

**e - c o m p a n i o n**

ONLY AVAILABLE IN ELECTRONIC FORM

Electronic Companion—“Optimal Price and Product Quality  
Decisions in a Distribution Channel” by Xiaowei Xu,  
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## Online Appendix

This appendix is an online companion to the main text. We adopt all notation used in the main text.

### A1. Uniqueness of optimal product quality and price decisions

First, we provide sufficient conditions that guarantee the uniqueness of the optimal product quality and price decisions. Recall that we used the following assumptions in the main text.

**Assumption I.**  $k(u)$  is increasing in  $u \in [0, +\infty)$  and  $\lim_{u \rightarrow +\infty} k(u) > 1$ .

**Assumption II.**  $\bar{C}(Q)$  is positive and decreasing in  $Q \in [0, Q_o]$ , but increasing in  $Q \in [Q_o, +\infty)$ , where  $Q_o > 0$ .

We consider two examples of  $\alpha(Q)$ ,  $\beta(Q)$  and  $C(Q)$  that satisfy Assumption II.

**Example I.** We let  $\alpha(Q) = \alpha_0 Q^{-\alpha_1}$ ,  $\beta(Q) = \beta_0 Q^{\beta_1} - \beta_2$  and  $C(Q) = c_0 Q^{c_1}$ , where  $\{\alpha_i\}_{i=0}^1$ ,  $\{\beta_i\}_{i=0}^2$  and  $\{c_i\}_{i=0}^1$  are positive,  $\beta_1 \leq 1$  and  $c_1 \geq 1$ . Hence,  $\bar{C}(Q) = \alpha_0 c_0 Q^{c_1 - \alpha_1} - \beta_0 Q^{\beta_1} + \beta_2$ . If  $c_1 - \alpha_1 = \beta_1$  and  $\alpha_0 c_0 > \beta_0$ , then Assumption II holds and  $Q_o = 0$ . If  $c_1 - \alpha_1 > \beta_1$  and  $\beta_2 > \frac{\beta_0(c_1 - \alpha_1 - \beta_1)}{c_1 - \alpha_1} [\frac{\beta_0 \beta_1}{\alpha_0 c_0 (c_1 - \alpha_1)}]^{c_1 - \alpha_1 - \beta_1}$ , then Assumption II holds and  $Q_o = [\frac{\beta_0 \beta_1}{\alpha_0 c_0 (c_1 - \alpha_1)}]^{c_1 - \alpha_1 - \beta_1}$ .

Second, we let  $\alpha(Q) = \alpha_0 e^{-\alpha_1 Q}$ ,  $\beta(Q) = \beta_0(1 - e^{-\beta_1 Q})$  and  $C(Q) = c_0 e^{c_1 Q}$ , where  $\{\alpha_i\}_{i=0}^1$ ,  $\{\beta_i\}_{i=0}^1$  and  $\{c_i\}_{i=0}^1$  are positive and  $\frac{\beta_1}{c_1 - \alpha_1} > \frac{\alpha_0 c_0}{\beta_0} > 1$ . Hence,  $\bar{C}(Q) = \alpha_0 c_0 e^{(c_1 - \alpha_1)Q} + \beta_0 e^{-\beta_1 Q} - \beta_0$ , which satisfies Assumption II and implies  $Q_o = \frac{1}{c_1 - \alpha_1 + \beta_1} \log\left(\frac{\beta_0 \beta_1}{\alpha_0 c_0 (c_1 - \alpha_1)}\right)$ .

### I. THE DIRECT-SELL CASE

The optimization procedure has two steps. First, we find the optimal direct-sell price  $p_d^*(Q)$  to maximize the profit function  $\Pi(p, Q)$  given product quality  $Q$ , where  $Q \geq Q_o$ .

