Supplementary Document

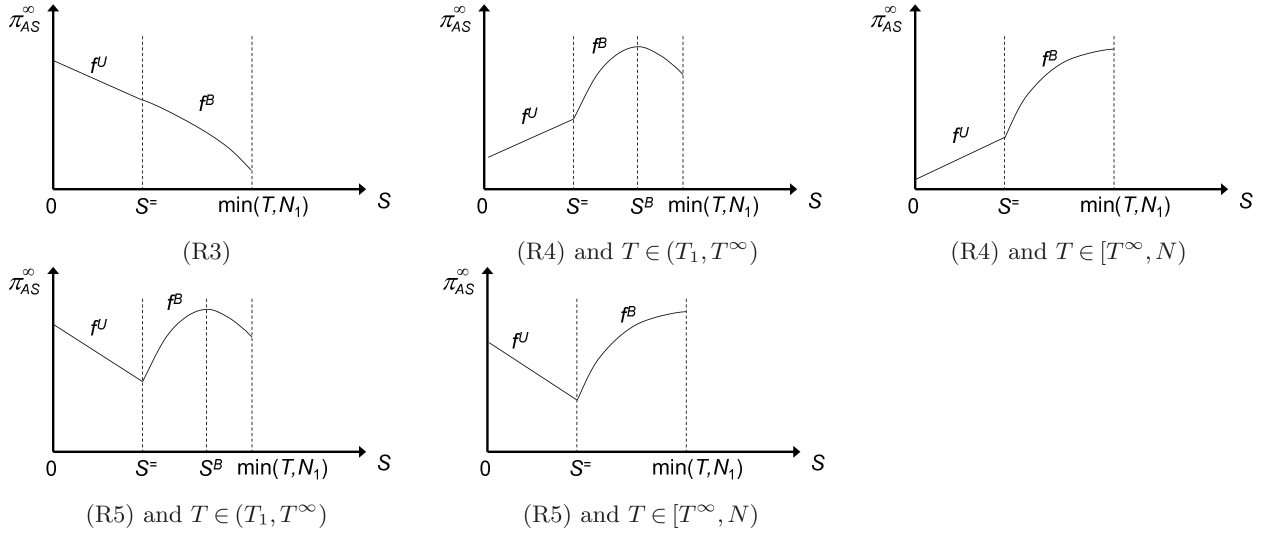
This document contains the omitted proofs for results in Sections 5 of the paper, and three extensions (not intending for publication). Section SA is for Section 5 and contains the complete proof of Theorem 5 (a sketch in the appendix). Section SB.1 shows that our main results and insights remain the same under three extensions on endorgenous capacity, multiple valuation distributions over time, and convenience in advance selling. Section SC covers the proofs for the results in Section SB.1. We replicated the statements of the results so that this supplementary document can be independently read and used as a reference.

Section SA Additional Proofs for Results in Section 5

This chapter contains a complete proof of Theorem 5.

Theorem 5 [The optimal capacity rationing in the ∞-group valuation model]
There exist two thresholds, T_1 and T^∞, where $0 \le T_1 \le T^\infty \le N$, and a switching curve $c^\infty(T)$, defined for $T > T_1$, such that
(i) if $T \ge T^\infty$ and $c \le c^\infty(T)$, then $S^\infty = \min(T, N_1)$ [**full advance selling**],
(ii) if $T \in (T_1, T^\infty)$ and $c \le c^\infty(T)$, then $0 < S^\infty < \min(T, N_1)$ [**limited advance selling**], *and*
(iii) otherwise, $S^\infty = 0$ [**no advance selling**].
Further, for a given $T < N$, if $S^\infty > 0$, then $p_1^{\max,\infty}(S^\infty) < p_2^\infty(S^\infty)$. That is, a seller with limited capacity always offers a discount in the advance period if he sells in advance.

Proof of Theorem 5 As described in the appendix, we examine five cases based on combinations of c and T as illustrated in Figure 10 (i).

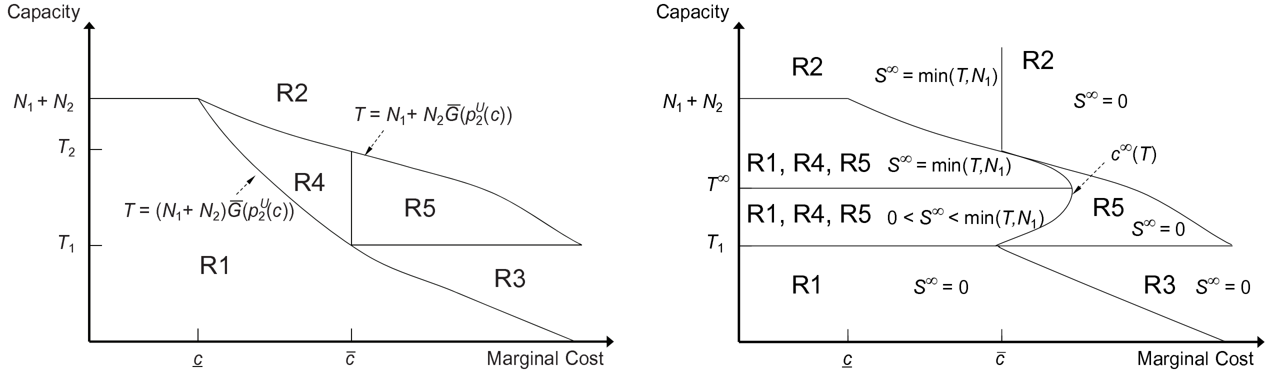


(R3) (R4) and $T \in (T_1, T^\infty)$ (R4) and $T \in [T^\infty, N)$

(R5) and $T \in (T_1, T^\infty)$ (R5) and $T \in [T^\infty, N)$

Figure 9 Illustration of $\pi_{AS}^\infty(S)$ for cases (R3), (R4), and (R5)

