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Electronic Companion—"Innovation Contests, Open Innovation, and Multiagent Problem Solving" by Christian Terwiesch and Yi Xu, *Management Science*, DOI 10.1287/mnsc.1080.0884.

Electronic Companion – "Innovation Contests, Open Innovation, and Multi-agent Problem Solving" by Christian Terwiesch and Yi Xu

Appendix A: Extended Literature Review

Following a period of off-shoring manufacturing jobs to low wage countries in the 1980s, off-shoring of white-collar jobs, including engineering and development, has been a major trend since the 1990s. There exists recent research (e.g. Joglekar and Rosenthal 2003, Joglekar and Anderson 2005, Sosa *et al.* 2002, Gomes *et al.* 2005, Anderson *et al.* 2006) on this topic and the interested reader is referred to Anderson *et al.* (2006) for an excellent review. Open Innovation in the context of innovation tournaments is different from outsourcing and off-shoring. It is different from off-shoring as participants in the innovation tournament can well be in the same geographic area as the company hosting the tournament. It is different from outsourcing, as it does not *ex ante* specify the participating innovators. For example, outsourcing a software development project creates a 1:1 relationship between the customer and the developer similar to a classical contracting model. In contrast, at the beginning of the innovation tournament, the seeking company does not know which solvers will participate. Green and Stokey (1983) conduct a comparison between labor tournaments and contracts.

We need to turn to auction models to capture heterogeneous solvers. Moldovanu and Sela (2001) study the optimal allocation of awards in deterministic contests. They show that when the contestants' effort cost functions are linear and concave, it is optimal to allocate the entire prize to a single first prize. When the cost functions are convex, it may be optimal to offer multiple prizes. Moldovanu and Sela (2006) study the optimal contest architecture with fixed prizes. They find that when the objective is to maximize expected total effort, the optimal contest architecture is a single round static contest, while it is optimal to split the contestants into two sub-contests and run a final contest between the sub-contest winners, if the objective is to maximize expected highest effort. Che and Gale (2003) develop an auction-based model to study the optimal mechanism for deterministic research contests. Similar to Taylor (1995) and Fullerton and McAfee (1999), they also suggest limiting the pool of solvers, potentially all the way down to two to improve the efficiency of the contest. Snir and Hitt (2003) model a procurement auction for software development services - a firm seeking external help posts a project description and then chooses the coder that offers an attractive price. In these auction models, solvers are heterogenous. Unfortunately, the solver heterogeneity comes at the price of a deterministic performance function. To the best of our knowledge, no model in the Economics literature includes both heterogeneity in solver expertise and a stochastic relationship between effort and performance.

Our paper differs from this prior work along three important dimensions. First, we consider different problem types, a prominent feature identified in the product development literature, which has not been studied in the Economics literature. Specifically, we show that the type of problem has a major impact on the optimal design of the innovation contest. Second, the above mentioned papers focus on competition as the only benefit of the tournament. We show how solver heterogeneity is another important benefit of Open Innovation. Specifically, we show that there exist problems in which the seeker benefits from increasing the pool of potential solvers (i.e. true Open Innovation similar to the case of Innocentive) as opposed to limiting the pool of solvers. Third, we compare the innovation contest with the case of internal innovation and identify which types of innovation we expect to be most appropriate for an innovation contest. This allows us to create interesting insights on which parts of the innovation process are most likely / most attractive to move from in-house innovation to the open innovation process.

Appendix B: Proofs

We will use two key properties of the Gumbel distribution frequently:

Property 1 *If ξ_i are i.i.d. according to the Gumbel distribution with scale parameter μ* and *mean zero, for any given constant a, the random variable $\overline{x} = \max_i \{x_i = a + \xi_i,\ i = 1, 2, ..., n\}$ is also a Gumbel distributed random variable with mean $a + \mu \ln n$.*

Property 2 (Luce and Suppes 1965) *If ξ_i are i.i.d. according to the Gumbel distribution with scale parameter μ, and a_i are constants, then*

$$\Pr\left\{a_i + \xi_i = \max_j \{a_j + \xi_j,\ j = 1, 2, ..., n\}\right\} = \frac{\exp\left(\frac{a_i}{\mu}\right)}{\sum_{j=1}^{n} \exp\left(\frac{a_j}{\mu}\right)}.$$

Proof of Theorem 1a: For a given amount of award A, suppose the seeker splits A into

A_1 as the first prize and A_2 as the second prize such that $A_1 \geq A_2 \geq 0$ and $A_1 + A_2 = A$.

For an expertise-based project, we derive the equilibrium strategy for solvers in two steps. In this first step, we derive the equilibrium participation and effort exerting strategy for solvers for the special case without the fixed cost, i.e., $c_f = 0$. In the second step, based on the equilibrium strategy we obtained for the special case, we then derive the impact of a positive fixed cost c_f on that equilibrium strategy to obtain the general equilibrium participation and effort exerting strategy for solvers. Similar process is widely used in literature (e.g., Moldovanu and Sela 2001).

We take the perspective of solver i with endowed expertise β_i. Let's assume solver i expects all other solvers exert effort according to a strategy $e_j = y(\beta_j)$ for all $j \neq i$, where $y(\cdot)$ is an increasing and differentiable function. Define $g(\beta_j) = \beta_j + r(y(\beta_j))$, which is also an increasing and differentiable function. We now consider the case with $c_f = 0$. Without a fixed cost in this case, all solvers would participate because the worse thing a solver could do is to exert zero effort (occur zero cost). The equilibrium we derive will indeed verify this conjecture. Therefore, conditional on his expectation on other solvers' efforts, solver i would win A_1 only if

$$v_i > v_j = g(\beta_j), \forall j \neq i,$$

which happens with probability $F(g^{-1}(v_i))^{n-1}$ because g is assumed to be increasing. Similarly, the probability that solver i would win A_2 is $(n-1)(1 - F(g^{-1}(v_i)))F(g^{-1}(v_i))^{n-2}$. So, solver i's problem without a fixed cost can be written as

$$\begin{aligned}\max_{v_i \geq 0} \pi_i^e(v_i|\beta_i, g(\beta_j)) &= A_1 F\left(g^{-1}(v_i)\right)^{n-1} + A_2(n-1)\left(1 - F\left(g^{-1}(v_i)\right)\right) F\left(g^{-1}(v_i)\right)^{n-2} \\ &\quad -c_1 r^{-1}(v_i - \beta_i),\end{aligned}$$

where r^{-1} is the reverse function of the return function r. We look for a symmetric Bayesian equilibrium in which all solvers including solver i play the same strategy $y(\cdot)$. The first-order condition of solver i's problem is

$$\begin{aligned}&A_1(n-1)F\left(g^{-1}(v_i)\right)^{n-2} f\left(g^{-1}(v_i)\right)\frac{1}{g'(g^{-1}(v_i))} \\ &+A_2(n-1)F\left(g^{-1}(v_i)\right)^{n-3}\left[n-2-(n-1)F\left(g^{-1}(v_i)\right)\right] f\left(g^{-1}(v_i)\right)\frac{1}{g'(g^{-1}(v_i))} \\ =\ &\frac{c_1}{r'(r^{-1}(v_i - \beta_i))}.\end{aligned}$$