**Proposition I.** If Assumptions I and II hold, then: (1).  $\Pi(p, Q)$  has a unique maximizer  $p_d^*(Q) = \frac{\bar{p}^*(\bar{C}(Q)) + \beta(Q)}{\alpha(Q)}$ , where  $\bar{p}^*(u)$  is the unique solution of  $\bar{p} - u = \frac{1}{h(\bar{p})}$  and  $u > 0$ ; (2).  $p_d^*(Q)$  is increasing in  $Q \in [Q_o, +\infty)$ .

*Proof.* For Claim 1, notice that  $\frac{\partial}{\partial p} \log(\Pi(p, Q)) = \frac{1}{p - C(Q)} - h(\alpha(Q)p - \beta(Q))\alpha(Q) = \alpha(Q) \left[ \frac{1}{\bar{p} - \bar{C}(Q)} - h(\bar{p}) \right]$ , where  $\bar{p} = \alpha(Q)p - \beta(Q)$  and  $\bar{C}(Q) = \alpha(Q)C(Q) - \beta(Q)$ . The first order condition is

equivalent to  $\bar{p} - \bar{C}(Q) = \frac{1}{h(\bar{p})}$  and  $1 = \frac{\bar{C}(Q)}{\bar{p}} + \frac{1}{k(\bar{p})}$ . By Assumption II,  $\bar{C}(Q) > 0$ . Since  $p > C(Q)$ , by Assumption II,  $\bar{p} > 0$ . By Assumption I,  $\frac{\bar{C}(Q)}{\bar{p}} + \frac{1}{k(\bar{p})}$  is decreasing in  $\bar{p}$  and  $\lim_{\bar{p} \rightarrow +\infty} k(\bar{p}) > 1$ . Hence,  $1 = \frac{\bar{C}(Q)}{\bar{p}} + \frac{1}{k(\bar{p})}$  has a unique solution  $\bar{p}^*(\bar{C}(Q))$ . Let  $p_d^*(Q) = \frac{\bar{p}^*(\bar{C}(Q)) + \beta(Q)}{\alpha(Q)}$ . Then Claim 1 holds.. For Claim 2, notice that  $\bar{p}^*(u)$  is increasing in  $u$ ,  $\alpha(Q)$  is decreasing in  $Q$ ,  $\beta(Q)$  is increasing in  $Q$  and  $\bar{C}(Q)$  is increasing in  $Q \in [Q_o, +\infty)$ .  $\square$

By Proposition I, a high quality product is associated with a high price. We let  $\Pi^*(Q) = \Pi(p_d^*(Q), Q)$  and make the following assumption of  $A(Q)$ ,  $C(Q)$ ,  $\alpha(Q)$  and  $\beta(Q)$ .

**Assumption III.**  $[\frac{d}{dQ} \log(\bar{A}(Q))]/[\frac{d}{dQ} \log(\bar{C}(Q))]$  is decreasing in  $Q \in [Q_o, +\infty)$ .

Assumption III requires that the ratio of the relative increase rate of  $\bar{A}(Q)$  to the relative increase rate of  $\bar{C}(Q)$  is decreasing in  $Q$ . We consider two examples that satisfy Assumption III.

**Example II.** We let  $A(Q) = a_0 Q^{a_1}$ ,  $\alpha(Q) = \alpha_0 Q^{-\alpha_1}$ ,  $\beta(Q) = \beta_0 Q^{\beta_1}$  and  $C(Q) = c_0 Q^{c_1}$ , where  $\{a_i\}_{i=0}^1$ ,  $\{\alpha_i\}_{i=0}^1$ ,  $\{\beta_i\}_{i=0}^1$  and  $\{c_i\}_{i=0}^1$  are positive,  $a_1 \leq 1$ ,  $\beta_1 \leq 1$  and  $c_1 \geq 1$ . Hence,  $\bar{A}(Q) = \frac{a_0}{\alpha_0} Q^{a_1 + \alpha_1}$  and  $\bar{C}(Q) = \alpha_0 c_0 Q^{c_1 - \alpha_1} - \beta_0 Q^{\beta_1}$ . If  $c_1 - \alpha_1 = \beta_1$  and  $\alpha_0 c_0 > \beta_0$ , then  $[\frac{d}{dQ} \log(\bar{A}(Q))]/[\frac{d}{dQ} \log(\bar{C}(Q))] = \frac{a_1 + \alpha_1}{c_1 - \alpha_1}$ , which implies Assumption III.