(R1) $T \le N\overline{G}(p_2^U(c))$ and (R2) $T \ge N_1 + N_2\overline{G}(p_2^U(c))$:
In (R1), $S^= \le 0$ and $\pi_{AS}^\infty(S) = f^B(S)$ for all feasible S. Hence, $S^\infty = S^B$. The result then immediately follows from Lemma E.1. In (R2), $S^= \ge N_1$ and $\pi_{AS}^\infty(S) = f^U(S)$ for all feasible S. As in the unlimited-capacity case, $S^\infty = N_1$ for $c \le \overline{c}$ and $S^\infty = 0$ for $c > \overline{c}$.



(i) Definition of the five regions (ii) S^∞ in each region

Figure 10 Five regions in the capacity-cost space

Now we consider the remaining region:

$$\mathcal{R} = \{(c,T) | N\overline{G}(p_2^U(c)) < T < N_1 + N_2\overline{G}(p_2^U(c))\}$$

In this region, $0 < S^= < \min(T, N_1)$. We divide this region into three cases (R3), (R4), and (R5). Figure 9 illustrates the profit function $\pi_{AS}^\infty(S)$ for each of these cases.

(R3): $\mathcal{R} \cap \{(c,T) | T \le T_1 = (N_1 + N_2)\overline{G}(p_2^U(\overline{c})), c > \overline{c}\}$.

In this case, both $f^U(S)$ and $f^B(S)$ strictly decreases in S (the former is by $c > \overline{c}$, and the latter by $T \le T_1$). Therefore, $\pi_{AS}^\infty(S)$ strictly decreases in S and maximized at $S^\infty = 0$.

(R4): $\mathcal{R} \cap \{(c,T) | T > T_1, c \le \overline{c}\}$.

Since $c \le \overline{c}$, $f^U(S)$ is nondecreasing in S. To characterize optimal S^∞, it suffices to show $S^\infty = S^B$. From quasi-concavity of f^B (Lemma E.1), it suffices to show $S^= \le S^B$. Prove by contradiction. Suppose $S^= > S^B$. This implies $p_2^B(S^B) < p_2^U$ by definition of $S^=$. Recall that p_2^U is the unique maximizer of $(p_2 - c)\overline{G}(p_2)$ (Theorem 2 (i)). Hence, $(p_2^B(S^B) - c)\overline{G}(p_2^B(S^B)) < (p_2^U - c)\overline{G}(p_2^U)$. By these facts and the definitions of $p_2^B(S^B)$ and $S^=$, we have

$$\begin{aligned} f^B(S^B) &= (\mathrm{E}[\min(p_2^B(S^B), \alpha)] - c)\, S^B + (N - S^B)\, (p_2^B(S^B) - c)\, \overline{G}(p_2^B(S^B)) \\ &< (\mathrm{E}[\min(p_2^U, \alpha)] - c)\, S^B + (N - S^B)\, (p_2^U - c)\, \overline{G}(p_2^U) \qquad [p_2^U \text{ increases both profits}] \\ &= f^U(S^B) \le f^U(S^=) = f^B(S^=) \end{aligned}$$

where the last inequality follows from the facts that $S^B < S^=$ and $f^U(S)$ is nondecreasing in S when $c \le \overline{c}$. This result, however, contradicts with the optimality of S^B.

(R5): $\mathcal{R} \cap \{(c,T) | T > T_1, c > \overline{c}\}$.

As in case (R3), $f^U(S)$ is decreasing in S. By Lemma E.1, it suffices to show that for this case, there exists a function $c^\infty(T)$ such that $S^\infty = S^B$ if and only if $\overline{c} \le c \le c^\infty(T)$. To this end, we first note that $S^\infty = S^B$ if and only if $S^= \le S^B$ and $f^B(S^B) \ge f^U(0)$. Hence, to show the function $c^\infty(T)$, it suffices to prove that **(a)** $f^B(S^B) \ge f^U(0)$ implies $S^= \le S^B$ (and thus it is sufficient to verify $f^B(S^B) - f^U(0) \ge 0$); and **(b)** $f^B(S^B) - f^U(0)$ is decreasing in c. **(a)** can be proved similarly to proof of (R4). To show **(b)**, define $f^B(S^B) - f^U(0)$ as a function of c: $h(c) = f^B(S^B) - f^U(0) = \{(\mathrm{E}[\min(p_2^B(S^B), \alpha)] - c)S^B + (T - S^B)(p_2^B(S^B) - c)\} - N\overline{G}(p_2^U(c))(p_2^U(c) - c)$. By

Lemma E.1, S^B is independent of c. Hence, so is $p_2^B(S^B)$. By envelop theorem, $\frac{dh(c)}{dc} = -T + N\overline{G}(p_2^U(c)) < 0$. Now, for given $T \in (T_1, T_2)$, define $c^\infty(T) = \max\{c \geq \bar{c} : h(c) \geq 0\}$.

