MANAGEMENT SCIENCE
DOI 10.1287/mnsc.1090.1008ec

e-companion

ONLY AVAILABLE IN ELECTRONIC FORM

Electronic Companion—"Quality Improvement Incentives and Product Recall Cost Sharing Contracts" by Gary H. Chao, Seyed M. R. Iravani, and R. Canan Savaskan, *Management Science*, DOI 10.1287/mnsc.1090.1008.

ON-LINE APPENDIX A

Detailed Description of Modeling Assumptions

A1: We assume that M products are manufactured and sold, and either are with the customers, or are in the distribution channel when a recall is issued. Once a particular problem has revealed itself, and the recall is issued, the manufacturer fixes the particular quality problem in all M products.

A recall can be initiated either by a manufacturer or a governmental agency if there is enough evidence that the quality issue is in fact due to a manufacturing or a design problem rather than a consumer usage problem. In most cases, the collection of product failure data, its preliminary investigation, and the decision to issue a recall, occurs long after products are sold. In a recent study of the automotive recall data between 1978-1998, Rupp and Taylor (2002) report that the initial investigation stage takes on average 140 days, which is followed by vehicle testing if the recall is initiated by the National Highway Safety Association (NHSA). The testing stage can take up to a year to complete (357 days). From the latest vehicle recall data published by the NHSA, we also find that the average time for a recall of a vehicle model is 3 years while in 80% of the cases, the recalls are for 1+ year-old models. In other words, a recall for a 2008 car model is expected to occur in 2011. By that time, most of the 2008 models (i.e., most of M cars) have been sold and new 2009 and 2010 models have been introduced. This is consistent with our Assumption A.1.

A2: We denote the recall cost *per unit* as ω and assume it to be independent of the root cause of the quality problem. At the time of contracting, both the manufacturer and the supplier agree on an estimate of ω.

Note that there are three major cost components that constitute ω: (i) the unit direct cost of fixing the quality problem in the product upon recall (ω_d), (ii) the unit indirect overhead cost associated with handling a recall process (ω_o), and (iii) the unit indirect cost due to loss of goodwill and firm value (ω_l). (See Baiman et al. 2001 for a similar definition of external quality costs). While ω_d can sometimes be root cause dependent, current research (Jarrell and Peltzman 1985, Barber and Darrough 1996, Rupp 2004) shows that the indirect costs of a recall (in the form of overhead ω_o and goodwill loss ω_l) are independent of the root cause of the problem, and in fact are much larger than the direct costs such as repair or replacement costs. Hence, the assumption that ω ($= \omega_d + \omega_0 + \omega_l$) is independent of the root cause of a recall is reasonable in many cases. Also note that when the manufacturer and the supplier sign the contract, the general type of quality problem that may result in a recall is often known (e.g., the failure in the air bag system when the supplier provides airbag systems). This allows the parties to get an estimate for the overhead cost ω_0 and the loss of goodwill cost ω_l. However, until the failure occurs and root cause analysis is performed, the nature of the type of repair operations needed to fix the problem is not clear. Therefore, when negotiating a contract, the manufacturer and the supplier usually use an estimate for ω_d (independent of the root cause of the recall, which is unknown at the time when the contract is signed).

In practice, unit recall cost can range between hundreds and thousands of dollars depending on the complexity of the product and the quality problem. In the auto industry, the total average cost of a recall varies between \$5M to \$20M, excluding customer goodwill loss and any liabilities (Sherefkin and Armstrong 2003).

A3: We denote the *unit cost* of root cause analysis by $c_r = \frac{C_r}{M}$, where C_r is the relevant fixed cost of the root cause analysis and M is the total number of products subject to a recall.

We assume that the root cause analysis perfectly identifies the component that caused the quality problem.

When a recall is initiated, the root cause analysis is performed on failed products to identify *which* component fell short of performing its intended function, and *whose* responsibility the product failure was. As discussed under Assumption A.1, recalls often happen after the product has been in the market for extended periods of time, during which new models are introduced. Therefore, when a quality problem reveals itself leading to a recall, the defective product/component is often replaced with a newer unit, rather than being repaired at the point of return. Therefore, the recall may not initially involve a detailed root cause analysis to identify the party whose process has been responsible for the quality problem. For instance, consider the Lenovo battery recall. In this case, each customer with the defective battery received a new replacement one. At the time of the recall, neither Lenovo nor its supplier knew what caused the over-heating of the batteries. Only after Lenovo wanted to share the recall cost with Sanyo, was a detailed root cause analysis performed to determine if the defect was Sanyo's or Lenovo's fault (Nystedt 2007). A further point is that when the recall involves older product models, unless there is a cost sharing contract requiring root cause analysis information, manufacturers may initially have fewer incentives to identify the part at fault since the new component/product model has already undergone design changes and any information gained from the root cause analysis may not be useful for the current product design. Therefore, in this paper, c_r models the unit root cause analysis cost incurred when the cost sharing contract requires further analysis of the failed product to identify the party at fault for the quality problem.

A4: The manufacturer and the supplier have inherent process capabilities modeled by the initial failure rate of their components due to a design related quality problem. The initial failure rates are common knowledge to both parties and are denoted by $\lambda_m'^0$ and $\lambda_s'^0$ for the manufacturer and the supplier, respectively.

The failure rates are assumed to be time homogeneous (i.e., a constant failure rate). Most components, particularly electronics, exhibit a "bathtub" shaped failure rate function (Barlow and Proschan 1996). The initial down slope of the curve corresponds to the production and testing phase at the supplier's or the manufacturer's site. The failure rate in that part of the curve (also called the infant mortality rate) is reduced through better design, production, and quality control. Even when a product gets out of the assembly line, manufacturers often use "burn-in tests" to detect early failures. A burn-in test is designed to eliminate or reduce infant mortality failures prior to selling the product (Ebeling 1997). Hence, when a customer receives the product, as modeled in our paper, the product is at the beginning of its useful life, with a constant failure rate.

A constant failure rate for each component allows us to model the product's time to a *failure that results in a recall* by the exponential distribution. Modeling product failures by the exponential lifetime distribution is also widely studied in the reliability literature (Barlow and Proschan 1996).

A5: In our model, the contract negotiated between the manufacturer and the supplier covers external quality costs (recall costs) for a duration of T periods, which will denote the duration that the product is in use by the consumers. Without loss of generality, we normalize T to 1.

A6: Improvements in the product design failure rate are costly to both parties. More specifically, the efforts of η_s and η_m result in an effort cost of $C_s(\eta_s)$ and $C_m(\eta_m)$ (per unit component) for the supplier and the manufacturer, respectively.

In practice, for a particular product line, the *total* design improvement related cost depends on the amount of effort spent during the design process. Let $D_s(\eta_s)$ and $D_m(\eta_m)$ denote such design related costs as a function of the supplier's and the manufacturer's effort. Then, the per component unit costs are $C_s(\eta_s) = \frac{D_s(\eta_s)}{M}$ and $C_m(\eta_m) = \frac{D_m(\eta_m)}{M}$, respectively, for the supplier and the manufacturer.

ON-LINE APPENDIX B

Proofs of Analytical Results: Symmetric Information Case

Before we present our proofs, in the following table we list additional notation which will be used frequently to simplify presentation of the proofs.

Notations	Sign	Notations	Sign
$G = \frac{\lambda_m^o(1-\eta_m)}{\lambda_m^o(1-\eta_m)+\lambda_s^o(1-\eta_s)}$	+	$H = 1 - e^{-[\lambda_m^o(1-\eta_m)+\lambda_s^o(1-\eta_s)]}$	+
$1 - G = \frac{\lambda_s^o(1-\eta_s)}{\lambda_m^o(1-\eta_m)+\lambda_s^o(1-\eta_s)}$	+	$H_m' = -\lambda_m^0 e^{-(\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s))}$	$-$
$K = \frac{\lambda_m^o}{\lambda_m^o(1-\eta_m^C)+\lambda_s^o(1-\eta_s^C)}$	+	$H_s' = -\lambda_s^0 e^{-(\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s))}$	$-$
$G_m' = -K(1-G)$	$-$	$H_{mm}^{''} = -(\lambda_m^0)^2 e^{-(\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s))}$	$-$
$G_s^{'} = GK\frac{\lambda_s^o}{\lambda_m^o}$	+	$H_{ss}^{''} = -(\lambda_s^0)^2 e^{-(\lambda_m^0(1-\eta_m^c)+\lambda_s^0(1-\eta_s^c))}$	$-$
$G_{sm}^{''} = -K^2\frac{\lambda_s^o}{\lambda_m^o}(1-2G)$	+/−	$H_{ms}' = -\lambda_m^0\lambda_s^0 e^{-(\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s))}$	$-$
$G_{mm}^{''} = -2K^2(1-G)$	$-$	$K_s' = \frac{\lambda_s^o}{\lambda_m^o}K^2$	+
$G_{ss}^{''} = 2(\frac{\lambda_s^o}{\lambda_m^o})^2K^2G$	+	$K_m' = K^2$	+

PROOF OF PROPOSITION 1

This proposition presents a sufficient condition on the second order derivatives of the manufacturer's and the supplier's effort cost functions to ensure concavity of the central planner's profit function.

The central planner's problem is given by:

$$\underset{\eta_m\ ,\ \eta_s}{Max}\ \Pi^C = r - u_s - u_m - \omega\Big[1 - e^{-[\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)]}\Big] - C_m(\eta_m) - C_s(\eta_s.)$$

To ensure concavity of the central planner's profit function, we investigate the signs of the second order derivatives of the central planner's objective function, which are given by:

$$\frac{\partial^2\Pi^C}{\partial\eta_m^2} = \omega(\lambda_m^0)^2 e^{-[\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)]}\ -\ C_m''(\eta_m) < 0. \tag{6}$$

$$\frac{\partial^2\Pi^C}{\partial\eta_m^2} = \omega(\lambda_s^0)^2 e^{-[\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)]}\ -\ C_s''(\eta_s) < 0 \tag{7}$$

Below, we prove that $C_m''(\eta_m) > \omega(\lambda_m^0)^2$ and $C_s''(\eta_s) > \omega(\lambda_s^0)^2$ for $\forall(\eta_s, \eta_m) \in [0,1]\times[0,1]$ guarantee concavity of the central planner's profit function and consequently a unique solution to her optimization problem.

Notice that the LHS of inequality (6) has its largest value when $\eta_s = 1$. Therefore, it is sufficient to have:

$$\omega(\lambda_m^0)^2 e^{-\lambda_m^0(1-\eta_m)} - C_m''(\eta_m) < 0 \quad \text{for} \quad 0 \leq \eta_m \leq 1,$$

or equivalently,

$$\omega(\lambda_m^0)^2 e^{-\lambda_m^0(1-\eta_m)}\ < C_m''(\eta_m) \quad \text{for} \quad 0 \leq\ \eta_m\ \leq 1.$$

Similarly, the LHS of inequality (7) assumes its largest value when $\eta_m = 1$. Therefore, it is sufficient to have

$$\omega(\lambda_s^0)^2 e^{-\lambda_s^0(1-\eta_s)}\ -\ C_s''(\eta_s) < 0 \quad \text{for} \quad 0 \leq\ \eta_s \leq 1,$$

or equivalently,

$$\omega(\lambda_s^0)^2 e^{-\lambda_s^0(1-\eta_s)}\ < C_s''(\eta_s) \quad \text{for} \quad 0 \leq\ \eta_s\ \leq 1.$$

Since $e^{-\lambda_s^0(1-\eta_s)} \leq 1$ and $e^{-\lambda_m^0(1-\eta_m)} \leq 1$, to ensure concavity of the centralized profits, it is sufficient to have $C''_m(\eta_m) > \omega(\lambda_m^0)^2$ and $C''_s(\eta_s) > \omega(\lambda_s^0)^2$. Therefore, to ensure concavity of the central planner's optimization problem, we require that the second order derivatives of the manufacturer's and the supplier's effort cost functions are greater than a threshold defined by the initial failure rates λ_m^0, λ_s^0 and the unit recall cost, ω. When the initial failure rates and/or unit recall costs increase, the sufficient condition for concavity becomes more restrictive.

PROOF OF PROPOSITION 2

The manufacturer's and the supplier's optimization problems are given, respectively, by:

$$\underset{\eta_m}{Max} : \Pi_m^N = r - u_m - p_0 - \omega(1 - e^{-[\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)]}) - C_m(\eta_m)$$

$$\underset{\eta_s}{Max} : \Pi_s^N = p_0 - u_s - C_s(\eta_s)$$

Since the effort cost function of the supplier is increasing in η_s (i.e., $C'_s(\eta_s) > 0$), her profit function is decreasing in effort η_s. Therefore, $\eta_s^{*N} = 0$.

To show that $\eta_m^{*N} < \eta_m^{*C}$, we first evaluate the first order condition of the manufacturer's objective function at $\eta_m^{*N} = \eta_m^{*C}$ and $\eta_s^{*N} = 0$.

Note that from the centrally coordinated supply chain case, we know that:

$$\frac{\partial \Pi^C}{\partial \eta_m^C} = \omega\lambda_m^0 e^{-[\lambda_m^0(1-\eta_m^{*C})+\lambda_s^0(1-\eta_s^{*C})]} - C'_m(\eta_m^{*C}).$$

Since,

$$\frac{\partial \Pi^C}{\partial \eta_m^C}\Big|_{\eta_m^{*C},\eta_s^{*C}} = 0 \qquad \Longrightarrow \qquad \omega\lambda_m^0 e^{-[\lambda_m^0(1-\eta_m^{*C})+\lambda_s^0(1-\eta_s^{*C})]} = C'_m(\eta_m^{*C})$$

and $\eta_m^{*C} > 0$, then one can rewrite the first order optimality condition of the manufacturer's profit function under No Cost Sharing evaluated at $\eta_m^{*N} = \eta_m^{*C}$ and $\eta_s^{*N} = 0$ as follows:

$$\begin{aligned} \omega\lambda_m^0 e^{-[\lambda_m^0(1-\eta_m^{*C})+\lambda_s^0]} - \omega\lambda_m^0 e^{-[\lambda_m^0(1-\eta_m^{*C})+\lambda_s^0(1-\eta_s^{*C})]} &= \\ \omega\lambda_m^0 e^{-\lambda_m^0(1-\eta_m^{*C})}\Big(e^{-\lambda_s^0} - e^{-\lambda_s^0(1-\eta_s^{*C})}\Big) &< 0. \end{aligned}$$

Negativity of the above expression and concavity of the manufacturer's objective function ensure that $\eta_m^{*N} < \eta_m^{*C}$. Therefore, compared with the first best effort level, the manufacturer underinvests in effort under No Cost Sharing, and the supplier exerts zero effort.

PROOF OF LEMMA 1

The intuition underlying our proof is as follows. When root cause analysis cost is zero, the summation of the manufacturer's and retailer's objective function (i.e., the total supply chain profit in the decentralized system) is equivalent to the objective function in the centralized system. Thus, the first best effort levels maximize the total expected supply chain profits in the decentralized system and result in the first best profit and quality. More specifically, one can also easily show the following.

