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Optimal Market Intelligence Strategy When Management Attention is Scarce

Technical Appendix

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Proof of Proposition 1

Proposition 1 and condition (7) follow directly from the second-order conditions (6b) and setting t=1–t =½. The first part of the two brackets in (6b) is always positive and the second-order conditions for a profit maximum can only hold when $\phi''>0$. ♦

Proof of Proposition 2

Using the definition of $\sigma_{\mathrm{ki}} = \phi(t_{ki};\, \eta)$, where $\partial\phi(t;\, \eta)/\partial\eta<0$, the effect of research effectiveness η follows from condition (7) and $\partial r/\partial\eta<0$. The effect of uncertainty u follows immediately from condition (7) and $\partial r/\partial u>0$. ♦

Proof of Propositions 3, 4 & 5

To simplify the notation, we assume A_1=A_2=A. This has no implications for the results. In a symmetric Bertrand duopoly, the unique Bayesian-Nash equilibrium price decision by firm i for market k is $p_{\mathrm{ki}}^* = B_0 + B_{\mathrm{ki}}(x_{\mathrm{ki}}-A)$, where

$$B_0 = \frac{A + bc}{b(2-d)} \text{ and } B_{\mathrm{ki}} = w_{\mathrm{ki}}\frac{2 + d\cdot w_{\mathrm{kj}}}{b\cdot \xi_{\mathrm{k}}}, \tag{A.1}$$

where $\xi_{\mathrm{k}} = 4 - d^2 w_{\mathrm{ki}} w_{\mathrm{kj}}$ (i=1, 2, j=3–i and k=1, 2).

The expected profit is $\Pi_i = E_x\left[E\left[\pi_i \middle| x_i\right]\right] = \frac{1}{b}\left[\Pi_0 + Z_i(t_i)\right]$, with

$$\Pi_0 = 2\left(\frac{A-(1-d)b\cdot c}{2-d}\right)^2 \text{ and} \tag{A.2}$$

$$Z_{\mathrm{i}} = b\left(B_{1i}^2\left(u+\phi(t_{\mathrm{i}})\right) + B_{2i}^2\left(u+\phi(1-t_{\mathrm{i}})\right)\right). \tag{A.3}$$

Defining $\zeta_{\mathrm{k}} = 4 + d^2 w_{\mathrm{ki}} w_{\mathrm{kj}}$ leads to the following first-order and second-order partials:

$$\frac{\partial Z_i}{\partial t_{\mathrm{i}}} = b\left(-B_{1i}^2\frac{\zeta_1}{\xi_1}\phi'(t_{\mathrm{i}}) + B_{2i}^2\frac{\zeta_2}{\xi_2}\phi'(1-t_{\mathrm{i}})\right), \tag{A.4}$$

$$\frac{\partial^2 Z_i}{\partial t_i^2} = b\sum_{k=1}^{2}\frac{B_{\mathrm{ki}}^2}{\xi_k^2}\left(\frac{8\left(\zeta_k + d^2 w_{\mathrm{ki}} w_{\mathrm{kj}}\right)\phi'(t_{ki})^2}{u+\phi(t_{ki})} - \xi_k\zeta_k\phi''(t_{ki})\right), \text{ and} \tag{A.5}$$

$$\frac{\partial^2 Z_i}{\partial t_i \partial t_j} = b\sum_{\mathrm{k}=1}^{2}\frac{2d w_{\mathrm{ki}} w_{\mathrm{kj}} B_{\mathrm{ki}}}{b\xi_k^2}\left(\zeta_k\left(1+bdB_{\mathrm{ki}}\right) + 4bdB_{\mathrm{ki}}\right)\frac{\phi'(t_{ki})\phi'(t_{kj})}{u+\phi(t_{kj})}. \tag{A.6}$$

For a symmetric Cournot duopoly, the unique Bayesian-Nash equilibrium quantity decision is $q_{\mathrm{ki}}^* = K_0 + K_{\mathrm{ki}}(x_{\mathrm{ki}}-A)$, where

$$\mathrm{K}_0 = \frac{A - \mathrm{c}}{\beta(2+\delta)} \text{ and } \mathrm{K}_{\mathrm{ki}} = \mathrm{w}_{\mathrm{ki}} \frac{2 - \delta \cdot \mathrm{w}_{\mathrm{kj}}}{\beta \cdot \xi_{\mathrm{k}}}, \tag{A.7}$$

with $\xi_{\mathrm{k}} = 4 - \delta^2 \mathrm{w}_{\mathrm{ki}} \mathrm{w}_{\mathrm{kj}}$ (i=1, 2, j=3–i and k=1, 2).