Figure 10 (ii) summarizes the optimal solution S^∞ for all regions. Note that the domain of $c^\infty(T)$ can be extended to $T \geq T_2$ by setting $c^\infty(T) = \bar{c}$ for $T \geq T_2$. The structure of the optimal ration S^∞ immediately follows.

Now, to show $p_1^{\max,\infty}(S^\infty) < p_2^\infty(S^\infty)$ whenever $T < N$ and $S^\infty > 0$, prove by contradiction. Suppose $p_1^{\max,\infty}(S^\infty) = p_2^\infty(S^\infty)$ for some $T < N$ and $S^\infty > 0$. By Theorem 4, $p_1^{\max,\infty}(S^\infty) = p_2^\infty(S^\infty)$ if and only if $p_2^\infty(S^\infty) = l$, which, by Lemma 1 and the fact $T < N$, occurs if and only if $p_2^\infty(S^\infty) = p_2^U$ and $c \leq \underline{c}$. However, by the proof above, $S^\infty = S^B$ for all $c \leq \underline{c}$ and $T < N$, as it is fully contained in (R1). Hence, $p_2^\infty(S^\infty) = p_2^B(S^\infty) > l$ by definition of $p_2^B(S)$. This implies that $p_1^{\max,\infty}(S^\infty) = p_2^\infty(S^\infty)$ is unlikely to occur for any $T < N$ and $S^\infty > 0$.

Section SB Extensions

SB.1 Seller's capacity choice

Changing capacity often requires significant time and resources, often making it hard to modify or revoke once it is set. In some situations, however, the seller is able to choose or adjust her capacity before pricing decisions. One might think that adding capacity for free can only increase the seller's profit. It, however, is not always the case when selling to strategic consumers: a larger capacity can decrease the profit even if it is free. To see why, note that a seller with a smaller capacity can usually charge a higher spot price due to low availability in spot. Anticipating a higher price and smaller capacity in spot, consumers in advance are willing to pay more, leading to a higher advance price and, eventually, increased total profit. Hence, a seller may intentionally choose a capacity level lower than the total market size N, even if capacity is free. We formalize this result in the following theorem, where the optimal capacity and selling strategy is characterized for the ∞-group and 1-group models. Let $\kappa > 0$ be the per-unit capacity cost and T^* be the (largest) maximizer of the profit function $\pi_{AS}^k(S^k|T) - \kappa T$ over $T \in (0, N]$.

Theorem 7 [The seller's optimal capacity]

(a) Under the ∞-group valuation model, $T^* \leq T_1$ or $T^* \geq T^\infty$. Thus, the seller choosing the optimal capacity will not use the limited advance selling. Furthermore, $T^* < N$ for any capacity cost $\kappa > 0$.

(b) Under the 1-group valuation model, the optimal capacity T^* is either 0, or N_1, or N. Furthermore, for sufficiently small κ, $T^* < N$ only if the proportion of consumers arriving in advance N_1/N is sufficiently large.

From the results of Section 5.2 and also from part (b) of Theorem 7, the limited selling is not optimal in the 1-group model at any capacity level. Rather surprisingly, part (a) of the above theorem implies that, when the valuations are diverse and independent as in our ∞-model, the seller will never choose intermediate capacity level at which he has to use the limited advance selling. In fact, in our numerical results we did not observe limited advance selling for any level of k when the seller can choose the initial capacity. The result suggests that the limited advance selling is a consequence of suboptimal capacity level and weak (or no) interdependence of valuations. Another interesting finding is how the valuation interdepdence affects the profit and prices. Under the ∞-group model, the optimal capacity is strictly less than the total market size. This is because if the seller has too much capacity, he has to sell to all (or almost all) consumers, thus he is not able to raise spot and/or advance price (recall that the seller always offer discounted advance price in the ∞-model. If valuations are highly interdependent as in the 1-group model, capacity has little effect on spot price (see Lemma 2). Thus, its effect on profit is primarily through creating scarcity and increasing the advance price. Interestingly, the more consumers arrive in advance, the larger the effect of capacity on the profit. This incentivizes the seller to decrease capacity below N. That is because the more consumers attempt to buy in advance, the more the seller gains from raising the advance price through decreasing total capacity. Consequently, with bigger portion of consumers arriving in advance the optimal capacity tends to decrease in 1-group model. We observe similar patterns under other valuation models.

SB.2 Different valuation distributions in spot and advance periods

So far, we have assumed that, while consumers can realize different valuations, all consumers draw their valuations from the same distribution. We now relax this assumption. Specifically, we assume that the valuation distribution for consumers in the advance period, denoted by $G_A(\cdot)$, is different (higher or lower) than the valuation distribution for consumers in the spot period, denoted by $G_S(\cdot)$. We numerically evaluate how the interdependence of valuations influences the seller's optimal strategy and profit from offering advance selling in such a scenario.

We note that, while this set-up seems similar to the settings of several papers in the revenue management (c.f., Littlewood, 1972; Robinson, 1995; Talluri and van Ryzin, 2004 and references therein), there is a major difference in how consumers make decisions. In these papers, consumers can buy only in the period they arrive (they cannot strategically determine whether they should buy now or wait to purchase in later period). Consequently, price in one period does not affect the consumer behavior and demand in another period. In our models, since consumers are strategic and can decide whether to buy or wait for later period, the price in one period influences the demand in another period.