Under Contract S, the manufacturer sets p, R and $\overline{T}$ in the contract offered to the supplier. Given p, R and $\overline{T}$, the supplier solves

$$\underset{\eta_s}{Max}\ \Pi_s(\eta_s, R, \overline{T}, \eta_m) = p - S_W(\eta_s, R, \overline{T}, \eta_m) - C_s(\eta_s)$$

where $S_W(\eta_s, R, \overline{T}, \eta_m)$ is the supplier's share of expected recall cost given the contract terms and the manufacturer's effort. Let η_s^* $(R\ ,\overline{T}$ and $\eta_m)$ denote $\mathrm{argmax}_{\eta_s}\{\Pi_s\}$ and be the reaction function of the supplier given the contract terms. The manufacturer, being the Stackelberg leader in the supply chain, offers a take-it or leave-it offer, which leaves the supplier indifferent between Contract S and her outside profit option which is normalized to zero. In other words, contract terms are set such that $p - S_W(\eta_s^*, R, \overline{T}, \eta_m) - C_s(\eta_s^*) = 0$ which implies $p = S_W(\eta_s^*, R, \overline{T}, \eta_m) + C_s(\eta_s^*)$.

The manufacturer determines his optimal effort and the optimal contract parameters R and $\overline{T}$ that would maximize his profits given the reaction (η_s^*) of the supplier to the contract terms.

The manufacturer solves:

$$\underset{\eta_m,\ R \text{ and } \overline{T}}{\Pi_m} = r - p - [\omega(1 - e^{-\Lambda_T(\eta_m, \eta_s)}) - S_W(\eta_s^*, R, \overline{T}, \eta_m)] - C_m(\eta_m).$$

Replacing p with $S_W(\eta_s^*, R, \overline{T}, \eta_m) + C_s(\eta_s^*)$, we get:

$$\underset{\eta_m,\ R \text{ and } \overline{T}}{\Pi_m} = r - [\omega(1 - e^{-\Lambda_T(\eta_m, \eta_s^*(R, \overline{T}, \eta_m))}] - C_m(\eta_m) - C_s(\eta_s^*).$$

Note that choosing $(R$ and $\overline{T})$ is equivalent to choosing η_s^*. Therefore solving the manufacturer's problem in terms of $(R$ and $\overline{T})$ is equivalent to solving it in terms of η_s^*. In other words, the manufacturer's problem can be restated as follows: $\underset{\eta_m, \eta_s^*}{\Pi_m} = r - [w(1 - e^{-\Lambda_T(\eta_m, \eta_s^*)})] - C_m(\eta_m) - C_s(\eta_s^*)$.

Note that the first order optimality conditions of the manufacturer's problem is equivalent to the first order optimality conditions of the centrally coordinated system. Hence, manufacturer's profits are optimized at $\eta_m^* = \eta_m^{*FB}$, $\eta_s^* = \eta_s^{*FB}$. Furthermore, the contract that attains first best effort from the supplier is in fact the effort coordinating contract.

PROOF OF PROPOSITION 3

Before we present the proof of Proposition 3, we present Proposition 3a which ensures supermodularity of the effort game under the Selective Root Cause Analysis Contract.

Proposition 3a: *Under the Selective Root Cause Analysis Contract, to ensure that the effort game played between the supplier and the manufacturer is supermodular, it is sufficient to have* $\lambda_s^0 = \ell \lambda_m^0$*, where* $\ell \in [0.36, 2.73]$.

Proof of Proposition 3a: Our goal is to identify two sufficient conditions under the Selective Root Cause Analysis Contract, that ensure non-negative cross partial derivatives of the manufacturer's and the supplier's profit functions with respect to η_s and η_m. This enables us to use the properties of supermodular game theory in characterizing the equilibrium outcome of the effort game played between the manufacturer and the supplier.

Consider the supplier's profit function under the Selective Root Cause Analysis Contract:

$$\Pi_s^S = p - u_s - (\omega + c_r)(\frac{\lambda_s^0(1 - \eta_s^S)}{\Lambda_T})[1 - e^{-\Lambda_T \overline{T}}] - R\omega[e^{-\Lambda_T \overline{T}} - e^{-\Lambda_T}] - C_s(\eta_s)$$

where $\Lambda_T = \lambda_m^0(1 - \eta_m) + \lambda_s^0(1 - \eta_s)$, $0 \le \overline{T} \le 1$, $0 \le R \le 1$, $0 \le \eta_m \le 1$ and $0 \le \eta_s \le 1$.

The cross partial derivative of the supplier's profit function with respect to η_m and η_s is given by

$$\begin{aligned}\frac{\partial^2 \Pi_s^S}{\partial \eta_m \partial \eta_s} &= e^{-\Lambda_T} R\omega\lambda_m^0\lambda_s^0 + \frac{e^{-\Lambda_T\overline{T}}(\omega + c_r)\overline{T}^2\lambda_m^0(1-\eta_s^S)(\lambda_s^0)^2}{\Lambda_T} \\ &\quad -\frac{(1-e^{-\Lambda_T\overline{T}})(\omega+c_r)\lambda_m^0\lambda_s^0}{\Lambda_T^2}\left(\frac{(\lambda_s^0)^2(1-\eta_s)}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0}\right) \\ &\quad +\frac{e^{-\Lambda_T\overline{T}}(\omega+c_r)\overline{T}\lambda_m^0\lambda_s^0}{\Lambda_T}\left(\frac{2(1-\eta_s^S)\lambda_s^0}{\Lambda_T}+1\right)\end{aligned} \tag{8}$$

The first two terms and the last term in (8) are non-negative, since $0 \leq \eta_m, \eta_s \leq 1$, $0 \leq \overline{T} \leq 1$, and $0 \leq R \leq 1$. The third term is non-negative when $\frac{(\lambda_s^0)^2(1-\eta_s)}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \leq 0$. The LHS of the inequality has its largest value when $\eta_s = 0$. Therefore, a sufficient condition for (8) to be non-negative is $\frac{(\lambda_s^0)^2}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \leq 0$.

Now consider the manufacturer's profit function under the Selective Root Cause Analysis Contract:

$$\Pi_m^S = r - u_m - p - (\omega + c_r)\left(\frac{\lambda_m^o(1-\eta_m^S)}{\Lambda_T}\right)[1-e^{-\Lambda_T\overline{T}}] - (1-R)\omega[e^{-\Lambda_T\overline{T}} - e^{-\Lambda_T}] - C_m(\eta_m)$$

where $\Lambda_T = \lambda_m^0(1-\eta_m) + \lambda_s^0(1-\eta_s)$, $\overline{T} \leq 1$, $0 \leq R \leq 1$, $0 \leq \eta_m \leq 1$ and $0 \leq \eta_s \leq 1$. The cross partial derivative of Π_m^S with respect to η_s and η_m is given by

$$\begin{aligned}\frac{\partial^2 \Pi_s^S}{\partial \eta_m \partial \eta_s} &= e^{-\Lambda_T}(1-R)\omega\lambda_m^0\lambda_s^0 + \frac{e^{-\Lambda_T\overline{T}}(\omega + c_r)\overline{T}^2\lambda_s^0(1-\eta_m^S)(\lambda_m^0)^2}{\Lambda_T} \\ &\quad -\frac{(1-e^{-\Lambda_T\overline{T}})(\omega+c_r)\lambda_m^0\lambda_s^0}{\Lambda_T^2}\left(\frac{(\lambda_m^0)^2(1-\eta_m)}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0}\right) \\ &\quad +\frac{e^{-\Lambda_T\overline{T}}(\omega+c_r)\overline{T}\lambda_m^0\lambda_s^0}{\Lambda_T}\left(\frac{2(1-\eta_m^S)\lambda_m^0}{\Lambda_T}+1\right)\end{aligned} \tag{9}$$

The first two terms and the last term in (9) are non-negative, since $0 \leq \eta_m \leq 1$, $0 \leq \overline{T} \leq 1$, and $0 \leq R \leq 1$. The third term is non-negative when $\frac{(\lambda_m^0)^2(1-\eta_m)}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$. The left hand side of the inequality has its largest value when $\eta_m = 0$. Therefore, a sufficient condition for (9) to be non-negative is $\frac{(\lambda_m^0)^2}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$.

Consequently, we find that $\frac{(\lambda_s^0)^2}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \leq 0$ and $\frac{(\lambda_m^0)^2}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$ are two sufficient conditions that ensure positive cross partial derivatives of the manufacturer's and the supplier's objective functions, respectively. Positive cross partial derivatives result in a supermodular effort game in which effort decisions are strategic complements (i.e., higher effort exerted by one player leads to higher effort exerted by the other player).

If we define $\lambda_s^0 = \ell\lambda_m^0$, then the above two conditions can be rewritten as $\ell^2 - 2(\ell+1) < 0$ (instead of $\frac{(\lambda_s^0)^2}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \leq 0$) and $2\ell^2 + 2\ell - 1 > 0$ (instead of $\frac{(\lambda_m^0)^2}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$).

Using some algebra, we find that when $\ell \in [\frac{1}{2}(-1+\sqrt{3}), 1+\sqrt{3}] = [0.36, 2.73]$ the two conditions hold simultaneously. This implies that when the manufacturer's and the supplier's initial failure rates are not drastically different, the supermodularity is satisfied. This ends the proof of Proposition 3a.

Note that the above condition is a sufficient condition that guarantees the supermodularity of the game. In our numerical study we have observed several cases that violated this condition and the game was still supermodular.

We now return to the proof of Proposition 3. First note that $\frac{\partial^2\Pi_m^S}{\partial\eta_m\partial\eta_s} > 0$ and $\frac{\partial^2\Pi_m^S}{\partial\eta_m\partial\eta_s} > 0$ ensure the supermodularity of the profit functions of the manufacturer and the supplier on lattice [0, 1]×[0, 1]. Theorems 2.2 (i.e., Tarski's Fixed Point Theorem) and 2.5 of Vives (1999) (i.e., Topkis' Theorem about equilibria in supermodular games) establish parts (i) and (ii).

Part (iii) follows from the complementarity of the effort choices, i.e., the best response function of the manufacturer (supplier) is increasing in the effort choice of the supplier (manufacturer). Specifically, define a pair of equilibria $(\widetilde{\eta}_s, \widetilde{\eta}_m)$ and $(\widehat{\eta}_s, \widehat{\eta}_m)$ and let $\widetilde{\eta}_s \leq \widehat{\eta}_s$ then, $\widetilde{\eta}_m = \eta_m^*(\widetilde{\eta}_s) \leq \eta_m^*(\widehat{\eta}_s) = \widehat{\eta}_m$.

For Part (iv), define a pair of equilibria $(\widetilde{\eta}_s, \widetilde{\eta}_m)$ and $(\widehat{\eta}_s, \widehat{\eta}_m)$ such that $\widetilde{\eta}_s \leq \widehat{\eta}_s$ and $\widetilde{\eta}_m \leq \widehat{\eta}_m$. From $\frac{\partial\Pi_s}{\partial\eta_m} > 0$ (i.e., supplier's profits are increasing with the manufacturer's quality improvement effort), it follows that $\Pi_s^*(\widetilde{\eta}_s, \widetilde{\eta}_m) \leq \Pi_s^*(\widetilde{\eta}_s, \widehat{\eta}_m) \leq \Pi_s^*(\eta_s^*(\widehat{\eta}_m), \widehat{\eta}_m) = \Pi_s^*(\widehat{\eta}_s, \widehat{\eta}_m)$. A similar argument can be made for the manufacturer's profits under the equilibria $(\widetilde{\eta}_s, \widetilde{\eta}_m)$ and $(\widehat{\eta}_s, \widehat{\eta}_m)$. Consequently, both the manufacturer and the supplier attain higher profits at the equilibrium in which both show high effort. This completes the proof of Proposition 3.

In what follows, we focus on this equilibrium outcome, which is the most preferred equilibrium (the Pareto optimal outcome) for both parties.

PROOF OF PROPOSITION 4

Under the Selective Root Cause Analysis Contract, the manufacturer's and the supplier's optimization problems are given by

$$\begin{aligned}
\underset{\eta_m}{Max} \ &: \ \Pi_m^S = r - u_m - p - (\omega + c_r)G[1 - e^{-\Lambda_T\overline{T}}] - \omega(1-R)[e^{-\Lambda_T\overline{T}} - e^{-\Lambda_T}] - C_m(\eta_m) \\
\underset{\eta_s}{Max} \ &: \ \Pi_s^S = p - u_s - (\omega + c_r)(1-G)[1 - e^{-\Lambda_T\overline{T}}] - R\omega[e^{-\Lambda_T\overline{T}} - e^{-\Lambda_T}] - C_s(\eta_s)
\end{aligned}$$

where $\Lambda_T = \lambda_m^0(1-\eta_m) + \lambda_s^0(1-\eta_s)$ and $G = \frac{\lambda_m^0(1-\eta_m)}{\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)}$.

(i) Coordinating Contract $(\overline{T}^*, R^*)$ for the Asymmetric Game When $c_r \geq 0$:

The coordinating values of $\overline{T}^*$ and R^* are given by the simultaneous solution of the following equations which follow from $\frac{\partial\Pi_m^S}{\partial\eta_m}(\overline{T}^*, R^*) \mid_{\eta_m^{*C},\eta_s^{*C}} = \frac{\partial\Pi^C}{\partial\eta_m} \mid_{\eta_m^{*C},\eta_s^{*C}} = 0$ and $\frac{\partial\Pi_s^S}{\partial\eta_s}(\overline{T}^*, R^*) \mid_{\eta_m^{*C},\eta_s^{*C}} = \frac{\partial\Pi^C}{\partial\eta_s} \mid_{\eta_m^{*C},\eta_s^{*C}} = 0$. One can easily show that

$$\begin{aligned}
\frac{\partial\Pi_m^S}{\partial\eta_m}\frac{1}{\lambda_m^0} &= -(\omega + c_r)\Big[\frac{G_m'^{*C}}{\lambda_m^0}(1 - e^{-\Lambda_T^{*C}\overline{T}}) - G^{*C}\overline{T}e^{-\Lambda_T^{*C}\overline{T}}\Big] - (\omega)(1-R)]\Big[\overline{T}e^{-\Lambda_T^{*C}\overline{T}} - e^{-\Lambda_T^{*C}}\Big] - \frac{C_m^{'}(\eta_m^{*C})}{\lambda_m^0} = 0 \\
\frac{\partial\Pi_s^S}{\partial\eta_s}\frac{1}{\lambda_s^0} &= -(\omega + c_r)\Big[\frac{-G_s'^{*C}}{\lambda_s^0}(1 - e^{-\Lambda_T^{*C}\overline{T}}) - (1 - G^{*C})\overline{T}e^{-\Lambda_T^{*C}\overline{T}}\Big] - (\omega)(R)]\Big[\overline{T}e^{-\Lambda_T^{*C}\overline{T}} - e^{-\Lambda_T^{*C}}\Big] - \frac{C_s(\eta_s^{*C})}{\lambda_s^0} = 0
\end{aligned}$$

Unfortunately, the complexity of the above equations do not allow us to derive closed-form solutions for R^* and $\overline{T}^*$. However, we can show that there exists a unique R^* and $\overline{T}^*$ that solves the above first order conditions (FOCs) at the first best effort level. To show this, we add the two FOCs and obtain

$$(\frac{G_s'^{*C}}{\lambda_s^0} - \frac{G_m'^{*C}}{\lambda_m^0})(\omega + c_r)(1 - e^{-\Lambda_T^{*C}\overline{T}}) - (\omega + c_r)(-\overline{T}e^{-\Lambda_T^{*C}\overline{T}}) - \omega(\overline{T}e^{-\Lambda_T^{*C}\overline{T}} - e^{-\Lambda_T^{*C}\bar{T}}) - \frac{C_m^{'}(\eta_m^{*C})}{\lambda_m^0} - \frac{C_s^{'}(\eta_s^{*C})}{\lambda_s^0} = 0$$