Second, we let  $A(Q) = a_0(1 - e^{-a_1 Q})$ ,  $\alpha(Q) = \alpha_0 e^{-\alpha_1 Q}$ ,  $\beta(Q) = 0$  and  $C(Q) = c_0 e^{c_1 Q}$ , where  $\{a_i\}_{i=0}^1$ ,  $\{\alpha_i\}_{i=0}^1$  and  $\{c_i\}_{i=0}^1$  are positive and  $c_1 - \alpha_1 > 0$ . Hence,  $\bar{A}(Q) = \frac{a_0}{\alpha_0} e^{\alpha_1 Q} (1 - e^{-a_1 Q})$  and  $\bar{C}(Q) = \alpha_0 c_0 e^{(c_1 - \alpha_1) Q}$ . Notice that  $[\frac{d}{dQ} \log(\bar{A}(Q))]/[\frac{d}{dQ} \log(\bar{C}(Q))] = \frac{1}{c_1 - \alpha_1} (\alpha_1 + \frac{a_1}{e^{a_1 Q} - 1})$ , which is decreasing in  $Q$ . Hence, Assumption III holds.

Next, we find the optimal product quality  $Q_d^*$  to maximize the manufacturer's profit  $\Pi^*(Q)$ . We let the optimal profit  $\Pi_d^* = \Pi^*(Q_d^*)$  and optimal price  $p_d^* = p_d^*(Q_d^*)$ .

**Proposition II.** If Assumptions I-III hold, then  $\Pi^*(Q)$  has a unique maximizer  $Q_d^*$ , where  $Q_d^* = \sup\{Q \geq Q_o \mid [\frac{d}{dQ} \log(\bar{A}(Q))] \geq [\frac{d}{dQ} \bar{C}(Q)] h(\bar{p}^*(\bar{C}(Q)))\}$ .

*Proof.* It is equivalent to maximize the profit function  $\bar{\Pi}^*(Q) = \bar{\Pi}(\bar{p}^*(\bar{C}(Q)), Q)$ , where  $\bar{\Pi}(\bar{p}, Q) = \bar{A}(Q)[\bar{p} - \bar{C}(Q)]e^{-H(\bar{p})}$  and  $\bar{p}^*(\bar{C}(Q))$  is the unique solution of  $\bar{p} - \bar{C}(Q) = \frac{1}{h(\bar{p})}$ . By the envelope

theorem,

$$\begin{aligned}
\frac{d}{dQ} \log(\bar{\Pi}^*(Q)) &= \left[ \frac{d}{dQ} \log(\bar{A}(Q)) \right] - \left[ \frac{d}{dQ} \bar{C}(Q) \right] / \left[ \bar{p}^*(\bar{C}(Q)) - \bar{C}(Q) \right] \\
&= \left[ \frac{d}{dQ} \log(\bar{A}(Q)) \right] - \left[ \frac{d}{dQ} \bar{C}(Q) \right] h(\bar{p}^*(\bar{C}(Q))) \\
&= \left[ \frac{d}{dQ} \log(\bar{C}(Q)) \right] \left( \left[ \frac{d}{dQ} \log(\bar{A}(Q)) \right] / \left[ \frac{d}{dQ} \log(\bar{C}(Q)) \right] - \bar{C}(Q) h(\bar{p}^*(\bar{C}(Q))) \right).
\end{aligned}$$