At the symmetric equilibrium, $v_i = v = g(\beta)$, the above condition becomes

$$\left[A_1(n-1)F(\beta)^{n-2} + A_2(n-1)F(\beta)^{n-3}[n-2-(n-1)F(\beta)]\right]f(\beta)\frac{1}{g'(\beta)} = \frac{c_1}{r'(r^{-1}(v-\beta))},$$

which is a differential equation with separation of variables. Since function g is assumed to be increasing, the solver with the lowest endowed expertise, $\underline{\beta}$, will not win for sure and will exert zero effort in equilibrium, that is, $y\left(\underline{\beta}\right) = 0$, or $g\left(\underline{\beta}\right) = \underline{\beta}$. Solving the differential equation with this boundary condition, we have the symmetric Bayesian equilibrium strategy for a solver with endowed expertise $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$ as

$$\begin{aligned} c_1 r^{-1}(v-\beta) &= c_1 e(\beta, A_1, A_2) \\ &= A_1(n-1)\int_{\underline{\beta}}^{\beta} F(x)^{n-2} f(x)\,dx \\ &\quad + A_2(n-1)\int_{\underline{\beta}}^{\beta} F(x)^{n-3}[n-2-(n-1)F(x)]f(x)\,dx, \end{aligned}$$

which implies that the equilibrium solver effort conditional on award (A_1, A_2) is

$$\begin{aligned} e(\beta, A_1, A_2) &= \frac{A_1(n-1)}{c_1}\int_{\underline{\beta}}^{\beta} F(x)^{n-2} f(x)\,dx \\ &\quad + \frac{A_2(n-1)}{c_1}\int_{\underline{\beta}}^{\beta} F(x)^{n-3}[n-2-(n-1)F(x)]f(x)\,dx \\ &= \frac{A_1 F(\beta)^{n-1} + A_2(n-1)\left(F(\beta)^{n-2} - F(\beta)^{n-1}\right)}{c_1}. \end{aligned} \tag{1}$$

We need to verify that $e(\beta, A_1, A_2)$ is indeed increasing and differentiable which are clearly true because $F(\beta)$ is continuous and increasing in β. Thus, $e(\beta, A_1, A_2)$ constitutes a symmetric Bayesian equilibrium for solvers. From $e(\beta, A_1, A_2)$, we can see that solvers with different β react differently to A_1 and A_2.

Based on the above equilibrium for the case with $c_f = 0$, we now consider the case with a positive c_f. In the presence of a fixed cost c_f, participating solvers would have to adjust efforts down such that they can at least recover c_f in expectation. Because in the equilibrium without a fixed cost, the solver with the lowest expertise makes zero effort already, this solver will not participate in the presence of c_f (effort cannot be negative). Thus, for a given award (A_1, A_2), let's assume that solvers with expertise in the interval of $\left[\beta_f, \overline{\beta}\right]$ participate and exert effort according to the function $e_f(\beta, A_1, A_2) = e(\beta, A_1, A_2) - k$, where $e(\beta, A_1, A_2)$ is given in (1), and k is a constant. The solver with expertise β_f

will exert zero effort and earn zero expected profit. Therefore, β_f and k are determined by the zero-effort and zero-profit conditions as

$$e_f\left(\beta_f, A_1, A_2\right) = 0,$$

$$A_1 F\left(\beta\right)^{n-1} + A_2\left(n-1\right) F\left(\beta\right)^{n-2}\left(1 - F\left(\beta\right)^{n-1}\right) - c_1 e_f\left(\beta_f, A_1, A_2\right) - c_f = 0,$$

or

$$e\left(\beta_f, A_1, A_2\right) - k = 0,$$

$$A_1 F\left(\beta\right)^{n-1} + A_2\left(n-1\right) F\left(\beta\right)^{n-2}\left(1 - F\left(\beta\right)^{n-1}\right) - c_f = 0.$$

Solving the above two-equation system, we have

$$A_1 F\left(\beta_f\right)^{n-1} + A_2\left(n-1\right) F\left(\beta_f\right)^{n-2}\left(1 - F\left(\beta_f\right)^{n-1}\right) = c_f, \tag{2}$$

$$k = \frac{c_f}{c_1}.$$

Hence, the equilibrium solver effort for a participating solver conditional on award $\left(A_1, A_2\right)$ with c_f is

$$e^*\left(\beta, A_1, A_2\right) = \frac{A_1 F\left(\beta\right)^{n-1} + A_2\left(n-1\right)\left(F\left(\beta\right)^{n-2} - F\left(\beta\right)^{n-1}\right) - c_f}{c_1}. \tag{3}$$

To simplify notation, define $A_1 = \left(1-\alpha\right) A$ and $A_2 = \alpha A$, where $\alpha \in [0, 1/2]$. So, we can re-write the equilibrium solver effort as

$$e^*\left(\beta, A, \alpha\right) = \frac{A\left[F\left(\beta\right)^{n-1} + \alpha\left[\left(n-1\right) F\left(\beta\right)^{n-2} - nF\left(\beta\right)^{n-1}\right]\right] - c_f}{c_1}.$$

Given the equilibrium strategy for solvers, the expected performance of the best solution among solutions submitted by n solvers is

$$\int_{\beta_f}^{\overline{\beta}} \left[\beta + r\left(e^*\left(\beta, A, \alpha\right)\right)\right] nF\left(\beta\right)^{n-1} f\left(\beta\right) d\beta,$$

where $nF\left(\beta\right)^{n-1} f\left(\beta\right)$ is the pdf of the highest endowment expertise among n solvers, and the average performance of all solutions is

$$\int_{\beta_f}^{\overline{\beta}} \left[\beta + r\left(e^*\left(\beta, A, \alpha\right)\right)\right] f\left(\beta\right) d\beta.$$

Therefore, the seeker's expected payoff from a contest with award A and allocation α is

$$V(\alpha) = \rho \int_{\beta_f}^{\overline{\beta}} \left[\beta + r\left(e^*\left(\beta, A, \alpha\right)\right)\right] nF(\beta)^{n-1} f(\beta)\, d\beta + (1-\rho) \int_{\beta_f}^{\overline{\beta}} \left[\beta + r\left(e^*\left(\beta, A, \alpha\right)\right)\right] f(\beta)\, d\beta.$$

The value of α that maximizes $V(\alpha)$ depending on the functional form of r and the CDF F. If conditions

$$V''(\alpha) < 0$$

and

$$V'(\alpha)\,|_{\alpha=0} < 0$$

can be satisfied, $V(\alpha)$ is maximized at $\alpha = 0$. In this case, a winner-takes-all contest $(\alpha = 0)$ is optimal for the seeker.