We consider the ∞-group model with $N_1 = N_2 = 10$. We gradually change the valuation distribution of advance consumers while keeping that of spot consumers the same. Specifically, we assume $G_A \sim U[0.5+\delta, 1.5+\delta]$ and $G_S \sim U[0.5, 1.5]$ and vary δ gradually from -0.3 to 0.3. The results are illustrated in Figure S.1.

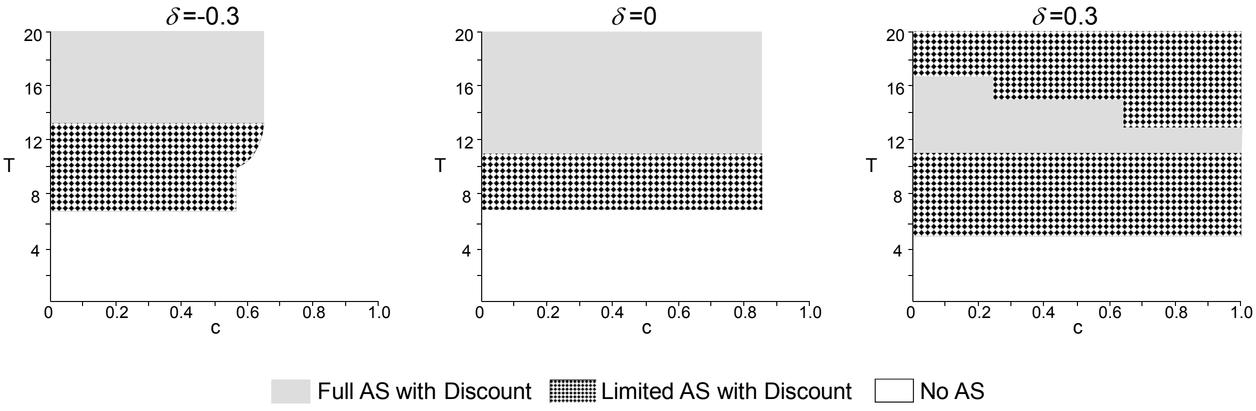


Figure S.1 Optimal policy as a function of heterogeneity factor δ, under the ∞-group valuation model: $N_1 = 10$, $N_2 = 10$, $G_A \sim U[0.5+\delta, 1.5+\delta]$ and $G_S \sim U[0.5, 1.5]$.

The seller is increasingly likely to offer advance selling, as the valuation of advance customers increases. Higher advance valuations allow the seller to increase the price at which the product can be sold in the advance period.

Note, however, that, when cost is low and capacity is high, the seller is increasingly likely to limit the advance sales, rather than selling all units in advance. This is because, when a bigger portion of advance customers is shifted to the spot, the spot price, and consequently, advance price, is increased.

We also observe that the higher the interdependence of valuation, the higher benefit of advance selling, similarly to the cases when valuations are the same in advance and spot. Figure S.2 depicts the seller's total profit from offering advance selling under different capacity T, valuation model k, and valuation heterogeneity factor δ. Recall that we showed in Section 6 that the effect of interdependence diminishes as the capacity becomes large. Figure S.2 illustrates that this continues to hold when consumers draw distributions from multiple distributions. As before, the effect of interdependence weakens when the seller has sufficiently large capacity.

