Technical Appendix

Proof of Proposition 1

If the market is fully covered and the monopolist uses umbrella branding, we must have:

$$p_{1H} = r_s - \beta\lambda_s - t_s \quad (A1)$$

$$p_{1L} = r_f + \lambda_f - t_f \quad (A2)$$

The corresponding profits of the monopolist are:

$$\Pi_1(1) = (r_s - \beta\lambda_s - t_s - c) + \beta(r_f + \lambda_f - t_f) \quad (A3)$$

If the firm uses individual branding, then the profits are:

$$\Pi_1(0) = (r_s - t_s - c) + \beta(r_f - t_f) \quad (A4)$$

We have:

$$\Pi_1(1) - \Pi_1(0) = \beta(\lambda_f - \lambda_s) \quad (A5)$$

which is positive only if $\lambda_f > \lambda_s$. □

Analysis for Monopoly with partially covered market.

Consider the case when the monopolist uses umbrella branding and the markets are not covered. The utility of a snob at θ_s who is indifferent between buying and not buying is given by:

$$U_s^1 = r_s - t_s\theta_s - \beta\lambda_s y_1^e - p_{1H} = 0 \quad (A6)$$

where y_1^e is the expected number of followers purchasing the product. This implies that the marginal consumer is defined by:

$$\theta_s = \frac{r_s - \beta\lambda_s y_1^e - p_{1H}}{t_s} \quad (A7)$$

Similarly, the consumer in the follower segment who is indifferent between buying and not buying is given by:

$$\theta_f = \frac{r_f + \lambda_f x_1^e - p_{1L}}{t_f} \quad (A8)$$

where x_1^e is the expected number of snobs who are expected to buy the product. Rational expectations imply that:

$$x_1^e = F_s(\theta_s) \quad (A9)$$

$$y_1^e = F_f(\theta_f) \quad (A10)$$

Using (A7) and (A8), we obtain:

$$\Omega_s(x_1^e) = F_s\left(\frac{r_s - p_{1H} - \beta\lambda_s F_f\left(\frac{r_f+\lambda_f x_1^e - p_{1L}}{t_f}\right)}{t_s}\right) - x_1^e = 0 \quad (A11)$$

$$\Omega_f(y_1^e) = F_f\left(\frac{r_f - p_{1L} + \lambda_f F_s\left(\frac{r_s-\beta\lambda_s y_1^e - p_{1H}}{t_s}\right)}{t_f}\right) - y_1^e = 0 \quad (A12)$$

Note that $\Omega_s(0) > 0$ and $\Omega_s(1) < 0$. This implies that there exists a x_1^e which solves the rational expectation condition. We will prove that $\Omega_s' < 0$ which will establish that the solution is unique. Similar results hold for Ω_f proving the existence of a unique y_1^e. We have:

$$\Omega_s' = -\left[\frac{\beta\lambda_s\lambda_f f_s f_f + t_s t_f}{t_s t_f}\right] \quad (A13)$$

where we use f_s to denote $f_s(\theta_s)$ and f_f to denote $f_f(\theta_f)$. Also, note that:

$$\frac{\partial\Omega_s}{\partial\lambda_s} = -\frac{\beta f_s y_1^e}{t_s} \quad (A14)$$

Also,

$$\frac{\partial \Omega_s}{\partial \lambda_f} = -\frac{\beta \lambda_s f_s f_f x_1^e}{t_s t_f} \tag{A15}$$

Similarly,

$$\frac{\partial \Omega_s}{\partial p_{1H}} = -\frac{f_s}{t_s} \tag{A16}$$

$$\frac{\partial \Omega_s}{\partial p_{1L}} = \frac{\beta \lambda_s f_s f_f}{t_s t_f} \tag{A17}$$

Using the implicit function theorem, we obtain:

$$\frac{\partial x_1^e}{\partial \lambda_s} = \frac{\beta f_s y_1^e}{t_s \Omega_s'} < 0 \tag{A18}$$

Similarly,

$$\frac{\partial x_1^e}{\partial \lambda_f} = \frac{\beta \lambda_s f_s f_f x_1^e}{t_s t_f \Omega_s'} < 0 \tag{A19}$$

Also:

$$\frac{\partial x_1^e}{\partial p_{1H}} = \frac{f_s}{t_s \Omega_s'} \tag{A20}$$

$$\frac{\partial x_1^e}{\partial p_{1L}} = \frac{-\beta \lambda_s f_s f_f}{t_s t_f \Omega_s'} \tag{A21}$$

Similarly,

$$\Omega_f' = -\left[\frac{\beta \lambda_s \lambda_f f_s f_f + t_s t_f}{t_s t_f}\right] = \Omega_s' \equiv \Omega' \tag{A22}$$

Using the implicit function theorem, we have:

$$\frac{\partial y_1^e}{\partial \lambda_s} = \frac{\beta \lambda_f f_f f_s y_1^e}{t_s t_f \Omega'} < 0 \tag{A23}$$

Also:

$$\frac{\partial y_1^e}{\partial \lambda_f} = \frac{-f_f t_s x_1^e}{t_s t_f \Omega'} > 0. \tag{A24}$$