We next consider ideation projects. Recall that in an ideation project, all solvers have the same endowed expertise β. We still take solver i's perspective and look for a symmetric Nash equilibrium. Suppose solver i expects all other solvers to exert effort e in equilibrium. Conditional on this expectation, solver i optimizes his strategy in exerting effort e_i. Using Property 2, solver i's probabilities of winning the first prize A_1 and the second prize A_2 are

$$\begin{aligned} \Pr\{\text{Solver } i \text{ wins } A_1\} &= \Pr\{\beta + e_i + \xi_i > \beta + e + \xi_j,\ \forall j \neq i\} \\ &= \Pr\{e_i + \xi_i > e + \xi_j,\ \forall j \neq i\} \\ &= \frac{1}{1 + (n-1)\exp\left(\frac{r(e) - r(e_i)}{\mu}\right)} \end{aligned}$$

and

$$\begin{aligned} \Pr\{\text{Solver } i \text{ wins } A_2\} &= (n-1)\Pr\{e_i + \xi_i < e + \xi_k,\ k \neq i\} \Pr\{e_i + \xi_i > e + \xi_j,\ \forall j \neq i, k\} \\ &= (n-1)\left[\frac{1}{1 + (n-2)\exp\left(\frac{r(e) - r(e_i)}{\mu}\right)} - \frac{1}{1 + (n-1)\exp\left(\frac{r(e) - r(e_i)}{\mu}\right)}\right], \end{aligned}$$

respectively. Solver i's problem can be written as

$$\begin{aligned} \max_{e_i \geq 0} \pi_i^i(e_i|e) &= A_1 \frac{1}{1 + (n-1)\exp\left(\frac{r(e) - r(e_i)}{\mu}\right)} \\ &\quad + A_2 (n-1)\left[\frac{1}{1 + (n-2)\exp\left(\frac{r(e) - r(e_i)}{\mu}\right)} - \frac{1}{1 + (n-1)\exp\left(\frac{r(e) - r(e_i)}{\mu}\right)}\right] \\ &\quad - c_1 e_i - c_f. \end{aligned} \tag{4}$$

It is not difficult to verify that the condition $r'' + r'^2/\mu \leq 0$, which is satisfied by logarithmic return functions, is sufficient to ensure that payoff function $\pi_i^i(e_i|e)$ is concave in solver i's strategy e_i. According to the Theorem 1.2 in Fudenberg and Tirole (2000), pure strategy equilibrium exists in this contest game. Using symmetry $e = e_i = e^*$ in solver i's first-order condition, it reduces to

$$\frac{r'(e^*)}{\mu n^2 (n-1)} \left[A_1 (n-1)^2 + A_2 \left(n^2 - 3n + 1\right)\right] - c_1 = 0.$$

Because $r'(e^*)$ is decreasing in e^* and $(n-1)^2 > n^2 - 3n + 1$, the above condition implies that it is optimal to set $A_1 = A$ and $A_2 = 0$, that is, winner-takes-all is optimal.

Similarly, for trial and error projects, suppose solver i expects all other solvers to exert effort m in equilibrium. Conditional on this expectation, solver i optimizes his strategy in exerting effort m_i. Using Property 2, solver i's probabilities of winning the first prize A_1 and the second prize A_2 are

$$\begin{aligned}
\Pr\{\text{Solver } i \text{ wins } A_1\} &= \frac{m_i \exp\left(\frac{\beta}{\mu}\right)}{m_i \exp\left(\frac{\beta}{\mu}\right) + (n-1)\, m \exp\left(\frac{\beta}{\mu}\right)} \\
&= \frac{m_i}{m_i + (n-1)\, m}
\end{aligned}$$

and

$$\Pr\{\text{Solver } i \text{ wins } A_2\} = (n-1)\left[\frac{m_i}{m_i + (n-2)\, m} - \frac{m_i}{m_i + (n-1)\, m}\right],$$

respectively. So, solver i's problem can be written as

$$\begin{aligned}
\max_{m_i \geq 1} \pi_i^t(m_i|m) = \; & A_1 \frac{m_i}{m_i + (n-1)\, m} + A_2 (n-1) \left[\frac{m_i}{m_i + (n-2)\, m} - \frac{m_i}{m_i + (n-1)\, m}\right] \\
& - c_2 m_i - c_f.
\end{aligned} \tag{5}$$

Because $A_1 \geq A_2$, it is not difficult to show that the payoff function $\pi_i^t(m_i|m)$ is concave in solver i's strategy m_i. According to the Theorem 1.2 in Fudenberg and Tirole (2000), pure strategy equilibrium exists in this contest game. Using symmetry $m = m_i = m^*$ in solver i's first-order condition, it reduces to

$$A_1 \frac{(n-1)}{n^2 m^*} + A_2 \frac{n^2 - 3n + 1}{(n-1)\, n^2 m^*} - c_2 = 0.$$

Because $n - 1 > (n^2 - 3n + 1)/(n-1)$, the above condition implies that it is optimal to set $A_1 = A$ and $A_2 = 0$, that is, winner-takes-all is optimal. ■

Proof of Theorem 1b: We consider expertise-based projects first. Setting $A_1 = A^e$ and $A_2 = 0$ in (2) and solving the equation, we have

$$\beta_f = F^{-1}\left(\left(\frac{c_f}{A^e}\right)^{\frac{1}{n-1}}\right). \tag{6}$$

Solvers with higher expertise than β_f would participate in the contest. Thus, for n solvers, the expected number of those solvers who would participate the contest is

$$\begin{aligned} n^{e*} &= n\left(1 - F\left(\beta_f\right)\right) \\ &= n\left(1 - \left(\frac{c_f}{A^e}\right)^{\frac{1}{n-1}}\right). \end{aligned}$$

Setting $A_1 = A^e$ and $A_2 = 0$ in (3), we have the equilibrium solver effort conditional on award A^e as

$$e^*\left(\beta, A^e\right) = \frac{A^e F\left(\beta\right)^{n-1} - c_f}{c_1}. \tag{7}$$