From the FOCs of the first best case, we know that $\frac{C_m^{'}(\eta_m^{*C})}{\lambda_m^0} = -\omega e^{-\Lambda_T^{*C}}$ and $\frac{C_s(\eta_s^{*C})}{\lambda_s^0} = -\omega e^{-\Lambda_T^{*C}}$. Furthermore, $G_s' = GK\frac{\lambda_s^0}{\lambda_m^0}$ and $G_m' = -(1-G)K$ (where $K = \frac{\lambda_m^0}{\Lambda_T^0}$). Consequently, the above equation simplifies to

$$\frac{\omega + c_r}{\Lambda_T^{*C}} - \omega e^{-\Lambda_T^{*C}} = \Big[c_r\overline{T} + \frac{(\omega + c_r)}{\Lambda_T^{*C}}\Big]e^{-\Lambda_T^{*C}\overline{T}}.$$

Note that the LHS is positive and is independent of $\overline{T}$ and the RHS is a continuous convex decreasing function of $\overline{T}$. Hence, to show that a unique $\overline{T}$ exists, we show that the RHS expression has a value greater (smaller) than the LHS when $T = 0$ (when $T = 1$). When $T = 0$, the RHS expression equals $\frac{\omega+c_r}{\Lambda_T^{*C}}$, which is greater than $\frac{\omega+c_r}{\Lambda_T^{*C}} - \omega e^{-\Lambda_T^{*C}}$, the value of the left hand side. When $T = 1$, we need to show that $\frac{\omega+c_r}{\Lambda_T^{*C}} - \omega e^{-\Lambda_T^{*C}} > \Big[c_r + \frac{(\omega+c_r)}{\Lambda_T^{*C}}\Big]e^{-\Lambda_T^{*C}}$, which simplifies to showing $(\omega + c_r)\frac{1}{\Lambda_T^{*C}} > (\omega + c_r)\frac{1}{\Lambda_T^{*C}}[\Lambda_T^{*C} + 1]e^{-\Lambda_T^{*C}}$, which is equivalent to showing $e^{\Lambda_T^{*C}} > 1 + \Lambda_T^{*C}$. The inequality $e^{\Lambda_T^{*C}} > 1 + \Lambda_T^{*C}$ is satisfied by $e^{\Lambda_T^{*C}} = \sum_{k=0}^{\infty} \frac{(\Lambda_T^{*C})^k}{k!}$.

Let $\overline{T}$ that solves $\frac{\omega+c_r}{\Lambda_T^{*C}} - \omega e^{-\Lambda_T^{*C}} = \Big[c_r\overline{T}^* + \frac{(\omega+c_r)}{\Lambda_T^{*C}}\Big]e^{-\Lambda_T^{*C}\overline{T}^*}$ be denoted by $0 < \overline{T}^* < 1$. To show that R^* exists, we subtract $\frac{\partial \Pi_s^S}{\partial \eta_s}\frac{1}{\lambda_s^0}$ from $\frac{\partial \Pi_m^S}{\partial \eta_m}\frac{1}{\lambda_m^0}$. We obtain:

$$-(\omega + c_r)\Big[\frac{(1 - e^{-\Lambda_T^{*C}\overline{T}})}{\Lambda_T^{*C}} - \overline{T}e^{-\Lambda_T^{*C}\overline{T}}\Big](1 - 2G^{*C}) - \omega(1 - 2R)\Big[\overline{T}e^{-\Lambda_T^{*C}}\overline{T} - e^{\Lambda_T^{*C}}\Big] = 0$$

Since the above equation is linear in R, we can solve for R^* as follows:

$$R^* = \frac{1}{2}(1 - \frac{(\omega + c_r)\Big[\overline{T}e^{-\Lambda_T^{*C}\overline{T}} - \frac{(1-e^{-\Lambda_T^{*C}\overline{T}})}{\Lambda_T^{*C}}\Big](1 - 2G^{*C})}{\omega\Big[\overline{T}e^{-\Lambda_T^{*C}\overline{T}} - e^{\Lambda_T^{*C}}\Big]}).$$

Hence, when $0 < R^* < 1$, then there exists a unique R^* and $\overline{T}^*$ that coordinate the effort decisions in the asymmetric game with $c_r \geq 0$.

(ii) Coordinating Contract ($\overline{T}^*, R^*$) for the Asymmetric Game When $c_r = 0$:

We evaluate the first order conditions of the manufacturer's and the supplier's profit functions at the first best level of effort. Consequently, we obtain:

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial \eta_m} &= -\omega\Big[G_m'^{*C}(1 - e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}\lambda_m^0 e^{-\Lambda_T^{*C}\overline{T}}\omega[G^{*C} - (1 - R)] - (1 - R)\lambda_m^0 e^{-\Lambda_T^{*C}}\Big] - C_m'(\eta_m^{*C}) = 0 \\
\frac{\partial \Pi_s^S}{\partial \eta_s} &= -\omega\Big[- G_s'^{*C}(1 - e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}\lambda_s^0 e^{-\Lambda_T^{*C}\overline{T}}[(1 - G^{*C}) - R] - R\omega\lambda_s^0 e^{-\Lambda_T^{*C}}\Big] - C_s'(\eta_s^{*C}) = 0
\end{aligned}$$

where $C_m'(\eta_m^{*C}) = \omega\lambda_m^0 e^{-\lambda_T^{*C}}$, $C_s'(\eta_s^{*C}) = \omega\lambda_s^0 e^{-\Lambda_T^{*C}}$ (please refer to the first best case presented in the main body of the paper). One can write:

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial \eta_m} &= -\omega\Big[G_m'^{*C}(1 - e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}\lambda_m^0 e^{-\Lambda_T^{*C}\overline{T}}[-G^{*C} + (1 - R)] - (1 - R)\lambda_m^0 e^{-\Lambda_T^{*C}}\Big] - \omega\lambda_m^0 e^{-\Lambda_T^{*C}} = 0 \\
\frac{\partial \Pi_s^S}{\partial \eta_s} &= -\omega\Big[- G_s'^{*C}(1 - e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}\lambda_s^0 e^{-\Lambda_T^{*C}\overline{T}}[G^{*C} - (1 - R)] - R\lambda_s^0 e^{-\Lambda_T^{*C}}\Big] - \omega\lambda_s^0 e^{-\lambda_T^{*C}} = 0
\end{aligned}$$

Let $(1 - R) = G^{*C}$, then the above equations are simplified as:

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial \eta_m} &= -\omega\Big[G_m'^{*C}(1 - e^{-\Lambda_T^{*C}\overline{T}}) - (1 - R)\lambda_m^0 e^{-\Lambda_T^{*C}}\Big] - \omega\lambda_m^0 e^{-\lambda_T^{*C}} = 0 \\
\frac{\partial \Pi_s^S}{\partial \eta_s} &= -\omega\Big[- G_s'^{*C}(1 - e^{-\Lambda_T^{*C}\overline{T}}) - R\lambda_s^0 e^{-\lambda_T^{*C}}\Big] - \omega\lambda_s^0 e^{-\Lambda_T^{*C}} = 0
\end{aligned}$$

Using $G'_m = -K(1-G)$, $G'_s = GK\frac{\lambda_s^0}{\lambda_m^0}$ and $R^* = 1-G^{*C}$, we can further simply the above equations into:

$$\begin{aligned}\frac{\partial \Pi_m^S}{\partial \eta_m} &= \omega(1-G^{*C})\lambda_m^0\Big[\frac{(1-e^{-\Lambda_T^{*C}\overline{T}})}{\lambda_T^{*C}} - e^{-\Lambda_T^{*C}}\Big] = 0\\ \frac{\partial \Pi_s^S}{\partial \eta_s} &= \omega G^{*C}\lambda_s^0\Big[\frac{(1-e^{-\Lambda_T^{*C}\overline{T}})}{\lambda_T^{*C}} - e^{-\Lambda_T^{*C}}\Big] = 0\end{aligned}$$

Solving for $\overline{T}$ from $\frac{(1-e^{-\Lambda_T^{*C}\overline{T}})}{\Lambda_T^{*C}} - e^{-\Lambda_T^{*C}} = 0$ we obtain $\overline{T} = -\frac{Ln(1-\Lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}}$.

(iii) Coordinating Contract in the Symmetric Game When $c_r \geq 0$:

We parameterize $c_r = d\omega$ where $d \geq 0$ and derive the first order conditions in the symmetric game as follows. Note that in the symmetric game $G^{*C} = \frac{1}{2}$.

$$\begin{aligned}\frac{\partial \Pi_m^S}{\partial \eta_m} &= (1+d)\frac{1}{2\Lambda_T}(1-e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}e^{-\lambda_T^{*C}\overline{T}}\Big[\frac{(1+d)}{2} - (1-R)\Big] - Re^{-\lambda_T^{*C}} = 0\\ \frac{\partial \Pi_s^S}{\partial \eta_s} &= (1+d)\frac{1}{2\Lambda_T}(1-e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}e^{-\lambda_T^{*C}\overline{T}}\Big[\frac{(1+d)}{2} - R)\Big] - (1-R)e^{-\Lambda_T^{*C}} = 0\end{aligned}$$

Since the first expression of both equations are the same, we can write:

$$\begin{aligned}\overline{T}e^{-\Lambda_T^{*C}\overline{T}}\Big[\frac{(1+d)}{2} - (1-R)\Big] - Re^{-\Lambda_T^{*C}} &= \overline{T}e^{-\lambda_T^{*C}\overline{T}}\Big[\frac{(1+d)}{2} - R\Big] - (1-R)e^{-\lambda_T^{*C}}\\ -\overline{T}e^{-\Lambda_T^{*C}\overline{T}}(1-2R) &= -(1-2R)e^{-\Lambda_T^{*C}}\\ (1-2R)(e^{-\Lambda_T^{*C}} - \overline{T}e^{-\Lambda_T^{*C}\overline{T}}) &= 0\end{aligned}$$

Hence, when $R^* = (1-R^*) = \frac{1}{2}$ then $\overline{T}^*$ can be calculated from the FOC as: $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}}$.

PROOF OF COROLLARY 1

We show that (i) $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} > 0$, and (ii) $\overline{T}^* = 0$ leads to underinvestment in effort by the manufacturer and the supplier. This implies that the Fixed Share Rate Contract, which is a special case of the Selective Root Cause Analysis Contract when $\overline{T}^* = 0$, cannot achieve the first best effort level from the supply chain partners.

(i) From $\lim_{\eta_s \to 1} C'_s(\eta_s) = \infty$ and $\lim_{\eta_m \to 1} C'_m(\eta_m) = \infty$ and $\lambda_m^0 > 0$ and $\lambda_s^0 > 0$, it follows that $\Lambda_T^{*C} > 0$ and $1 - \lambda_T^{*C}e^{-\Lambda_T^{*C}} < 1$. Consequently, $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} > 0$.

(ii) When $\overline{T}^* = 0$, we obtain the Fixed Share Rate Contract as the special case of Selective Root Cause Analysis Contract. In this case, the manufacturer's and the supplier's optimization problems are given by:

$$\begin{aligned}\underset{\eta_m}{Max} &: \Pi_m^S = r - u_m - p - \omega(1-R)[1-e^{-\Lambda_T}] - C_m(\eta_m)\\ \underset{\eta_s}{Max} &: \Pi_s^S = p - u_s - \omega R[1-e^{-\Lambda_T}] - C_s(\eta_s)\end{aligned}$$

The first order conditions of the manufacturer's and the supplier's optimization problems are, respectively, given by:

$$\begin{aligned}\frac{\partial \Pi_m}{\partial \eta_m} &= \omega(1-R)\lambda_m^0 e^{-[\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)]} - C_m'(\eta_m) = 0 \\ \frac{\partial \Pi_s}{\partial \eta_s} &= \omega R \lambda_s^0 e^{-[\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)]} - C_s'(\eta_s) = 0\end{aligned}$$

Since $C_m'(\eta_m^{*C}) = \omega\lambda_m^0 e^{-[\lambda_m^0(1-\eta_m^{*C})+\lambda_s^0(1-\eta_s^{*C})]}$ and $C_s'(\eta_s^{*C}) = \omega\lambda_s^0 e^{-[\lambda_m^0(1-\eta_m^{*C})+\lambda_s^0(1-\eta_s^{*C})]}$ and $C_s'(\eta_s) > 0$, $C_s'(\eta_s) > 0$, $C_m''(\eta_m) < 0$ and $C_s''(\eta_s) < 0$, it follows that the left hand side of the FOCs evaluated at $(\eta_m^{*C}, \eta_s^{*C})$ are negative. Therefore, $\eta_s^{*F} < \eta_s^{*C}$ and $\eta_m^{*F} < \eta_m^{*C}$, where F denotes the Fixed Share Rate Contract.

PROOF OF COROLLARY 2

We show that (i) $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C} e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} < 1$ and (ii) $\overline{T}^* = 1$ leads to overinvestment in effort by the manufacturer and the supplier. This implies that the total cost allocation contract, which is a special case of the Selective Root Cause Analysis Contract when $\overline{T}^* = 1$, cannot achieve the first best effort levels.

(i) Note that $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C} e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} < 1$ when $Ln(1-\lambda_T^{*C} e^{-\Lambda_T^{*C}}) > -\Lambda_T^{*C}$. This condition can be restated as $1-\lambda_T^{*C} e^{-\Lambda_T^{*C}} > e^{-\Lambda_T^{*C}}$. Notice that when $\frac{1+\lambda_T^{*C}}{e^{\Lambda_T^{*C}}} < 1$ then $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C} e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} < 1$. Since $e^{\Lambda_T^{*C}} = 1 + \Lambda_T^{*C} + \frac{(\Lambda_T^{*C})^2}{2!} + \cdots$, it follows that $\frac{1+\lambda_T^{*C}}{e^{\Lambda_T^{*C}}} < 1$. Consequently, the optimal root cause analysis threshold time that attains first best effort level is less than 1. Therefore, sharing the external quality costs based always on root cause analysis information (where $R = 0$) cannot attain the centrally coordinated (first best) quality.

(ii) When $\overline{T}^* = 1$, the manufacturer's and the supplier's optimization problems are given by:

$$\begin{aligned}\underset{\eta_m}{Max} &: \Pi_m^S = r - u_m - p - (\omega + c_r)G[1-e^{-\Lambda_T}] - C_m(\eta_m) \\ \underset{\eta_s}{Max} &: \Pi_s^S = p - u_s - (\omega + c_r)(1-G)[1-e^{-\Lambda_T}] - C_s(\eta_s)\end{aligned}$$

where $G = \frac{\lambda_m^0(1-\eta_m)}{\lambda_m^0(1-\eta_m)+\lambda_s^0(1-\eta_s)}$. The respective first order conditions evaluated at the first best effort levels are given by:

$$\begin{aligned}\frac{\partial \Pi_m}{\partial \eta_m} &= -(\omega + c_r)(G_m'(1-e^{-\Lambda_T^{*C}}) - G\lambda_m^0 e^{-\Lambda_T^{*C}}) - C_m'(\eta_m^{*C}) \\ \frac{\partial \Pi_s}{\partial \eta_s} &= -(\omega + c_r)(-G_s'(1-e^{-\Lambda_T^{*C}}) - (1-G)\lambda_s^0 e^{-\Lambda_T^{*C}}) - C_s'(\eta_s^{*C})\end{aligned}$$

where $G_m' = \frac{\partial G}{\partial \eta_m}$ and $G_s' = \frac{\partial G}{\partial \eta_s}$.