The expected value of information for a Cournot duopoly is:

$$Z_i = b\left(\mathrm{K}_{1i}^2\left(u + \phi(\mathrm{t}_\mathrm{i})\right) + \mathrm{K}_{2i}^2\left(u + \phi(1 - \mathrm{t}_\mathrm{i})\right)\right), \tag{A.8}$$

Defining $\zeta_{\mathrm{k}} = 4 + \delta^2 \mathrm{w}_{\mathrm{ki}} \mathrm{w}_{\mathrm{kj}}$ leads to the following first- and second-order partials:

$$\frac{\partial Z_i}{\partial \mathrm{t}_\mathrm{i}} = \beta\left(-K_{1i}^2 \frac{\zeta_1}{\xi_1} \phi'(\mathrm{t}_\mathrm{i}) + K_{2i}^2 \frac{\zeta_2}{\xi_2} \phi'(1 - \mathrm{t}_\mathrm{i}) \right), \tag{A.9}$$

$$\frac{\partial^2 Z_i}{\partial t_i^2} = \beta \sum_{\mathrm{k}=1}^{2} \frac{\mathrm{K}_{\mathrm{ki}}^2}{\xi_{\mathrm{k}}^2} \left(\frac{8\left(\zeta_k + \delta^2 \mathrm{w}_{\mathrm{ki}} \mathrm{w}_{\mathrm{kj}}\right)\phi'(t_{ki})^2}{u + \phi(t_{ki})} - \xi_k \zeta_k \phi''(t_{ki}) \right), \text{ and} \tag{A.10}$$

$$\frac{\partial^2 Z_i}{\partial t_i \partial t_j} = -\beta \sum_{\mathrm{k}=1}^{2} \frac{4\delta \mathrm{w}_{\mathrm{ki}} \mathrm{K}_{\mathrm{ki}}}{\xi_k^2} \left(\zeta_k \mathrm{K}_{\mathrm{kj}} - 2\delta \mathrm{w}_{\mathrm{kj}} \mathrm{K}_{\mathrm{ki}}\right) \frac{\phi'(t_{ki})\phi'(t_{kj})}{u + \phi(t_{kj})} \tag{A.11}$$

Due to their symmetry, the allocation t_i=½, i=1, 2, always satisfies the first-order conditions (A.4) and (A.9), respectively. Irrespective of the type of competition, a sufficient and (almost) necessary condition for the allocation t_i=½, i=1, 2, to be a locally stable equilibrium solution is (see Varian 1992):

$$\begin{vmatrix} \dfrac{\partial^2 \mathrm{Z}_\mathrm{i}(t)}{\partial t_i^2} & \dfrac{\partial^2 \mathrm{Z}_\mathrm{i}(t)}{\partial t_i \partial t_j} \\ \dfrac{\partial^2 \mathrm{Z}_\mathrm{j}(t)}{\partial t_j \partial t_i} & \dfrac{\partial^2 \mathrm{Z}_\mathrm{j}(t)}{\partial t_j^2} \end{vmatrix} > 0. \tag{A.12}$$

Conditions (8) for price competition and (9) for quantity competition follow immediately from (A.12). Importantly, an increase in the degree of quantity competition, as measured by δ, makes a focused market intelligence strategy more likely, which means that Proposition 4 does not depend on the type of competition.

Existence and uniqueness of the equilibrium market intelligence strategies follow from the properties of the payoff and the best response functions. The approach is the same for both types of competition. Details of the proof are provided here for price competition. When condition (8) holds, the payoff function, $\mathrm{Z}_\mathrm{i}(t)$, is bounded and concave. The properties of the best response functions $\mathrm{r}_\mathrm{i}(t_j)$, i=1, 2, j=3–i, follow from

$$\frac{\partial F_i}{\partial t_i}\mathrm{d}t_i + \frac{\partial F_i}{\partial t_j}\mathrm{d}t_j = 0, \tag{A.13}$$

where $F_i = \partial Z_i(t)/\partial t_i$ are the first-order conditions (A.4), respectively. First, the slope of the best response functions is determined by the cross partial derivatives, $\partial F_i(t)/\partial t_j$ i.e., (A.6). The parts within the summation of (A.6) are always positive. Thus, the entire expression and the slope of the best response functions are always positive, i.e., *upward* sloping. Existence and uniqueness of the equilibrium, $t_i^*=½$, $i=1, 2$ follow from this together with the fact that the best response functions, $t_i = r_i(t_j)$, are monotone increasing (Huang and Li 1990).