The seeker's problem can be written as

$$\begin{aligned} \max_{A^e \geq 0} \Pi^e\left(A^e\right) &= \rho \int_{\beta_f}^{\overline{\beta}} \left[\beta + r\left(e^*\left(\beta, A^e\right)\right)\right] nF\left(\beta\right)^{n-1} f\left(\beta\right) d\beta \\ &\quad + \left(1-\rho\right) \int_{\beta_f}^{\overline{\beta}} \left[\beta + r\left(e^*\left(\beta, A^e\right)\right)\right] f\left(\beta\right) d\beta - A^e. \end{aligned}$$

Assuming the above problem has a unique solution, the optimal prize A^{e*} satisfies the following first-order condition

$$\frac{\partial \Pi^e\left(A^{e*}\right)}{\partial A^{e*}} = 1. \tag{8}$$

Substituting A^{e*} into (6) and (7), we have

$$\beta_f = F^{-1}\left(\left(\frac{c_f}{A^{e*}}\right)^{\frac{1}{n-1}}\right), \tag{9}$$

$$e^*\left(\beta\right) = \frac{A^{e*} F\left(\beta\right)^{n-1} - c_f}{c_1}. \tag{10}$$

Therefore, the equilibrium fixed price contest $\{A^{e*}, \beta_f, e^*\left(\beta\right)\}$ is defined implicitly by (8), (9) and (10). With this equilibrium strategy $e^*\left(\beta\right)$, the expected profit for a solver with endowed expertise β is

$$\begin{aligned} \pi^e\left(\beta\right) &= A^{e*} F\left(\beta\right)^{n-1} - c_1\left(\frac{A^{e*} F\left(\beta\right)^{n-1}}{c_1}\right) \\ &= 0. \end{aligned}$$

Substituting $c_f = 0$ into (9), we have $\beta_f = \underline{\beta}$. For a logarithmic return function $r(e) = \theta \ln e$ and $c_f = 0$, from (8) with $\beta_f = \underline{\beta}$, we have the equilibrium award as

$$A^{e*} = \theta.$$

Substituting $A^{e*} = \theta$ and $c_f = 0$ into (10), we have the equilibrium improvement effort for solvers as

$$e^*(\beta) = \frac{\theta F(\beta)^{n-1}}{c_1}.$$

Substituting A^{e*}, $e^*(\beta)$, $c_f = 0$, and $r(e) = \theta \ln e$ into the seeker's objective function and simplifying, we have the equilibrium expected profit for the seeker as

$$\Pi^{e*} = \rho \int_{\underline{\beta}}^{\overline{\beta}} \beta n F(\beta)^{n-1} f(\beta)\, d\beta + (1-\rho) \int_{\underline{\beta}}^{\overline{\beta}} \beta f(\beta)\, d\beta + \theta \left(\ln \frac{\theta}{c_1} - \frac{(n-1)(\rho + (1-\rho) n) + n}{n} \right).$$

We next consider ideation projects. Setting $A_1 = A^i$ and $A_2 = 0$ in (4), solver i's problem can be written as

$$\max_{e_i \geq 0} \pi_i^i(e_i|e) = A^i \frac{1}{1 + (n-1)\exp\left(\frac{r(e)-r(e_i)}{\mu}\right)} - c_1 e_i - c_f.$$

Solver i's first-order condition is

$$A^i \frac{\frac{r'(e_i)}{\mu}(n-1)\exp\left(\frac{r(e_i)}{\mu}\right)\exp\left(\frac{r(e)}{\mu}\right)}{\left[\exp\left(\frac{r(e_i)}{\mu}\right) + (n-1)\exp\left(\frac{r(e)}{\mu}\right)\right]^2} - c_1 = 0.$$

Using symmetry $e = e_i = e(A^i)$, the first-order condition reduces to

$$A^i \frac{r'(e(A^i))(n-1)}{\mu n^2} - c_1 = 0, \tag{11}$$

which has a unique solution that satisfies

$$r'\left(e\left(A^i\right)\right) = \frac{c_1 \mu n^2}{A^i (n-1)}. \tag{12}$$

Given the equilibrium strategy $e(A^i)$ for solvers, using Property 1, the seeker's problem can be written as

$$\begin{aligned} \max_{A^i \geq 0} \Pi^i\left(A^i\right) &= \rho\left(\beta + r\left(e\left(A^i\right)\right) + \mu \ln n\right) + (1-\rho)\left(\beta + r\left(e\left(A^i\right)\right)\right) - A^i \\ &= \beta + r\left(e\left(A^i\right)\right) + \rho\mu \ln n - A^i, \end{aligned}$$

where the term $\mu \ln n$ is the expected maximum value of n realizations of the zero mean Gumbel distributed random noise variable ξ. It is not difficult to verify that the condition $r'r''' - 2r''^2 \leq 0$, which is satisfied by logarithmic return functions, is sufficient to ensure that the seeker's objective function to be concave in A^i. Therefore, the first-order condition determines the optimal award A^{i*} as

$$\frac{\partial e^*}{\partial A^{i*}} = \frac{A^{i*}(n-1)}{c_1 \mu n^2}. \tag{13}$$

Substituting A^{i*} from (13) into (14), we have the equilibrium effort for the solvers as

$$r'(e^*) = \frac{c_1 \mu n^2}{A^{i*}(n-1)}. \tag{14}$$

Therefore, the equilibrium fixed price contest $\{A^{i*}, e^*\}$ for ideation projects is defined implicitly by (13) and (14).

For the logarithmic return function $r(e) = \theta \ln e$, (13) and (14) determine the equilibrium award for the seeker and the equilibrium effort for solvers as

$$A^{i*} = \theta,$$

and

$$e^* = \frac{\theta^2 (n-1)}{c_1 \mu n^2}.$$

Substituting A^{i*}, e^*, and $r(e) = \theta \ln e$ into the seeker's objective function, we have the equilibrium expected profit for the seeker as

$$\Pi^{i*} = \beta + \theta \left(\ln \frac{\theta^2 (n-1)}{c_1 \mu n^2} - 1 \right) + \rho \mu \ln n.$$

Furthermore, the equilibrium profit for a solver is

$$\begin{aligned} \pi^{i*} &= \frac{A^{i*}}{n} - c_1 e^* - c_f \\ &= \frac{\mu \theta n - \theta^2 (n-1)}{\mu n^2} - c_f \\ &\approx \frac{\mu \theta - \theta^2}{\mu n} - c_f. \end{aligned}$$

Setting $\pi^{i*} = 0$ and solving for n, we have

$$n^{i*} \approx \frac{\theta(\mu - \theta)}{\mu c_f},$$

which is the maximum number of solvers that a free-entry fixed price contest can accommodate such that all participating solvers can earn a non-negative expected profit. For $c_f = 0$, to ensure the equilibrium profits for solvers are greater than zero, it is not difficult to verify that the condition

$$\mu \geq \theta \tag{15}$$

is sufficient.