Let $c_r = 0$ (since the optimal effort is increasing in $(\omega + c_r)$ the overinvestment result still holds when $c_r > 0$). From $C_m'(\eta_m^{*C}) = \omega\lambda_m^0 e^{-[\lambda_m^0(1-\eta_m^{*C})+\lambda_s^0(1-\eta_s^{*C})]}$, the FOC of the manufacturer's optimization problem evaluated at the first best effort level can be restated as $\frac{\partial \Pi_m}{\partial \eta_m} = -\omega(G_m'(1-e^{-\Lambda_T^{*C}})) + \omega(1-G)\lambda_m^0 e^{-\Lambda_T^{*C}}$. Since $G_m' < 0$, it follows that $\frac{\partial \Pi_m}{\partial \eta_m} |_{\eta_m^{*C}, \eta_s^{*C}} > 0$. Therefore, when $\overline{T}^* = 1$, the manufacturer overinvests in quality improvement effort relative to the first best effort level. A similar reasoning is used to prove the supplier's overinvestment in effort result.

PROOF OF PROPOSITION 5

The manufacturer's and the supplier's objective functions under the Partial Cost Allocation Contract are given by:

$$\begin{aligned}
\underset{\eta_m}{Max} &: \Pi_m^P = r - u_m - p - (\omega + c_r)((1-R_m)\frac{\lambda_m^o(1-\eta_m^P)}{\Lambda_T} + (1-R_s)\frac{\lambda_s^o(1-\eta_s)}{\Lambda_T})(1-e^{-\Lambda_T}) - C_m(\eta_m) \\
\underset{\eta_s}{Max} &: \Pi_s^P = p - u_s - (\omega + c_r)(R_m\frac{\lambda_m^o(1-\eta_m)}{\Lambda_T} + R_s\frac{\lambda_s^o(1-\eta_s)}{\Lambda_T})(1-e^{-\Lambda_T}) - C_s(\eta_s)
\end{aligned}$$

The supermodularity of the symmetric effort game under partial cost allocation ensures the existence of at least one Nash equilibrium. Furthermore, similar to the Selective Root Cause Analysis Contract, one can easily show that the equilibria are rankable and both the supplier and the manufacturer attain higher profits in the equilibrium where they both show high effort. Therefore, in what follows, we focus on this most preferred (Pareto optimal) equilibrium point and discuss the effort coordinating contract.

Next, we show the supermodularity result for the symmetric effort game under Partial Cost Allocation Contract. This is followed by the derivation of the effort coordinating contract parameters.

Supermodularity of the Effort Game

We restate the manufacturer's and the supplier's optimization problems as follows. First we define the unit cost of root cause analysis as $c_r = d\omega$ where $d \geq 0$ and ω is the unit recall cost. Note that since the parameter d always appears in the expression $(1+d)$, which cancels out in the derivation of the supermodularity condition, our proof holds for all non-negative values of d, i.e including the case when $c_r = 0$.

$$\begin{aligned}
\underset{\eta_m}{Max} \ : \ \Pi_m &= r - u_m - p - \omega(1+d)[(1-R_m)G + (1-R_s)(1-G)]H - C_m(\eta_m) \\
\underset{\eta_s}{Max} \ : \ \Pi_s &= p - u_s - \omega(1+d)[R_mG + R_s(1-G)]H - C_s(\eta_s)
\end{aligned}$$

where R_m is the supplier's share of total recall cost when the manufacturer is at fault and R_s is the supplier's share of total recall cost when the supplier is at fault. The first order derivatives of the manufacturer's and the supplier's objective functions are, respectively, given by:

$$\begin{aligned}
\frac{\partial \Pi_m}{\partial \eta_m} &= -\omega(1+d)\big[((1-R_m)G_m^{'} - (1-R_s)G_m^{'})H + [(1-R_m)G + (1-R_s)(1-G)]H_m^{'}\big] - C_m^{'}(\eta_m) \\
\frac{\partial \Pi_s}{\partial \eta_s} &= -\omega(1+d)\big[[R_mG_s^{'} - R_sG_s^{'}]H + [(R_mG + R_s(1-G)]H_s^{'}\big] - C_s^{'}(\eta_s)
\end{aligned}$$

The cross partial derivatives of the manufacturer's and the supplier's objective functions are, respectively, given by:

$$\begin{aligned}
\frac{\partial \Pi_m}{\partial \eta_m \partial \eta_s} &= -\omega(1+d)\Big[((1-R_m)G_{ms}^{'} - (1-R_s)G_{ms}^{'})H + ((1-R_m)G_m^{'} - (1-R_s)G_m^{'})H_s^{'} \\
&\quad +((1-R_m)G_s^{'} - (1-R_s)G_s^{'})H_m^{'} + [(1-R_m)G + (1-R_s)(1-G)]H_{ms}^{'}\Big] \\
\frac{\partial \Pi_s}{\partial \eta_s \partial \eta_m} &= -\omega(1+d)\Big[[(R_m)G_{sm}^{'} - (R_s)G_{sm}^{'}]H + [(R_m)G_s^{'} - (R_s)G_s^{'}]H_m^{'} \\
&\quad +[(R_m)G_m^{'} - (R_s)G_m^{'}]H_s^{'} + [(R_m)G + (R_s)(1-G)]H_{sm}^{'}\Big]
\end{aligned}$$

Note that in the symmetric effort game where the supplier and the manufacturer have identical initial process failure rates and effort cost functions, $G^{'}_{ms} = 0$, $G^{'}_{m} = -G^{'}_{s}$, and $H^{'}_{s} = H^{'}_{m}$. Consequently, the above expressions simplify to:

$$\begin{aligned}\frac{\partial \Pi_m}{\partial \eta_m \partial \eta_s} &= -\omega(1+d)[(1-R_m)G + (1-R_s)(1-G)]H^{'}_{ms}\\ \frac{\partial \Pi_s}{\partial \eta_s \partial \eta_m} &= -\omega(1+d)[(1-R_m)G + (1-R_s)(1-G)]H^{'}_{ms}\end{aligned}$$

Since $H^{'}_{ms} < 0$, it follows that $\frac{\partial \Pi_m}{\partial \eta_m \partial \eta_s} \geq 0$ and $\frac{\partial \Pi_m}{\partial \eta_s \partial \eta_m} \geq 0$, which ensures the supermodularity of the symmetric effort game under Partial Cost Allocation Contract.

Effort Coordinating Contract P

(i) In the asymmetric effort game with $c_r = 0$, the manufacturer's and the supplier's objective functions are given by:

$$\begin{aligned}\underset{\eta_m}{Max} \ : \ \Pi_m &= r - u_m - p - \omega[(1-R_m)G + (1-R_s)(1-G)]H - C_m(\eta_m)\\ \underset{\eta_s}{Max} \ : \ \Pi_s &= p - u_s - \omega[R_m G + R_s(1-G)]H - C_s(\eta_s)\end{aligned}$$

The first order optimality conditions for η_s and η_m are given by

$\frac{\partial \Pi_m}{\partial \eta_m} = -\omega\{[(1-R_m)G^{'}_{m} - (1-R_s)G^{'}_{m})]H + [(1-R_m)G + (1-R_s)(1-G)]H^{'}_{m}\} - C^{'}_{m}(\eta_m) = 0$
$\frac{\partial \Pi_s}{\partial \eta_s} = -\omega\{[R_m G^{'}_{s} - R_s G^{'}_{s})]H + [R_m G + R_s(1-G)]H^{'}_{s}\} - C^{'}_{s}(\eta_s) = 0$

We want to solve for R_m and R_s such that these optimality conditions are satisfied at the coordinated effort levels $\eta_s = \eta_s^{*C}$ and $\eta_m = \eta_m^{*C}$. From the centrally coordinated system first order optimality conditions, we know that $C'_m(\eta_m^{*C}) = \omega\lambda_m^0(1 - e^{-\lambda_T^{*C}})$ and $C'_s(\eta_s^{*C}) = \omega\lambda_s^0(1 - e^{-\lambda_T^{*C}})$. Solving for R_m and R_s, from $\frac{\partial \Pi_m}{\partial \eta_m} |_{\eta_s = \eta_s^{*C}, \eta_m = \eta_m^{*C}} = 0$ and $\frac{\partial \Pi_s}{\partial \eta_s} |_{\eta_s = \eta_s^{*C}, \eta_m = \eta_m^{*C}} = 0$ where $C'_m(\eta_m^{*C}) = \omega\lambda_m^0(1 - e^{-\lambda_T^{*C}})$ and $C'_s(\eta_s^{*C}) = \omega\lambda_s^0(1 - e^{-\lambda_T^{*C}})$ we obtain

$$R_m^* = \frac{e^{2\lambda_T^{'}}}{(-e^{\lambda_T^0} + e^{\lambda_T^{'}})(\lambda_T^{*C})}\{e^{\lambda_T^0}\lambda_m^{*C} + e^{\lambda_T^{'}}[(\lambda_s^{*C})^2 - \lambda_m^{*C}(1-\lambda_s^{*C})] + e^{\lambda_T^0}(\lambda_s^{*C}\lambda_s^0 - \lambda_m^{*C}\lambda_m^0) - e^{\lambda_T^{'}}Z\}$$

$$R_s^* = R_m^* + \frac{\lambda_T^{*C}[1-\lambda_T^0]}{e^{\lambda_T^{*C}}}$$

where $\lambda_T^0 = \lambda_m^0 + \lambda_s^0$, $\lambda_s^{*C} = \lambda_s^0(1-\eta_s^{*C})$, $\lambda_m^{*C} = \lambda_m^0(1-\eta_m^{*C})$, $\lambda_T^{*C} = \lambda_s^{*C} + \lambda_m^{*C}$, $\lambda_T^{'} = \lambda_T^0 - \lambda_T^{*C}$ and $Z = \lambda_s^{*C}(2 - \eta_m^{*C} - \eta_s^{*C})\lambda_m^0\lambda_s^0 + \lambda_s^{*C}(1+\lambda_s^{*C})\lambda_s^0 - \lambda_m^{*C}\lambda_m^0(1-\lambda_s^{*C})$.

(ii) In the symmetric effort game with $c_r \geq 0$, the first order derivatives of the manufacturer's and the supplier's objective functions that can be simplified as follows:

$$\begin{aligned}\frac{\partial \Pi_m}{\partial \eta_m} &= -\omega(1+d)\Big[\Delta G^{'}_{m}H + (\Delta G H^{'}_{m} + (1-R_s)H^{'}_{m})\Big] - C^{'}_{m}(\eta_m)\\ \frac{\partial \Pi_s}{\partial \eta_s} &= -\omega(1+d)\Big[(-\Delta G^{'}_{s})H + (R_s H^{'}_{s} - \Delta G H^{'}_{s})\Big] - C^{'}_{s}(\eta_s)\end{aligned}$$

where $\Delta = R_s^* - R_m^*$. We evaluate $\frac{\partial \Pi_m}{\partial \eta_m}$ and $\frac{\partial \Pi_s}{\partial \eta_s}$ at the first best effort levels $(\eta_s^{*C}, \eta_m^{*C})$. Then, we solve for R_m^* and Δ^* that satisfy $\frac{\partial \Pi_m}{\partial \eta_m}(\eta_s^{*C}, \eta_m^{*C}, R_m^*, \Delta^*) = \frac{\partial \Pi^C}{\partial \eta_m^C}(\eta_s^{*C}, \eta_m^{*C})$ and $\frac{\partial \Pi_s^P}{\partial \eta_s}(\eta_s^{*C}, \eta_m^{*C}, R_s^*, \Delta^*) =$

$\frac{\partial \Pi^C}{\partial \eta_s^C}(\eta_s^{*C}, \eta_m^{*C})$ simultaneously to obtain:

$$\begin{aligned}
R_m^* &= \frac{A^*D^* - B^*\lambda_m^0 e^{-\lambda_T^{*C}} + A^*\lambda_s^0 e^{-\lambda_T^{*C}}}{B^*C^* + A^*D^*} \\
R_s^* &= R_m^* + \frac{A^*}{(A^* - C^*)} - \frac{\lambda_m^0 e^{-\lambda_T^{*C}}}{(A^* - C^*)}
\end{aligned}$$

where $A^* = -(1 + \frac{c_r}{\omega})(G_m^{'} H - G H_m^{'})$, $B^* = (1 + \frac{c_r}{\omega})(G_s^{'} H - G H_s^{'})$, $C^* = (1 + \frac{c_r}{\omega})H_m^{'}$ and $D^* = (1 + \frac{c_r}{\omega})H_m^{'}$ are evaluated at $(\eta_m^{*C}, \eta_s^{*C})$.

PROOF OF COROLLARY 3

Under the selective root cause analysis, at the first best effort level, the likelihood of performing a root cause analysis is given by $(1 - e^{-\Lambda_T \overline{T}^*})$, where $\Lambda_T = \lambda_m^0(1 - \eta_m^C) + \lambda_s^0(1 - \eta_s^C)$ and $\overline{T}^* < 1$. The expected root cause analysis cost of the supply chain is given by $c_r(1 - e^{-\Lambda_T \overline{T}^*})$.

Under the partial cost allocation scheme, at the first best effort level, the root cause analysis is performed for all failures. Consequently, the expected root cause analysis cost of the supply chain is given by $c_r(1 - e^{-\Lambda_T})$.

Since $\overline{T}^* < 1$, at the first best effort level, the expected root cause analysis cost under Selective Root Cause Analysis Contract is less than the expected root cause analysis cost under Partial Cost Allocation Contract. As the threshold time $\overline{T}^*$ gets smaller, the cost difference between the two contracts gets larger.

ON-LINE APPENDIX C

Proofs of Analytical Results: Asymmetric Information Case

In this appendix, we first present our discussion of the asymmetric information case. The proofs of the corresponding lemmas, propositions and remarks are presented at the end of our discussion.

As a benchmark case, let us define the centrally coordinated (first best) effort level under complete information about supplier quality. In other words, if the supplier's initial failure rate were known, and his effort level was observable, the first best effort level of the supplier would be given by the solution to the following optimization problem defined for each type supplier,

$$\max_{\eta^C_{s_j}} \Pi^C_j = r - u_m - u_s - \omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_j}(1-\eta^C_{s_j})]}) - C_s(\eta^C_{s_j}),$$

where $\eta^{*C}_{s_j}$ denotes the first best effort level of supplier type j, ($j \in \{H, L\}$). The optimal $\eta^{*C}_{s_j}$ trade-offs the expected recall cost with the cost of exerting quality improvement effort.

The next section characterizes the optimal menu of Contract S. This is followed by a numerical study that investigates the value of implementing a menu relative to the complete information model in terms of its impact on supply chain cost (profits) and on final product quality.

