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Electronic Companion

EC.1. Supplemental discussion of modelling assumptions

EC.1.1. Deterministic but unknown valuations as an alternative modelling approach

An alternative to our specifications of consumer product choice would be to consider deterministic but unknown valuations. Such models have indeed been considered in economics and RM literature using a mechanism design approach for the monopoly case (see Harris and Raviv (1981) and Gallien (2006)). They assume that a seller controls how sales proceed (e.g., full control of a demand fill rate up to the individual consumer level). Along with Gallego and Hu (2006) and Lin and Sibdari (2007) who model sales as a stochastic process, we do not assume that the firms can explicitly control the demand fill rate. However, a competitive mechanism design approach has not been examined in RM and presents an interesting open problem.

EC.1.2. Example of specifications of the basics model elements

Consider two identical firms and two distinct segments: business (segment 1) and leisure (segment 2) travelers. Business consumers are myopic ($\beta_1 = 0$) and, at time t, their valuations of the two products are independent and distributed exponentially with equal means that increase over the course of the selling season. That is B_{t1j}, $j = 1, 2$ are exponentially distributed with equal means $a_{t1j} = \mu_1^0 + \mu_1^1 t/T$, $j = 1, 2$ where $\mu_1^1 > 0$, Leisure consumers are fully strategic ($\beta_2 = 1$) and their valuations are independent and exponentially distributed with constant mean. That is B_{t2j}, $j = 1, 2$ are exponentially distributed with equal means $a_{t2j} = \mu_2^0$, $j = 1, 2$. The distribution of $\mathbf{B}_{tr}$ captures random preferences (or uncertain perceptions of value) of segment r consumers at time t. These preferences change over time in a deterministic fashion.

EC.1.3. Discussion of the perfect information assumption in a practical context

Contexts where the perfect information assumption is relevant are becoming increasingly common with expanding capabilities in internet and other information technologies. For example, in modern online booking systems for airlines and other travel services it has become commonplace to present consumers with layouts or counts of the remaining availability of assets, and firms can keep track of the number of unique hits to their web-sites. Other examples of such systems include online ticket sales for sports and entertainment events, and online retailers. In many cases, third-party brokers (for example, travel agents) can impart knowledge about inventory levels and historical pricing patterns to consumers, and market size to companies. In addition, such agents often work with consumers individually and have a reasonable understanding of consumers' valuation levels, preferences, and purchasing behavior, and can supply this information as well. There is also the recent emergence of 'price advisor' web-sites that track and report the patterns of company pricing and other policies. This raises the interesting possibility that such price advisors could become sufficiently large and technically sophisticated to deploy 'purchasing cost management' systems to counter firms' RM systems.

EC.2. The case of high product supply

Here, we discuss the 'high product supply' situation, in which each firm has sufficient capacity to satisfy the purchasing requirements of all consumers. That is, capacity is no longer a constraint on available resources, and the only limited resource is the consumer population. We also consider only one consumer segment ($s=1$) and omit the consumer segment index in our notation. The situation discussed here provides a limiting case most favorable for strategic consumers: there is a plentiful product supply, and there is no interplay between market segments that could be exploited by the firms. All results, except for the asymptotic one, hold for either choice model.

The first result shows that the game at each time t and state $(\mathbf{y},n)$ essentially decomposes into n identical games played simultaneously between the firms and each of the remaining consumers individually.

PROPOSITION EC.1. *Let $s=1$ and suppose that there exists a unique equilibrium in pure strategies. For any t and $(\mathbf{y},n)$ such that $y_j \geq n$, $j=1,\ldots,m$ the following hold:*

$$U(t,\mathbf{y},n) = U(t,\mathbf{1},1), \tag{EC.1}$$

$$R_j(t,\mathbf{y},n) = nR_j(t,\mathbf{1},1),\ j=1,\ldots,m, \tag{EC.2}$$

$$D_j(t,\mathbf{y},n,\mathbf{p}) = nD_j(t,\mathbf{1},1,\mathbf{p}),\ \textit{for all}\ \mathbf{p},\ j=1,\ldots,m, \tag{EC.3}$$

$$p_j^*(t,\mathbf{y},n) = p_j^*(t,\mathbf{1},1),\ j=1,\ldots,m, \tag{EC.4}$$

where $\mathbf{1}=(1,1,\ldots,1)^\top$.

We can view the game as decomposed into plays of firms in the 1-consumer markets corresponding to individual consumers because the expected utilities do not depend on n (see (EC.1)) and the expected revenues are additive over the consumers (see (EC.2)). This result also shows that each consumer gains so much market power in this setting that the prices are insensitive to the remaining market size n.

A corollary of the above proposition is a particularly simple expression for the expected utility:

Corollary EC.1. *For any t and $(\mathbf{y},n)$ such that $y_j\ge n$, $j=1,\ldots,m$ we have*

$$U(t,\mathbf{y},n,\mathbf{p}) = \bar\lambda E_{\mathbf{B}_t}\big[\mathbf{x}^*(t,\mathbf{y},n,\mathbf{p},\mathbf{B}_t)^\top\big(\mathbf{B}_t-\mathbf{p}-\mathbf{1}\beta U(t+1,\mathbf{y},n)\big)\big]+\beta U(t+1,\mathbf{1},1). \tag{EC.5}$$

This results underscores the independent role of model parameters $\bar\lambda$ and β. The maximum shopping intensity $\bar\lambda$ determines how quickly consumers can build up expected utility, while $1-\beta$ determines the speed of its deterioration.

Expression (EC.5) immediately implies natural monotonic properties for the expected utility (decreasing in time) and the demand intensity (increasing in time, given that prices are at the same level) when consumers are fully strategic:

Corollary EC.2. *If $\beta=1$, then for any t and $(\mathbf{y},n)$ such that $y_j\ge n$, $j=1,\ldots,m$ we have $U(t,\mathbf{y},n)\ge U(t+1,\mathbf{y},n)$ and $D_j(t,\mathbf{y},n,\mathbf{p})\le D_j(t+1,\mathbf{y},n,\mathbf{p})$, $j=1,\ldots,m$.*

Finally, we obtain a uniform bound on the expected utility of a consumer with limited strategic behavior $\beta<1$ in the stationary valuation distribution case:

Proposition EC.2. *Let $\beta<1$ and suppose that the distribution of $\mathbf{B}_t$ does not depend on time. Then, for any t and $(\mathbf{y},n)$ such that $y_j\ge n$, $j=1,\ldots,m$:*

$$U(t,\mathbf{y},n,\mathbf{p}) \le \bar U = \frac{\bar\lambda\sum_{j=1}^m E[B_j]}{1-\beta}. \tag{EC.6}$$

Parameter $\bar{\lambda}$ determines how quickly an eager consumer can complete a purchase, while $1-\beta$ measures a relative loss of utility per decision period due to waiting. Thus, when $\beta < 1$, the ratio $\bar{\lambda}/(1-\beta)$ measures the effect of procrastination by consumers in the context of their strategic behavior. A consumer with a high ratio (less procrastination) is more likely to obtain a high utility value within his/her effective planning horizon which is determined by the magnitude of discount factor β. A consumer whose effective planning horizon is too short to encounter a significant number of genuine purchase opportunities is, in effect, myopic. As a consequence, the above proposition can be used to evaluate the potential importance of consumer strategic behavior for given distributions and parameter values. Recall that certainty equivalent $Q(t,\mathbf{y},n)$ of a future purchase by a strategic customer appears inside the comparison of the form $B_j \geq p_j + Q(t,\mathbf{y},n)$ in the purchase probability $D_j(t,\mathbf{y},n,\mathbf{p})$. Thus, if the magnitude of $Q(t,\mathbf{y},n)$ is comparable to 'typical values' of B_j, then the effects of strategic behavior cannot be ignored. If consumer valuations of different products are identically distributed, then, if we use $E[B]$ as a typical valuation value, the condition under which strategic behavior is important can be expressed as

$$E[B] \leq \beta\bar{U} = \frac{\beta\bar{\lambda}mE[B]}{1-\beta},$$

or, equivalently, as

$$\beta \geq \frac{1}{1+\bar{\lambda}m}.$$

The condition has the form of a lower bound on β which increases as $\bar{\lambda}m$ decreases. This can be interpreted as follows: since $\bar{\lambda}m$ is an upper bound on the probability that a particular consumer purchases a unit of any of the products in the given time period, β has to be sufficiently large for an appreciable utility to be accumulated during an effective planning horizon of a consumer.