When $t_i \neq ½$, $i=1, 2$, it follows from the symmetry in the equilibrium price, p_i^*, and the payoff function, $Z_i(t)$, that either $t_i^*=t_j^*$ or $t_i^*=1-t_j^*$. However, $Z_i(t_i, t_i) > Z_i(t_i, 1-t_i)$, $\forall\ t_i \neq ½$, $i=1, 2$, since B_{ki} is an increasing function of the competitor's allocation, t_j, to information about market k. The best response functions $r_i(t_j)$, $i=1, 2, j=3-i$, can cross the equilibrium line, $t_i^*=t_j^*$, at most once for $0 \leq t_j < ½$ and once for $½ < t_j \leq 1$ since they are convex for $0 \leq t_j < ½$ and concave for $½ < t_j \leq 1$. This follows from

$$\frac{\mathrm{d}^2 t_i}{\mathrm{d}t_j^2} = \frac{\partial^2 F_i}{\partial t_j^2} \Bigg/ \left(-\frac{\partial F_i}{\partial t_i}\right) + \frac{\partial F_i}{\partial t_j} \cdot \frac{\partial^2 F_i}{\partial t_i^2} \Bigg/ \left(-\frac{\partial F_i}{\partial t_i}\right)^2 . \tag{A.14}$$

Both denominators are always positive since they contain the second-order condition. From (A.6) follows that $\partial F_i(t)/\partial t_j$ is always positive. It can be shown that for $0 \leq t_j < ½$ both $\partial^2 F_i(t)/\partial t_j^2$ and $\partial^2 F_i(t)/\partial t_i^2$ are positive and negative for $½ < t_j \leq 1$. These two factors yield similar expressions as those in (A.6). A sufficient condition for these signs to hold is $\phi''' < 0$. However, in many cases it is not needed at all. As a result, (A.14) is positive for $0 \leq t_j < ½$ and the best response functions convex. For $t_j = ½$ (A.14) is 0 and for $½ < t_j \leq 1$ the signs of $\partial^2 F_i(t)/\partial t_j^2$ and $\partial^2 F_i(t)/\partial t_i^2$ reverse so that (A.14) becomes negative and the best response functions concave. Hence, when condition (8) does not hold, the pricing game has a unique pair of symmetric equilibrium allocations $t_i^*=t_j^* \neq ½$, $i, j=1, 2$. Whether the reaction functions cross the equilibrium line $t_i^*=t_j^*$ resulting in an interior focused allocation or not and thus resulting in a corner solution $t_i^*=t_j^*=0, 1$, depends on the function ϕ.

For quantity competition, it again follows from the symmetry in the equilibrium output, q_i^*, and the payoff function, $Z_i(t)$, that either $t_i^*=t_j^*$ or $t_i^*=1-t_j^*$ when $t_i \neq ½$, $i=1, 2$. Because K_{ki} is a decreasing function of the competitor's allocation, t_j, we have $Z_i(t_i, t_i) < Z_i(t_i, 1-t_i)$, $\forall\ t_i \neq ½$, $i=1, 2$. The best response functions $r_i(t_j)$, $i=1, 2, j=3-i$, are *downward* sloping and can cross the equilibrium line,

$t_i^*=1-t_j^*$, at most once for $0\leq t_j<½$ and once for $½<t_j\leq 1$ since they are convex for $0\leq t_j<½$ and concave for $½<t_j\leq 1$. Hence, when condition (9) does not hold, the Cournot game has a pair of asymmetric equilibrium allocations $t_i^*=1-t_j^*\neq ½$, $i, j=1, 2$.

The effect of competition, i.e., the effect of an increase in the parameter d for the pricing game and δ for the quantity game, on the likelihood of a broad market intelligence strategy as proposed in Proposition 4, is found from $\partial R_P/\partial d>0$, from condition (8) and from $\partial R_Q/\partial\delta>0$ from condition (9). ♦

Comparative Static Analysis

The effect of changes in uncertainty u and a firm's research effectiveness η can be determined directly from condition (8) for price competition and condition (9) for quantity competition. A change in uncertainty u increases the numerators in both conditions, whereas an increase in effectiveness η desreases the numerator through decreases in s_2.