Finally, we consider trial and error projects. Setting $A_1 = A^t$ and $A_2 = 0$ in (5), solver i's problem can be written as

$$\max_{m_i \geq 1} \pi_i^t(m_i|m) = A^t \frac{m_i}{m_i + (n-1)\, m} - c_2 m_i - c_f.$$

Solver i's first-order condition is

$$A^t \frac{(n-1)\, m}{(m_i + (n-1)\, m)^2} - c_2 = 0.$$

Using symmetry $m = m_i = m^*(A^t)$, the above condition becomes

$$A^t \frac{(n-1)}{n^2 m^*(A^t)} - c_2 = 0, \tag{16}$$

which has a unique symmetric solution

$$m^*\left(A^t\right) = \frac{A^t (n-1)}{n^2 c_2}.$$

Then, using Property 1, the seeker's problem can be written as

$$\begin{aligned} \max_{A^t \geq 0} \Pi^t\left(A^t\right) &= \rho\left(\beta + \mu \ln n m^*\left(A^t\right)\right) + (1-\rho)\left(\beta + \mu \ln m^*\left(A^t\right)\right) - A^t \\ &= \beta + \rho\mu \ln n + \mu \ln m^*\left(A^t\right) - A^t, \end{aligned}$$

which implies the optimal award is

$$A^{t*} = \mu. \tag{17}$$

With A^{t*}, the symmetric equilibrium effort for solvers is

$$m^* = \frac{\mu\,(n-1)}{n^2 c_2}. \tag{18}$$

Hence, the equilibrium fixed price contest $\{A^{t*}, m^*\}$ for trial and error projects is defined by (17) and (18). The seeker's equilibrium expected profit is

$$\Pi^{t*} = \beta + \mu\left(\ln\frac{\mu(n-1)}{nc_2} - 1\right) - (1-\rho)\mu\ln n,$$

and the equilibrium expected profit for a solver is

$$\begin{aligned}\pi^{t*} &= \frac{A^{t*}}{n} - c_2 m^* - c_f \\ &= \frac{\mu}{n^2} - c_f.\end{aligned}$$

Setting $\pi^{t*} = 0$ and solving for n, we have

$$n^{t*} = \sqrt{\frac{\mu}{c_f}},$$

which is the maximum number of solvers that a free-entry fixed price contest can accommodate such that all participating solvers can earn a non-negative expected profit. ■

Proof of Theorem 1c: For an expertise-based project with Gumbel distributed solver expertise with scale parameter λ, the equilibrium expected profit for the seeker is

$$\Pi^{e*} = \beta_o + \rho\lambda\ln n + +\theta\left(\ln\frac{\theta}{c_1} - \frac{(n-1)(\rho+(1-\rho)n)+n}{n}\right),$$

where β_o is the mean of the Gumbel distribution. Differentiating Π^{e*} with respect to n, we have

$$\frac{\partial\Pi^{e*}}{\partial n} = \frac{\rho(\lambda n - \theta)}{n^2} - \theta(1-\rho).$$

If $\lambda \geq \theta/2$, because $n \geq 2$ for a contest, $\partial\Pi^{e*}/\partial n$ is increasing in ρ, strictly greater than 0 at $\rho = 1$, and strictly less than 0 at $\rho = 0$. Therefore, for expertise-based projects, if $\lambda \geq \theta/2$, free entry open innovation contest is optimal for high enough ρ and always optimal for $\rho = 1$, while it is not optimal for sufficiently low ρ, especially for $\rho = 0$.

For an ideation project with the logarithmic return function $r(e) = \theta\ln e$, the equilibrium expected profit for the seeker is

$$\Pi^{i*} = \beta + \theta\left(\ln\frac{\theta^2(n-1)}{c_1\mu n^2} - 1\right) + \rho\mu\ln n.$$

Differentiating Π^{i*} with respect to n, we obtain

$$\frac{\partial\Pi^{i*}}{\partial n} = -\frac{\theta(n-2)}{n(n-1)} + \frac{\rho\mu}{n}.$$

Clearly, $\partial \Pi^{i*}/\partial n$ is increasing in ρ. By condition (15), $\partial \Pi^{i*}/\partial n > 0$ at $\rho = 1$, and $\partial \Pi^{i*}/\partial n < 0$ at $\rho = 0$.

For a trial and error project, the seeker's equilibrium expected profit is

$$\Pi^{t*} = \beta + \mu \left(\ln \frac{\mu (n-1)}{n c_2} - 1 \right) - (1-\rho) \mu \ln n.$$

Differentiating Π^{i*} with respect to n, we have

$$\frac{\partial \Pi^{t*}}{\partial n} = \frac{\mu}{n (n-1)} - \frac{(1-\rho) \mu}{n},$$

which is clearly increasing in ρ, and $\partial \Pi^{t*}/\partial n > 0$ at $\rho = 1$ and $\partial \Pi^{i*}/\partial n < 0$ at $\rho = 0$.

Therefore, for ideation projects and trial and error projects, free entry open innovation contest is optimal for high enough ρ and always optimal for $\rho = 1$, while it is not optimal for sufficiently low ρ, especially for $\rho = 0$. ■

Proof of Corollary: Let c_e denote the entry fee for each participating solver. Let's consider the case with zero fixed cost, $c_f = 0$. Let's also assume that n is large enough so that for a given entry fee, there are some solvers would not participate. From a solver's perspective, an entry fee just behaves like a fixed cost.

When $\rho = 1$, according to Theorem 1b, for an ideation project, the seeker's expected profit by charging an entry fee c_e is

$$\Pi^i (c_e) = \beta + \theta \left(\ln \frac{\theta^2 (n^{i*} (c_e) - 1)}{\mu c_1^2 n^{i*} (c_e)} - 1 \right) + \mu \ln n^{i*} (c_e) + c_e n^{i*} (c_e),$$

where

$$n^{i*} (c_e) \approx \frac{\theta (\mu - \theta)}{\mu c_e}.$$

Simplifying the expression, we have

$$\Pi^i (c_e) \approx \beta + \theta \left(\ln \frac{\theta^2 (n^{i*} (c_e) - 1)}{\mu c_1^2 n^{i*} (c_e)} - 1 \right) + \mu \ln n^{i*} (c_e) + \frac{\theta (\mu - \theta)}{\mu}.$$

We have shown in the proof of Theorem 1c that the above expression is increasing in $n^{i*} (c_e)$ which is decreasing in c_e. Hence, $\Pi^i (c_e)$ is decreasing in c_e, which implies it is optimal for the seeker to set $c_e = 0$.