Similarly,

$$\frac{\partial y_1^e}{\partial p_{1H}} = \frac{f_f f_s \lambda_f}{t_s t_f \Omega'} \tag{A25}$$

$$\frac{\partial y_1^e}{\partial p_{1L}} = \frac{f_f}{t_f \Omega'} \tag{A26}$$

Note that rational expectations imply that $x_1^e(\cdot) = x_1(\cdot)$ where x_1 is the actual demand from the snob segment and $y_1^e(\cdot) = y_1$ where y_1 is the demand among the followers. The profit function for the monopolist is :

$$\Pi_1 = x_1(p_{1H} - c) + \beta y_1 p_{1L} \tag{A27}$$

First order condition for p_{1L} implies that:

$$\frac{\partial \Pi_1}{\partial p_{1L}} = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{1L}} + \beta y_1 + \beta p_{1L}^* \frac{\partial y_1}{\partial p_{1L}} \tag{A28}$$

$$0 = \frac{-\left[(p_{1H}^* - c) f_s \lambda_s - t_s p_{1L}^*\right] \beta f_f}{t_s t_f \Omega'} + \beta y_1 \tag{A29}$$

Hence,

$$\left[t_s p_{1L}^* - (p_{1H}^* - c)\lambda_s f_s\right] = \frac{-y_1 t_s t_f \Omega'}{f_f} \tag{A30}$$

Similarly, the first-order condition for p_{1H} implies that:

$$\frac{\partial \Pi_1}{\partial p_{1H}} = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{1H}} + x_1 + \beta p_{1L}^* \frac{\partial y_1}{\partial p_{1H}} \tag{A31}$$

$$0 = x_1 + \frac{\left[(p_{1H}^* - c) t_f + \beta f_f \lambda_f p_{1L}^*\right] f_s}{t_s t_f \Omega'} \tag{A32}$$

This implies that:

$$[(p_{1H}^* - c)t_f + \beta f_f \lambda_f p_{1L}^*] = \frac{-t_s t_f \Omega' x_1}{f_s} \tag{A33}$$

Using the Envelope theorem, we get:

$$\frac{\partial \Pi_1^*}{\partial \lambda_s} = (p_{1H}^* - c)\frac{\partial x_1}{\partial \lambda_s} + \beta p_{1L}^* \frac{\partial y_1}{\partial \lambda_s} \tag{A34}$$

$$= \left(\frac{-\beta y_1 f_s}{t_s t_f + \beta \lambda_s \lambda_f f_s f_f}\right) \cdot ((p_{1H}^* - c)t_f + \beta p_{1L}^* \lambda_f f_f) \tag{A35}$$

$$= -\beta x_1 y_1 < 0 \tag{A36}$$

where the last inequality follows from (A33). Also, using the Envelope theorem, we have:

$$\frac{\partial \Pi_1^*}{\partial \lambda_f} = (p_{1H}^* - c)\frac{\partial x_1}{\partial \lambda_f} + \beta p_{1L}^* \frac{\partial y_1}{\partial \lambda_f} \tag{A37}$$

$$= \left[\frac{\beta x_1 f_f}{t_s t_f + \beta \lambda_s \lambda_f f_s f_f}\right] \cdot [t_s p_{1L}^* - (p_{1H}^* - c)\lambda_s f_s] \tag{A38}$$

$$= \beta x_1 y_1 > 0 \tag{A39}$$

where the last inequality follows from (A30) and by noting that $t_s t_f \Omega' = -[t_s t_f + \beta \lambda_s \lambda_f f_s f_f]$.

Thus, we have established that as λ_s increases the monopolist's profit decreases, whereas as λ_f increases monopolist's profits increase. This implies that umbrella branding is attractive for the firm as long as $\lambda_f > \lambda_f^*(\lambda_s)$ where the critical value is increasing in λ_s.

If we impose the condition that $f_f(\cdot)$ and $f_s(\cdot)$ are uniform with range $(0, 1)$ then we can directly solve for the optimal profits. Consider the case when $\lambda_s = \lambda$ and $\lambda_f = \lambda$. Solving this case when the firm uses umbrella branding, we find that the optimal prices and sales are:

$$x_1^* = \frac{r_s - c}{2t_s} \tag{A40}$$

$$y_1^* = \frac{r_f}{2t_f} \tag{A41}$$

$$p_{1H}^* = \frac{t_f(r_s + c) - r_f \beta \lambda}{2t_f} \tag{A42}$$

$$p_{1L}^* = \frac{t_s r_f + \lambda r_s - \lambda c}{2t_s} \tag{A43}$$

Substituting into (A27), we find that the optimal profits are:

$$\Pi_1^* = \frac{\beta r_f^2 t_s r_s^2 t_f + c^2 t_f - 2c t_f r_s}{4 t_s t_f} \tag{A44}$$

Note that this term is independent of λ and equals the profits when there are no social effects or when the firm uses individual branding. From (A36) and (A38) it follows that umbrella branding is profitable iff $\lambda_s > \lambda_f$, which is the result we have for the case when the markets are completely covered.