Since the formal proof of the effect of changes in model parameters for the entire parameter space $0<t_i<1$ is similar for any of the parameter with either type of competition, we limit the proof to the effect of changes in prior uncertainty u with Cournot competition. The overall effect of changes in uncertainty, u, follows from

$$\left[\frac{\partial^2 \mathrm{Z_i}(t)}{\partial t_i^2}+\frac{\partial^2 \mathrm{Z_i}(t)}{\partial t_i \partial t_j}\frac{\partial t_j^*}{\partial t_i}\right]\frac{\mathrm{d}t_i}{\mathrm{d}u}+\left[\frac{\partial^2 \mathrm{Z_i}(t)}{\partial t_i \partial u}+\frac{\partial^2 \mathrm{Z_i}(t)}{\partial t_i \partial t_j}\frac{\partial t_j^*}{\partial u}\right]=0. \tag{A.15}$$

From the stability condition (A.12) and $\partial t_j^*/\partial t_i=-1$ follows that the first bracket is always negative. The sign of the total effect of a change in uncertainty u, $\mathrm{d}t_i/\mathrm{d}u$, is determined by the sign of the second bracket. The sign of $\partial^2\mathrm{Z_i}(t)/\partial t_i\partial t_j$ is negative (see A.11). The sign of the first part of the second bracket is determined by

$$\begin{aligned}\frac{\partial^2 \mathrm{Z_i}(\mathrm{t})}{\partial t_i \partial u}=&-\frac{K_{1i}\phi'(t_i)}{\xi_1^2}\left[2\xi_1\zeta_1\frac{\partial K_{1i}}{\partial u}+8\delta^2 K_{1i}\left(w_{1i}\frac{\partial w_{1j}}{\partial u}+w_{1j}\frac{\partial w_{1i}}{\partial u}\right)\right]\\&-\frac{K_{2i}\phi'(1-t_i)}{\xi_2^2}\left[2\xi_2\zeta_2\frac{\partial K_{2i}}{\partial u}+8\delta^2 K_{2i}\left(w_{2i}\frac{\partial w_{2j}}{\partial u}+w_{2j}\frac{\partial w_{2i}}{\partial u}\right)\right].\end{aligned} \tag{A.16}$$

When $t_i=t_j=½$, the two parts are identical and $\partial^2\mathrm{Z_i}(t)/\partial t_i\partial u=0$. The first part is positive and of larger magnitude when $t_i^*<½$. The second part is negative and of larger magnitude when $t_i^*>½$. Thus, $\partial^2\mathrm{Z_i}(t)/\partial t_i\partial u>0$ when $t_i^*<½$ and $\partial^2\mathrm{Z_i}(t)/\partial t_i\partial u<0$ when $t_i^*>½$. The sign for $\partial t_j^*/\partial u$ is found similarly. However, from the discussion above follows that when $t_i^*<½$, then $t_j^*>½$. In other words, in equilibrium when $\partial^2 Z_i(t)/\partial t_i \partial u > 0$, then $\partial t_j^*/\partial u < 0$, and vice versa. As a result, both parts of the second bracket of (A.16) are positive when $t_i^* < ½$, negative when $t_i^* > ½$, and zero when $t_i^* = ½$, showing that the result presented in Proposition 2 for a monopolist fully extends to a duopoly. In sum, as uncertainty, u, increases, the equilibrium market research strategies, t_i^*, $i=1, 2$, become less focused, i.e., move towards ½. ♦

Extension to Oligopoly Markets

To examine the optimal market intelligence strategy for an oligopoly, we consider a quantity game wit homogeneous products. The analysis for a pricing game would require differentiated products but follow the same approach as shown below and lead to the same insights about the effect of the number of competitors n and the number of markets K. In contrast to quantity competition, the equilibrium would be symmetric.

We assume the following stochastic linear inverse demand function: $P_k = \tilde{A} - bQ_k$, where $Q_k = \Sigma q_{ki}$, $b>0$. The intercept term, $\tilde{A}$, is an additive function of $K > 1$ independent stochastic factors, i.e., $\tilde{A} = \Sigma a_k$. Each factor, a_k, is distributed with finite mean A_k and variance u. Firms must decide how much capacity, t_{ki}, to allocate to information about each market, where $\Sigma_k t_{ki} = 1$. With a broad market intelligence strategy, firm i allocates $t_{ki}=1/K$ to each market k.