When $\rho = 1$, according to Theorem 1b, for a trial and error project, the seeker's expected profit by charging an entry fee c_e is

$$\Pi^t (c_e) = \beta + \mu \left(\ln \frac{\mu (n^{t*} (c_e) - 1)}{n^{t*} (c_e) c_2} - 1 \right) + c_e n^{t*} (c_e),$$

where

$$n^{t*}(c_e) = \sqrt{\frac{\mu}{c_e}}.$$

Plugging $n^{t*}(c_e)$ and simplifying, we have

$$\Pi^t(c_e) = \beta + \mu\left(\ln\frac{\mu\left(\sqrt{\frac{\mu}{c_e}}-1\right)}{c_2\sqrt{\frac{\mu}{c_e}}} - 1\right) + \sqrt{\mu c_e}.$$

Differentiating $\Pi^t(c_e)$ with respect to c_e, we have

$$\begin{aligned}\frac{\partial \Pi^t(c_e)}{\partial c_e} &= -\frac{1}{2\sqrt{\frac{c_e}{\mu}}\left(1-\sqrt{\frac{c_e}{\mu}}\right)} + \frac{\mu}{2\sqrt{\mu c_e}} \\ &= \frac{1}{2}\sqrt{\frac{\mu}{c_e}}\left(1 - \frac{1}{1-\sqrt{\frac{c_e}{\mu}}}\right) \leq 0.\end{aligned}$$

Hence, $\Pi^t(c_e)$ is decreasing in c_e, which implies it is optimal for the seeker to set $c_e = 0$.

■

Proof of Theorem 2: In an ideation project, using Properties 1 and 2, for a given performance contingent award ϕ, solver i's problem is

$$\begin{aligned}\max_{e_i \geq 0} \pi_i^i(e_i|e) &= \phi\left[\beta + \ln\left(\exp\left(r(e_i)/\mu\right) + (n-1)\exp\left(r(e)/\mu\right)\right)\right]\frac{\exp\left(\frac{r(e_i)}{\mu}\right)}{\exp\left(\frac{r(e_i)}{\mu}\right) + (n-1)\exp\left(\frac{r(e)}{\mu}\right)} \\ &\quad - c_1 e_i.\end{aligned}$$

At the symmetric equilibrium $e = e_i = e^*(\phi)$, the first-order condition is

$$\frac{\phi r'(e^*(\phi))}{\mu n^2} + \phi\left(\beta + r(e^*(\phi)) + \mu \ln n\right)\frac{r'(e^*(\phi))(n-1)}{\mu n^2} - c_1 = 0. \qquad (19)$$

For any given fixed price award A^i, we can adjust ϕ such that $\phi(\beta + r(e^*(\phi)) + \mu \ln n) = A^i$, where $e^*(\phi)$ satisfies (19), that is, we can find a performance contingent award that pays the winner the same amount as the fixed award. With $\phi(\beta + r(e^*(\phi)) + \mu \ln n) = A^i$, by comparing first-order conditions (11) and (19), we can see that $e^*(\phi) \geq e^*(A^i)$ because the first term in (19) is positive and r is concave. Therefore, for any given fixed price award, there exists at least a performance contingent award that can induce solvers to exert higher effort in equilibrium without paying the winner more. As a result, using performance contingent award improves the seeker's expected profit for ideation projects.

In a trial and error project, for a given performance contingent award ϕ, solver i's problem is

$$\max_{m_i \geq 1} \pi_i^t(m_i|m) = \phi\left[\beta + \mu \ln\left(m_i + (n-1)\,m\right)\right] \frac{m_i}{m_i + (n-1)\,m} - c_2 m_i.$$

At the symmetric equilibrium $m = m_i = m^*(\phi)$, the first-order condition is

$$\frac{\phi\mu}{n^2 m^*(\phi)} + \phi\left[\beta + \mu \ln n m^*(\phi)\right] \frac{(n-1)}{n^2 m^*(\phi)} - c_2 = 0. \tag{20}$$

For any given fixed price award A^t, we can adjust ϕ such that $\phi\left[\beta + \mu \ln n m^*(\phi)\right] = A^t$, where $m^*(\phi)$ satisfies (20), that is, we can find a performance contingent award that pays the winner in the same amount as the fixed award in equilibrium. With $\phi\left[\beta + \mu \ln n m^*(\phi)\right] = A^t$, by comparing first-order conditions (16) and (20), we can see that $m^*(\phi) \geq m^*(A^t)$ because the first term in (20) is positive. Therefore, using performance contingent award improves the seeker's expected profit for trial and error projects.■

Proof of Theorem 3a: Recall that we consider the case with $\rho = 1$ and $c_f = 0$. For an expertise-based project, let β_s be the seeker's endowed expertise on the project. If the seeker chooses to conduct the project internally, she solves the following problem

$$\max_{e \geq 0} \Pi_I^e(e) = \beta_s + r(e) - c_{s1} e,$$

which has the optimal effort satisfying $r'(e^*) = c_{s1}$. For a logarithmic return function $r(e) = \theta \ln e$, the optimal effort is $e^* = \frac{\theta}{c_{s1}}$, and the optimal profit for the seeker is

$$\Pi_I^{e*} = \beta_s + \theta\left(\ln \frac{\theta}{c_{s1}} - 1\right). \tag{21}$$

For an expertise-based project with a logarithmic return function $r(e) = \theta \ln e$, the seeker's problem for a self-administrated open innovation contest is

$$\max_{n \geq 2} \Pi_S^e(n) = \beta_o + \lambda \ln n + \theta\left(\ln \frac{\theta}{c_1} - \frac{2n-1}{n}\right) - c_s n,$$

which has the optimal solver pool size

$$n_S^{e*} = \frac{\lambda + \sqrt{\lambda^2 - 4 c_s \theta}}{2 c_s}.$$

Substituting n_S^{e*}, we have

$$\Pi_S^{e*} = \beta_o + \lambda \ln n_S^{e*} + \theta\left(\ln \frac{\theta}{c_1} - \frac{2n_S^{e*} - 1}{n_S^{e*}}\right) - c_s n_S^{e*}.$$

For an intermediary-administrated open innovation contest, from Theorem 1b, we have

$$\Pi_O^{e*} = \beta_o + \lambda \ln n_o + \theta \left(\ln \frac{\theta}{c_1} - \frac{2n_o - 1}{n_o} \right) - p.$$