Asymptotic analysis of a symmetric oligopolistic equilibrium

We now examine the effect of a long time horizon in relation to the number of firms m. This analysis applies to situation in which the market consists of a single segment, each firm can supply the entire market, the firms are identical, $\beta < 1$, consumers behave in agreement with the multiple choice model, and the distribution of the valuations is stationary exponential: $f_t(b) = \frac{1}{a}e^{-\frac{b}{a}}$. With stationarity, longer time horizons can be described either by increasing T or by letting t be negative (and go to $-\infty$ in the limit). To simplify notation, we use the second possibility. Based on the results of Proposition EC.1, it is enough to consider a situation with $N = Y_1 = \ldots = Y_m = 1$. Thus, to simplify notation we also drop n and $\mathbf{y}$. Moreover, due to the symmetry of the equilibrium, we can drop the index j from the equilibrium prices and revenues. By Proposition EC.2, $U(t) \leq \bar{U}$. Since $\Delta_j R_j(t+1, \mathbf{y}, n) = R(t+1, \mathbf{1}, 1) = R(t+1) \geq 0$, the equilibrium prices can be expressed as $p^*(t) = a + R(t+1) + c$, $j = 1, \ldots, m$, where c is the unit cost (identical for all firms). Let us now turn to the equilibrium revenues, which can be expressed, after substituting the equilibrium prices, as

$$R(t) = R(t+1) + \bar{\lambda} e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}} (a - (m-1)R(t+1)).$$

The following proposition summarizes the asymptotic results.

Proposition EC.3. *For the case $m > 1$, under the conditions of this subsection, the sequence $R(t)$ is decreasing in time and $R(t) \leq \frac{a}{m-1}$. Moreover $\lim_{t\to-\infty} R(t) = \frac{a}{m-1}$. The equilibrium prices $p^*(t)$ are also decreasing in time and $p^*(t) \leq \frac{ma}{m-1} + c$. Moreover, $\lim_{t\to-\infty} p^*(t) = \frac{ma}{m-1} + c$. For the case $m = 1$, while $R(t)$ and $p^*(t)$ are decreasing, we also have $R(t) \to \infty$ and $p^*(t) \to \infty$ as $t \to -\infty$.*

The above proposition shows that monopoly and competitive situations result in markedly different asymptotic behavior. Given a sufficiently long selling horizon, a monopolist firm can start with much higher prices than firms in an oligopoly. This result also shows that strategic behavior by consumers who have limited foresight ($\beta < 1$) becomes less important as the length of a selling season increases.

EC.3. Proofs

Proof of Proposition 2 According to "perfect foresight" assumption, consumers can compute the probability of every possible event in the current information state and their expected utility in the corresponding states. It then is a simple matter to compute the expected utility in the current information state. Let

$$f_{tr}^{j}(\mathbf{b}) = \begin{cases} \frac{f_{tr}(\mathbf{b})}{P(\mathbf{B}_{tr} \in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}))} & \text{if } \mathbf{b} \in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}) \\ 0 & \text{otherwise} \end{cases}$$

be the conditional valuation density for a segment r consumer at time t given that this consumer prefers an item j. According to the assumption that "consumer perceptions are conditional on purchase", consumers update their valuation distribution to this density if a purchase does occur. Given that a consumer prefers an item j and acquires it in the current decision period, he/she derives the conditional expected utility

$$\int_{\mathbf{b}\in\mathbb{R}^m} (b_j - p_j) f_{tr}^{j}(\mathbf{b}) d\mathbf{b} = \frac{1}{P(\mathbf{B}_{tr} \in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}))} \int_{\mathbf{b}\in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})} (b_j - p_j) f_{tr}(\mathbf{b}) d\mathbf{b}.$$

Recall that $A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})$ is, by definition (2), the set of all valuation vectors for which the purchase of product j is preferred to a purchase of any other product or a purchase delay by a segment r consumer. The outcome of the customer purchase of item j occurs with probability $\lambda_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})$ given by (3).

Other possible events include a sale of item j to a different consumer of some segment r' (perhaps coinciding with r) with probability $\lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p})$. In this case, the expected present value of utility of a segment r consumer at time $t+1$ is $\beta_r U_r(t+1,(y_j-1,\mathbf{y}_{-j}),(n_{r'}-1,\mathbf{n}_{-r'}))$. It is also possible that no sale occurs, and the expected present value at time $t+1$ in this case is $\beta_r U_r(t+1,\mathbf{y},\mathbf{n})$. Adding over all these possibilities, we find the expected utility in the current information state

$$\begin{aligned} U_r(t,\mathbf{y},\mathbf{n},\mathbf{p}) = \bar{\lambda} \sum_{j=1}^{m} & \int_{\mathbf{b}\in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})} (b_j - p_j) f_{tr}(\mathbf{b}) d\mathbf{b} \\ & + \beta_r \sum_{r'=1}^{s} (n_{r'} - I(r=r')) \sum_{j=1}^{m} \lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p}) U_r(t+1,(y_j-1,\mathbf{y}_{-j}),(n_{r'}-1,\mathbf{n}_{-r'})) \\ & \quad + \beta_r \Big(1 - \lambda_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}) - \sum_{r'=1}^{s} (n_{r'} - I(r=r')) \sum_{j=1}^{m} \lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p})\Big) U_r(t+1,\mathbf{y},\mathbf{n}), \end{aligned}$$

where $I(A)=1$ when A holds and 0 otherwise. After collecting terms corresponding to the same pairs (r',j), we obtain the recursive relation (4) for the expected utility.

Proof of Theorem 1. The proof is by backward induction on t. The payoffs are uniquely defined for T due to the boundary conditions. Suppose that for any $(\mathbf{y},\mathbf{n})$ the payoffs of firms, $R_j(t+1,\mathbf{y},\mathbf{n})$, $j=1,\ldots,m$, and expected utilities of consumers, $U_r(t+1,\mathbf{y},\mathbf{n})$, in the subsequent decision period $t+1$ are uniquely defined. We can assume that the consumers respond as described in Proposition 1, and they need not be treated explicitly in this reasoning. Thus, the pure strategy payoffs in $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ are defined by (7). Moreover, expression (7) is continuous in $\mathbf{p}$ and the strategy sets of game $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ are compact due to assumption (A) (closed intervals of the form $[0,\bar{p}]$). We next draw upon Theorem 1.3 on page 35 in Fudenberg and Tirole (1991) which is restated here for completeness:

THEOREM EC.1. *Consider a strategic-form game whose strategy spaces are nonempty compact subsets of a metric space. If the payoff functions are continuous, then there exists a Nash equilibrium in mixed strategies.*

By the above theorem, game $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ has an equilibrium in mixed strategies. By assumption (B), even if there are multiple equilibria, only one of them can be implemented. Thus, the resulting equilibrium payoffs in $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ are uniquely defined.