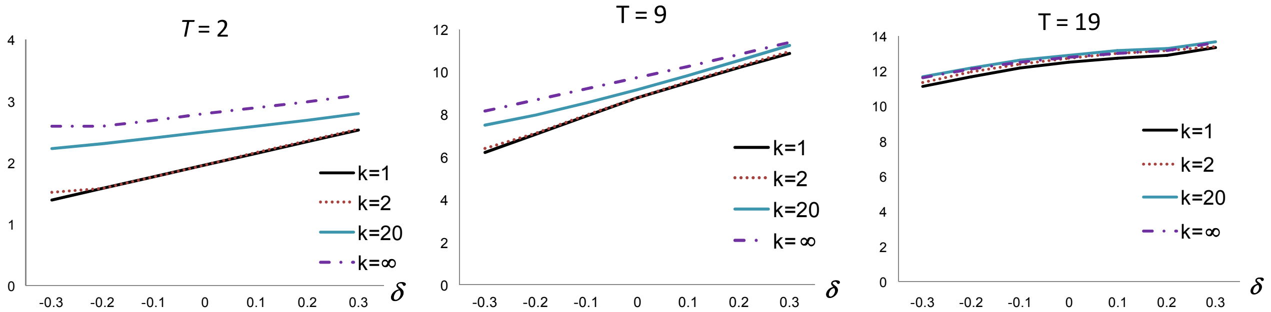


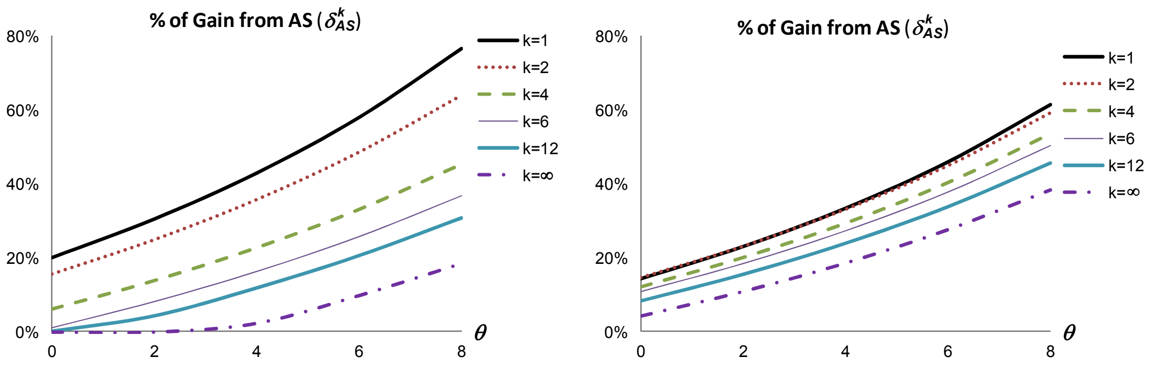
Figure S.2 Seller's total profit $\pi^k_{AS}(S^k)$ as a function of heterogeneity factor δ and consumer groups k: $N_1 = 10$, $N_2 = 10$, $G_A \sim U[0.5+\delta, 1.5+\delta]$ and $G_S \sim U[0.5, 1.5]$.

SB.3 The value of convenience in advance selling

One of main reasons that advance selling is attractive to consumers is convenience in buying compared to spot buying: In some cases,consumers can advance-purchase through the internet or phone while they have to be physically present for spot sales (e.g., tickets of Broadway shows can be purchased online in advance, but in spot they are purchased at ticket offices). We interpret the difference as consumers incurring additional transaction inconvenience in spot, denoted by $\theta > 0$. The next proposition characterizes how the optimal strategy changes as the spot selling becomes more inconvenient for consumers, compared to advance selling.

Proposition 2 [The effect of transaction cost in spot on the seller's pricing and capacity rationing]
Under both the ∞-group and 1-group models, the seller increases the quantity sold in advance as buying in spot becomes inconvenient compared to buying in advance. That is, the optimal capacity ration in the advance period is nondecreasing in θ.

As illustrated on Figure S.3, the benefit of advance selling increases as the spot selling becomes inconvenient. For given level of valuation interdependence, the larger the difference in shopping convenience is, the larger the benefit of advance selling is. This result is most pronounced when valuations are highly interdependent as the seller is more likely to use advance selling as the interdependence grows. One interesting observation is that making spot sales more inconvenient can increase the seller's profit. This happens when the seller's capacity is not small and the majority of consumers are present from the advance period. In such a case, while the



$T=2$ $T=7$

Figure S.3 **The seller's gain from advance selling δ_{AS}^{k} w.r.t k and T: $N_1=6$, $N_2=6$, $\alpha \sim U[25,35]$, $c=0$.**

inconvenience in spot lowers the margin, it increases the advance price (thus the gain from advance selling increases). This observation, together with Proposition 2, implies that the seller with reasonably large capacity could benefit when spot purchase becomes more inconvenient. For example, the seller can sell tickets for an event at multiple outlets or online in advance, but only at the event venue on the event day at which consumers have to wait in a long line.

Section SC Proofs for Results in Section SB.1

This chapter contains the proof of Theorem 7.

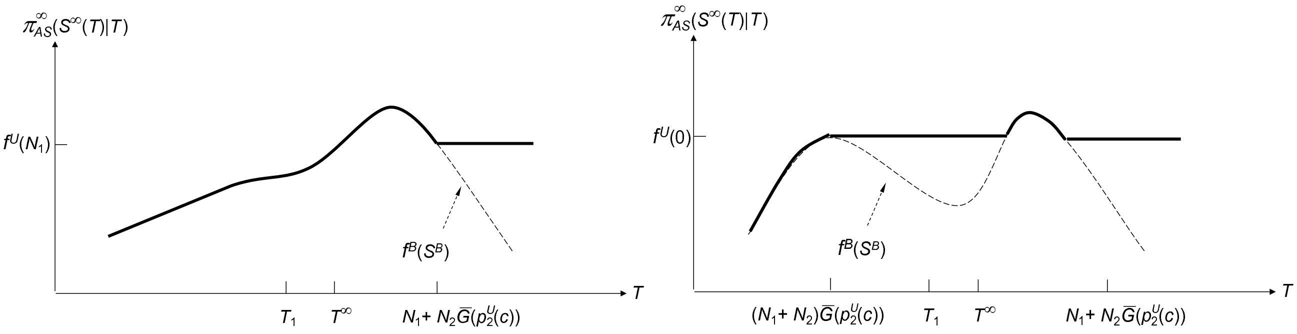
THEOREM 7 [The seller's optimal capacity]
(a) Under the ∞-group model, $T^* \leq T_1$ or $T^* \geq T^\infty$. Thus, the seller choosing the optimal capacity will not use the limited advance selling. Furthermore, $T^* < N$ for arbitrarily small capacity cost κ.
(b) Under the 1-group model, the optimal capacity T^* is either 0, or N_1, or N. Furthermore, for sufficiently small κ, $T^* < N$ only if the proportion of consumers arriving in advance N_1/N is sufficiently large.