Next, we prove that  $\bar{C}(Q)h(\bar{p}^*(\bar{C}(Q)))$  is increasing in  $Q \in [Q_o, +\infty)$ . Let  $r = \frac{\bar{p}}{\bar{C}(Q)} - 1$ . The equation of  $\bar{p} - \bar{C}(Q) = \frac{1}{h(\bar{p})}$  can be rewritten as  $r = \frac{1}{\bar{C}(Q)h(\bar{p})}$  and  $\frac{r}{r+1} = \frac{1}{k(\bar{C}(Q)(r+1))}$ . By Assumption II,  $\bar{C}(Q)$  is increasing in  $Q \in [Q_o, +\infty)$ . By Assumption I,  $\bar{r}^*(Q)$  is decreasing in  $Q \in [Q_o, +\infty)$ , where  $\bar{r}^*(Q) = \frac{\bar{p}^*(\bar{C}(Q))}{\bar{C}(Q)} - 1$ . Since  $\bar{r}^*(Q) = \frac{1}{\bar{C}(Q)h(\bar{p}^*(\bar{C}(Q)))}$ ,  $\bar{C}(Q)h(\bar{p}^*(\bar{C}(Q)))$  is increasing in  $Q \in [Q_o, +\infty)$ . By Assumption III,  $\left[ \frac{d}{dQ} \log(\bar{A}(Q)) \right] / \left[ \frac{d}{dQ} \log(\bar{C}(Q)) \right] - \bar{C}(Q)h(\bar{p}^*(\bar{C}(Q)))$  is decreasing in  $Q \in [Q_o, +\infty)$ . Hence, the claim holds.  $\square$

By Propositions I and II, we can calculate the optimal product quality and price decisions  $(Q_d^*, p_d^*)$ .

## II. THE INDIRECT-SELL CASE

In Proposition III, we find the optimal retail price  $p_{id}^*(w, Q)$  to maximize the retailer's profit given the manufacturer's wholesale price  $w$  and product quality  $Q$ , where  $w > C(Q)$ .

**Proposition III.** *If Assumptions I and II hold, then: (1).  $\Pi_R(p, w, Q)$  has a unique maximizer  $p_{id}^*(w, Q) = \frac{\bar{p}^*(\alpha(Q)w - \beta(Q)) + \beta(Q)}{\alpha(Q)}$ , where  $\bar{p}^*(u)$  is the unique solution of  $\bar{p} - u = \frac{1}{h(\bar{p})}$  and  $u > 0$ ; (2).  $p_{id}^*(w, Q)$  is increasing in  $w$ .*

*Proof.* Notice that  $\frac{\partial}{\partial p} \log(\Pi_R(p, w, Q)) = \frac{1}{p-w} - h(\alpha(Q)p - \beta(Q))\alpha(Q) = \alpha(Q) \left[ \frac{1}{p-w} - h(\bar{p}) \right]$ , where  $\bar{p} = \alpha(Q)p - \beta(Q)$  and  $\bar{w} = \alpha(Q)w - \beta(Q)$ . The first order condition is equivalent to  $\bar{p} - \bar{w} = \frac{1}{h(\bar{p})}$  and  $1 = \frac{\bar{w}}{\bar{p}} + \frac{1}{k(\bar{p})}$ . Since  $w > C(Q)$ , by Assumption II,  $\bar{w} > 0$ . By Assumption I,  $\frac{\bar{w}}{\bar{p}} + \frac{1}{k(\bar{p})}$  is decreasing in  $\bar{p}$  and  $\lim_{\bar{p} \rightarrow +\infty} k(\bar{p}) > 1$ . Hence,  $1 = \frac{\bar{w}}{\bar{p}} + \frac{1}{k(\bar{p})}$  has a unique solution  $\bar{p}^*(\bar{w})$ . This implies Claim 1 holds. For Claim 2, notice that  $\bar{p}^*(u)$  is increasing in  $u$ , where  $u > 0$ .  $\square$

We prove that the indirect demand function has an increasing price elasticity function under Assumption IV.

**Assumption IV.**  $u \frac{d}{du} [\frac{1}{k(u)}]$  is increasing in  $u$ , where  $u > 0$ .

Since  $u \frac{d}{du} [\frac{1}{k(u)}] = \frac{d}{du} [\frac{1}{h(v)}] - \frac{1}{k(u)}$ , Assumption I together with Case 3.1 (or 3.2) in the main text implies Assumption IV. Hence, linear, exponential, logit and probit demand functions satisfy Assumption IV.