Optimal Menu of Selective Root Cause Analysis Contracts

The manufacturer offers a menu of Selective Root Cause Analysis Contracts $\{S_L, S_H\}$, where $S_L = (p_L, R_L, \overline{T}_L)$ denotes the contract designed for the low failure type supplier and $S_H = (p_H, R_H, \overline{T}_H)$ is the contract designed for the high failure type supplier. Laffont and Martimort (2002) show that, in a mixed modeling framework, the revelation principle (Kreps 1990) still applies, and therefore, one can focus on the menu of contracts that induces a truthful revelation of the supplier type. After a contract is accepted from the menu, the supplier chooses the investment in quality improvement effort that maximizes her profits.

Let $\Pi_{s_j}(S_a)$ denote the profits of supplier type j, where $j = \{L, H\}$ when she chooses Contract S_a from the menu and where the contract type $a \in \{L, H\}$ and the effort level $b \in \{L, H\}$. Then,

$$\Pi_{s_j}(S_a) = p_a - u_s - Z(\lambda^0_{s_j}, R_a, \overline{T}_a, \eta^b_{s_j}) - C_s(\eta^b_{s_j}),$$

where $Z(\lambda^0_{s_j}, R_a, \overline{T}_a, \eta^b_{s_j})$ represents the supplier's share of expected recall cost under Contract S_a with parameters $R_a, \overline{T}_a$ when her initial failure rate is $\lambda^0_{s_j}$ and she exerts effort $\eta^b_{s_j}$ and $b \in \{L, H\}$. This is given by:

$$\begin{aligned} Z(\lambda^0_{s_j}, R_a, \overline{T}_a, \eta^b_{s_j},) \quad &= \quad (\omega + c_r)\frac{\lambda^0_{S_j}(1-\eta^b_{s_j})}{\lambda^0_m + \lambda^0_{s_j}(1-\eta^b_{s_j})}\Big(1 - e^{-[\lambda^0_m + \lambda^0_{s_j}(1-\eta^b_{s_j})]\overline{T}_a}\Big) \\ &\quad + \omega R_a\Big(e^{-[\lambda^0_m + \lambda^0_{S_j}(1-\eta^b_{s_j})]\overline{T}_a} - e^{-[\lambda^0_m + \lambda^0_{s_j}(1-\eta^b_{s_j})]}\Big). \end{aligned}$$

To ensure truthful revelation of the supplier type, the menu of contracts should be *incentive compatible.* More specifically, given an incentive compatible menu, the supplier of type λ_{s_L} weakly prefers $S_L = (p_L, R_L, \overline{T}_L)$ over $S_H = (p_H, R_H, \overline{T}_H)$, and the supplier of type $\lambda^0_{s_H}$ weakly prefers

$S_H = (p_H, R_H, \overline{T}_H)$ over $S_L = (p_L, R_L, \overline{T}_L)$. These requirements are formalized by the following *Incentive Compatibility (I.C.) Constraints.*

$$
\begin{aligned}
p_H - u_s - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H}) - C_s(\eta^b_{s_H}) &\geq p_L - u_s - Z(\lambda^0_{s_H}, R_L, \overline{T}_L, \eta^b_{s_H}) - C_s(\eta^b_{s_H}) \quad (10)\\
p_L - u_s - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^b_{s_L}) - C_s(\eta^b_{s_L}) &\geq p_H - u_s - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_L}) - C_s(\eta^b_{s_L}) \quad (11)
\end{aligned}
$$

Secondly, for a contract to be accepted, it must yield to each type supplier at least his outside profit opportunity, which we normalize to zero. Therefore, the following *Participation Constraints* must be satisfied for the high and the low failure type supplier:

$$
\begin{aligned}
\Pi_{s_H}(S_H) &\geq 0 \quad (12)\\
\Pi_{s_L}(S_L) &\geq 0 \quad (13)
\end{aligned}
$$

Lastly, to induce optimal effort in equilibrium, the two *Moral Hazard Incentive Constraints* must hold depending on the effort level that the manufacturer wants to induce from each supplier type. For instance, if it is optimal to induce low effort level from the high failure rate supplier and high effort level from the low failure rate supplier, then the moral hazard constraints are given by:

$$
\begin{aligned}
p_H - u_s - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^H_{s_H}) - C_s(\eta^H_{s_H}) &< p_H - u_s - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^L_{s_H}) - C_s(\eta^L_{s_H}) \quad (14)\\
p_L - u_s - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^H_{s_L}) - C_s(\eta^H_{s_L}) &> p_L - u_s - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^L_{s_L}) - C_s(\eta^L_{s_L}) \quad (15)
\end{aligned}
$$

which simplify to:

$$
\begin{aligned}
Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^L_{s_H}) - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^H_{s_H}) &< C_s(\eta^H_{s_H}) - C_s(\eta^L_{s_H})\\
Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^L_{s_L}) - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^H_{s_L}) &> C_s(\eta^H_{s_L}) - C_s(\eta^L_{s_L}).
\end{aligned}
$$

Besides designing the optimal menu to screen the supplier type, the manufacturer's problem also involves solving for the optimal effort level to induce from each type. Constraints (10) to (15) characterize the set of incentive feasible menus. Below, we first characterize the condition under which a separating menu of Contract S exists. Next, we impose the moral hazard constraints and discuss how to integrate them into the manufacturer's choice of the optimal menu.

Separation of Supplier Types

In this section, we present a sufficient condition under which a menu of Contract S that ensures the separation of supplier types exists. We prove Lemma 2 and Propositon 6.

Lemma 2: *A sufficient condition to ensure the separation of supplier types in equilibrium under a menu of Selective Root Cause Analysis contracts is given by* $\lambda^0_m + \lambda^0_{s_H}(1 - \eta^L_{s_H}) < 1$.

Proposition 6: *In an incentive compatible menu of Selective Root Cause Analysis Contracts, which ensures separation of supplier types, it is sufficient that the high failure rate supplier has a lower fixed share rate ($R_H < R_L$) and a lower root cause analysis threshold ($\overline{T}_H < \overline{T}_L$) than the low failure rate type supplier.*

Proposition 6 shows that the manufacturer uses a higher root cause analysis threshold time (which, *ceteris paribus*, results in a higher likelihood of performing root cause analysis) and a higher cost share rate for the low failure rate supplier to deter the high failure rate supplier from mimicking him/her. Thus, the manufacturer makes it more costly for the high failure rate (low quality) supplier to misrepresent his/her type while not imposing a huge cost burden on the low failure rate (high quality) supplier.

Next, we rewrite equations (10) and (11) as follows:

$$\begin{aligned} \Pi_{s_L}(S_L) &\geq \Pi_{s_H}(S_H) + Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H}) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_L}) \qquad (16) \\ \Pi_{s_H}(S_H) &\geq \Pi_{s_L}(S_L) + Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^b_{s_L}) - Z(\lambda^0_{s_H}, R_L, \overline{T}_L, \eta^b_{s_L}) \end{aligned}$$

In constraint (16), the expression $Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{S_H}) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_L})$ denotes the *information rent* given to the low failure rate supplier to ensure separation of supplier types. The manufacturer's challenge is to determine the least costly way to give up rent to the low failure rate supplier provided by any incentive compatible contract. As will be shown later, since the manufacturer's profits are decreasing in $\Pi_{s_L}(S_L)$ and $\Pi_{s_H}(S_H)$, in equilibrium, the manufacturer sets $\Pi_{s_L}(S_L) = Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^H_s) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^H_s)$ and $\Pi_{s_H}(S_H) = 0$. Also, note that, consistent with the general theory of adverse selection, constraint (16) implies constraint (12); therefore the latter is ignored in characterizing the optimal contract.

Manufacturer's Choice of Optimal Menu

Let Π_m denote the manufacturer's expected profits, then Π_m is given by

$$\begin{aligned} \Pi_m &= r - u_m - \alpha p_H - (1-\alpha)p_L - \alpha\Big[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^b_{s_H})]}) - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H})\Big] \\ &\quad -(1-\alpha)\Big[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-\eta^b_{s_L})]}) - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^b_{s_L})\Big] \end{aligned}$$

The manufacturer solves

$$\begin{aligned} &\max_{\eta^b_{s_L},\, \eta^b_{s_H},\, S_L,\, S_H} \quad \Pi_m \\ &\textit{subject to} : \\ &\qquad \text{Constraints} \quad (10),\ (11),\ (12),\ (13),\ (14),\ \text{and } (15) \end{aligned}$$

Since the manufacturer's profits are decreasing in $\Pi_{s_H}(S_H)$ and $\Pi_{s_L}(S_L)$, in equilibrium constraint (12) and (16) are binding, which results in the following optimization problem for the manufacturer:

$$\begin{aligned} \max_{\eta^b_{s_L},\, \eta^b_{s_H},\, S_L,\, S_H} \Pi_m &= r - u_m - u_s - (1-\alpha)\Big[Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H}) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_L})\Big] \\ &\quad -\omega\Big[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^b_{s_H})]})\Big] - \omega\Big[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-\eta^b_{s_L})]})\Big] \\ &\quad -\alpha C_s(\eta^b_{s_H}) - (1-\alpha)C_s(\eta^b_{s_L}). \end{aligned}$$

In the manufacturer's problem stated above, his profit function consists of the following terms: (i) the unit revenue, (ii) the expected information rent given to the low failure type supplier, (iii) the expected total recall cost, and (iv) the expected supplier quality improvement effort cost. The manufacturer's challenge is to choose the optimal supplier effort ($\eta^b_{s_L}$, $\eta^b_{s_H}$) for each type as well as to determine (R^*_H,$\overline{T}^*_H$) and (R^*_L,$\overline{T}^*_L$) such that $R^*_H < R^*_L$ and $\overline{T}^*_H < \overline{T}^*_L$ to satisfy incentive compatibility constraints and thus screen supplier type.

A couple of interesting observations can be made regarding the manufacturer's optimization problem. First, note that it is independent of (R_L, $\overline{T}_L$). While the contract parameters (R_L, $\overline{T}_L$) do not drive the profits of the manufacturer directly, they have an indirect effect through the moral hazard constraint (15), which induces optimal quality improvement effort from the low failure rate supplier. Secondly, note that, if we ignore the expected information rent, the manufacturer's optimization problem is separable into two sub-problems, each of which solves the centrally coordinated effort level under complete information about each type of supplier. This implies that, if there were no information rent, then the first best effort level would be optimal from each supplier type. However, with nonzero information rent, the manufacturer distorts the optimal supplier effort from the first best.

Below we list our insights regarding the optimal solution to the manufacturer's optimization problem.

Remark 1: It is optimal to induce high effort from a high failure rate supplier (i.e., $\eta^H_{s_H}$) when the information rent given to the low failure rate supplier and the incremental effort cost is less than the savings in expected recall cost incurred at the higher effort level. More specifically, $\eta^H_{s_H}$ is optimal when

$$\begin{aligned} & \frac{(1-\alpha)}{\alpha}\Big[Z(\lambda^0_{s_H}, R^*_H, \overline{T}^*_H, \eta^H_{s_H}) - Z(\lambda^0_{s_L}, R^*_H, \overline{T}^*_H, \eta^b_{s_L})\Big] + \Big[C_s(\eta^H_{s_H}) - C_s(\eta^L_{s_H})\Big] \qquad (17) \\ > \; & \omega\Big(e^{-[\lambda^0_m+\lambda^0_{s_H}(1-\eta^H_{s_H})]} - e^{-[\lambda^0_m+\lambda^0_{s_H}(1-\eta^L_{s_H})]}\Big) \end{aligned}$$

holds and when R^*_H and T^*_H exist that solve the following moral hazard constraint:

$$Z(\lambda^0_{s_H}, R^*_H, \overline{T^*}_H, \eta^L_{s_H}) - Z(\lambda^0_{s_H}, R^*_H, \overline{T^*}_H, \eta^H_{s_H}) = C_s(\eta^H_{s_H}) - C_s(\eta^L_{s_H}) \qquad (18)$$

Otherwise, $R^*_H = 0$ and $T^*_H = 0$.

Notice that, when α is close to zero, the first term in (17) approaches infinity and it becomes optimal to induce low effort from a high failure rate supplier. When α is close to zero, the likelihood of having a low failure rate supplier $(1-\alpha)$ is close to 1. This translates into the probability of paying the information rent close to one. The information rent increases in the effort exerted by the high failure rate supplier. Therefore, to minimize the expected information rent, the manufacturer sets the effort level of the high failure rate supplier to η^L. To induce low effort from the high failure rate supplier, it is optimal to set $R^*_H = 0$ and $\overline{T}^*_H = 0$.

Remark 2: When it is optimal for the manufacturer to induce low effort from the high failure rate supplier (i.e., $R^*_H = 0$ and $\overline{T}^*_H = 0$), then it is optimal to induce the centrally coordinated (first best) effort level ($\eta^{*C}_{s_L}$) from the low failure rate supplier.

Suppose $\eta^{*C}_{s_L} = \eta^H_{s_L}$, then the optimal R^*_L and $\overline{T}^*_L$ is chosen to satisfy the following moral hazard constraint.

$$Z(\lambda^0_{s_L}, R^*_L, \overline{T}^*_L, \eta^L_{s_L}) - Z(\lambda^0_{s_L}, R^*_L, \overline{T}^*_L, \eta^H_{s_L}) = C_s(\eta^H_{s_L}) - C_s(\eta^L_{s_L}) \qquad (19)$$

Furthermore, a solution (R^*_L, $\overline{T}^*_L$) which satisfies (19) also exists.

Remark 3: When $R^*_H > 0$ and $\overline{T}^*_H > 0$ (i.e., when it is optimal to induce high effort from the high failure rate supplier) then it is optimal to induce high effort from the low failure rate supplier when the savings in external quality costs dominate the information rent and the incremental effort cost incurred due to high effort. Specifically, high effort is optimal from the low failure rate supplier when

$$\begin{aligned} & \Big[Z(\lambda^0_{s_L}, R^*_H, \overline{T}^*_H, \eta^L_{s_L}) - Z(\lambda^0_{s_L}, R^*_H, \overline{T}^*_H, \eta^H_{s_L})\Big] + \Big[C_s(\eta^H_{s_L}) - C_s(\eta^L_{s_L})\Big] \\ < \; & \omega\Big(e^{-[\lambda^0_m+\lambda^0_{s_L}(1-\eta^H_{s_L})]} - e^{-[\lambda^0_m+\lambda^0_{s_L}(1-\eta^L_{s_L})]}\Big), \end{aligned}$$

where $(R_H^*, \overline{T}_H^*)$ satisfies (18) and $R_L^* > R_H^*$ and $\overline{T}_L^* > \overline{T}_H^*$ satisfies

$$Z(\lambda^0_{s_L}, R_L^*, \overline{T}_L^*, \eta^L_{s_L}) - Z(\lambda^0_{s_L}, R_L^*, \overline{T}_L^*, \eta^H_{s_L}) = C_s(\eta^H_{s_L}) - C_s(\eta^L_{s_L})$$

Furthermore, one can also show that $R_L^* > R_H^*$ and $\overline{T}_L^* > \overline{T}_H^*$ exists (please see the proofs below).

PROOF OF LEMMA 2:

We start the proof by proving a condition which ensures separation of supplier type under the Spence-Mirrlees single crossing property. Next, for the Selective Root Cause Analysis menu of contracts, we prove the monotonicity constraints on the contract parameters (R_H, T_H) and (R_L, T_L).