Setting $\Pi_S^{e*} = \Pi_O^{e*}$ and simplifying, we have θ_{SO}^e as the solution of the following equation

$$\lambda \ln n_S^{e*} - \frac{\theta \left(2n_S^{e*} - 1\right)}{n_S^{e*}} - c_s n_S^{e*} = \lambda \ln n_o - \frac{\theta \left(2n_o - 1\right)}{n_o} - p.$$

If $\theta < \theta_{SO}^e$, a self-administrated open innovation contest is preferred to an intermediary-administrated open innovation contest. Otherwise, an intermediary-administrated open innovation contest is preferred to a self-administrated open innovation contest. Setting $\Pi_I^{e*} = \Pi_O^{e*}$ and solving for c_1, we have

$$c_1 = g^e(\theta) = c_{s1} (n_o)^{\frac{\lambda}{\theta}} \exp \left(\frac{\beta_o - \beta_s - p}{\theta} + \frac{1}{n_o} - 1 \right).$$

For any given θ, if $c_1 > g^e(\theta)$, internal R&D is preferred to an intermediary-administrated open innovation contest. Otherwise, an intermediary-administrated open innovation contest is preferred to internal R&D. Setting $\Pi_I^{e*} = \Pi_S^{e*}$ and solving for c_1, we have

$$c_1 = f^e(\theta) = c_{s1} \left(n_S^{e*}\right)^{\frac{\lambda}{\theta}} \exp \left(\frac{\beta_o - \beta_s - c_s n_S^{e*}}{\theta} + \frac{1}{n_S^{e*}} - 1 \right).$$

For any given θ, if $c_1 > f^e(\theta)$, internal R&D is preferred to a self-administrated open innovation contest. Otherwise, a self-administrated open innovation contest is preferred to internal R&D.

For an ideation project, the seeker (unlike an external solver) knows his subjective taste. She therefore will exert both improvement effort (e) and experimentation effort (m). Thus, he solves the following problem

$$\max_{e \geq 0} \Pi_I^i(e) = \beta + r(e) + \mu \ln m - c_{s1} e - c_{s2} m,$$

which has the optimal effort level satisfying $r'(e^*) = c_{s1}$ and $m^* = \frac{\mu}{c_{s2}}$. For a logarithmic return function $r(e) = \theta \ln e$, the optimal improvement effort is $e^* = \frac{\theta}{c_{s1}}$, and the optimal profit for the seeker is

$$\Pi_I^{i*} = \beta + \theta \left(\ln \frac{\theta}{c_{s1}} - 1 \right) + \mu \left(\ln \frac{\mu}{c_{s2}} - 1 \right). \tag{22}$$

For an ideation project with a logarithmic return function $r(e) = \theta \ln e$, the seeker's problem for a self-administrated open innovation contest is

$$\begin{aligned}\max_{n\geq 2} \Pi_S^i(n) &= \beta + \theta \left(\ln \frac{\theta^2 (n-1)}{\mu c_1 n^2} - 1 \right) + \mu \ln n - c_s n \\ &\approx \beta + \theta \left(\ln \frac{\theta^2}{\mu c_1 n} - 1 \right) + \mu \ln n - c_s n,\end{aligned}$$

which has the optimal solver pool size

$$n_S^{i*} \approx \frac{\mu - \theta}{c_s}.$$

Substituting n_S^{i*}, we have

$$\Pi_S^{i*} \approx \beta + \theta \left(\ln \frac{c_s \theta^2}{\mu c_1 (\mu - \theta)} - 1 \right) + \mu \ln \frac{\mu - \theta}{c_s} - (\mu - \theta).$$

For an intermediary-administrated open innovation contest, from Theorem 1b, we have

$$\begin{aligned}\Pi_O^{i*} &= \beta + \theta \left(\ln \frac{\theta^2 (n_o - 1)}{\mu c_1 n_o^2} - 1 \right) + \mu \ln n_o - p \\ &\approx \beta + \theta \left(\ln \frac{\theta^2}{\mu c_1 n_o} - 1 \right) + \mu \ln n_o - p.\end{aligned}$$

Setting $\Pi_S^{i*} = \Pi_O^{i*}$ and simplifying, we have θ_{SO}^i as the solution of the following equation

$$\theta \ln \frac{c_s}{(\mu - \theta)} + \mu \ln \frac{\mu - \theta}{c_s} - (\mu - \theta) = \theta \ln \frac{1}{n_o} + \mu \ln n_o - p.$$

If $\theta < \theta_{SO}^i$, a self-administrated open innovation contest is preferred to an intermediary-administrated open innovation contest. Otherwise, an intermediary-administrated open innovation contest is preferred to a self-administrated open innovation contest. Setting $\Pi_I^{i*} = \Pi_O^{i*}$ and solving for c_1, we have

$$c_1 = g^i(\theta) = \frac{\theta c_{s1} (n_o)^{\frac{\mu-\theta}{\theta}} \exp\left(\frac{\beta - \beta_s - p}{\theta}\right)}{\mu}.$$

For any given θ, if $c_1 > g^i(\theta)$, internal R&D is preferred to an intermediary-administrated open innovation contest. Otherwise, an intermediary-administrated open innovation contest is preferred to internal R&D. Setting $\Pi_I^{i*} = \Pi_S^{i*}$ and solving for c_1, we have

$$c_1 = f^i(\theta) = \frac{\theta c_s c_{s1} \left(\frac{\mu-\theta}{c_s}\right)^{\frac{\mu}{\theta}} \exp\left(\frac{\beta - \beta_s - (\mu - \theta)}{\theta}\right)}{\mu (\mu - \theta)}.$$

For any given θ, if $c_1 > f^i(\theta)$, internal R&D is preferred to a self-administrated open innovation contest. Otherwise, a self-administrated open innovation contest is preferred to internal R&D.