**Example III.** If customer heterogeneity  $\varepsilon$  follows a gamma distribution  $\Phi(u) = \int_0^u \phi(z) dz$ , where  $\phi(u) = \frac{u^{a-1} e^{-u}}{\Gamma(a)}$  and  $a > 0$ , then  $\frac{1}{k(u)} = \int_1^{+\infty} z^{a-1} e^{u(1-z)} dz$ . Since  $u \frac{d}{du} [\frac{1}{k(u)}] = - \int_1^{+\infty} a z^{a-2} (z-1 + \frac{1}{a}) e^{u(1-z)} dz$ , Assumption IV holds.

If customer heterogeneity  $\varepsilon$  follows a Weibull distribution  $\Phi(u) = \int_0^u \phi(z) dz$ , where  $\phi(u) = a u^{a-1} e^{-u^a}$  and  $a > 0$ , then  $\frac{1}{k(u)} = \frac{1}{a u^a}$ . Since  $u \frac{d}{du} [\frac{1}{k(u)}] = -u^{-a}$ , Assumption IV holds.

**Lemma I.** If Assumptions I and IV hold, then  $f(u)$  is increasing in  $u$  and  $\lim_{u \rightarrow +\infty} f(u) > 1$ .

*Proof.* By the proof of Lemma 1 in the main text,  $g(u) = \frac{h(v)}{1 - \frac{d}{dv} [\frac{1}{h(v)}]}$  and  $f(u) = \frac{u h(v)}{1 - \frac{d}{dv} [\frac{1}{h(v)}]}$ , where  $v = \bar{p}^*(u)$  and  $v - u = \frac{1}{h(v)}$ . Since  $1 = \frac{u}{v} + \frac{1}{k(v)}$ ,  $f(u) = \frac{u h(v)}{\frac{u}{v} + \frac{1}{k(v)} - \frac{d}{dv} [\frac{1}{h(v)}]} = \frac{k(v)}{1 + \frac{v}{u} (\frac{1}{k(v)} - \frac{d}{dv} [\frac{1}{h(v)}])}$ . Since  $v$  is increasing in  $u$  and  $\frac{u}{v} = 1 - \frac{1}{k(v)}$ , by Assumption I,  $k(v)$  and  $\frac{u}{v}$  are increasing in  $u$ . Notice that  $\frac{1}{k(v)} - \frac{d}{dv} [\frac{1}{h(v)}] = -v \frac{d}{dv} [\frac{1}{k(v)}] \geq 0$ . By Assumption IV,  $\frac{1}{k(v)} - \frac{d}{dv} [\frac{1}{h(v)}]$  is decreasing in  $u$ . Hence,  $\frac{v}{u} (\frac{1}{k(v)} - \frac{d}{dv} [\frac{1}{h(v)}])$  is decreasing in  $u$ . This implies that  $f(u)$  is increasing in  $u$ .

Since  $v > u$ ,  $G(u) > H(u)$ . Hence,  $\int_0^{+\infty} e^{-G(u)} du < \int_0^{+\infty} e^{-H(u)} du$ , which is the mean of  $\Phi(u)$ . By Assumption I and Theorem 4 of Lariviere (2006),  $\lim_{u \rightarrow +\infty} f(u) > 1$ .  $\square$

By Lemma I, the indirect demand function has an increasing price elasticity function  $f(\alpha(Q)p)$  if  $\beta(Q) = 0$ . Hence, we can apply all results obtained in Section I for maximizing the manufacturer's profit function  $\Pi_M(w, Q)$ .