Proof of Theorem 7 To explicitly recognize the dependence of strategies and profits on T, within this proof we write all of them as functions of T.
(a) We first consider the ∞-group model. Recall the function $f^B(S)$ defined in the proof of Theorem 5 and that S^B is the unique maximizer of $f^B(S)$ on $[0, \min(T, N_1)]$. The following lemma characterizes $f^B(S^B(T)|T)$, where T_1 and T^∞ are defined in Theorem 5.

Lemma SC.1 *[Proof available upon request]*
$f^B(S^B(T)|T)$ is continuous in T and is concave for $0 < T \leq T_1$, convex for $T_1 < T < T^\infty$, and concave for $T^\infty \leq T < N$. Furthermore, $f^B(S^B(T)|T)$ is differentiable in T at $T = T_1$ and $T = T^\infty$.

We consider two cases to prove the theorem: (a-i) when $c \leq \bar{c}$ and (a-ii) when $c > \bar{c}$.
(a-i) $c \leq \bar{c}$: In such a case, by the proof of Theorem 5,

$$\pi_{AS}^{\infty}(S^\infty(T)|T) - \kappa T = \begin{cases} f^B(S^B(T)|T) - \kappa T & \text{if } T < N_1 + N_2\overline{G}(p_2^U) \\ f^U(N_1) - \kappa T & \text{if } T \geq N_1 + N_2\overline{G}(p_2^U) \end{cases} \tag{S.1}$$



when $c \le \bar{c}$ when $c > \bar{c}$

Figure S.4 Illustration of $\pi^D_{AS}(S^*(T)|T)$ with respect to T

To show $T^* \le T_1$ or $T^* \ge T^{\infty}$, it suffices to show that $\pi^{\infty}_{AS}(S^{\infty}(T)|T) - \kappa T$ is continuous and convex for $T_1 < T < T^{\infty}$. Since κT is linear, it then suffices to show $\pi^{\infty}_{AS}(S^{\infty}(T)|T)$ is continuous and convex for $T_1 < T < T^{\infty}$. (see Figure S.4 left).

To show the continuity of $\pi^{\infty}_{AS}(S^*(T)|T)$, since $f^B(S^B(T)|T)$ is continuous in T (by Lemma SC.1) and $f^U(N_1)$ is independent of T, it suffices to show that when $T = N_1 + N_2\overline{G}(p_2^U)$, $f^B(S^B(T)|T) = f^U(N_1)$. By Theorem 5, when $T = N_1 + N_2\overline{G}(p_2^U)$ and $c \le \bar{c}$, $S^B(T) = N_1$ and $p_2^B(S^B(T)|T) = p_2^U$. As a result, $f^B(S^B(T)|T) = f^U(N_1)$.

To show that $\pi^{\infty}_{AS}(S^{\infty}(T)|T)$ is convex for $T_1 < T < T^{\infty}$, note that $T^{\infty} \le N_1 + N_2\overline{G}(p_2^U)$ for $c \le \bar{c}$ by the proof of Theorem 5. Hence, the convexity of $\pi^{\infty}_{AS}(S^{\infty}(T)|T)$ directly follows the convexity of $f^B(S^B(T)|T)$ by equation (S.1) and Lemma SC.1.

Now, to show $T^* < N$ for $\kappa > 0$, note that $f^U(N_1)$ is independent of T and $\pi^{\infty}_{AS}(S^{\infty}(T)|T) - \kappa T$ strictly decreases in T for $T \ge N_1 + N_2\overline{G}(p_2^U)$. Hence, $T^* < N$.

(a-ii) $c > \bar{c}$: By the proof of Theorem 5, as T increases we move across (R1), (R3), (R5), (R2), resulting in

$$\pi^{\infty}_{AS}(S^{\infty}|T) - \kappa T = \begin{cases} f^B(S^B(T)) = f^B(0|T) - \kappa T & \text{if } T < N\overline{G}(p_2^U) \\ f^U(0) - \kappa T & \text{if } N\overline{G}(p_2^U) \le T \le T_1 \\ \max(f^B(S^B(T)|T), f^U(0)) - \kappa T & \text{if } T_1 < T < N_1 + N_2\overline{G}(p_2^U) \\ f^U(0) - \kappa T & \text{if } T \ge N_1 + N_2\overline{G}(p_2^U) \end{cases} \tag{S.2}$$

Note that $f^U(0)$ is independent of T and it is easy to show that when $T = N\overline{G}(p_2^U)$, $f^B(0|T) = f^U(0)$ and $\frac{df^B(0|T)}{dT} = 0$. $\pi^{\infty}_{AS}(S^*(T)|T)$ is illustrated in Figure S.4 right.

Recall that by proof of Theorem 5, $f^B(S^B(T)|T) - f^U(0)$ strictly decreases in c for given $T > N\overline{G}(p_2^U)$. Hence, when c is extremely high, $f^B(S^B(T)|T) < f^U(0)$ for all $T > T_1$ and $\pi^{\infty}_{AS}(S^{\infty}|T) - \kappa T$ decreases in T for all $T > T_1$, implying $T^* \le T_1$. If, however, c is sufficiently low such that $f^B(S^B(T)|T) \ge f^U(0)$ for some $T > T_1$, since $f^B(S^B(T))$ is convex in $T \in (T_1, T^{\infty})$ and $f^B(S^B(T)|T = T_1) \le f^U(0)$, we have $T^* \le T_1$ or $T^* \ge T^{\infty}$.