Proof of Proposition 2

Consider the case when both firms use umbrella branding strategy. If the consumer located at θ in the snob market buys product 1, she gets the following (indirect) utility:

$$U_s^1 = r_s - p_{1H} - t_s\theta - \beta\lambda_s y_1^e \tag{A45}$$

The utility derived by the consumer on buying firm 2's product is given by:

$$U_s^2 = r_s - p_{2H} - t_s(1-\theta) - \beta\lambda_s(1 - y_1^e) \tag{A46}$$

Consumer θ_s who is indifferent between the two firms in the snob market is given by:

$$\theta_s = \frac{p_{2H} - p_{1H} + t_s + \beta\lambda_s - 2\beta\lambda_s y_1^e}{2t_s} \tag{A47}$$

Similarly, consumer θ_f who is indifferent between the two brands in the follower segment is given by:

$$\theta_f = \frac{p_{2L} - p_{1L} + t_f - \lambda_f + 2\lambda_f x_1^e}{2t_f} \tag{A48}$$

Rational expectations imply:

$$y_1^e = F_f(\theta_f) \tag{A49}$$

$$x_1^e = F_s(\theta_s) \tag{A50}$$

Using these rational expectation conditions, we have:

$$\Omega_f(y_1^e) \equiv y_1^e - F_f\left(\frac{p_{2L} - p_{1L} + t_f - \lambda_f + 2\lambda_f x_1^e}{2t_f}\right) \tag{A51}$$

$$= y_1^e - F_f\left(\frac{p_{2L} - p_{1L} + t_f - \lambda_f + 2\lambda_f F_s(\theta_s)}{2t_f}\right) \tag{A52}$$

Note that since θ_s is a function of y_1^e, (A52) defines y_1^e. As $\Omega_f(0) < 0$ and $\Omega_f(1) > 0$, existence is guaranteed. Furthermore,

$$\Omega_f'(y_1^e) = 1 + \frac{\beta f_f(\theta_f) f_s(\theta_s)\lambda_s\lambda_f}{t_s t_f} > 0 \tag{A53}$$

This establishes uniqueness. Also,

$$\frac{\partial \Omega_f(y_1^e)}{\partial p_{1L}} = \frac{f_f(\theta_f)}{2t_f} \tag{A54}$$

Using implicit function theorem, it follows that:

$$\frac{\partial y_1^e}{\partial p_{1L}} = \frac{-f_f(\theta_f)}{2t_f \Omega_f'(y_1^e)} \tag{A55}$$

We have:

$$\frac{\partial \Omega_f(y_1^e)}{\partial p_{1H}} = \frac{f_f(\theta_f) f_s(\theta_s)\lambda_f}{2t_s t_f} \tag{A56}$$

Therefore,

$$\frac{\partial y_1^e}{\partial p_{1H}} = \frac{-f_f(\theta_f) f_s(\theta_s)\lambda_f}{2t_s t_f \Omega_f'(y_1^e)} \tag{A57}$$

We also have:

$$\Omega_s(x_1^e) \equiv x_1^e - F_s\left(\frac{p_{2H} - p_{1H} + t_s + \beta\lambda_s - 2\beta\lambda_s y_1^e}{2t_s}\right) \tag{A58}$$

$$= x_1^e - F_s\left(\frac{p_{2H} - p_{1H} + t_s + \beta\lambda_s - 2\beta\lambda_s F_f(\theta_f)}{2t_s}\right) \tag{A59}$$

Existence is assured by the fact that $\Omega_s(0) < 0$ and $\Omega_s(1) > 0$, and uniqueness is also ensured because:

$$\Omega_s'(x_1^e) = 1 + \frac{\beta\lambda_s\lambda_f f_s(\theta_s) f_f(\theta_f)}{t_s t_f} > 0 \tag{A60}$$

Using the implicit function theorem, we obtain:

$$\frac{\partial x_1^e}{\partial p_{1L}} = \frac{\beta f_f(\theta_f) f_s(\theta_s)\lambda_s}{2t_s t_f \Omega_s'(x_1^e)} \tag{A61}$$

$$\frac{\partial x_1^e}{\partial p_{1H}} = \frac{-f_s(\theta_s)}{2t_s \Omega_s'(x_1^e)} \tag{A62}$$

The first order conditions imply that:

$$0 = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{1H}} + x_1 + \beta p_{1L}^* \frac{\partial y_1}{\partial p_{1H}} \quad \text{(A63)}$$

$$0 = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{1L}} + \beta y_1 + \beta p_{1L}^* \frac{\partial y_1}{\partial p_{1L}} \quad \text{(A64)}$$

Note that rational expectations imply that the derivatives with respect to expected sales is the same as the derivatives with respect to the actual sales. Also, symmetry implies that $x_1^e = y_1^e = \frac{1}{2}$ and $\theta_s = \theta_f = \frac{1}{2}$. Also, $\Omega'_s = \Omega'_f \equiv \Omega'$. Denote $f_s(\frac{1}{2})$ by f_s and $f_f(\frac{1}{2})$ by f_f. Therefore, we have:

$$0 = (p_{1H}^* - c)\left[\frac{-f_s}{2t_s\Omega'}\right] + \frac{1}{2} + \beta p_{1L}^* \left[\frac{-f_s f_f \lambda_f}{2t_s t_f \Omega'}\right] \quad \text{(A65)}$$

$$0 = (p_{1H}^* - c)\left[\frac{\beta f_s f_f \lambda_s}{2t_s t_f \Omega'}\right] + \frac{\beta}{2} + \beta p_{1L}^* \left[\frac{-f_f}{2t_f\Omega'}\right] \quad \text{(A66)}$$