Proof of Proposition 3. Recall that, by assumption, consumers average their shopping intensity and the expected utility over their valuation distribution. Suppose that a consumer in segment r, in addition to the state information, $(\mathbf{y},\mathbf{n},\mathbf{p})$ also knows the valuation vector $\mathbf{B}_{tr}$. Let the expected utility for this consumer be then denoted as $U_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr})$. The other consumers follow equilibrium responses denoted as $\boldsymbol{\lambda}_{r'}(t,\mathbf{y},\mathbf{n},\mathbf{p})$. While the responses of other consumers are not observable, the consumer under consideration can assume that other consumers act in the same way as he does, given their segment, and that their valuations follow known probability distributions. The optimal expected utility in case of known valuation is then computed by taking expectations over all possible purchase events:

$$U_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr}) = \max_{\mathbf{x}\geq\mathbf{0},\,\|\mathbf{x}\|_q\leq\bar{\lambda}} \Bigg\{ \bar{\lambda}\sum_{j=1}^{m} x_j(B_{trj}-p_j)$$

$$+\beta_r \sum_{r'=1}^{s}(n_{r'}-I(r=r'))\sum_{j=1}^{m}\lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p})U_r(t+1,(y_j-1,\mathbf{y}_{-j}),(n_{r'}-1,\mathbf{n}_{-r'}))$$
$$+\beta_r\Big(1-\bar{\lambda}\sum_{j=1}^{m}x_j-\sum_{r'=1}^{s}(n_{r'}-I(r=r'))\sum_{j=1}^{m}\lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p})\Big)U_r(t+1,\mathbf{y},\mathbf{n})\Big\}.$$

Since the expression under the max is linear in the strategies of all consumers, we can collect the terms depending on the consumer's strategy and take all other terms outside the maximization operation resulting in

$$U_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr})=\bar{\lambda}\max_{\mathbf{x}\geq\mathbf{0},\,\|\mathbf{x}\|_q\leq\bar{\lambda}}\big\{\mathbf{x}^\top\big(\mathbf{B}_{tr}-\mathbf{p}-\mathbf{1}\beta_r U_r(t+1,\mathbf{y},\mathbf{n})\big)\big\}$$
$$-\beta_r\sum_{r'=1}^{s}(n_{r'}-I(r=r'))\boldsymbol{\lambda}_{r'}(t,\mathbf{y},\mathbf{n},\mathbf{p})^\top\boldsymbol{\Delta}U_r(t+1,\mathbf{y},\mathbf{n})+\beta_r U_r(t+1,\mathbf{y},\mathbf{n}),\quad\text{(EC.7)}$$

where we change to vector notation instead of summation over j. We see that the maximizer (the optimal intensity for given valuation vector) does not depend on the responses of other consumers, and is given by (8) for $q=\infty$ as a solution vector for a linear maximization problem with simple bound constraints. For $q=1$, the solution is given by (9) in all information states where $\operatorname{Argmax}\{\tilde{b}_j-p_j : j=1,\dots,m\}$ is a singleton. The set of states where $\operatorname{Argmax}\{B_{trj}-p_j : j=1,\dots,m\}$ is not a singleton has measure 0. While the optimal $\mathbf{x}$ may not be unique due to ties in the coefficients of the linear objective, the expected values do not depend on this nonuniqueness because the distribution of valuations is continuous. The expression (10) for the expected utility is obtained by taking the expected value of (EC.7) over $\mathbf{B}_{tr}$.

Proof of Theorem 2. When $q=\infty$, the expected future payoff of firm j given by the expression (7) is separable in the components of the price vector $\mathbf{p}$. The equilibrium strategy of firm j is to maximize the quantity

$$g_j(p_j)=\sum_{r=1}^{s}D_{rj}^{M}(t,\mathbf{y},\mathbf{n},p_j)(p_j-\Delta_{rj}R_j(t+1,\mathbf{y},\mathbf{n})-c_j)\qquad\text{(EC.8)}$$

with respect to p_j and is independent of the strategies of other firms at the same time instance. Since $g_j(p_j)$ is a continuous function on a closed interval $[0,\bar{p}]$, it attains its maximum value. Thus, there is a pure strategy equilibrium in $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ and the resulting equilibrium payoffs are uniquely defined.

Proof of Theorem 3. We first establish the following lemma that describes the equilibrium strategies of the firms and the corresponding equilibrium payoffs in the multiple choice case:

LEMMA EC.1. *Let $q = \infty$ and $s = 1$, and consider decision period t. Suppose that there exists a unique equilibrium in all subsequent periods $t+1, t+2, \ldots, T-1$, and the distribution of valuations $\mathbf{B}_t$ satisfies the logconcavity and regularity assumptions (C) and (D). Then there exists an equilibrium in $\mathcal{G}(t, \mathbf{y}, n)$ such that firm j strategy is*

$$p_j^*(t, \mathbf{y}, n) = \underset{p_j \geq 0}{\arg\max}\, D_j^M(t, \mathbf{y}, n, p_j)(p_j - \Delta_j R_j(t+1, \mathbf{y}, n) - c_j). \tag{EC.9}$$

This strategy selection is unique if there is $p_j > \Delta_j R_j(t+1, \mathbf{y}, n) + c_j$ such that $D_j^M(t, \mathbf{y}, n, p_j) > 0$. Moreover, the equilibrium payoffs of firms are defined uniquely by $R_j(t, \mathbf{y}, n) = R_j(t, \mathbf{y}, n, \mathbf{p}^*(t, \mathbf{y}, n))$.

The condition of the lemma that there is a $p_j > \Delta_j R_j(t+1, \mathbf{y}, n) + c_j$ such that $D_j^M(t, \mathbf{y}, n, p_j) > 0$ means that either there is no price $\bar{p}_j$ (typically called a *shutdown price*) such that $D_j^M(t, \mathbf{y}, n, \bar{p}_j) = 0$, or $\bar{p}_j$ exceeds $\Delta_j R_j(t+1, \mathbf{y}, n) + c_j$.

When $q = \infty$ and $s = 1$ the equilibrium strategy of each firm is to maximize the quantity $g_j(p_j) = D_j^M(t, \mathbf{y}, n, p_j)(p_j - \Delta_j R_j(t+1, \mathbf{y}, n) - c_j)$, which is a special case of (EC.8). Note that no firm will set its price below $p_j = \Delta_j R_j(t+1, \mathbf{y}, n) + c_j$. Because of the regularity condition, $\lim_{p_j \to \infty} g_j(p_j) = 0$ and, consequently, there exists a maximizer of $g_j(p_j)$ on $[0, +\infty)$. Note also that $g_j(p_j)$ is a logconcave function of p_j for $p_j \geq \Delta_j R_j(t+1, \mathbf{y}, n) + c_j$ as a product of logconcave functions. Moreover, it is strictly logconcave where it is positive. If there is a $p_j > \Delta_j R_j(t+1, \mathbf{y}, n) + c_j$ such that $D_j^M(t, \mathbf{y}, n, p_j) > 0$, then $g_j(p_j)$ will have a unique maximizer.

Now, combining Propositions 3 and EC.1 with the fact that payoffs are uniquely defined as 0 in all 'terminal' states, we obtain the claim of Theorem 3.