For a trial and error project, the internal effort is optimized based on:

$$\max_{m\geq 0} \Pi_I^t(m) = \beta + \mu \ln m - c_{s2} m,$$

which has the optimal effort as $m^* = \frac{\mu}{c_{s2}}$. The optimal profit for the seeker is

$$\Pi_I^{t*} = \beta + \mu \left(\ln \frac{\mu}{c_{s2}} - 1 \right). \tag{23}$$

For a trial and error project, the seeker's problem for a self-administrated open innovation contest is

$$\max_{n\geq 2} \Pi_S^t(n) = \beta + \mu \left(\ln \frac{\mu (n-1)}{n c_2} - 1 \right) - c_s n,$$

which has the optimal solver pool size

$$n_S^{t*} = \sqrt{\frac{\mu}{c_s}}.$$

Substituting n_S^{t*}, we have

$$\Pi_S^{t*} = \beta + \mu \left(\ln \frac{\mu \left(\sqrt{\frac{\mu}{c_s}} - 1 \right)}{c_2 \sqrt{\frac{\mu}{c_s}}} - 1 \right) - \sqrt{\mu c_s}.$$

For an intermediary-administrated open innovation contest, from Theorem 1b, we have

$$\Pi_O^{t*} = \beta + \mu \left(\ln \frac{\mu (n_o - 1)}{n_o c_2} - 1 \right) - p.$$

Setting $\Pi_S^{t*} = \Pi_O^{t*}$ and simplifying, we have μ_{SO}^t as the solution of the following equation

$$\ln \frac{\sqrt{\frac{\mu}{c_s}} - 1}{\sqrt{\frac{\mu}{c_s}}} - \sqrt{\frac{c_s}{\mu}} = \ln \frac{n_o - 1}{n_o} - p,$$

whose left hand-side is increasing in μ. If $\mu \geq \mu_{SO}^t$, a self-administrated open innovation contest is preferred to an intermediary-administrated open innovation contest. Otherwise, an intermediary-administrated open innovation contest is preferred to a self-administrated open innovation contest. Setting $\Pi_I^{t*} = \Pi_O^{t*}$ and solving for c_2, we have

$$c_2 = c_2^t = \frac{(n_o - 1)\, c_{s2} \exp\left(-\frac{p}{\mu}\right)}{n_o}.$$

For any given μ, if $c_2 > c_2^t$, internal R&D is preferred to an intermediary-administrated open innovation contest. Otherwise, an intermediary-administrated open innovation contest is preferred to internal R&D. Setting $\Pi_I^{t*} = \Pi_S^{t*}$ and solving for c_2, we have

$$c_2 = g^t(\mu) = \frac{c_{s2}\left(\mu - \sqrt{\mu c_s}\right)\exp\left(-\sqrt{\frac{c_s}{\mu}}\right)}{\mu}.$$

For any given μ, if $c_2 > g^t(\mu)$, internal R&D is preferred to a self-administrated open innovation contest. Otherwise, a self-administrated open innovation contest is preferred to internal R&D. ■

Proof of Theorem 3b: For an expertise-based project with a logarithmic return function $r(e) = \theta \ln e$, and the seeker's optimal profit from internal R&D is

$$\Pi_I^{e*} = \beta_s + \theta\left(\ln\frac{\theta}{c_{s1}} - 1\right),$$

and the seeker's optimal expected profit in a fixed price open innovation contest is

$$\Pi_O^{e*} = \int_{\underline{\beta}}^{\overline{\beta}} \beta n_o F(\beta)^{n_o-1} f(\beta)\,d\beta + \theta\left(\ln\frac{\theta}{c_1} - \frac{2n_o - 1}{n_o}\right) - p.$$

Because the term $(2n_o - 1)/n_o$ inside the parenthesis, which is increasing in n_o, converges to 2 as $n_o \to \infty$, the difference between the seeker's expected profits under open innovation and internal R&D is bounded below as

$$\Pi_O^{e*} - \Pi_I^{e*} > \int_{\underline{\beta}}^{\overline{\beta}} \beta n_o F(\beta)^{n_o-1} f(\beta)\,d\beta + \theta\left(\ln\frac{\theta}{c_1} - \ln\frac{\theta}{c_{1s}}\right) - p - \beta_s - \theta.$$

If the support of the distribution function F is not bounded above, that is, $\overline{\beta} = \infty$, the first term on the right-hand side of the above inequality converges to ∞ as $n \to \infty$. Thus, for any given p, β_s, θ, c_1, and c_{1s}, there exists a high enough $n_o = \overline{n^e}$ such that the right-hand side of the above inequality is zero, which implies that open innovation is preferred to internal R&D for $n_o \geq \overline{n^e}$.

If the support of the distribution function F is bounded above, that is, $\overline{\beta} < \infty$, the difference between the seeker's expected profits under open innovation and internal R&D is bounded above as

$$\Pi_O^{e*} - \Pi_I^{e*} < \overline{\beta} + \theta\left(\ln\frac{\theta}{c_1} - \ln\frac{\theta}{c_{1s}}\right) - p - \beta_s - \frac{\theta(n_o - 1)}{n_o}.$$

Hence for any given n_o, p, θ, c_1, and c_{1s}, there exists a high enough $\beta_s = \overline{\beta_s}$ such that the right-hand side of the above inequality is zero, which implies that open innovation is preferred to internal R&D for $\beta_s \geq \overline{\beta_s}$.

For an ideation project with the logarithmic return function $r(e) = \theta \ln e$, the seeker's optimal expected profit from a fixed price open innovation contest is

$$\Pi_O^{i*} = \beta + \theta \left(\ln \frac{\theta^2 (n_o - 1)}{c_1 \mu n_o^2} - 1 \right) + \mu \ln n_o - p.$$

In the proof of Theorem 1b, we have shown that Π_O^{i*} is always increasing in n_o. Clearly, Π_O^{i*} converges to ∞ as $n_o \to \infty$. Therefore, there exists a high enough $n_o = \overline{n^i}$, such that Π_O^{i*} is equal to the seeker's optimal profit under internal R&D, Π_I^{i*}, which is independent of n_o. It implies that open innovation is preferred to internal R&D for $n_o \geq \overline{n^i}$.

For a trial and error project, the difference between the seeker's expected profits under open innovation and internal R&D is

$$\Pi_O^{t*} - \Pi_I^{t*} = \mu \left(\ln \frac{\mu (n_o - 1)}{n_o c_2} - \ln \frac{\mu}{c_{2s}} \right) - p.$$

Note that $(n_o - 1)/n_o < 1$ and $(n_o - 1)/n_o$ converges to 1 as $n_o \to \infty$. Therefore, if $c_{2s} > c_2$, there exists a high enough $n_o = \overline{n^t}$ such that the above difference is positive for $n_o \geq \overline{n^t}$, which implies that open innovation is preferred to internal R&D. However, if $c_{2s} \leq c_2$, $\Pi_O^{t*} - \Pi_I^{t*}$ is always negative, which implies that internal R&D is preferred to open innovation. ■

Proof of Corollary: The corollary follows directly from that for the logarithmic return function $r(e) = \theta \ln e$, the seeker's total R&D costs under internal R&D are θ, $\theta + \mu$, and μ for expertise-based, ideation, and trial and error projects, respectively, and the optimal awards the seeker offers in fixed price open innovation contests for expertise-based, ideation, and trial and error projects are θ, μ, and μ, respectively. ■

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