Solving (A65) and (A66), we get:

$$p_{1H}^* = c + \frac{t_s - \beta\lambda_f f_s}{f_s} \quad \text{(A67)}$$

$$p_{1L}^* = \frac{t_f + \lambda_s f_f}{f_f}. \quad \text{(A68)}$$

This leads to the profit function in the main body of the paper, which is

$$\Pi_1^*(1,1) = \left[\frac{t_s}{2f_s} + \frac{\beta t_f}{2f_f}\right] + \frac{\beta(\lambda_s - \lambda_f)}{2} \quad \text{(A69)}$$

and the result follows. □

Proof of Proposition 3

First note that the parameters λ_s and λ_f do not affect the equilibrium profits when both firms use individual branding. Furthermore, when both firms offer umbrella branding, we can use (21) to obtain:

$$\frac{d}{d\lambda_s}\Pi_1^*(1,1) = \frac{\beta}{2} \quad \text{(A70)}$$

$$\frac{d}{d\lambda_f}\Pi_1^*(1,1) = \frac{-\beta}{2} \quad \text{(A71)}$$

So, we only need to establish what happens when only one firm offers umbrella branding. We will first consider the impact of λ_s and then consider the impact of λ_f.

Impact of λ_s on $\not{b}_{10}$ and $\not{b}_{11}$.

Suppose firm 1 uses umbrella branding and firm 2 continues to use individual branding. We have:

$$\frac{d}{d\lambda_s}\Pi_1^*(1,0) = \frac{\partial \Pi_1(1,0)}{\partial \lambda_s} + \frac{\partial \Pi_1(1,0)}{\partial p_{2H}} \cdot \frac{\partial p_{2H}^*}{\partial \lambda_s} + \frac{\partial \Pi_1(1,0)}{\partial p_{2L}} \cdot \frac{\partial p_{2L}^*}{\partial \lambda_s} \quad \text{(A72)}$$

Furthermore,

$$\frac{\partial \Pi_1(1,0)}{\partial \lambda_s} = (p_{1H}^* - c)\frac{\partial x_1}{\partial \lambda_s} + \beta p_{1L}^* \frac{\partial y_1}{\partial \lambda_s} \quad \text{(A73)}$$

The indifferent consumers in this case are given by:

$$\theta_f = \frac{p_{2L} - p_{1L} + t_f + \lambda_f x_1^e}{2t_f} \quad \text{(A74)}$$

$$\theta_s = \frac{p_{2H} - p_{1H} + t_s - \beta\lambda_s y_1^e}{2t_s} \quad \text{(A75)}$$

Using rational expectations, we get the following:

$$\Omega_f(y_1^e) \quad = \quad y_1^e - F_f\left(\frac{p_{2L} - p_{1L} + t_f + \lambda_f F_s(\theta_s)}{2t_f}\right) \tag{A76}$$

$$\Omega_f(x_1^e) \quad = \quad x_1^e - F_s\left(\frac{p_{2H} - p_{1H} + t_s - \beta\lambda_s F_f(\theta_f)}{2t_s}\right) \tag{A77}$$

It follows from (A77) and (A76) that:

$$\frac{\partial x_1}{\partial \lambda_s} = \frac{-\beta f_s(\theta_s^*) y_1^*}{2t_s\Omega'} \tag{A78}$$

$$\frac{\partial y_1}{\partial \lambda_s} = \frac{f_f(\theta_f^*) f_s(\theta_s^*)\beta\lambda_f y_1^*}{4t_s t_f\Omega'} \tag{A79}$$

$$\frac{\partial x_1}{\partial p_{1H}} = \frac{-f_s(\theta_s^*)}{2t_s\Omega'} = -\frac{\partial x_1}{\partial p_{2H}} \tag{A80}$$

$$\frac{\partial y_1}{\partial p_{1H}} = \frac{-f_f(\theta_f^*) f_s(\theta_s^*)\lambda_f y_1^*}{4t_s t_f\Omega'} = -\frac{\partial y_1}{\partial p_{2H}} \tag{A81}$$

$$\frac{\partial x_1}{\partial p_{1L}} = \frac{-\beta f_s(\theta_s^*) f_f(\theta_f^*)\lambda_s}{4t_s t_f\Omega'} = -\frac{\partial x_1}{\partial p_{2L}} \tag{A82}$$

$$\frac{\partial y_1}{\partial p_{1L}} = \frac{-f_f(\theta_f^*)}{2t_f\Omega'} = -\frac{\partial y_1}{\partial p_{2L}} \tag{A83}$$

Note that for small λ_s and λ_f, $\Omega' \to 1$, θ_s^* is close to $\frac{1}{2}$.

$$\lim_{\lambda_s\to 0}\lim_{\lambda_f\to 0}\frac{\partial \Pi_1(1,0)}{\partial \lambda_s} = \frac{-\beta(p_{1H}^* - c)f_s}{4t_s} \tag{A84}$$

where we use the notation f_s to denote $f_s(\frac{1}{2})$.