Proof of Proposition EC.1. The proof is by backward induction on t. Relations (EC.1) and (EC.2) hold for $t = T$. Suppose that (EC.1)-(EC.2) hold for $t+1$ and all $(\mathbf{y}, n)$ such that

$y_j \geq n$, $j = 1, \ldots, m$. We now prove that (EC.1)-(EC.4) for t. Relation (EC.1) for $t+1$ implies that $\mathbf{\Delta} U(t+1, \mathbf{y}, n) = 0$, resulting in

$$U(t, \mathbf{y}, n, \mathbf{p}) = E_{\mathbf{B}_t}\left[\mathbf{x}^*(t, \mathbf{y}, n, \mathbf{p}, \mathbf{B}_t)^\top \left(\mathbf{B}_t - \mathbf{p} - \mathbf{1}\beta U(t+1, \mathbf{y}, n)\right)\right] + \beta U(t+1, \mathbf{y}, n)$$

for all $\mathbf{y}, n$ such that $y_j \geq n$, $j = 1, \ldots, m$. Moreover, since $U(t+1, \mathbf{y}, n) = U(t+1, \mathbf{1}, 1)$, we also have

$$\mathbf{x}^*(t, \mathbf{y}, n, \mathbf{p}, \mathbf{B}_t) = \mathbf{x}^*(t, \mathbf{1}, 1, \mathbf{p}, \mathbf{B}_t), \tag{EC.10}$$

and it follows that

$$U(t, \mathbf{y}, n, \mathbf{p}) = \bar{\lambda} E_{\mathbf{B}_t}\left[\mathbf{x}^*(t, \mathbf{1}, 1, \mathbf{p}, \mathbf{B}_t)^\top \left(\mathbf{B}_t - \mathbf{p} - \mathbf{1}\beta U(t+1, \mathbf{1}, 1)\right)\right] + \beta U(t+1, \mathbf{1}, 1) = U(t, \mathbf{1}, 1).$$

From (EC.10) we also conclude that (EC.3) holds. The relation $R_j(t+1, \mathbf{y}, n) = nR_j(t+1, \mathbf{1}, 1)$ for all $\mathbf{y}, n$ such that $y_j \geq n$, implies that, for any j', $\Delta_{j'} R_j(t+1, \mathbf{y}, n) = R_j(t+1, \mathbf{1}, 1) = \Delta_{j'} R_j(t+1, \mathbf{1}, 1)$. From (7) and the above, we get

$$\begin{aligned} R_j(t, \mathbf{y}, n, \mathbf{p}) &= nD_j(t, \mathbf{1}, 1, \mathbf{p})(p_j - \Delta_j R_j(t+1, \mathbf{1}, 1) - c_j) \\ &\quad - \sum_{j' \neq j} nD_{j'}(t, \mathbf{1}, 1, \mathbf{p}) \Delta_{j'} R_j(t+1, \mathbf{1}, 1) + nR(t+1, \mathbf{1}, 1) = nR(t, \mathbf{1}, 1, \mathbf{p}), \end{aligned}$$

and (EC.2) follows.

Proof of Proposition EC.2. Because of (EC.1), it is enough to prove the statement for $\mathbf{y} = \mathbf{1}$ and $n = 1$. Observe that, under either choice model, $x_j^*(t, \mathbf{1}, 1, \mathbf{p}, \mathbf{b}) \leq I(b_j - p_j \geq \beta U(t+1, \mathbf{1}, 1))$. It follows that

$$U(t, \mathbf{1}, 1, \mathbf{p}) \leq \bar{\lambda} \sum_{j=1}^{m} E[(B_{tj} - p_j - \beta U(t+1, \mathbf{1}, 1))^+] + \beta U(t+1, \mathbf{1}, 1).$$

Since $(B_{tj} - p_j - \beta U(t+1, \mathbf{1}, 1))^+ \leq B_{tj}$, we have

$$U(t, \mathbf{1}, 1, \mathbf{p}) \leq \bar{\lambda} \sum_{j=1}^{m} E[B_{tj}] + \beta U(t+1, \mathbf{1}, 1).$$

Recall now that the distribution of $\mathbf{B}_t$ is stationary, and consider the quantity $\bar{U}$ defined in the statement. We can prove by backward induction on time that $U(t,\mathbf{1},1,\mathbf{p}) \le \bar{U}$. Indeed, the statement holds for $t=T$ since $U(T,\mathbf{1},1,\mathbf{p})=0$. Suppose it holds for $t+1$. Then we have

$$U(t,\mathbf{1},1,\mathbf{p}) \le \bar{\lambda}\sum_{j=1}^{m} E[B_j] + \beta\frac{\bar{\lambda}\sum_{j=1}^{m} E[B_j]}{1-\beta} = \bar{U}.$$

Since this inequality holds for any prices, including the equilibrium ones, it follows that the equilibrium utility satisfies $U(t,\mathbf{1},1) \le \bar{U}$.

Proof of Proposition EC.3. Indeed, $R(T)=0$, therefore the bound holds for $t=T$ and $R(T-1) > R(T)$ since $a-(m-1)R(T) > 0$. Similarly, if $R(t+1) < \frac{a}{m-1}$, then $a-(m-1)R(t+1) > 0$ and $R(t) > R(t+1)$. On the other hand, since $(m-1)\bar{\lambda} < m\bar{\lambda} \le 1$, and $e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}} < 1$, it follows that

$$R(t) < \left(\frac{a}{m-1} - R(t+1)\right) + R(t+1) < \frac{a}{m-1}.$$

Finally, since $R(t+1)$ and $U(t+1)$ are bounded from above, the quantity $e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}}$ is bounded from below by some value v. Thus, $R(t)$ is closer to $\frac{a}{m-1}$ than $R(t+1)$ by no less than a fraction $\bar{\lambda}v$ of $\frac{a}{m-1} - R(t+1)$. Therefore, $\lim_{t\to-\infty} R(t) = \frac{a}{m-1}$. The implication for the equilibrium price is that $\lim_{t\to-\infty} p^*(t) = \frac{ma}{m-1} + c$. In the monopoly case, the recursion $R(t) = \bar{\lambda}e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}}a + R(t+1)$ leads to $R(t)\to\infty$ and $p^*(t)\to\infty$ as $t\to-\infty$.

EC.4. Demand functions and best-response mappings for the exponential valuation distribution and numerical issues

The choice of valuation distributions in the exponential form results in a very efficient numerical implementation (both for the multiple and the specific choice cases).

The purchase probabilities (demand functions) in the specific and multiple choice cases are obtained by substituting the exponential density (12) into expressions (6) and (11), respectively. The multiple consumer choice model results in segment r demand for product j of the form:

$$D^M_{rj}(t,\mathbf{y},\mathbf{n},p_j) = n_r\bar{\lambda}e^{-\frac{p_j+Q_r(t,\mathbf{y},\mathbf{n})}{a_{rj}}}.$$

For the case of a one-segment market, the resulting equilibrium prices are then given by the closed form expression $p_j^*(t,\mathbf{y},n) = \max\{a_j + \Delta_j R_j(t+1,\mathbf{y},n),\, 0\}$. This result can be obtained immediately by applying the first-order optimality conditions in expression (EC.9) established in Lemma EC.1. This solution is in closed form and can be computed very efficiently. In the multiple-segment case, a numerical maximization of the objective

$$\sum_{r=1}^{s} n_r \bar{\lambda} e^{-\frac{p_j + Q_r(t,\mathbf{y},\mathbf{n})}{a_{rj}}} (p_j - \Delta_j R_j(t+1,\mathbf{y},\mathbf{n}))$$

with respect to p_j is required. However, this can also be done efficiently using local search methods (like Newton-Armijo) from multiple starting points since the problem is one-dimensional. A simple search over sufficiently fine mesh also provides an acceptable approximate alternative.