Now, to show $T^* < N$ for $\kappa > 0$, note that $f^U(0)$ is independent of T and $\pi^{\infty}_{AS}(S^{\infty}(T)|T) - \kappa T$ strictly decreases in T for $T \ge N_1 + N_2\overline{G}(p_2^U)$. Hence, $T^* < N$.

(b) Now for the 1-group model, let $\pi^1_{AS}(S^1(T)|T)$ be the seller's optimal profit at capacity T under the 1-group model. To show that T^* equals to either 0 or N_1 or N, it suffices to show that $\pi^1_{AS}(S^1(T)|T) - \kappa T$ is (b-i) continuous in T, (b-ii) convex in T for $0 < T \le N_1$ and for $T > N_1$. In preparation, note that by Lemma 3, $\pi^1_{AS}(S^1(T)|T) = \max[\pi^1_{AS}(0|T), \pi^1_{AS}(\min(T, N_1)|T)]$. From the expression of $\pi^1_{AS}(S|T)$ in equation (15), we have

$$\pi^1_{AS}(0|T) - \kappa T = T[(p_2^U - c)\overline{G}(p_2^U) - \kappa]$$

$$\pi^1_{AS}(\min(T,N_1)|T) = \begin{cases} (\mathrm{E}[\alpha]-c-\kappa)\,T & \text{if } T \le N_1, \\ \left(\mathrm{E}[\alpha] - \frac{T-N_1}{N_2}\mathrm{E}\left[\max(\alpha - p_2^U, 0)\right] - c\right) N_1 + (T-N_1)(p_2^U - c)\overline{G}(p_2^U) - \kappa T & \text{if } T > N_1. \end{cases}$$

(b-i) By the continuity of $\pi^1_{AS}(S|T)$ in (S,T), both $\pi^1_{AS}(0|T)$ and $\pi^1_{AS}(\min(T,N_1)|T)$ are continuous in T. This immediately implies the continuity of $\pi^1_{AS}(S^1|T) - \kappa T$ in T.

(b-ii) For $0 < T \le N_1$ or $T > N_1$, both $\pi^1_{AS}(0|T) - \kappa T$ and $\pi^1_{AS}(\min(T,N_1)|T) - \kappa T$ are linear in T. That is, $\pi^1_{AS}(S^1(T)|T) - \kappa T$ equals to the maximum of two linear functions and is thus convex.

Now, assume capacity cost κ is sufficiently small: $\kappa < (p_2^U - c)\overline{G}(p_2^U)$. In such a case, $T^* > 0$ since $\pi^1_{AS}(0|T) - \kappa T > 0$ for any $T > 0$. To show when $T^* < N$, it suffices to compare the profits under $T = N$ and $T = N_1$. Note that

$$[\pi^1_{AS}(S^1(T)|T) - \kappa T]|_{T=N} = \max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^U, 0)] - c]N_1 + N_2(p_2^U - c)\overline{G}(p_2^U) - \kappa N$$

$$[\pi^1_{AS}(S^1(T)|T) - \kappa T]|_{T=N_1} = \max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - c]N_1 - \kappa N_1$$

That is,

$$\begin{aligned} &[\pi^1_{AS}(S^1(T)|T) - \kappa T]|_{T=N} - [\pi^1_{AS}(S^1(T)|T) - \kappa T]|_{T=N_1} \\ &= \left\{\max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^U, 0)] - c] - \max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - c]\right\} N_1 + N_2[(p_2^U - c)\overline{G}(p_2^U) - \kappa] \end{aligned}$$

Note that $\max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^U, 0)] - c] \le \max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - c]$. Hence, $T^* < N$ if and only if $[\pi^1_{AS}(S^1(T)|T) - \kappa T]|_{T=N} \le [\pi^1_{AS}(S^1(T)|T) - \kappa T]|_{T=N_1}$, or

$$N_1 \ge N_2 \frac{(p_2^U - c)\overline{G}(p_2^U) - \kappa}{\max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - c] - \max[(p_2^U - c)\overline{G}(p_2^U), \mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^U, 0)] - c]}$$

That is equivalent to requiring N_1/N sufficiently large.

Proof of Proposition 2 Within this proof we focus on the case with limited capacity. We start with **the ∞-group valuation model**. After incorporating the factor θ, we note: the optimal spot price $p_2^*(S,\theta) = \max(p_2^U(\theta), p_2^B(S,\theta))$, where $p_2^U(\theta)$ maximizes $(p_2 - c)\overline{G}(p_2 + \theta)$ and $p_2^B(S,\theta)$ satisfies $\overline{G}(p_2^B(S,\theta) + \theta) = \frac{T-S}{N_1+N_2-S}$; the optimal advance price $p_1^{\max,D}(S,\theta) = \mathrm{E}[\min(p_2^*(S,\theta) + \theta, \alpha)]$, and the total profit function $\pi^\infty_{AS}(S,\theta) = (p_1^{\max,D}(S,\theta) - c)S + (p_2^*(S,\theta) - c)\min(T - S, (N_1 + N_2 - S)\overline{G}(p_2^*(S,\theta) + \theta))$. Furthermore, we also redefine the two functions used in proof of Theorem 5: $f^U(S,\theta) = (\mathrm{E}[\min(p_2^U(\theta) + \theta, \alpha)] - c)S + (N_1 + N_2 - S)(p_2^U(\theta) - c)\overline{G}(p_2^U(\theta) + \theta)$, $f^B(S,\theta) = (\mathrm{E}[\min(p_2^B(S,\theta) + \theta, \alpha)] - c)S + (T - S)(p_2^B(S,\theta) - c)$.