Also, note that:

$$\lim_{\lambda_s\to 0}\lim_{\lambda_f\to 0}\frac{\partial \Pi_1(1,0)}{\partial p_{2H}} = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{2H}} + \beta p_{1L}^*\frac{\partial y_1}{\partial p_{2H}} = \frac{(p_{1H}^* - c)f_s}{2t_s} \tag{A85}$$

where the last equality follows from the assumption that λ_s and λ_f are small.

Similarly,

$$\lim_{\lambda_s\to 0}\lim_{\lambda_f\to 0}\frac{\partial \Pi_1(1,0)}{\partial p_{2L}} = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{2L}} + \beta p_{1L}^*\frac{\partial y_1}{\partial p_{2L}} = \frac{\beta f_f p_{1L}^*}{2t_f} \tag{A86}$$

Therefore:

$$\lim_{\lambda_s\to 0}\lim_{\lambda_f\to 0}\frac{d}{d\lambda_s}\Pi_1^*(1,0) = \frac{-\beta(p_{1H}^* - c)f_s}{4t_s} + \frac{(p_{1H}^* - c)f_s}{2t_s}\cdot\frac{\partial p_{2H}^*}{\partial \lambda_s} + \frac{\beta f_f p_{1L}^*}{2t_f}\cdot\frac{\partial p_{2L}^*}{\partial \lambda_s} \tag{A87}$$

We need to examine how equilibrium prices vary with λ_s for small values of the social parameters. The first order conditions which describe the equilibrium prices are:

$$0 = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{1H}} + x_1^* + \beta p_{1L}^*\frac{\partial y_1}{\partial p_{1H}} \tag{A88}$$

$$0 = (p_{1H}^* - c)\frac{\partial x_1}{\partial p_{1L}} + \beta y_1^* + \beta p_{1L}^*\frac{\partial y_1}{\partial p_{1H}} \tag{A89}$$

$$0 = -(p_{2H}^* - c)\frac{\partial x_1}{\partial p_{2H}} + (1 - x_1^*) - \beta p_{2L}^*\frac{\partial y_1}{\partial p_{2H}} \tag{A90}$$

$$0 = -(p_{2H}^* - c)\frac{\partial x_1}{\partial p_{2L}} + \beta(1 - y_1^*) - \beta p_{2L}^*\frac{\partial y_1}{\partial p_{2L}} \tag{A91}$$

To derive the impact of λ_s on equilibrium prices, we use these first-order conditions, the implicit function theorem and the assumption that λ_s and λ_f are small. After simplification, the relevant equation becomes:

$$\begin{pmatrix} -\frac{f_s}{t_s} & 0 & \frac{f_s}{2t_s} & 0 \\ 0 & \frac{-\beta f_f}{t_f} & 0 & \frac{\beta f_f}{2t_f} \\ \frac{f_s}{2t_s} & 0 & \frac{-f_s}{t_s} & 0 \\ 0 & \frac{\beta f_f}{2t_f} & 0 & \frac{-\beta f_f}{t_f} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial p_{1H}^*}{\partial \lambda_s} \\ \frac{\partial p_{1L}^*}{\partial \lambda_s} \\ \frac{\partial p_{2H}^*}{\partial \lambda_s} \\ \frac{\partial p_{2L}^*}{\partial \lambda_s} \end{pmatrix} = \begin{pmatrix} \beta f_s/(4t_s) \\ -\beta f_f/(4t_f) \\ -\beta f_s/(4t_s) \\ -\beta f_f/(4t_f) \end{pmatrix} \tag{A92}$$

Upon solving, we get:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \begin{pmatrix} \frac{\partial p_{1H}^*}{\partial \lambda_s} \\ \frac{\partial p_{1L}^*}{\partial \lambda_s} \\ \frac{\partial p_{2H}^*}{\partial \lambda_s} \\ \frac{\partial p_{2L}^*}{\partial \lambda_s} \end{pmatrix} = \begin{pmatrix} -\beta/6 \\ 1/2 \\ \beta/6 \\ 1/2 \end{pmatrix} \tag{A93}$$

Using the first-order conditions for equilibrium prices, we can also show that for small λ_s and λ_f, $p_{1H}^* \approx c + \frac{t_s}{f_s}$ and $p_{1L}^* \approx \frac{t_f}{f_f}$. Using these and substituting them in (A72), we get:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d}{d\lambda_s} \Pi_1^*(1,0) = -\frac{t_s}{f_s} \cdot \frac{f_s \beta}{4t_s} + \frac{t_s}{f_s} \cdot \frac{f_s}{2t_s} \cdot \frac{\beta}{6} + \frac{\beta}{2} \cdot \frac{1}{2} = \frac{\beta}{12} \tag{A94}$$

Note that:

$$\text{б}_{10} = \Pi_1(1,0) - \Pi_1(0,0) \tag{A95}$$

Since $\Pi_1(0,0)$ does not depend on λ_s, it follows from equation (A94) that б_{10} increases in λ_s. We know that:

$$\text{б}_{11} = \Pi_1(1,1) - \Pi_1(0,1) \tag{A96}$$

Note that:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d}{d\lambda_s} \Pi_1(1,1) = \frac{\beta}{2} \tag{A97}$$