In a Cournot oligopoly with n identical firms and K uncertain demand factors, a_k, the unique Bayesian-Nash equilibrium quantity decision is $q_{ki}^* = B_0 + B_{ki}(x_{ki} - A)$, where

$$B_0 = \frac{A - c}{(n+1)b} \text{ and } B_{ki} = \frac{w_{ki}}{X_{ki}}$$

$$\text{with } X_{ki} = 2b(1 + D_{kj}) - b w_{ki} D_{kj} \text{ and } D_{kj} = \sum_{j \neq i} \frac{w_{kj}}{2 - w_{kj}}. \qquad (A.17)$$

The optimal market intelligence strategy follows from the first- and second-order conditions:

$$\frac{\partial Z_i(t)}{\partial t_{ki}} = -w_{ki}^2 \frac{X_{ki}^3}{Y_{ki}} \phi'_{ki} - w_{Ki}^2 \frac{X_{Ki}^3}{Y_{Ki}} \phi'_{Ki} = 0,\ k, l = 1 \ldots K\text{-}1,\ i, j = 1 \ldots n,\ i \neq j, \qquad (A.18)$$

$$\left[\frac{\partial^2 Z_i}{\partial t_{ki} \partial t_{li}} \right] \text{ is negative semi definite,}$$

$$\frac{\partial^2 Z_i(t)}{\partial t_{ki}^2} = \gamma \cdot \frac{\mathrm{w}_{\mathrm{ki}}^2 \mathrm{X}_{\mathrm{ki}}^3}{u\mathrm{Y}_{\mathrm{ki}}} \left[\mathrm{w}_{\mathrm{ki}}\left(2 + \mathrm{w}_{\mathrm{ki}}\mathrm{Z}_{\mathrm{ki}}\left(3\mathrm{X}_{\mathrm{ki}} + \mathrm{Y}_{\mathrm{ki}}\right)\right)\phi'^{\,2}_{ki} - u\phi''_{ki}\right]$$
$$+ \frac{\mathrm{w}_{\mathrm{Ki}}^2 \mathrm{X}_{\mathrm{Ki}}^3}{u\mathrm{Y}_{\mathrm{Ki}}} \left[\mathrm{w}_{\mathrm{Ki}}\left(2 + \mathrm{w}_{\mathrm{Ki}}\mathrm{Z}_{\mathrm{Ki}}\left(3\mathrm{X}_{\mathrm{Ki}} + \mathrm{Y}_{\mathrm{Ki}}\right)\right)\phi'^{\,2}_{Ki} - u\phi''_{Ki}\right], \tag{A.19}$$

where $\gamma = 1$ if $k = l$ and $\gamma = 0$ otherwise, $\mathrm{Y}_{\mathrm{ki}} = 2b\left(1 + \mathrm{D}_{\mathrm{kj}}\right) + b\mathrm{w}_{\mathrm{ki}}\mathrm{D}_{\mathrm{kj}}$, $\mathrm{Z}_{\mathrm{ki}} = (b(2\mathrm{D}_{\mathrm{kj}}+\mathrm{w}_{\mathrm{kj}}(1-\mathrm{D}_{\mathrm{kj}})))/(2-\mathrm{w}_{\mathrm{kj}})$, and $\phi'_{\mathrm{ki}} = \partial\phi(t_{ki})/\partial t_{ki} < 0$, $\phi'_{\mathrm{Ki}} = \partial\phi(1\text{-}\Sigma_{\mathrm{k}} t_{ki})/\partial t_{ki} > 0$, $\phi''_{\mathrm{ki}} = \partial^2\phi(t_{ki})/\partial t_{ki}^2$ and $\phi''_{\mathrm{Ki}} = \partial^2\phi(1\text{-}\Sigma_{\mathrm{k}} t_{ki})/\partial t_{ki}^2$ (see also Li et al. 1987 or Hauk and Hurkens 2001). From (A.19) follows immediately that diminishing marginal returns to management attention ($\phi''>0$) are necessary for (A.18) to have an interior solution. If the marginal returns are non-diminishing ($\phi''\leq 0$), the game has only corner solutions.