For the specific consumer choice model we can use conditioning on the value of $B_{tj} = b_j$ in expression (6) to obtain (since we treat the case of $s = 1$, we can omit the r index) the demand functions of the form:

$$D_j^S(t,\mathbf{y},n,\mathbf{p}) = n\bar{\lambda} \int_{p_j + Q(t,\mathbf{y},n)}^{+\infty} \frac{1}{a_j} e^{-\frac{b}{a_j}} \prod_{j' \neq j} \left(1 - e^{-\frac{b - p_j + p_{j'}}{a_{j'}}}\right) db. \tag{EC.11}$$

Consider the duopoly case and let $\tilde{a} = \left(\frac{1}{a_1} + \frac{1}{a_2}\right)^{-1}$. The expression in the integral (EC.11) simplifies to

$$\begin{aligned}
D_j^S(t,\mathbf{y},n,\mathbf{p}) &= n\bar{\lambda} \int_{p_j + Q(t,\mathbf{y},n)}^{+\infty} \frac{1}{a_j} e^{-\frac{b}{a_j}} \left(1 - e^{-\frac{b - p_j + p_{j'}}{a_{j'}}}\right) db \\
&= \frac{n\bar{\lambda}}{a_j} \int_{p_j + Q(t,\mathbf{y},n)}^{+\infty} \left(e^{-\frac{b}{a_j}} - e^{-\frac{b}{\tilde{a}}} e^{\frac{p_j - p_{j'}}{a_{j'}}}\right) db \\
&= \frac{n\bar{\lambda}}{a_j} \left(a_j e^{-\frac{p_j + Q(t,\mathbf{y},n)}{a_j}} - \tilde{a} e^{-(p_j + Q(t,\mathbf{y},n))\left(\frac{1}{a_{j'}} + \frac{1}{a_j}\right)} e^{\frac{p_j - p_{j'}}{a_{j'}}}\right) \\
&= n\bar{\lambda} e^{-\frac{p_j + Q(t,\mathbf{y},n)}{a_j}} \left(1 - \frac{a_{j'}}{a_j + a_{j'}} e^{-\frac{p_{j'} + Q(t,\mathbf{y},n)}{a_{j'}}}\right), \; j' \neq j.
\end{aligned}$$

We now turn to derivation of the best response mapping in state $(t,\mathbf{y},n)$. The expected profits of firm j are obtained by substituting the derived demand functions into expression (7):

$$R_j(t,\mathbf{y},n,\mathbf{p}) = n\bar{\lambda} e^{-\frac{p_j + Q(t,\mathbf{y},n)}{a_j}} \left(1 - \frac{a_{j'}}{a_j + a_{j'}} e^{-\frac{p_{j'} + Q(t,\mathbf{y},n)}{a_{j'}}}\right) (p_j - \Delta_j R_j(t+1,\mathbf{y},n) - c_j)$$

$$-n\bar{\lambda}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\left(1-\frac{a_j}{a_j+a_{j'}}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\right)\Delta_{j'}R_j(t+1,\mathbf{y},n)+R(t+1,\mathbf{y},n).$$

To find the best response of firm j (the maximizer of this expression with respect to p_j for a given $p_{j'}$) we apply the first-order optimality condition. The derivative of the above expression with respect to p_j is equal to

$$\frac{\partial R_j}{\partial p_j}=n\bar{\lambda}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\left(1-\frac{a_{j'}}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\right)\left(1-\frac{p_j-\Delta_jR_j(t+1,\mathbf{y},n)-c_j}{a_j}\right)$$
$$-n\bar{\lambda}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\frac{1}{a_j+a_{j'}}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\Delta_{j'}R_j(t+1,\mathbf{y},n).$$

Setting this expression equal to 0 and dividing by

$$\frac{n\bar{\lambda}}{a_j}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\left(1-\frac{a_{j'}}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\right)$$

(a positive quantity), we obtain

$$-p_j+a_j+\Delta_jR_j(t+1,\mathbf{y},n)+c_j-\frac{\frac{a_j}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\Delta_{j'}R_j(t+1,\mathbf{y},n)}{1-\frac{a_{j'}}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t+1,\mathbf{y},n)}{a_{j'}}}}=0.$$

The solution of this equation in terms of p_j exists and is unique. The left-hand-side is positive (negative) for values of p_j less (more) than the solution value. Note also that if this solution is negative then the left-hand-side is negative for $p_j=0$. Since the sign of the left-hand-side coincides with the sign of the derivative of $R_j(t,\mathbf{y},n,\mathbf{p})$ with respect to p_j, a negative solution means that the derivative is negative for all $p_j\geq 0$. We conclude that the best response of firm j has the form

$$p_j(p_{j'})=\max\left\{a_j+\Delta_jR_j(t+1,\mathbf{y},n)-\frac{a_j\Delta_{j'}R_j(t+1,\mathbf{y},n)}{(a_j+a_{j'})e^{(p_{j'}+Q(t,\mathbf{y},n))/a_{j'}}-a_{j'}},0\right\},\ j'\neq j.$$

With best-response mappings in hand, we can use best-response iteration to find the equilibrium (it converged in our numerical experiments).

To solve for equilibrium we apply the dynamic programming principle and start in decision period $T-1$ (expected utilities and revenues are given as 0 at time T by the boundary conditions). Once the equilibrium prices for stage t are found, it is a matter of a simple application of equations (7) and (10) (where we substitute equilibrium prices for $\mathbf{p}$) to propagate calculations to stage $t-1$. CPU times for a single equilibrium calculation of the scale discussed in our numerical section are of the order of a few seconds to a few minutes on a modern workstation.

EC.5. Additional numerical illustrations

EC.5.1. Effects of asymmetry in consumer willingness to pay and firms' capacity on equilibrium

When we examine the effect of asymmetry in a_j in Table EC.1, we see that the first firm (whose product is, on average, valued by the consumers twice as much as the competing product) roughly doubles its revenues in the case of myopic consumers and more than doubles in the case of strategic consumers. In contrast, the second firm is affected only slightly by a better competitor product in the myopic consumer case, but it can be seen to suffer a significant loss in the strategic consumer case. This suggests that competition in quality could intensify in a market with strategic consumers. We can also see that higher capacity can have either positive or negative effects. When the population size $N \geq 50$ and the consumers are myopic, the second firm benefits from having a higher capacity. However, in the case of strategic consumers, the benefit only shows at $N = 70$ and 80, and it is much smaller than the benefit in the myopic case in relative terms. We can explain this as a negation of the positive effects of higher capacity by an increase in strategic behavior of consumers. The first firm (with a smaller capacity) usually suffers from the higher capacity of the competitor. The negative effects on its revenues are more pronounced (in relative terms) in the case of strategic consumers.

EC.5.2. Effects of strategic behavior on prices

Since the most profound differences in equilibrium are observed in the asymmetric situation of $a_1 = 2$, $a_2 = 1$, we conducted a pricing policy simulation to understand how this asymmetry affects the policies. Figure EC.3 shows representative price sample paths, and Figure EC.4 shows the averages of price sample paths over 10,000 simulations of the equilibrium policy. The presence of strategic consumers leads to less variability in prices of both firms – flatter graphs of prices over time. Moreover, the weak firm is more affected by this phenomenon and consistently avoids price increases. The pricing strategy of the strong firm is more flexible, but, in the strategic case, its price exhibits significant variability later in the selling season than in the myopic case. This is explained by the uncertainty in demand. In the strategic case, the strong firm may have an option of increasing the price in the end of the season, but only if its sales proceeded well up to that point. The weak firm does not have many opportunities to increase prices and usually carries too much product in the end of the season. (After one or two seasons like this, a weak firm would presumably lower production.)

EC.5.3. Effects of strategic behavior when only some consumers are strategic

Significant effects of strategic behavior persist even if some consumers are myopic. To model such situations we use two market segments: one with fully strategic consumers ($\beta = 1$) and another with myopic consumers ($\beta = 0$). The segments are otherwise identical. The contours of the percentage difference in revenues between the partially strategic and completely myopic cases are shown in Figure EC.2 for symmetric duopoly with $Y_1 = Y_2 = 20$ and monopoly with initial capacity $Y_1 = 40$. Again, we see that the effect of strategic behavior is strongest when the firms divide the market ($N = 40$). Given the fraction of myopic consumers, the effect of strategic behavior in the monopoly is independent of population size if $N \leq Y_1 = 40$. This occurs because the monopolist with a higher capacity than the market cannot induce competition between strategic consumers (while it is possible to do so in a duopoly for a lower-capacity firm in information states with highly asymmetric capacity).