As $\Pi_1(0,1) = \Pi_2(1,0)$, we have $\text{б}_{11} = \Pi_1(1,1) - \Pi_2(1,0)$. Further note that:

$$\frac{d}{d\lambda_s} \Pi_2^*(1,0) = \frac{\partial \Pi_2(1,0)}{\partial \lambda_s} + \frac{\partial \Pi_2(1,0)}{\partial p_{1H}} \cdot \frac{\partial p_{1H}^*}{\partial \lambda_s} + \frac{\partial \Pi_2(1,0)}{\partial p_{1L}} \cdot \frac{\partial p_{1L}^*}{\partial \lambda_s} \tag{A98}$$

and

$$\frac{\partial \Pi_2(1,0)}{\partial \lambda_s} = -(p_{2H}^* - c) \frac{\partial x_1}{\partial \lambda_s} - \beta p_{2L}^* \frac{\partial y_1}{\partial \lambda_s}. \tag{A99}$$

Using (A78) and (A79), we find that for small values of social effects:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{\partial \Pi_2(1,0)}{\partial \lambda_s} = \frac{\beta (p_{2H}^* - c) f_s}{4t_s} \tag{A100}$$

Also, note that:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{\partial \Pi_2(1,0)}{\partial p_{1H}} = -(p_{2H}^* - c) \frac{\partial x_1}{\partial p_{1H}} - \beta p_{2L}^* \frac{\partial y_1}{\partial p_{1H}} = \frac{(p_{2H}^* - c) f_s}{2t_s} \tag{A101}$$

where the last equality follows from the assumption that λ_s and λ_f are small.

Similarly,

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{\partial \Pi_2(1,0)}{\partial p_{1L}} = -(p_{2H}^* - c) \frac{\partial x_1}{\partial p_{1L}} - \beta f_f p_{2L}^* \frac{\partial y_1}{\partial p_{1L}} = \frac{\beta f_f p_{2L}^*}{2t_f} \tag{A102}$$

Therefore:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d}{d\lambda_s} \Pi_2^*(1,0) = \frac{\beta (p_{2H}^* - c) f_s}{4t_s} + \frac{(p_{2H}^* - c) f_s}{2t_s} \cdot \frac{\partial p_{1H}^*}{\partial \lambda_s} + \frac{\beta f_f p_{2L}^*}{2t_f} \cdot \frac{\partial p_{1L}^*}{\partial \lambda_s} \tag{A103}$$

Using(A93), we obtain:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d}{d\lambda_s} \Pi_2(1,0) = \frac{\beta}{4} + \frac{\beta}{4} - \frac{\beta}{6} \cdot \frac{1}{2} = \frac{5\beta}{12} \tag{A104}$$

Thus:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d\text{б}_{11}}{d\lambda_s} = \frac{\beta}{2} - \frac{5\beta}{12} = \frac{\beta}{12} \tag{A105}$$

Impact of λ_f on $\mathscr{b}_{10}$ and $\mathscr{b}_{11}$.

Now, let us consider the impact of λ_f. We will use the same approach as above. Suppose firm 1 uses umbrella branding and firm 2 continues to use individual branding. We have:

$$\frac{d}{d\lambda_f}\Pi_1^*(1,0) = \frac{\partial \Pi_1(1,0)}{\partial \lambda_f} + \frac{\partial \Pi_1(1,0)}{\partial p_{2H}} \cdot \frac{\partial p_{2H}^*}{\partial \lambda_f} + \frac{\partial \Pi_1(1,0)}{\partial p_{2L}} \cdot \frac{\partial p_{2L}^*}{\partial \lambda_f} \tag{A106}$$

Furthermore,

$$\frac{\partial \Pi_1(1,0)}{\partial \lambda_f} = (p_{1H}^* - c)\frac{\partial x_1}{\partial \lambda_s} + \beta p_{1L}^* \frac{\partial y_1}{\partial \lambda_f} \tag{A107}$$

It follows from (A77) and (A76) that for small social effects:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{\partial x_1}{\partial \lambda_f} = 0 \tag{A108}$$

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{\partial y_1}{\partial \lambda_f} = \frac{f_f}{4t_f} \tag{A109}$$

Therefore:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{\partial \Pi_1(1,0)}{\partial \lambda_f} = \frac{\beta}{4} \tag{A110}$$

To these first order conditions, we apply the implicit function theorem and assume that λ_s and λ_f are small to derive the impact of λ_f on equilibrium prices. After simplification, the relevant equation becomes:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \begin{pmatrix} -\frac{f_s}{t_s} & 0 & \frac{f_s}{2t_s} & 0 \\ 0 & \frac{-\beta f_f}{t_f} & 0 & \frac{\beta f_f}{2t_f} \\ \frac{f_s}{2t_s} & 0 & \frac{-f_s}{t_s} & 0 \\ 0 & \frac{\beta f_f}{2t_f} & 0 & \frac{-\beta f_f}{t_f} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial p_{1H}^*}{\partial \lambda_f} \\ \frac{\partial p_{1L}^*}{\partial \lambda_f} \\ \frac{\partial p_{2H}^*}{\partial \lambda_f} \\ \frac{\partial p_{2L}^*}{\partial \lambda_f} \end{pmatrix} = \begin{pmatrix} \beta f_s/(4t_s) \\ -\beta f_f/(4t_f) \\ \beta f_s/(4t_s) \\ \beta f_f/(4t_f) \end{pmatrix} \tag{A111}$$