Spence-Mirrlees Single Crossing Property

To simplify the exposition, in the derivative expressions, we omit the index $a \in \{L, H\}$, which denotes contract type in the menu. To ensure that Spence-Mirrlees single crossing property holds, given a particular contract from the menu, we require that $\frac{\partial Z}{\partial \lambda_{s_j} \partial R} = Z_{\lambda_{s_j} R} > 0$ and $\frac{\partial Z}{\partial \lambda_{s_j} \partial \overline{T}} = Z_{\lambda_{s_j} \overline{T}} > 0$ hold (Laffont and Martimort 2002). To this end, we investigate the sign of $\frac{\partial Z}{\partial \lambda_{s_j} \partial R} = Z_{\lambda_{s_j} R}$ and $\frac{\partial Z}{\partial \lambda_{s_j} \partial \overline{T}} = Z_{\lambda_{s_j} \overline{T}}$ where $Z(\lambda_{s_j}, R, \overline{T}, \eta^b_{s_j})$ is given by:

$$\begin{aligned} Z(\lambda_{s_j}, R, \overline{T}, \eta^b_{s_j}) &= w\frac{\lambda_{s_j}(1-\eta^b_{s_j})}{\lambda_m + \lambda_{s_j}(1-\eta^b_{s_j})}(1 - e^{-(\lambda_m + \lambda_{s_j}(1-\eta^b_{s_j}))\overline{T}}) \\ &\quad + wR(e^{-(\lambda_m + \lambda_{s_j}(1-\eta^b_{s_j}))\overline{T}} - e^{-(\lambda_m + \lambda_{s_j}(1-\eta^b_{s_j}))}) \end{aligned}$$

We calculate $\frac{\partial Z}{\partial \lambda_{s_j} \partial R} = Z_{\lambda_{s_j} R}$ and $\frac{\partial Z}{\partial \lambda_{s_j} \partial \overline{T}} = Z_{\lambda_{s_j} \overline{T}}$ as follows:

$$\begin{aligned} Z_{\lambda_{s_j} R} &= e^{-\overline{T}\lambda_T} w(1-\eta^b_{s_j})(e^{-(1-\overline{T})\lambda_T} - \overline{T}) \\ Z_{\lambda_{s_j} \overline{T}} &= e^{-\overline{T}\lambda_T} w(1-\eta^b_{s_j})[(1-R)(1-\overline{T}(1-\eta^b_{s_j})\lambda_{s_j}) + \overline{T}R\lambda_m] \end{aligned}$$

Now consider $Z_{\lambda_{s_j} R}$. Note that $Z_{\lambda_{s_j} R} > 0$ if $(e^{-(1-\overline{T})\lambda_T} - \overline{T}) > 0$. On the other hand, $(e^{-(1-\overline{T})\lambda_T} - \overline{T}) > 0$ holds if $\lambda_T < -\frac{Ln(\overline{T})}{(1-\overline{T})}$. Notice that $-\frac{Ln(\overline{T})}{(1-\overline{T})}$ is decreasing in $\overline{T}$, $\lim_{\overline{T} \to 0} -\frac{Ln(\overline{T})}{(1-\overline{T})} = \infty$ and $\lim_{\overline{T} \to 1} -\frac{Ln(\overline{T})}{(1-\overline{T})} = 1$. Hence, the most restrictive bound on λ_T is 1. Therefore, $Z_{\lambda_{s_j} R} > 0$ for $0 \le \overline{T} \le 1$ if $\lambda_T = \lambda^0_m + \lambda^0_{s_j}(1-\eta^b_{s_j}) < 1$ for $\eta^b_{s_j} \in \{\eta^L_{s_j}, \eta^H_{s_j}\}$. Hence, it is sufficient that $\lambda^0_m + \lambda^0_{s_H}(1-\eta^L_{s_H}) < 1$ holds, which ensures $Z_{\lambda_{s_j} R} > 0$.

Now consider $Z_{\lambda_{s_j} \overline{T}} > 0$. From $0 \le \overline{T} \le 1$, $Z_{\lambda_{s_j} R} > 0$ (i.e., $\lambda_T = \lambda^0_m + \lambda^0_{s_j}(1-\eta^b_{s_j}) < 1$) and $(1-\eta^b_{s_j})\lambda_{s_j} < 1$, it follows that $(1 - \overline{T}(1-\eta^b_{s_j})\lambda_{s_j}) > 0$. Consequently, $Z_{\lambda_{s_j} \overline{T}} > 0$. Therefore, $\lambda_m + \lambda_{s_H}(1-\eta^L_{s_H}) < 1$ is sufficient to guarantee that both $Z_{\lambda_{s_j} R} > 0$ and $Z_{\lambda_{s_j} \overline{T}} > 0$ hold.

PROOF OF PROPOSITION 6:

Monotonicity of (R_H , T_H) and (R_L , T_L)

To ensure truthful revelation of supplier type, the menu of contracts should be incentive compatible. More specifically, given an incentive compatible menu, the supplier of type $\lambda^0_{s_L}$ weakly prefers $S_L = (p_L, R_L, \overline{T}_L)$ over $S_H = (p_H, R_H, \overline{T}_H)$ and the supplier of type $\lambda^0_{s_H}$ weakly prefers $S_H = (p_H, R_H, \overline{T}_H)$ over $S_L = (p_L, R_L, \overline{T}_L)$. These requirements are formalized by the following Incentive Compatibility (I.C.) Constraints.

$$p_H - u_s - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H}) - C_s(\eta^b_{s_H}) > p_L - u_s - Z(\lambda^0_{s_H}, R_L, \overline{T}_L, \eta^b_{s_H}) - C_s(\eta^b_{s_H}) \quad (20)$$

$$p_L - u_s - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^b_{s_L}) - C_s(\eta^b_{s_L}) > p_H - u_s - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_L}) - C_s(\eta^b_{s_L}) \qquad (21)$$

By adding (20) and (21) we obtain

$$Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_j}) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_j}) < Z(\lambda^0_{s_H}, R_L, \overline{T}_L, \eta^b_{s_j}) - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^b_{s_j}) \qquad (22)$$

Given a particular contrat (R, T) from the menu, $Z_{\lambda_{s_j} R} > 0$, $Z_{\lambda_{s_j} \overline{T}} > 0$ and $Z_{\lambda_{s_j} R\overline{T}} > 0$ imply that the expression $Z(\lambda^0_{s_H}, R, \overline{T}, \eta^b_{s_j}) - Z(\lambda^0_{s_L}, R, \overline{T}, \eta^b_{s_j})$ is increasing with R and T. Therefore to satisfy (22), it is sufficient to have $R_L > R_H$ and $\overline{T}_L > \overline{T}_H$.

Note that $Z_{\lambda_{s_j} R\overline{T}} = e^{-\lambda_T} w(1 - \eta^b_{s_j})(1 - e^{(1-\overline{T})\lambda_T}\overline{T})$. On the other hand, $Z_{\lambda_{s_j} R\overline{T}}$ is non-negative if $(1 - e^{(1-\overline{T})\lambda_T}\overline{T}) \geq 0$. Furthermore, $(1 - e^{(1-\overline{T})\lambda_T}\overline{T}) \geq 0$ if $e^{(1-\overline{T})\lambda_T} \leq \frac{1}{\overline{T}}$. Notice that $0 \leq \overline{T} \leq 1$ and both sides of the inequality are decreasing in $\overline{T}$ with the LHS assuming values in the range [1, $e^{(1-\overline{T})\lambda_T}$] and the RHS assuming values in the range [1,∞). Consequently, $(1 - e^{(1-\overline{T})\lambda_T}\overline{T}) > 0$ and $Z_{\lambda_{s_j} R\overline{T}} > 0$.

Manufacturer's Optimization Problem

Let Π_m denote the manufacturer's expected profits, then Π_m is given by:

$$\begin{aligned} \Pi_m &= r - u_m - \alpha p_H - (1-\alpha) p_L - \alpha\Big[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^b_{s_H})]}) - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H})\Big] \\ & -(1-\alpha)\Big[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-\eta^b_{s_L})]}) - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^b_{s_L})\Big] \end{aligned}$$

The manufacturer solves:

$$\begin{aligned} & \max_{\eta^b_{s_L},\, \eta^b_{s_H},\, S_L,\, S_H} \Pi_m \\ \textit{subject to} \ : & \\ & \text{Constraints (10), (11), (12), (13), (14), and (15)} \end{aligned}$$

Notice that one can rewrite

$$\begin{aligned} p_H &= \Pi_{s_H}(S_H) + Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H}) + C_s(\eta^b_{s_H}) + u_s \\ p_L &= \Pi_{s_L}(S_L) + Z(\lambda^0_{s_L}, R_L, \overline{T}_L, \eta^b_{s_L}) + C_s(\eta^b_{s_L}) + u_s \end{aligned}$$

Substituting p_H and p_L into the manufacturer's objective function, we obtain:

$$\begin{aligned} \Pi_m &= r - u_m - u_s - \alpha \Pi_{s_H}(S_H) - (1-\alpha)\Pi_{s_L}(S_L) \\ & -\omega\Big[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^b_{s_H})]})\Big] - \omega\Big[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-\eta^b_{s_L})]})\Big] \\ & -\alpha C_s(\eta^b_{s_H}) - (1-\alpha) C_s(\eta^b_{s_L}) \end{aligned}$$

Since the manufacturer's profits are decreasing in $\Pi_{s_H}(S_H)$ and $\Pi_{s_L}(S_L)$, in equilibrium the individual rationality constraint for high failure rate type and the incentive compatibility constraint of low failure rate supplier are binding, which results in the following optimization problem for the manufacturer:

$$\begin{aligned} \max_{\eta^b_{s_L},\, \eta^b_{s_H},\, S_L,\, S_H} \Pi_m &= r - u_m - u_s - (1-\alpha)\Big[Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^b_{s_H}) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_L})\Big] \\ & -\omega\Big[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^a_{s_H})]})\Big] - \omega\Big[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-\eta^a_{s_L})]})\Big] \\ & -\alpha C_s(\eta^b_{s_H}) - (1-\alpha) C_s(\eta^b_{s_L}) \end{aligned}$$

PROOF OF REMARK 1

We first characterize the condition under which it is optimal to induce high effort from the high failure rate supplier (i.e., $\eta^H_{s_H}$). Fixing the low failure rate supplier's effort level at $\eta^b_{s_L}$, we compare the manufacturer's profits when the high failure rate supplier exerts high effort ($\widehat{\Pi}_m$) to his profits when the high failure rate supplier exerts low effort ($\widetilde{\Pi}_m$). This is formalized as follows:

$$\begin{aligned}\widehat{\Pi}_m &= r - u_m - u_s - (1-\alpha)[Z(\lambda^0_{s_H}, R_H, \overline{T}_H, \eta^H_{s_H}) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, \eta^b_{s_L})] \\ &\quad -\omega[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^H_{s_H})]})] - \omega[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-\eta^b_{s_L})]})] \\ &\quad -\alpha C_s(\eta^H_{s_H}) - (1-\alpha) C_s(\eta^b_{s_L})\end{aligned}$$

$$\begin{aligned}\widetilde{\Pi}_m &= r - u_m - u_s \\ &\quad -\omega[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^L_{s_H})]})] - \omega[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-\eta^b_{s_L})]})] \\ &\quad -\alpha C_s(\eta^L_{s_H}) - (1-\alpha) C_s(\eta^b_{s_L})\end{aligned}$$

The condition given below directly follows from simplification of the inequality $\widehat{\Pi}_m > \widetilde{\Pi}_m$.

$$\begin{aligned}&\frac{(1-\alpha)}{\alpha}[Z(\lambda^0_{s_H}, R^*_H, \overline{T}^*_H, \eta^H_{s_H}) - Z(\lambda^0_{s_L}, R^*_H, \overline{T}^*_H, \eta^b_{s_L})] + [C_s(\eta^H_{s_H}) - C_s(\eta^L_{s_H})] \\ > \; &\omega[e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^H_{s_H})]} - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-\eta^L_{s_H})]}]\end{aligned}$$

Note that a solution $R^*_H > 0$ and $T^*_H > 0$ to (23) exists.

$$Z(\lambda^0_{s_H}, R^*_H, \overline{T}^*_H, \eta^L_{s_H}) - Z(\lambda^0_{s_H}, R^*_H, \overline{T}^*_H, \eta^H_{s_H}) = C_s(\eta^H_{s_H}) - C_s(\eta^L_{s_H}) \tag{23}$$

The proof follows from (i) $\frac{\partial Z}{\partial \eta^b_{s_j}} < 0$, (ii) $\frac{\partial Z}{\partial \eta^b_{s_j} \partial R} < 0$ and $\frac{\partial Z}{\partial \eta^b_{s_j} \partial T} < 0$, and (iii) $C'_s(\eta^b_{s_j}) > 0$ which implies $C_s(\eta^H_{s_H}) - C_s(\eta^L_{s_H}) > 0$.

Condition (i) implies that Z is decreasing in the supplier's effort. Conditions (ii) and (iii) imply that as R_H and T_H increase, the difference $Z(\lambda^0_{s_H}, R^*_H, \overline{T}^*_H, \eta^L_{s_H}) - Z(\lambda^0_{s_H}, R^*_H, \overline{T}^*_H, \eta^H_{s_H})$ increases. Furthermore, since $Z(\lambda^0_{s_H}, 0, 0, \eta^L_{s_H}) = 0$, $Z(\lambda^0_{s_H}, 0, 0, \eta^H_{s_H}) = 0$ and $C_s(\eta^H_{s_H}) - C_s(\eta^L_{s_H})$ is positive, there exists $R^*_H > 0$ and $\overline{T}^*_H > 0$ that solves (23).

Note that $\frac{\partial Z}{\partial \eta^b_{s_j}} < 0$ because

$$\frac{\partial Z}{\partial \eta^b_{s_j}} = -\frac{(1 - e^{-\lambda_T \overline{T}})(c+\omega)\lambda^0_{s_j}\lambda^0_m}{\lambda^2_T} - \frac{e^{-\lambda_T \overline{T}}(c+\omega)\eta^b_{s_j}\overline{T}(\lambda^0_{s_j})^2}{\lambda_T} - R\omega\lambda^0_{s_j}(e^{-\lambda_T} - e^{-\lambda_T \overline{T}}\overline{T})$$

and $e^{-\lambda_T} - e^{-\lambda_T \overline{T}}\overline{T} > 0$ for $0 \le \overline{T} \le 1$, which implies that all of the terms in $\frac{\partial Z}{\partial \eta^b_{s_j}}$ expression are negative.

Furthermore, $Z_{\eta^b_{s_j} R} = -e^{-\overline{T}\lambda_T} w(e^{-(1-\overline{T})\lambda_T} - \overline{T})\lambda_T$ and $Z_{\eta^b_{s_j} R} = Z_{\eta^b_{s_j} \overline{T}} = e^{-\overline{T}\lambda_T} w\lambda[-(1-R) + \overline{T}(-R\lambda^0_m + (1-R)(1-\eta^b_{s_j})\lambda^0_s)]$. From $0 \le \overline{T} \le 1$ and $e^{-(1-\overline{T})\lambda_T} - \overline{T} > 0$, it follows that $Z_{\eta^b_{s_j} R} < 0$. From $0 \le \overline{T} \le 1$, $0 \le R \le 1$ and $\lambda_m(1-\eta_m) + (1-\eta_s)\lambda_s < 1$, it follows that $[-(1-R) + \overline{T}(-R\lambda^0_m + (1-R)(1-\eta^b_{s_j})\lambda^0_s)] < 0$. Consequently, $Z_{\eta^b_{s_j} \overline{T}} < 0$.