EC.5.4. Losses from deviating from equilibrium by assuming wrong competitor capacity

Our model assumes the ideal case of perfect information. It is also important to understand how departures from this assumption might affect revenues. Thus, we examine the effect of deviation from equilibrium in duopoly if one firm wrongly estimates competitor capacity. The capacity of a competitor may be the hardest to know exactly in practice. The same will likely be true for consumers; however, to focus on the consequences of a firm's estimation error, ceteris paribus, we continue to allow full information for consumers. Thus, we take the same base cases in terms of a_j's (and with $Y_1 = Y_2 = 20$) and compute the expected revenues in situations when one of the firms consistently underestimates or overestimates the current remaining capacity of the competitor by 5 units (25% of the initial capacity value). In this experiment, we use a multiple choice model. We present the loss in revenue relative to the equilibrium value as a function of the population size in Figure EC.5. The figure contains four plots corresponding to each possible combination of myopic/fully strategic consumers and underestimation/overestimation of the competitor capacity. The symmetric, weak and strong scenarios have the same meaning as in the previous subsection. In the case of myopic consumers, the relative loss in all three scenarios is identical. In the case of underestimation, the maximum loss occurs for $N = 30$ and reaches 5%. In the case of overestimation, the maximum loss occurs for $N = 40$ and is only 1.5%. On the other hand, we see a significant difference in losses when the consumers are strategic. The strong firm is the least affected by either under- or overestimation (the loss is well under 0.5% for all values of N), while the weak firm is the most affected (the loss of almost 2% in the worst case). We see that in case of error, it is somewhat better to underestimate than to overestimate when the population size equals or exceeds the combined capacity of the firms (40). In contrast, we see that overestimation is less serious than underestimation in the case of lower population size. Thus, ceteris paribus, if firms do not know competitor capacities exactly, they should prefer to overestimate when the population size is small and underestimate when it is large.

EC.5.5. The effects of strategic behavior on equilibrium in the case of two market segments

In this subsection, we illustrate the model in the case of symmetric duopoly with $Y_1 = Y_2 = 20$ and two equally sized market segments. In the reference case, segment 1 has $a_{1j} = 1$ and segment 2 – $a_{2j} = 2$. The following scenarios are studied: both segments are myopic, segment 2 (high valuation) is strategic while segment 1 (low valuation) is myopic, segment 2 is myopic while segment 1 is strategic, and both segments are strategic. Figure EC.6 shows the effect of segment size on equilibrium prices in the initial state, the strategic behavior effect on equilibrium revenues (% revenue difference), and the relative utility gain/loss for consumers of each segment when another segment becomes strategic. The top plot shows the graphs of initial price for each of the four scenarios. As the segment sizes grow, the initial prices start to increase in all four situations. However, this occurs for larger segment sizes when the high-valuation segment is strategic, and the prices are much lower when the high-valuation segment (or both) are strategic. When only low valuation consumers are strategic and the segment sizes are small, the initial price is higher than in the completely myopic case. This can be explained by attempts of the firms to initially price low-valuation consumers out of the market. On the second plot, the effects of high-valuation segment strategic behavior is much stronger across all market sizes than those of low-valuation segment strategic behavior, and the revenue drop is the largest for $N_1 = N_2 = 20$ which is consistent with our results for one-segment symmetric duopoly (where the ratio is the lowest for $N = 40$). Finally, the third plot shows that high-valuation consumers always gain when low-valuation consumers become strategic while low valuation consumers gain only if the segment sizes are sufficiently large.

In the final experiment, we study the properties of equilibrium policies for two distinct market segments in a symmetric duopoly by means of simulation. We examine the same four scenarios in terms of strategic behavior as above. Figure EC.7 shows plus/minus one standard deviation ranges around the averages for the price sample paths over 10,000 simulations for the case of $a_{1j} = 1$, $a_{2j} = 2$. A fundamental difference in policy realizations between the scenarios of strategic and myopic high valuation consumers is that the prices lose variability and become much flatter overall when the high valuation consumers are strategic. An additional reduction in variability can be seen when both segments are strategic. If high valuation consumers are myopic, we see a decreasing trend of an average price realization. An early deviation from the downward trend when only low valuation consumers are strategic can be explained by the attempts of firms to sell primarily to high valuation myopic consumers in the beginning of the selling season. The low valuation consumers are priced out of the market until the second half of the selling season when their behavior resemble myopic behavior (because of the lower expected utility).

EC.6. Tables and Figures

		Equilibrium with multiple choice			Equilibrium with specific choice		
N	β	Symmetric	$a_1 = 2a_2$	$Y_1 = Y_2/2$	Symmetric	$a_1 = 2a_2$	$Y_1 = Y_2/2$
20	0	17.4	(34.7, 17.4)*	(17.4, 17.4)	16.1	(33.2, 15.5)	(16.1, 16.1)
20	1	11.7	(31.7, 7.3)	(11.7, 11.7)	11.3	(31.3, 6.7)	(11.3, 11.3)
30	0	25.5	(51.0, 25.5)	(25.9, 25.7)	22.2	(48.8, 21.1)	(23.4, 23.6)
30	1	17.6	(47.9, 11.0)	(17.6, 17.6)	16.9	(47.4, 10.2)	(16.9, 16.9)
40	0	39.8	(79.6, 39.8)	(39.1, 38.7)	38.3	(78.0, 37.8)	(38.4, 37.3)
40	1	24.2	(66.3, 18.0)	(24.1, 23.7)	23.3	(65.9, 17.3)	(23.3, 22.8)
50	0	50.4	(100.8, 50.4)	(45.9, 54.4)	49.6	(99.8, 49.4)	(45.0, 52.9)
50	1	34.1	(80.8, 27.9)	(31.2, 32.7)	33.2	(80.2, 27.1)	(30.6, 31.9)
60	0	56.9	(113.8, 56.9)	(50.8, 69.9)	56.3	(113.0, 56.2)	(49.8, 68.4)
60	1	44.6	(96.5, 39.2)	(36.7, 43.0)	43.9	(95.6, 38.4)	(36.1, 42.1)
70	0	61.6	(123.3, 61.6)	(56.1, 84.7)	61.2	(122.7, 61.1)	(55.0, 83.5)
70	1	52.2	(109.6, 47.9)	(41.1, 53.4)	51.6	(108.9, 47.2)	(40.3, 52.5)
80	0	65.4	(130.9, 65.4)	(61.1, 95.6)	65.1	(130.4, 65.0)	(60.2, 94.7)
80	1	57.7	(119.6, 54.2)	(45.7, 65.6)	57.3	(119.1, 53.7)	(44.7, 64.4)

Table EC.1 Expected revenues for duopoly with multiple and specific consumer choice in the cases of symmetric ($a_1 = a_2 = 1$, $Y_1 = Y_2 = 20$) and asymmetric ($a_1 = 2$ and $Y_2 = 40$) equilibria (note: * – (firm1, firm2))

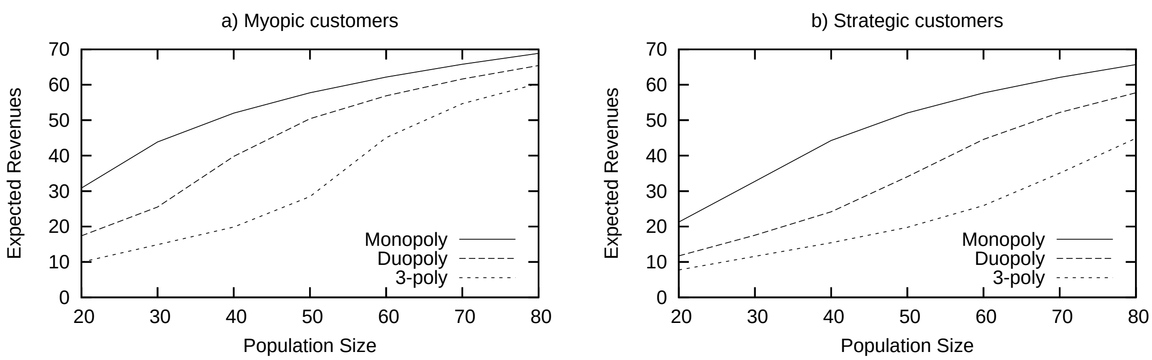

Figure EC.1 Equilibrium revenues under myopic and strategic consumer behavior and different levels of competition as a function of population size