Solving the above system on equations, we obtain:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \begin{pmatrix} \frac{\partial p_{1H}^*}{\partial \lambda_f} \\ \frac{\partial p_{1L}^*}{\partial \lambda_f} \\ \frac{\partial p_{2H}^*}{\partial \lambda_f} \\ \frac{\partial p_{2L}^*}{\partial \lambda_f} \end{pmatrix} = \begin{pmatrix} -\beta/2 \\ 1/6 \\ -\beta/2 \\ -1/6 \end{pmatrix} \tag{A112}$$

Using these, we get:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d}{d\lambda_f}\Pi_1^*(1,0) = \frac{\partial \Pi_1(1,0)}{\partial \lambda_f} + \frac{\partial \Pi_1(1,0)}{\partial p_{2L}} \cdot \frac{\partial p_{2L}^*}{\partial \lambda_f} + \frac{\partial \Pi_2(1,0)}{\partial p_{2H}} \cdot \frac{\partial p_{2H}^*}{\partial \lambda_f} \tag{A113}$$

$$= \frac{\beta}{4} + \frac{\beta}{2} \cdot \left(\frac{-1}{6}\right) + \frac{1}{2} \cdot \left(\frac{-\beta}{2}\right) = \frac{-\beta}{12} \tag{A114}$$

Note that:

$$\mathscr{b}_{10} = \Pi_1(1,0) - \Pi_1(0,0) \tag{A115}$$

Since $\Pi_1(0,0)$ does not depend on λ_f, it follows that $\mathscr{b}_{10}$ decreases in λ_f. Also, note that:

$$\mathscr{b}_{11} = \Pi_1(1,1) - \Pi_1(0,1) \tag{A116}$$

Next note that:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d}{d\lambda_f}\Pi_1(1,1) = \frac{-\beta}{2} \tag{A117}$$

As $\Pi_1(0,1) = \Pi_2(1,0)$, $\mathscr{b}_{11} = \Pi_1(1,1) - \Pi_2(1,0)$. Now we have:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d}{d\lambda_s}\Pi_2^*(1,0) = \frac{\partial \Pi_2(1,0)}{\partial \lambda_f} + \frac{\partial \Pi_2(1,0)}{\partial p_{1L}} \cdot \frac{\partial p_{1L}^*}{\partial \lambda_f} + \frac{\partial \Pi_2(1,0)}{\partial p_{1H}} \cdot \frac{\partial p_{1H}^*}{\partial \lambda_f} \tag{A118}$$

$$= \frac{-\beta}{4} + \frac{\beta}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \left(\frac{-\beta}{2}\right) = \frac{-5\beta}{12} \tag{A119}$$

Therefore:

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d\not{b}_{10}}{d\lambda_f} = \frac{-\beta}{12} \quad \text{(A120)}$$

$$\lim_{\lambda_s \to 0} \lim_{\lambda_f \to 0} \frac{d\not{b}_{11}}{d\lambda_f} = \frac{-\beta}{2} - \left(\frac{-5\beta}{12}\right) = -\frac{\beta}{12} \quad \text{(A121)}$$

This completes the proof. □

Proof of Proposition 4

We have already derived the profits when both firms use symmetric branding strategies. However, for the general case when firms are using different strategies, we cannot derive closed form solution. For the case when $f(\cdot)$ is uniform, we can solve for the prices directly, using the first order conditions.

Consider the case when firm 1 uses umbrella branding and firm 2 uses individual branding. First consider the snob segment. If a consumer at θ_s buys from firm 1 then his utility is given by:

$$U_1^s = r_s - p_{1H} - t_s\theta_s - \beta\lambda_s y_1^e \quad \text{(A122)}$$

where y_1^e is the expected number of followers who are expected to purchase firm 1's product. If this consumer buys from firm 2 then his utility is:

$$U_2^s = r_s - p_{2H} - t_s(1 - \theta_s) \quad \text{(A123)}$$

Note that since firm 2 uses individual branding, this consumer does not experience a negative externality due to followers buying from firm 2. The consumer in the snob segment who is indifferent between buying 1 and 2 is indexed by θ_s and is therefore given by:

$$\theta_s = \frac{p_{2H} - p_{1H} + t_s - \beta\lambda_s y_1^e}{2t_s} \quad \text{(A124)}$$

Similarly, the consumer in the follower segment who is indifferent between buying the two products is indexed by θ_f which is given by:

$$\theta_f = \frac{p_{2L} - p_{1L} + t_f + \lambda_f x_1^e}{2t_f} \quad \text{(A125)}$$