**Proposition IV.** If Assumptions I, II and IV hold, then: (1).  $\Pi_M(w, Q)$  has a unique maximizer  $w_{id}^*(Q) = \frac{\bar{w}^*(\bar{C}(Q)) + \beta(Q)}{\alpha(Q)}$ , where  $\bar{w}^*(u)$  is the unique solution of  $\bar{w} - u = \frac{1}{g(\bar{w})}$ ; (2).  $w_{id}^*(Q)$  is increasing in  $Q \in [Q_o, +\infty)$ .

*Proof.* Similar to the proof of Proposition I.  $\square$

Next, we find the optimal product quality  $Q_{id}^*$  to maximize the manufacturer's profit  $\Pi_M^*(Q)$ , where  $\Pi_M^*(Q) = \Pi_M(w_{id}^*(Q), Q)$ .

**Proposition V.** *If Assumptions I-IV hold, then  $\Pi_M^*(Q)$  has a unique maximizer  $Q_{id}^*$ , where  $Q_{id}^* = \sup\{Q \geq Q_o \mid [\frac{d}{dQ} \log(\bar{A}(Q))] \geq [\frac{d}{dQ} \bar{C}(Q)]g(\bar{w}^*(\bar{C}(Q)))\}$ .*

*Proof.* Similar to the proof of Proposition II. □

By Propositions IV and V, the optimal wholesale price for the manufacturer  $w_{id}^* = w_{id}^*(Q_{id}^*)$ . By Propositions III-V, the optimal retail price for the retailer  $p_{id}^* = p_{id}^*(w_{id}^*, Q_{id}^*)$ . By Propositions III-V, we can calculate the optimal product quality and price decisions  $(Q_{id}^*, w_{id}^*, p_{id}^*)$ . Finally, we denote the manufacturer's optimal profit as  $\Pi_M^* = \Pi_M(p_{id}^*, w_{id}^*, Q_{id}^*)$  and the retailer's optimal profit as  $\Pi_R^* = \Pi_R(p_{id}^*, w_{id}^*, Q_{id}^*)$  and define the total channel profit as  $\Pi_{id}^* = \Pi_R^* + \Pi_M^*$ .

**Example IV.** *For an exponential demand function (i.e.,  $D(p, Q) = A(Q)e^{\beta(Q) - \alpha(Q)p}$ ), customer heterogeneity  $\varepsilon$  follows an exponential distribution with the failure rate function  $h(u) = 1$ , where  $u > 0$ . By Proposition I,  $p_d^*(Q) = C(Q) + \frac{1}{\alpha(Q)}$ . By Proposition II,  $Q_d^* = \sup\{Q \geq Q_o \mid [\frac{d}{dQ} \log(\bar{A}(Q))] \geq [\frac{d}{dQ} \bar{C}(Q)]\}$ . The optimal profit is  $\Pi_d^* = \frac{A(Q_d^*)}{\alpha(Q_d^*)} e^{\beta(Q_d^*) - \alpha(Q_d^*)C(Q_d^*) - 1}$ . By Claim 2 of Propositions 1 and 2 in the main text,  $Q_d^* = Q_{id}^*$  and  $p_d^* = w_{id}^* = C(Q_d^*) + \frac{1}{\alpha(Q_d^*)}$ . By Proposition III,  $p_{id}^* = C(Q_d^*) + \frac{2}{\alpha(Q_d^*)}$ . The manufacturer and retailer's optimal profits are  $\Pi_M^* = \Pi_R^* = \frac{A(Q_d^*)}{\alpha(Q_d^*)} e^{\beta(Q_d^*) - \alpha(Q_d^*)C(Q_d^*) - 2}$  and the channel profit is  $\Pi_{id}^* = \frac{2A(Q_d^*)}{\alpha(Q_d^*)} e^{\beta(Q_d^*) - \alpha(Q_d^*)C(Q_d^*) - 2} = \frac{2}{e} \Pi_d^*$ .*