PROOF OF REMARK 2

Notice that when $R_H^* = 0$ and $\overline{T}_H^* = 0$, the information rent is zero. Consequently, the manufacturer chooses $\eta_{s_L}^b$ to minimize the following objective function: $\Pi_m = r - u_s - u_m - (1-\alpha)[\omega(1 - e^{-[\lambda_m^0 + \lambda_{s_L}^0(1-\eta_{s_L}^b)]}) + C_s(\eta_{s_L}^b)] - \alpha[\omega(1 - e^{-[\lambda_m^0 + \lambda_{s_H}^0]}) + C_s(\eta_{s_H}^L)]$. Since $\frac{\partial \Pi_m}{\partial \eta_{s_L}^b} = (1-\alpha)\frac{\partial \Pi_j^C}{\partial \eta_{s_L}^b}$, the manufacturer chooses the centrally coordinated first best effort level as the optimal effort for the low failure rate supplier.

Suppose $\eta_{s_L}^{*C} = \eta_{s_L}^H$ then the optimal R_L^* and $\overline{T}_L^*$ is chosen to satisfy the following moral hazard constraint.

$$Z(\lambda_{s_L}^0, R_L^*, \overline{T}_L^*, \eta_{s_L}^L) - Z(\lambda_{s_L}^0, R_L^*, \overline{T}_L^*, \eta_{s_L}^H) = C_s(\eta_{s_L}^H) - C_s(\eta_{s_L}^L) \tag{24}$$

R_L^* and T_L^* exists because (i) $\frac{\partial Z}{\partial \eta_{s_j}^b} < 0$, (ii) $\frac{\partial Z}{\partial \eta_{s_j}^b \partial R} < 0$ and $\frac{\partial Z}{\partial \eta_{s_j}^b \partial T} < 0$, and (iii) $C_s'(\eta_{s_j}^b) > 0$ which implies $C_s(\eta_{s_L}^H) - C_s(\eta_{s_L}^L) > 0$.

Condition (i) implies that Z is decreasing in the supplier's effort. Conditions (ii) and (iii) imply that as R_L and T_L increase, the difference $Z(\lambda_{s_L}^0, R_L^*, \overline{\overline{T}}_L^*, \eta_{s_L}^L) - Z(\lambda_{s_H}^0, R_L^*, \overline{\overline{T}}_L^*, \eta_{s_L}^H)$ increases. Furthermore, since $Z(\lambda_{s_L}^0, 0, 0, \eta_{s_L}^L) = 0$, $Z(\lambda_{s_L}^0, 0, 0, \eta_{s_L}^H) = 0$ and $C_s(\eta_{s_L}^H) - C_s(\eta_{s_L}^L)$ is positive, then there exists $R_L^* > 0$ and $\overline{T}_L^* > 0$ that solves (24).

PROOF OF REMARK 3

When $R_H^* > 0$ and $\overline{T}_H^* > 0$ (i.e., when it is optimal to induce high effort from the high failure rate supplier), then it is optimal to induce high effort from the low failure rate supplier when the savings in external quality costs dominate the information rent and the incremental effort cost incurred due to high effort. Specifically, high effort is optimal from the low failure rate supplier when:

$$\begin{aligned} & [Z(\lambda_{s_L}^0, R_H^*, \overline{T}_H^*, \eta_{s_L}^L) - Z(\lambda_{s_L}^0, R_H^*, \overline{T}_H^*, \eta_{s_L}^H)] + [C_s(\eta_{s_L}^H) - C_s(\eta_{s_L}^L)] \\ < \;\; & \omega[e^{-[\lambda_m^0 + \lambda_{s_L}^0(1-\eta_{s_L}^H)]} - e^{-[\lambda_m^0 + \lambda_{s_L}^0(1-\eta_{s_L}^L)]}] \end{aligned} \tag{25}$$

To prove the above inequality, we fix the effort of the high failure rate supplier to $\eta_{s_H}^H$. We define $\widehat{\Pi}_m$ as the manufacturer's profits when the low failure rate supplier shows high effort, and $\widetilde{\Pi}_m$ as the manufacturer's profits when the low failure rate supplier shows low effort. Specifically,

$$\begin{aligned} \widehat{\Pi}_m &= r - u_m - u_s - (1-\alpha)[Z(\lambda_{s_H}^0, R_H, \overline{T}_H, \eta_{s_H}^H) - Z(\lambda_{s_L}^0, R_H, \overline{T}_H, \eta_{s_L}^H)] \\ &\quad -\omega[\alpha(1 - e^{-[\lambda_m^0 + \lambda_{s_H}^0(1-\eta_{s_H}^H)]})] - \omega[(1-\alpha)(1 - e^{-[\lambda_m^0 + \lambda_{s_L}^0(1-\eta_{s_L}^H)]})] \\ &\quad -\alpha C_s(\eta_{s_H}^H) - (1-\alpha)C_s(\eta_{s_L}^H) \end{aligned}$$

$$\begin{aligned} \widetilde{\Pi}_m &= r - u_m - u_s \\ &\quad -\omega[\alpha(1 - e^{-[\lambda_m^0 + \lambda_{s_H}^0(1-\eta_{s_H}^H)]})] - \omega[(1-\alpha)(1 - e^{-[\lambda_m^0 + \lambda_{s_L}^0(1-\eta_{s_L}^L)]})] \\ &\quad -\alpha C_s(\eta_{s_H}^H) - (1-\alpha)C_s(\eta_{s_L}^L) \end{aligned}$$

The condition in (25) directly follows from simplification of the inequality $\widehat{\Pi}_m > \widetilde{\Pi}_m$.

Based on arguments presented in Remark 1 and Remark 2, one can show that $(R_H^*, \overline{T}_H^*)$ and $(R_L^*, \overline{T}_L^*)$ exist. Furthermore, $\frac{\partial Z}{\partial \eta_{s_j}^b \partial \lambda_j} < 0$ (i.e., the impact of effort on the expected recall cost is larger at higher initial failure rates) implies that

$$Z(\lambda_{s_H}^0, R_H^*, \overline{T}_H^*, \eta_{s_H}^L) - Z(\lambda_{s_H}^0, R_H^*, \overline{T}_H^*, \eta_{s_H}^H) > Z(\lambda_{s_L}^0, R_H^*, \overline{T}_H^*, \eta_{s_H}^L) - Z(\lambda_{S_L}^0, R_H^*, \overline{T}_H^*, \eta_{s_H}^H).$$

From $\frac{\partial Z}{\partial \eta^b_{s_j} \partial R} < 0$ and $\frac{\partial Z}{\partial \eta^b_{s_j} \partial T} < 0$, it follows that

$$Z(\lambda^0_{s_L}, R^*_H, \overline{T}^*_H, \eta^L_{s_H}) - Z(\lambda^0_{s_L}, R^*_H, \overline{T}^*_H, \eta^H_{s_H}) < Z(\lambda^0_{s_L}, R^*_L, \overline{T}^*_L, \eta^L_{s_H}) - Z(\lambda^0_{s_L}, R^*_L, \overline{T}^*_L, \eta^H_{s_H})$$

if $R^*_L > R^*_H$ and $\overline{T}^*_L > \overline{T}^*_H$. Therefore,

$$Z(\lambda^0_{s_L}, R^*_L, \overline{T}^*_L, \eta^L_{s_H}) - Z(\lambda^0_{s_L}, R^*_L, \overline{T}^*_L, \eta^H_{s_H}) = C_s(\eta^H_{s_L}) - C_s(\eta^L_{s_L})$$

is satisfied when $R^*_L > R^*_H$ and $\overline{T}^*_L > \overline{T}^*_H$.

Note that $\frac{\partial Z}{\partial \eta^b_{s_j} \partial \lambda^0_{s_j}}$ is given by:

$$\begin{aligned} \frac{\partial Z}{\partial \eta^b_{s_j} \partial \lambda_j} &= \frac{-2(1-e^{-\lambda_T \overline{T}})\omega(1-\eta^b_{s_j})^2(\lambda^0_{s_j})^2}{\lambda_T^3} - \frac{(1-e^{-\lambda_T \overline{T}})\omega}{\lambda_T} - \frac{3e^{-\lambda_T \overline{T}}\omega(1-\eta^b_{s_j})\overline{T}\lambda^0_{s_j}}{\lambda_T} \\ &\quad + R\omega(-e^{-\lambda_T}(1-(1-\eta^b_{s_j})\lambda^0_{s_j}) - e^{-\lambda_T \overline{T}}(1-(1-\eta^b_{s_j})\lambda^0_{s_j}\overline{T})) \end{aligned}$$

From Lemma 1 (i.e., $\lambda^0_m + (1-\eta^b_{s_j})\lambda^0_{s_j} < 1$) and $0 \leq \overline{T} \leq 1$, it follows that $\frac{\partial Z}{\partial \eta^b_{s_j} \partial \lambda^0_{s_j}} < 0$.

ON-LINE APPENDIX D
An Algorithm for Obtaining the Optimal Menu of Selective Root Cause Analysis Contracts

In this appendix we present our algorithm for obtaining the optimal menu of contracts. This algorithm is for general cases where both the manufacturer and the supplier can exert quality improvement efforts, which assume continuous values in the range of [0,1]. To calculate the optimal ($\overline{T}^*_L$, R^*_L) and ($\overline{T}^*_H$, R^*_H), we use the following sequence of steps:

Step 1: For a given $\overline{T}_H$ and R_H and manufacturer effort η_m, for each supplier type, we calculate the optimal effort levels and the optimal expected costs, which are, respective, denoted by $E_s\big[cost|\overline{T}_H,\, R_H, \lambda^0_{s_H}\big]$ and $E_s\big[cost|\overline{T}_H,\, R_H, \lambda^0_{s_L}\big]$ for high and low failure rate suppliers.

Step 2: We set the price of the high failure rate supplier to $p_H = E_s\big[cost|\overline{T}_H,\, R_H, \lambda^0_{s_H}\big]$, which makes her indifferent between accepting or rejecting the contract. We calculate the information rent (π) of the low failure rate supplier by $\pi = E_s\big[cost|\overline{T}_H, R_H, \lambda^0_{s_H}\big] - E_s\big[cost|\overline{T}_H, R_H, \lambda^0_{s_L}\big]$.

Step 3: We calculate $E_m\big[cost|\overline{T}_H,\, R_H, p_H, \lambda^0_{s_H}\big]$, the manufacturer's expected cost given that the supplier is the high failure rate supplier.

Step 4: We solve the following optimization problem to determine the optimal contract parameters for the low failure rate supplier, i.e. ($\overline{T}^*_L$, R^*_L). Notice from (27) that we leave the low failure rate supplier indifferent between accepting or rejecting the contract.

$$\underset{(\overline{T}_L, R_L)}{\mathbf{Min}}\; E_m\Big[cost|\overline{T}_L, R_L, p_2, \lambda^0_{s_L}\Big] \tag{26}$$

subject to:

$$p_L \;=\; \pi + E_s\Big[cost|\overline{T}_L, R_L, \lambda^0_{s_L}\Big] \tag{27}$$

Step 5: With the probability α, the supplier has a high failure rate and with probability $(1-\alpha)$, the supplier has a low failure rate. We can calculate the expected manufacturer's cost to optimize the manufacturer's profits by solving:

$$\begin{aligned} \underset{\eta_m}{Min} E_m\Big[cost|\overline{T}_L, R_L, p_L, \overline{T}_H, R_H, p_H\Big] \;&=\; \alpha E_m\Big[cost|\overline{T}_H, R_H, p_H, \lambda^0_{s_H}\Big] \\ &\quad + (1-\alpha) E_m\Big[cost|\overline{T}_L, R_L, p_L, \lambda^0_{s_L}\Big] \end{aligned}$$

Step 6: We change $\overline{T}_H$ and R_H (between 0 and 1) and repeat steps 1 to 5 until we obtain ($\overline{T}^*_L, R^*_L$) and ($\overline{T}^*_H, R^*_H$) that minimizes $E_m\big[cost|\overline{T}_L, R_L, p_L, \overline{T}_H, R_H, p_H\big]$.

ON-LINE APPENDIX E
Details of The Numerical Study

Numerical Study for Symmetric Information Case

To perform our numerical analysis, following the existing literature, we assume that the manufacturer's and the supplier's effort cost functions have the following functional forms (Corbett and DeCroix 2001): $C_m(\eta_m) = \gamma_m[-Ln(1-\eta_m) - \eta_m]$ and $C_s(\eta_s) = \gamma_s[-Ln(1-\eta_s) - \eta_s]$. The parameters γ_m and γ_s model the convexity of the effort cost functions, i.e., a larger value of γ_m (value of γ_s) is associated with a faster increasing effort cost function for the manufacturer (for the supplier).

Table E.1. Parameter Values for Numerical Analysis (Section 5)

Parameter	Values	Parameter	Values
ω	$100, 500, 1000$	c_r	0.1ω, 0.5ω, ω
γ_m	$10, 100, 500$	γ_s	$0.2\gamma_m$, $0.5\gamma_m$, γ_m, $2\gamma_m$, $5\gamma_m$,
λ_m^0	$0.4, 0.8, 1.2$	λ_s^0	$0.5\lambda_m^0$, λ_m^0, $2\lambda_m^0$,
M	$1000, 8000, 16000, 24000$		

Table E.1 lists the range of parameter values for our numerical analysis. These parameters are chosen such that they cover a wide range of cases, while creating scenarios in which the quality improvement efforts are not zero. We consider 12,960 combinations of parameter values while evaluating our cost and quality index for each contract. Tables E.2 and E.3 report a summary of C_I and Q_I across 12,960 parameter settings.

Table E.2. Cost Inefficiency Index (Section 5)

	Frequency		
Cost Inefficiency	Contract F	Contract S	Contract P
Less than 1%	25.93%	25.93%	0.00%
Less than 3%	40.25%	50.53%	5.60%
Less than 5%	47.16%	67.98%	12.43%
Less than 10%	86.67%	97.70%	36.13%
Less than 15%	99.26%	99.92%	41.56%
Less than 20%	100.00%	100.00%	49.30%
Less than 50%	100.00%	100.00%	81.89%
More than 50%	0.00%	0.00%	18.11%
Average	5.17%	3.64%	31.27%
Maximum	19.73%	16.39%	100.04%

Table E.3. Quality Inefficiency Index (Section 5)

	Frequency		
Quality Inefficiency	Contract F	Contract S	Contract P
Less than −10%	0.00%	0.00%	57.45%
Less than −5%	0.00%	0.00%	79.34%
Less than −3%	0.00%	0.00%	85.10%
Less than −1%	0.00%	0.00%	94.40%
Less than 0%	0.00%	0.00%	100.00%
Less than 1%	4.69%	4.69%	100.00%
Less than 3%	14.07%	14.07%	100.00%
Less than 5%	19.75%	19.75%	100.00%
Less than 10%	27.65%	35.64%	100.00%
Less than 15%	33.58%	49.30%	100.00%
Less than 20%	35.56%	55.39%	100.00%
Less than 50%	48.64%	82.39%	100.00%
More than 50%	51.36%	17.61%	0.00%
Average	54.57%	24.74%	−22.41%
Maximum	290%	121%	−0.04%

Numerical Study for Asymmetric Information Case

Our numerical study is based on the combination of the following set of parameter values that result in 5400 different experiments.