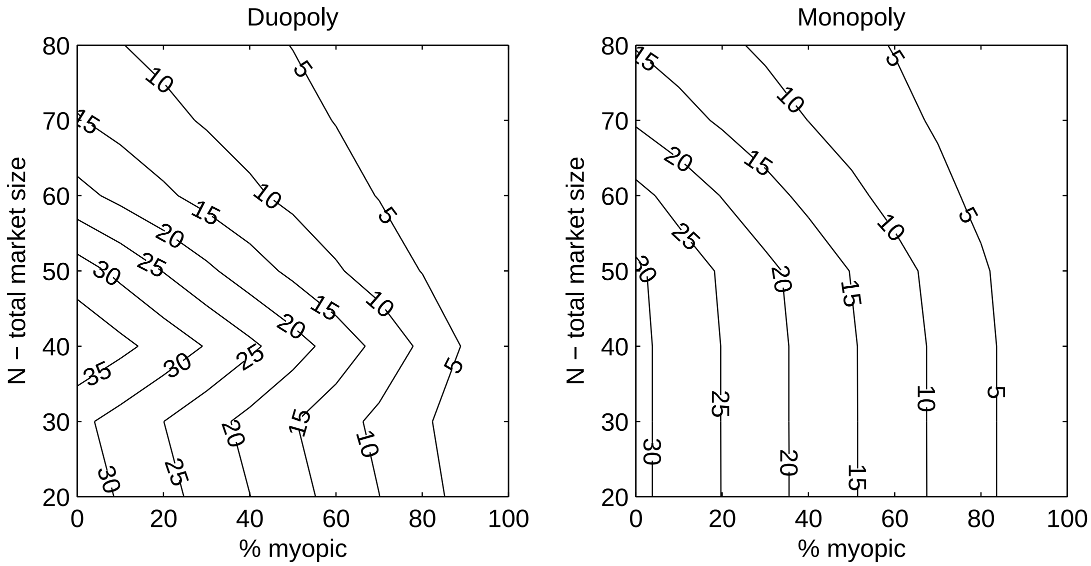

Figure EC.2 Levels of equal % revenue difference as a function of population size and the percentage of myopic consumers for symmetric duopoly and monopoly

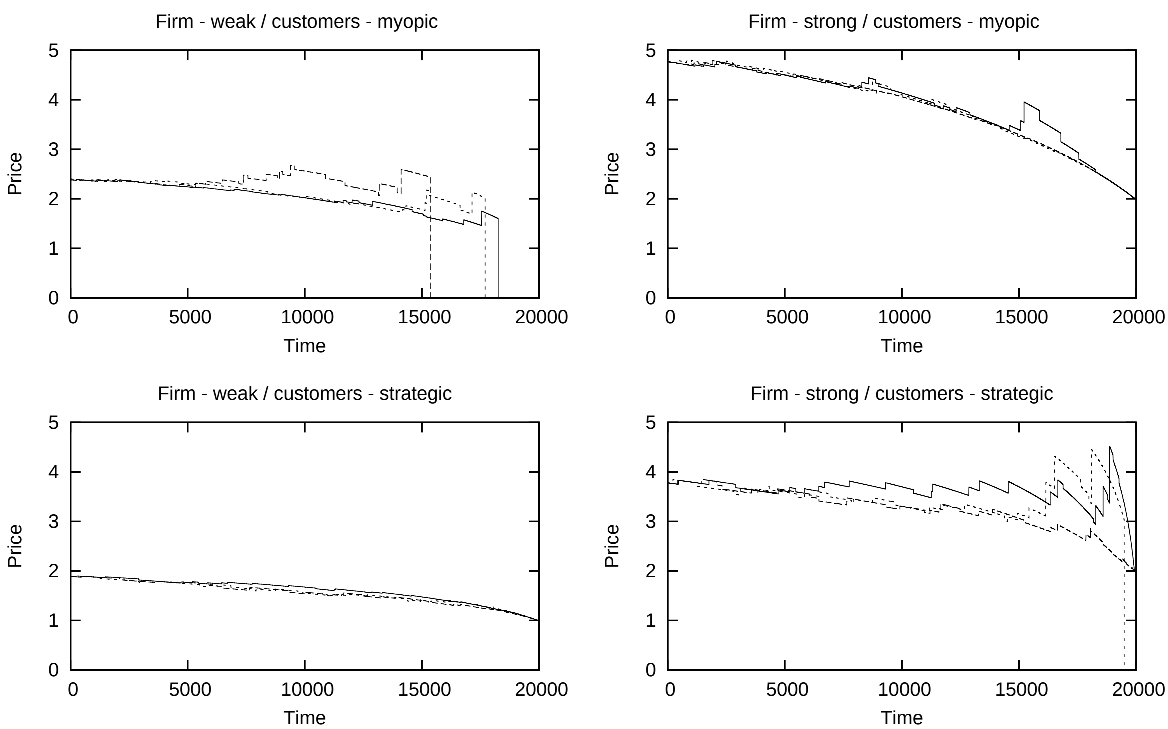

Figure EC.3 Representative simulated price paths under asymmetric ($a_1 = 1$, $a_2 = 2$) equilibrium pricing policies for $N = 40$

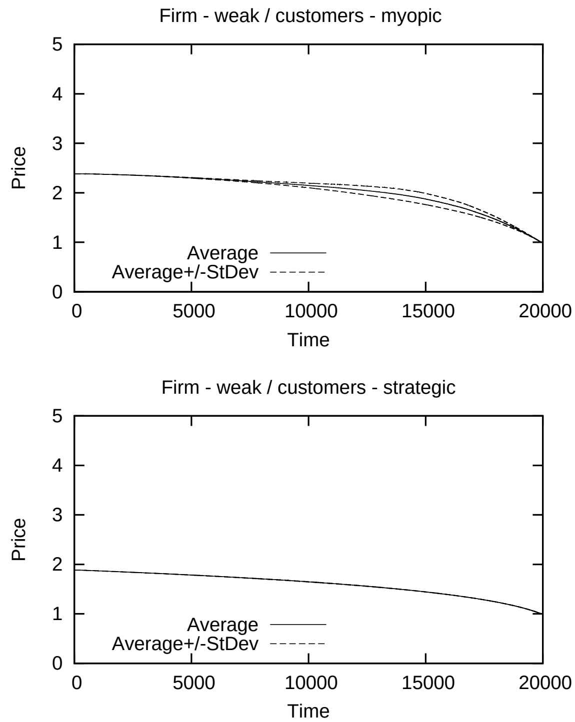

Figure EC.4 Averages of simulated price paths under asymmetric ($a_1 = 1$, $a_2 = 2$) equilibrium pricing policies for $N = 40$