*For a linear demand function (i.e.,  $D(p, Q) = \frac{A(Q)}{B} [B + \beta(Q) - \alpha(Q)p]$ ), customer heterogeneity  $\varepsilon$  follows a uniform distribution with the failure rate function  $h(u) = \frac{1}{B-u}$ , where  $B > u > 0$ . By Proposition I,  $p_d^*(Q) = 0.5(C(Q) + \frac{B+\beta(Q)}{\alpha(Q)})$ . By Proposition II,  $Q_d^* = \sup\{Q \geq Q_o \mid [\frac{d}{dQ} \log(\bar{A}(Q))] \geq \frac{2[\frac{d}{dQ} \bar{C}(Q)]}{B - \bar{C}(Q)}\}$ . The optimal profit is  $\Pi_d^* = \frac{A(Q_d^*)(B+\beta(Q_d^*) - \alpha(Q_d^*)C(Q_d^*))^2}{4B\alpha(Q_d^*)}$ . By Claim 2 of Propositions 1 and 2 in the main text,  $Q_d^* = Q_{id}^*$  and  $p_d^* = w_{id}^* = 0.5(C(Q_d^*) + \frac{B+\beta(Q_d^*)}{\alpha(Q_d^*)})$ . By Proposition III,  $p_{id}^* = 0.25(C(Q_d^*) + \frac{3(B+\beta(Q_d^*))}{\alpha(Q_d^*)})$ . The manufacturer's profit is  $\Pi_M^* = \frac{A(Q_d^*)(B+\beta(Q_d^*) - \alpha(Q_d^*)C(Q_d^*))^2}{8B\alpha(Q_d^*)}$ , the retailer's profit  $\Pi_R^* = \frac{A(Q_d^*)(B+\beta(Q_d^*) - \alpha(Q_d^*)C(Q_d^*))^2}{16B\alpha(Q_d^*)}$  and the channel profit  $\Pi_{id}^* = \frac{3A(Q_d^*)(B+\beta(Q_d^*) - \alpha(Q_d^*)C(Q_d^*))^2}{16B\alpha(Q_d^*)} = \frac{3}{4} \Pi_d^*$ .*

## A2. Numerical Studies

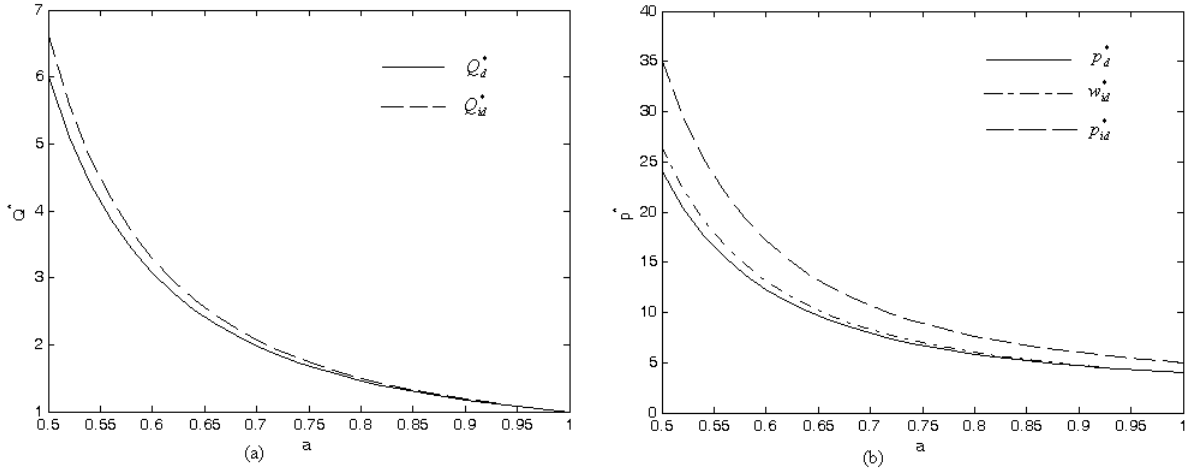


FIGURE 1. (a). Optimal quality levels  $Q_d^*$  versus  $Q_{id}^*$ ; (b). Optimal prices  $p_d^*$ ,  $w_{id}^*$  and  $p_{id}^*$ .