The best-response functions $\mathrm{r}_{\mathrm{ki}}(t_{lj})$ are downward sloping for any t_{lj}, $i, j=1\ldots n$, $\mathrm{i}\neq\mathrm{j}$ and $\mathrm{k}, \mathrm{l} = 1\ldots K-1$, which follows from

$$\frac{\partial^2 Z_i(t)}{\partial t_{ki}\partial t_{lj}} = -\gamma \cdot \frac{\mathrm{w}_{\mathrm{ki}}^2 \mathrm{w}_{\mathrm{lj}}^2 \mathrm{X}_{\mathrm{ki}}^3}{u\mathrm{Y}_{\mathrm{ki}}} \mathrm{R}_{\mathrm{lj}}\left[3\mathrm{X}_{\mathrm{ki}}\left(2 - \mathrm{w}_{\mathrm{ki}}\right) - \mathrm{Y}_{\mathrm{ki}}\left(2 + \mathrm{w}_{\mathrm{ki}}\right)\right]\phi'_{ki}\phi'_{lj}$$
$$+ \frac{\mathrm{w}_{\mathrm{Ki}}^2 \mathrm{w}_{\mathrm{lj}}^2 \mathrm{X}_{\mathrm{Ki}}^3}{u\mathrm{Y}_{\mathrm{Ki}}} \mathrm{R}_{\mathrm{lj}}\left[3\mathrm{X}_{\mathrm{Ki}}\left(2 - \mathrm{w}_{\mathrm{Ki}}\right) - \mathrm{Y}_{\mathrm{Ki}}\left(2 + \mathrm{w}_{\mathrm{Ki}}\right)\right]\phi'_{Ki}\phi'_{lj}, \tag{A.20}$$

where $\gamma = 1$ if $k = l$ and $\gamma = 0$ if $k \neq l$ and $\mathrm{R}_{\mathrm{lj}} = 2b/((2b-\mathrm{w}_{\mathrm{lj}})^2)$. This expression is negative for all k and l because both the leading factors and the parenthesis of both parts are always positive. The negative sign of the second part follows from $\phi'_{\mathrm{lj}} < 0$ and $\phi'_{\mathrm{Ki}} > 0$.

The sufficient condition for $t_{ki} = 1/\mathrm{K}$, $\forall$ i and k, to be an equilibrium of the game follows from the same stability condition as used in the proof of Proposition 2. For the Cournot oligopoly, the K-1 identical nxn-matrices $\left[\dfrac{\partial^2 \mathrm{Z}_{\mathrm{i}}}{\partial t_{ki}\partial t_{kj}}\right]$, $i, j = 1\ldots n$, $k = 1\ldots K$-1, must be negative semidefinite. When $t_{ki} = 1/K$ $\forall$ k, all the diagonal elements ($i = j$) are identical and all the off-diagonal elements ($i \neq j$) are identical. Thus, the determinants of the first two principal minors are necessary and sufficient conditions. The negative sign of the determinant of the first principal minor, $\partial^2\mathrm{Z}_{\mathrm{i}}/\partial t_{ki}^{2,}$ follows from the second-order conditions (A.19). The determinant of the second principal minor is positive when $\partial^2\mathrm{Z}_{\mathrm{i}}/\partial t_{ki}^2 - \partial^2\mathrm{Z}_{\mathrm{i}}/\partial t_{ki}\partial t_{kj} < 0$. Setting $\mathrm{w}_{\mathrm{ki}}(t_{ki} = 1/K) = \mathrm{w}$ $\forall i = 1\ldots n$ and $k = 1\ldots K$-1, immediately leads to the result that in a Cournot oligopoly a broad market intelligence strategy is only an equilibrium outcome when $\nu|_{t=1/K} < -\mathrm{R}_{\mathrm{Q}}\cdot r|_{t=1/K}$ where:

$$\mathrm{R}_{\mathrm{Q}'} = \frac{4\left[(2+\mathrm{w})(4+\mathrm{w}^2)+2(n-2)\left(\mathrm{w}(2+\mathrm{w})-\mathrm{w}^3\right)\right]}{(4-\mathrm{w}^2)\left(4+\mathrm{w}^2+(n-2)\mathrm{w}(2-\mathrm{w})\right)}. \tag{A.21}$$

When setting $n = 2$ and $\delta = 1$, it follows immediately that $\mathrm{R}_{\mathrm{Q}'} = 4/(2–\mathrm{w})$ as shown in (A.13). For $\delta<1$, one would need to replace w with δw in (A.21). $\mathrm{R}_{\mathrm{Q}'}$ is decreasing with K because $\mathrm{w}=u/(u+\phi(t))$ is decreasing in K. The noise in the information $\phi(t=1/\mathrm{K})$ increases as a firm spreads its attention capacity over more markets. In other words, the opportunity cost of focusing on single market increases, thereby making a focused allocation less attractive. As the number of competitors n increases, a focused allocation becomes more likely. This effect follows immediately from $\partial \mathrm{R}_{\mathrm{Q}'}/\partial n$. ♦

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