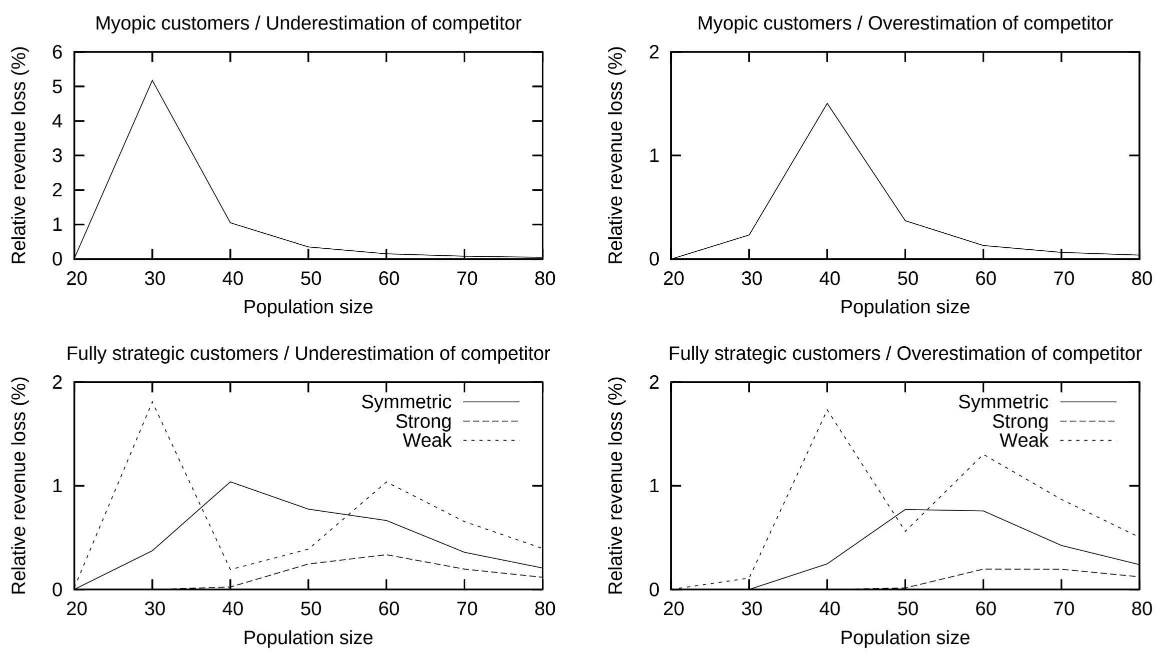

Figure EC.5 Relative revenue loss for deviation from equilibrium by assuming wrong competitor capacity, as a function of the population size

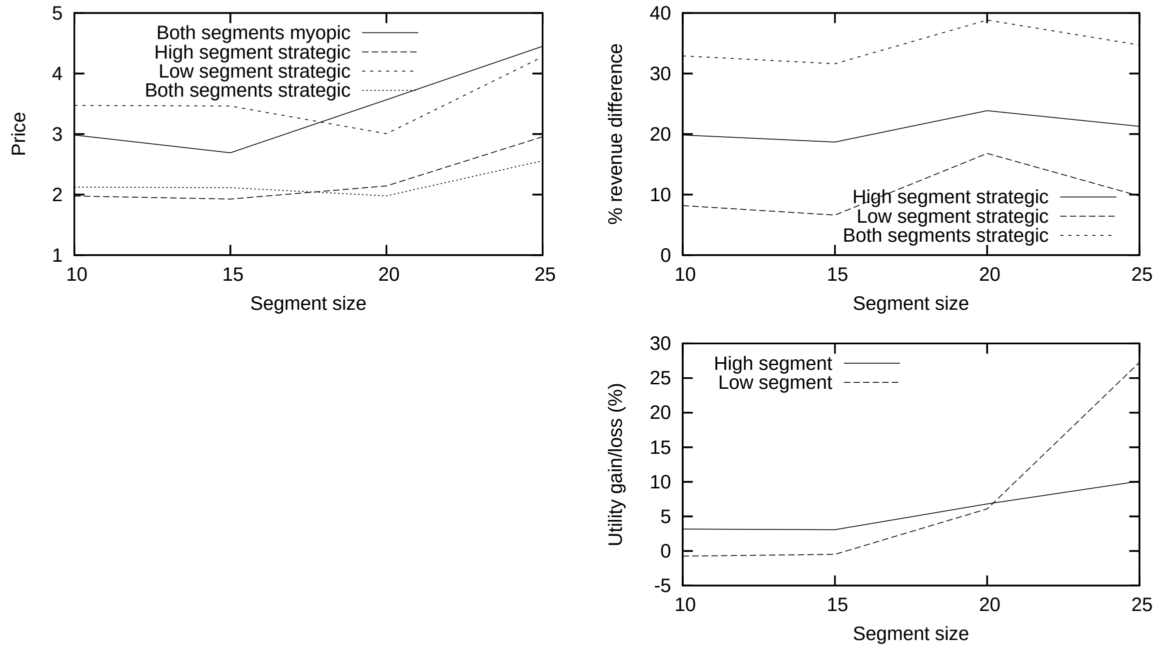

Figure EC.6 **Initial prices, effect of strategic behavior on revenues and utility gain/loss from strategic behavior of another segment in the case of symmetric duopoly and equally-sized segments with $a_{1j} = 1$ and $a_{2j} = 2$, as functions of segment size**

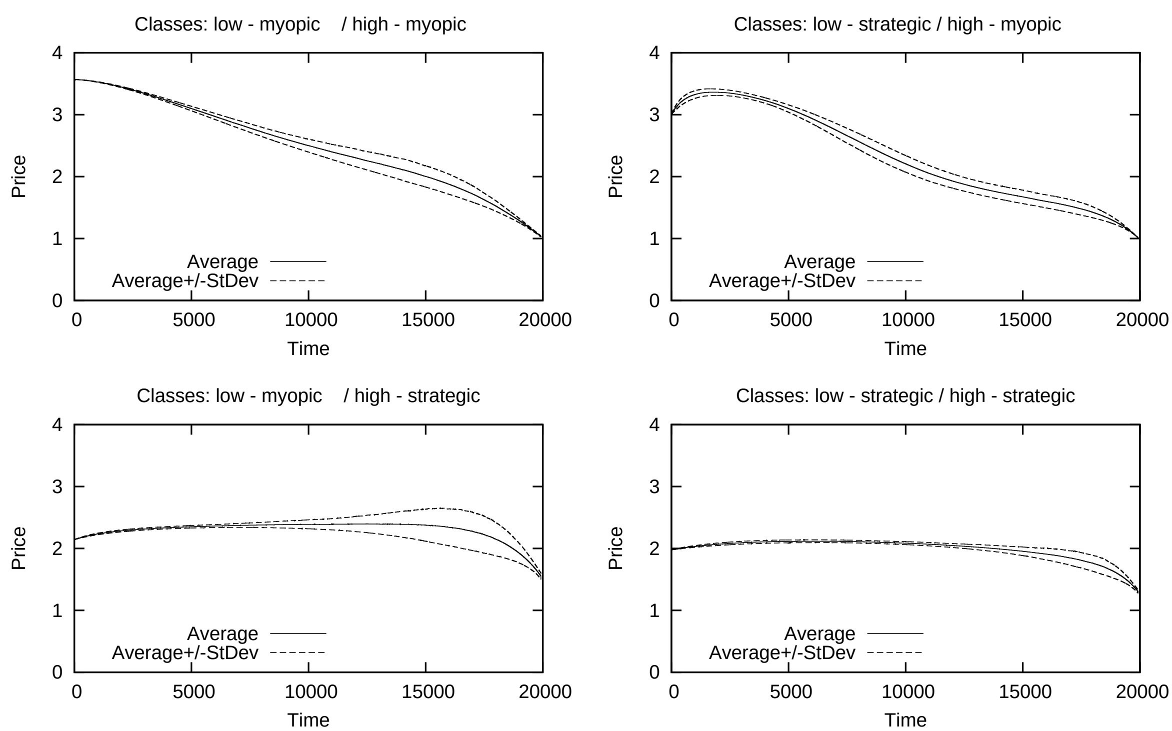

Figure EC.7 **Averages of simulated price paths under two segment ($a_{1j} = 1$, $a_{2j} = 2$) symmetric equilibrium pricing policies for $N_1 = N_2 = 20$**