$, the deviation is not profitable if

$$c \leq \frac{2(1 - \phi)(\beta\phi + 1 - \beta)}{4(\beta\phi + 1 - \beta) - 1}. \tag{13}$$

Another potential deviation would be an ε price cut which would attract only the $\beta\phi$ customers. This yields $\Pi' = 2\beta\phi c$ and is unprofitable if $2\beta\phi c \leq \frac{c}{2} \Leftrightarrow \beta\phi \leq \frac{1}{4}$. A deviation to $F' = S$ and $i' = 1$ yields $\mu_{10} = 1$ which is superfluous since any price cut would imply such a change in product as well. Thus, the conditions for the equilibrium remain the same: (13) if the price cut is below $3c - (1 - \phi)$ and $\beta\phi \leq \frac{1}{4}$ for an epsilon cut.

As we did in the proofs to both Propositions **??** and **??**, it is useful to note the role played by the off-path actions and beliefs. Our belief structure implies that any price cut to $P' < 3c$ implies that $\chi = \varnothing$: there no longer exists a shopping policy q that ensures the firm will not deviate on product quality. As a result, the firm is assumed to deviate to $A' = 10$ and the customer adjust her shopping policy to reflect this. Were we to, instead, assume that $\chi \neq \varnothing$ when $P < 3c$, then the customer would continue to randomize and a series of ε price cut would erode the profit margin. Similarly, were we to assume that $\mu_{10} < 1$ when $F' = S$ and $i' = 1$, then the firm might have additional opportunities to deviate profitably. In particular, were μ_{11} relatively high in this case, the firm would be able to deviate to $A' = 10$ and $F' = S$ with no change in price. We again point out, however, why such assumptions would be, in our view, unreasonable. These alternative belief structures would be tantamount to the customer believing the firm is producing a costly feature even when there is no way to verify it.

Similarly, the off-path actions that support this equilibrium are such that the customer does not randomize when her belief is such that $\mu_{10} = 1$. Were we to instead assume that she continues to randomize even when $\mu_{10} = 1$ – specifically, when $P' \leq 3c - (1 - \phi)$ – and that she places high enough probability on inspecting a_2 then the firm might be able to deviate to $A' = 00$ profitably. However, we rule this action out on the grounds that a customer shouldn't reasonably be expected to inspect a feature that she doesn't think is there and that, given the price, will not play a role in her decision to buy or not buy the product.□