To show the optimal S nondecreasing in θ, by the proof of Theorem 5, it suffices to show that (i) the slope of $f^U(S,\theta)$ is nondecreasing in θ; (ii) In Lemma E.1, S^B is nondecreasing in θ and both T_1 and T^∞ are nonincreasing in θ; (iii) When (c,T) lies in the region (R5) defined in the proof of Theorem 5, then $d[f^B(S^B,\theta) - f^U(0,\theta)]/d\theta \ge 0$ whenever $f^B(S^B,\theta) \ge f^U(0,\theta)$. This result implies that for given T, $c^\infty(T)$ is nondecreasing in θ.

(i) The slope of $f^U(S,\theta)$ equals to $\frac{df^U(S,\theta)}{dS} = \mathrm{E}[\min(p_2^U(\theta) + \theta, \alpha)] - c - (p_2^U(\theta) - c)\overline{G}(p_2^U(\theta) + \theta)$. To show that the slope is nondecreasing in θ, note that by envelop theorem, $(p_2^U(\theta) - c)\overline{G}(p_2^U(\theta) + \theta)$ is nonincreasing in θ. Meanwhile, since $p_2^U(\theta) + \theta$ maximizes $[p_2 - (c + \theta)]\overline{G}(p_2)$, by the IFR property of $G(\cdot)$, $p_2^U(\theta) + \theta$ is nondecreasing in θ. Thus, $\frac{df^U(S,\theta)}{dS}$ is nondecreasing in θ.

(ii) It is easy to show that $\frac{df^B(S,\theta)}{dS}$ increases in θ. Hence, S^B is nondecreasing in θ, implying that both T_1 and T^∞ are also nonincreasing in θ.

(iii) First note that similarly to the proof of Theorem 5 region (R5), we can show that when $f^B(S^B,\theta) \geq f^U(0,\theta)$, $S^= \leq S^B$, which is equivalent to

$$T - S^B \leq (N_1 + N_2 - S^B)\overline{G}(p_2^U(\theta) + \theta). \tag{S.3}$$

To derive $\frac{d[f^B(S^B,\theta)-f^U(0,\theta)]}{d\theta}$, by the envelop theorem and the fact that $p_2^B(S,\theta)+\theta$ is independent of θ, we have

$$\frac{d[f^B(S^B,\theta)-f^U(0,\theta)]}{d\theta} = -(T - S^B) - N(p_2^U(\theta) - c)\frac{\partial\overline{G}(p_2^U(\theta)+\theta)}{\partial\theta} \tag{S.4}$$

To show $\frac{d[f^B(S^B,\theta)-f^U(0,\theta)]}{d\theta} \geq 0$, consider two cases: if θ is large enough that $c \geq h - \theta$, we have $p_2^U(\theta) + \theta \equiv h$, which implies that $\overline{G}(p_2^U(\theta) + \theta) \equiv 0$ and $\frac{\partial\overline{G}(p_2^U(\theta)+\theta)}{\partial\theta} = 0$. Hence, equation (S.3) implies $S^B \geq T$, which further implies $S^B = T$ since S^B cannot exceed T. Hence, $\frac{d[f^B(S^B,\theta)-f^U(0,\theta)]}{d\theta} = 0$. If, however, θ is small enough that $c \in (\bar{c}, h-\theta)$, we have $(p_2^U(\theta)-c)g(p_2^U(\theta)+\theta) = \overline{G}(p_2^U(\theta)+\theta)$ and $\frac{\partial\overline{G}(p_2^U(\theta)+\theta)}{\partial\theta} = -g(p_2^U(\theta)+\theta)$. Thus, by equations (S.3) and (S.4), $\frac{d[f^B(S^B,\theta)-f^U(0,\theta)]}{d\theta} = -(T - S^B) + N(p_2^U(\theta) - c)g(p_2^U(\theta) + \theta) \geq -(N_1 + N_2 - S^B)\overline{G}(p_2^U(\theta) + \theta) + N\overline{G}(p_2^U(\theta) + \theta) = S^B\overline{G}(p_2^U(\theta) + \theta) \geq 0$.

Now, for **the 1-group valuation model**, to show that the optimal ration is nondecreasing in θ, it suffices to show that $\pi_{AS}^1(\min(T,N_1),\theta) - \pi_{AS}^1(0,\theta)$ is nondecreasing in θ. Incorporating the factor θ into equation (22): $\pi_{AS}^1(\min(T,N_1),\theta) - \pi_{AS}^1(0,\theta) = \min(T,N_1)\{\mathrm{E}[\alpha] - \frac{T-\min(T,N_1)}{N_1+N_2-\min(T,N_1)}\mathrm{E}[\max(\alpha - p_2^U(\theta) - \theta, 0)] - c - \overline{G}(p_2^U(\theta) + \theta)(p_2^U(\theta) - c)\}$. From part (i), $\overline{G}(p_2^U + \theta)(p_2^U - c)$ is nonincreasing in θ and $p_2^U(\theta) + \theta$ is nondecreasing in θ. Thus, $\pi_{AS}^1(\min(T,N_1),\theta) - \pi_{AS}^1(0,\theta)$ is nondecreasing in θ.