We let  $A(Q) = \sqrt{Q}$ ,  $C(Q) = 3Q$ ,  $\beta(Q) = 2.5Q$ ,  $\alpha(Q) = 1$  and customer heterogeneity  $\varepsilon$  follow a Weibull distribution (i.e.  $\phi(u) = au^{a-1}e^{-u^a}$ ).

First, we vary  $a \in [0.5, 1]$ . As shown in Figure 1(a), the manufacturer's optimal product quality  $Q_{id}^*$  is higher than the first-best quality  $Q_d^*$ . As shown in Figure 1(b), the optimal retail price  $p_{id}^*$  and wholesale price  $w_{id}^*$  are higher than the optimal direct-sell price  $p_d^*$  due to the higher product quality and the double marginalization effect. These are consistent with Claim 3 of Propositions 1 and 2 in the main text.

Second, we vary  $a \in [1, 15]$ . As shown in Figure 2(a), the manufacturer's optimal product quality  $Q_{id}^*$  is lower than the first-best quality  $Q_d^*$ . This is consistent with Claim 1 of Proposition 2 in the main text. As shown in Figure 2(b), the optimal retail price  $p_{id}^*$  is higher than the optimal direct-sell price  $p_d^*$  due to the double marginalization effect when  $a$  is low. However, when  $a$  is high, the optimal retail price  $p_{id}^*$  becomes lower than the optimal direct-sell price  $p_d^*$ . This is because the product quality deterioration in the indirect-sell channel causes the retailer to lose the pricing power.

## REFERENCES

Lariviere, M.A. 2006. A note on probability distributions with increasing generalized failure rates. *Operations Research* 54(3) 602-604.

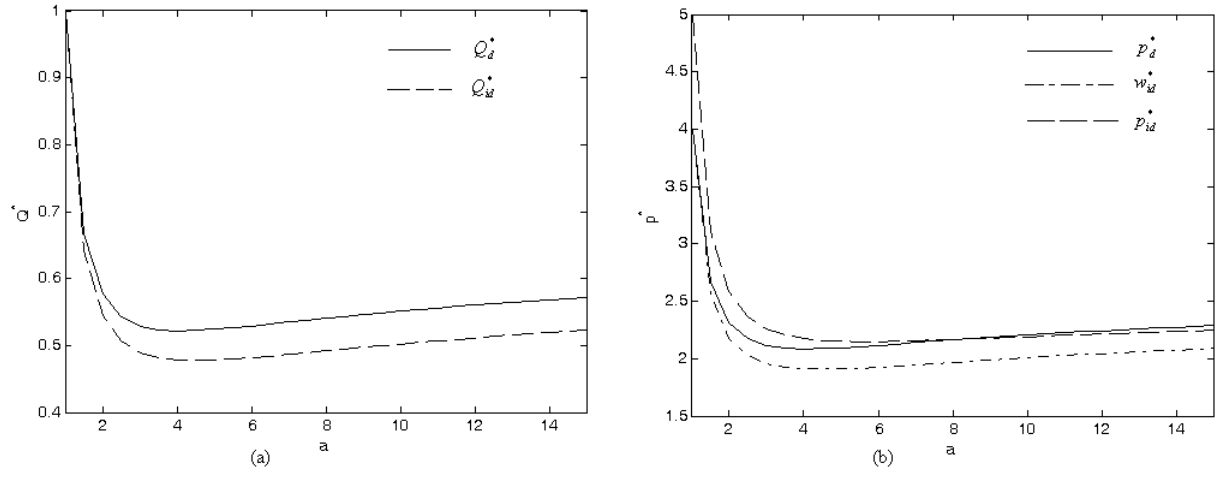


FIGURE 2. (a). Optimal quality levels  $Q_d^*$  versus  $Q_{id}^*$ ; (b). Optimal prices  $p_d^*$ ,  $w_{id}^*$  and  $p_{id}^*$ .