Table E.4. Parameter Values for Numerical Analysis (Section 7)

Parameter	Values	Parameter	Values
ω	$10, 100, 1000$	c_r	$0,\ 0.1\omega,\ 0.5\omega,\ \omega$
γ_m	$10, 100, 500$	γ_s	$0.2\gamma_m,\ 0.5\gamma_m,\ \gamma_m,\ 2\gamma_m,\ 5\gamma_m,\ 10\gamma_m$
λ_m^0	$0.05, 0.4, 0.8$	λ_{s1}^0	$0.2\lambda_m^0,\ 0.5\lambda_m^0,\ \lambda_m^0,\ 2\lambda_m^0,\ 5\lambda_m^0$
λ_{s2}^0	$0.2\lambda_{s1}^0,\ 0.8\lambda_{s1}^0$	α	$0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 0.9$

We also assume the same functional forms for the effort cost functions $C_s(\eta_s)$ and $C_m(\eta_m)$ as in our numerical study for symmetric information case.

Table E.5. Detail Data for Asymmetric Information (Section 7)

	Frequency Value of Information		Frequency Value of Menu	
Value Observed	Cost	Quality	Cost	Quality
Less than −10%	0.00%	0.33%	0.00%	0.00%
Less than −5%	0.00%	1.91%	0.00%	0.00%
Less than −3%	0.00%	3.37%	0.00%	0.00%
Less than −1%	0.00%	8.98%	0.00%	0.00%
Less than 0%	0.00%	30.74%	0.00%	3.31%
Less than 1%	5.03%	76.81%	5.93%	28.61%
Less than 3%	30.80%	87.01%	21.84%	41.94%
Less than 5%	45.24%	89.86%	38.21%	47.29%
Less than 10%	62.69%	94.88%	68.51%	58.75%
Less than 15%	74.97%	97.92%	79.91%	69.49%
Less than 20%	83.48%	98.83%	87.62%	74.47%
Less than 50%	100%	99.91%	100%	94.27%
More than 50%	0%	0.09%	0%	5.63%
Average	10.14%	1.26%	9.35%	13.79%
Maximum	46.28%	89.06%	46.12%	102.93%

From our numerical study we obtained several insights that we presented in Section 5. Those insights were based on 12,960 cases that we analyzed. Since we cannot present the details of all those 12,960 cases here, we have chosen several *representative* examples that show the typical behavior that we observed in 12,960 cases of our numerical study. These representative examples are shown in Figures 1 to 9. The detailed explanation of each figure can be found in its caption.

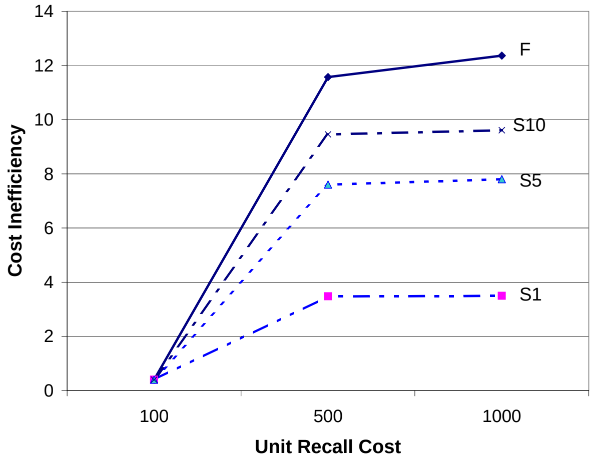

Figure 1: The gap between the cost efficiency of *Contract S* and that of *Contract F* increases as the gap between the unit recall cost (ω) and the unit root cause analysis cost (c_r) increases. In this figure (F: Contract F); (S1: Contract S with $c_r = 0.1\omega$); (S5: Contract S with $c_r = 0.5\omega$); and (S10: Contract S with $c_r = \omega$). The example in this figure is for a case with $\gamma_m = 100$, $\gamma_s = 100, \lambda_m^o = 0.8$, $\lambda_s^o = 1.6$, and $M = 8000$.

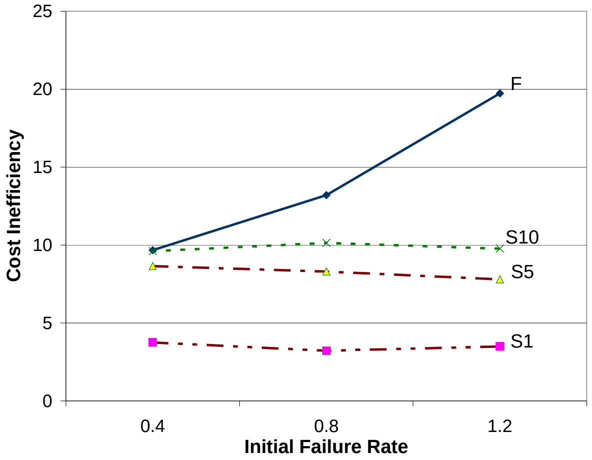

Figure 2: The gap between the cost efficiency of *Contract S* and that of *Contract F* increases as the initial failure rate (λ_m^o) increases. In this figure (F: Contract F); (S1: Contract S with $c_r = 0.1\omega$); (S5: Contract S with $c_r = 0.5\omega$); and (S10: Contract S with $c_r = \omega$). The example in this graph is for a case with $\omega = 100$, $\gamma_m = 10$, $\gamma_s = 20$, $\lambda_s^o = 2\lambda_m^o$, and $M = 8000$.

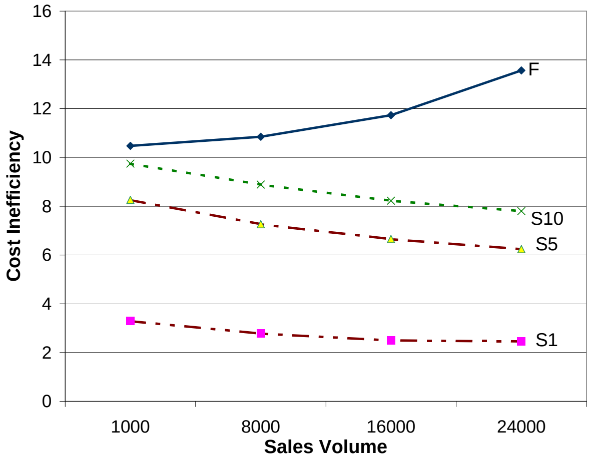

Figure 3: The gap between the cost efficiency of *Contract S* and that of *Contract F* increases as the sales volume of the product (M) increases. In this figure (F: Contract F); (S1: Contract S with $c_r = 0.1\omega$); (S5: Contract S with $c_r = 0.5\omega$); and (S10: Contract S with $c_r = \omega$). The example in this figure is for a case with $\omega = 1000$, $\gamma_m = 100$, $\gamma_s = 50, \lambda_m^o = 0.00005$, and $\lambda_s^o = 2\lambda_m^o$.

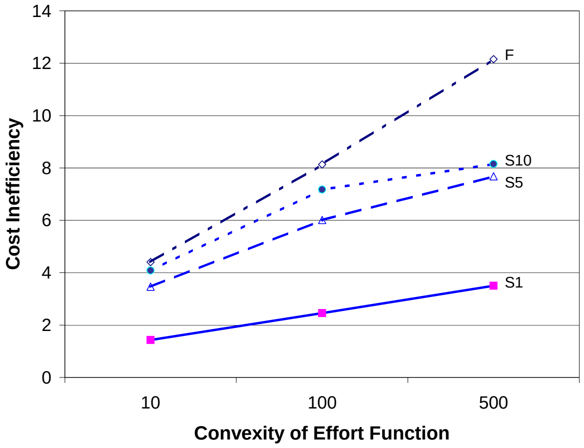

Figure 4: The gap between the cost efficiency of *Contract S* and that of *Contract F* increases as the convexity of the effort cost function (γ_m) increases. In this Figure, (F: Contract F); (S1: Contract S with $c_r = 0.1\omega$); (S5: Contract S with $c_r = 0.5\omega$); and (S10: Contract S with $c_r = \omega$). The example in this figure is for a case with $\omega = 1000$, $\gamma_m = 0.2\gamma_s, \lambda_m^o = \lambda_s^o = 1.2$, and $M = 8000$.

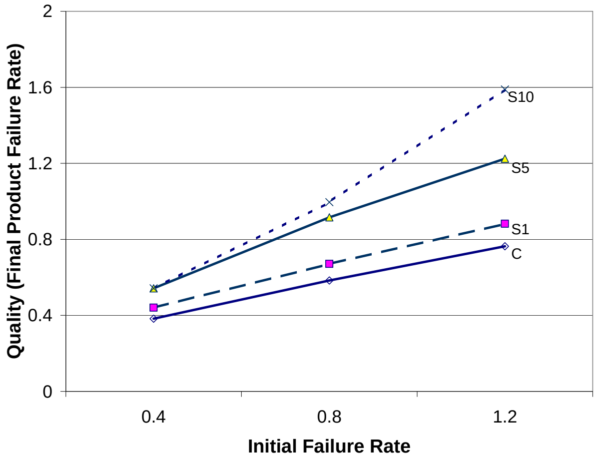

Figure 5: The gap between the product quality under *Contract S* in a *decentralized* system and that in the *centralized system* increases as the root causes analysis cost and the initial failure rates of the manufacturer (λ_m^o) increase. In this Figure (C: centralized system); (S1: Contract S with $c_r = 0.1\omega$); (S5: Contract S with $c_r = 0.5\omega$); and (S10: Contract S with $c_r = \omega$). The example in this figure is for a case with $\omega = 1000$ $\gamma_m = 500$, $\gamma_s = 100$, $\lambda_s^o = 2\lambda_m^o$, and $M = 8000$.

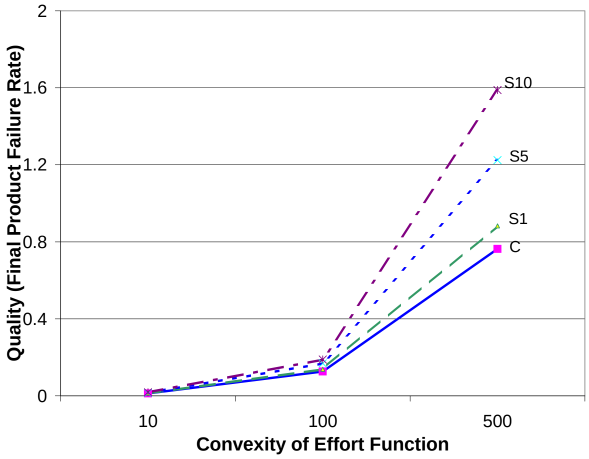

Figure 6: The gap between the product quality under *Contract S* in a *decentralized* system and that in the *centralized system* increases as the convexity of the effort cost function (γ_m) increases. In this figure (C: centralized system); (S1: Contract S with $c_r = 0.1\omega$); (S5: Contract S with $c_r = 0.5\omega$); and (S10: Contract S with $c_r = \omega$). The example in this figure is for a case with $\omega = 1000$, $\gamma_s = 0.2\gamma_m$, $\lambda_m^o = 1.2$, $\lambda_s^o = 2.4$, and $M = 8000$.

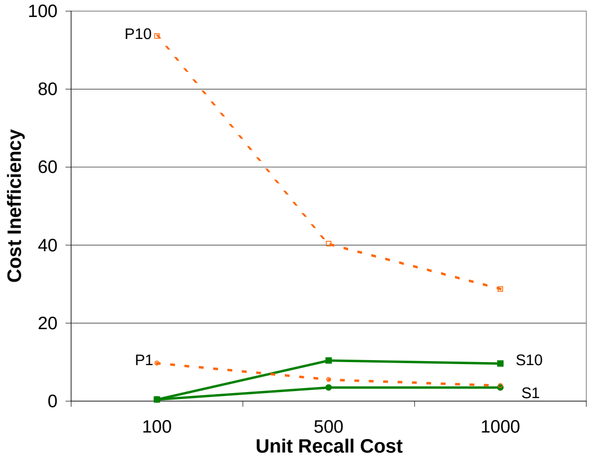

Figure 7: The gap between the cost efficiency of *Contract S* and that of *Contract P* decreases as the unit recall cost (ω) increases. In this figure (S1: Contract S with $c_r = 0.1\omega$); (S10: Contract S with $c_r = \omega$); (P1: Contract P with $c_r = 0.1\omega$); and (P10: Contract P with $c_r = \omega$). The example in this figure is for a case with $\gamma_m = 100$, $\gamma_s = 200$, $\lambda_m^o = 0.8$, $\lambda_s^o = 1.6$, and $M = 8000$.

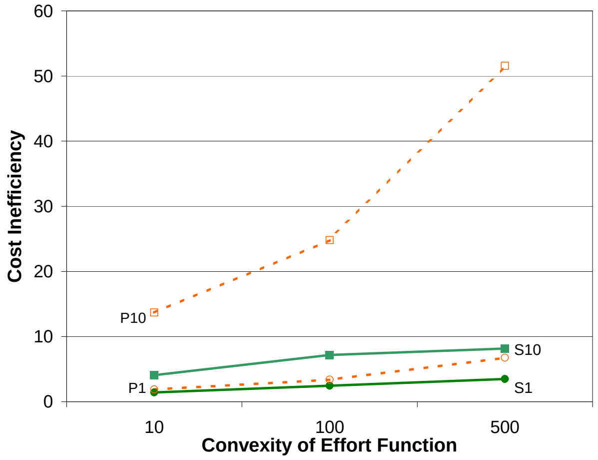

Figure 8: The gap between the cost efficiency of *Contract S* and that of *Contract P* decreases as the convexity of the effort cost function (γ_m) decreases. In this figure (S1: Contract S with $c_r = 0.1\omega$); (S10: Contract S with $c_r = \omega$); (P1: Contract P with $c_r = 0.1\omega$); and (P10: Contract P with $c_r = \omega$). The example in this figure is for a case with $\omega = 1000$, $\gamma_s = 0.2\gamma_m$, $\lambda_m^o = \lambda_s^o = 1.2$, and $M = 8000$.

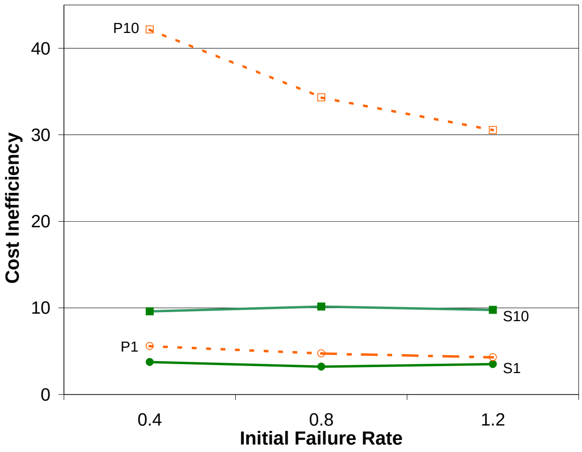

Figure 9: The gap between the cost efficiency of *Contract S* and that of *Contract P* decreases as the initial failure rate of the manufacturer (λ_m^o) increases. In this figure (S1: Contract S with $c_r = 0.1\omega$); (S10: Contract S with $c_r = \omega$); (P1: Contract P with $c_r = 0.1\omega$); and (P10: Contract P with $c_r = \omega$). This example is for a case with $\omega = 100$, $\gamma_m = 10$, $\gamma_s = 20$, $\lambda_s^o = 2\lambda_m^o$, and $M = 8000$.