Rational expectations and the assumption of uniform distribution implies that:

$$x_1^e = \theta_s \quad \text{(A126)}$$

$$y_1^e = \theta_f \quad \text{(A127)}$$

Solving, this we get the demand functions which are:

$$x_1 = \frac{2t_f(p_{2H} - p_{1H}) + \beta\lambda_s(p_{1L} - p_{2L}) + (2t_s - \lambda_s\beta)t_f}{4t_s t_f + \beta\lambda_s\lambda_f} \quad \text{(A128)}$$

$$y_1 = \frac{\lambda_f(p_{2H} - p_{1H}) + 2t_s(p_{2L} - p1L) + t_s(\lambda_f - 2t_f)}{4t_s t_f + \beta\lambda_s\lambda_f} \quad \text{(A129)}$$

The profits for firm 1 are given by:

$$\Pi_1(1,0) = x_1(p_{1H} - c) + \beta y_1 p_{1L} \quad \text{(A130)}$$

Similarly, the profits for firm 2 are given by:

$$\Pi_2(1,0) = (1 - x_1)(p_{2H} - c) + \beta(1 - y_1)p_{2L} \quad \text{(A131)}$$

Using the first order conditions for both firms and solving, we get the equilibrium prices which are:

$$p_{1L}^* = \frac{(-2t_f\beta\lambda_f^2 - 3t_f\lambda_s^2\beta + 36t_st_f^2 + 4t_f\lambda_s\beta\lambda_f + 18t_st_f\lambda_s + 6\lambda_ft_st_f - \beta\lambda_s^3 + 2\lambda_f\beta\lambda_s^2)}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \tag{A132}$$

$$p_{1H}^* = c + \frac{(\lambda_f^3\beta^2 - 2\beta^2\lambda_f^2\lambda_s - 3\beta\lambda_f^2t_s - 18\beta\lambda_ft_st_f + 4\lambda_s\beta\lambda_ft_s - 2\beta\lambda_s^2t_s - 6t_s\lambda_s\beta t_f + 36t_s^2t_f)}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \tag{A133}$$

$$p_{2L}^* = \frac{-2t_f\beta\lambda_f^2 - t_f\lambda_s^2\beta + 36t_st_f^2 + 6t_f\lambda_s\beta\lambda_f + 18t_st_f\lambda_s - 6\lambda_ft_st_f - \beta\lambda_s^3 - 2\lambda_s\beta\lambda_f^2 + 3\lambda_f\beta\lambda_s^2}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \tag{A134}$$

$$p_{2H}^* = c + \frac{\lambda_f^3\beta^2 - 3\beta^2\lambda_f^2\lambda_s + 2\lambda_s^2\beta^2\lambda_f + 36t_s^2t_f - \beta\lambda_f^2t_s - 2\beta\lambda_s^2t_s + 6\lambda_s\beta\lambda_ft_s - 18\beta\lambda_ft_st_f + 6t_s\lambda_s\beta t_f}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \tag{A135}$$

Using this and the equilibrium demand functions, we obtain the following equilibrium profits:

$$\begin{aligned}\Pi_1^*(1,0) &= \frac{-\beta^2\lambda_f^2\lambda_s - \beta^2t_f\lambda_f^2 + \lambda_s^2\beta^2\lambda_f + 3\beta^2t_f\lambda_s\lambda_f + 3\lambda_s\beta\lambda_ft_s}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \\ &\quad + \frac{-3\beta\lambda_ft_st_f - \beta\lambda_s^2t_s + 3t_s\lambda_s\beta t_f + 18\beta t_st_f^2 + 18t_s^2t_f}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)}\end{aligned} \tag{A136}$$

$$\begin{aligned}\Pi_2^*(1,0) &= \frac{\lambda_f^3\beta^2 - 3\beta^2\lambda_f^2\lambda_s - \beta^2t_f\lambda_f^2 + 3\lambda_s^2\beta^2\lambda_f + 2\beta^2t_f\lambda_s\lambda_f - \beta^2\lambda_s^3 - \beta^2t_f\lambda_s^2 - \beta\lambda_f^2t_s}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \\ &\quad + \frac{2\lambda_s\beta\lambda_ft_s - 15\beta\lambda_ft_st_f - \beta\lambda_s^2t_s + 15t_s\lambda_s\beta t_f + 18\beta t_st_f^2 + 18t_s^2t_f}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)}\end{aligned} \tag{A137}$$

Note that we have already derived $\Pi(0,0)$ in the base case. The incentives for firm 1 to deviate from $(0,0)$ to $(1,0)$, namely β_{10}, are follows:

$$\beta_{10} = \frac{\beta(-2\lambda_s\beta\lambda_f^2 + 2\lambda_f\beta\lambda_s^2 + t_f\lambda_s\beta\lambda_f + 2t_f\lambda_s^2\beta + 2t_s\lambda_f^2 + \lambda_ft_s\lambda_s - 6\lambda_ft_st_f + 6t_st_f\lambda_s)}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \tag{A138}$$

The incentives for firm 2 to deviate from $(1,0)$ to $(1,1)$, namely β_{11}, are given by:

$$\beta_{11} = \frac{\beta(-\lambda_s\beta\lambda_f^2 + t_f\lambda_s\beta\lambda_f + \lambda_f\beta\lambda_s^2 + \lambda_ft_s\lambda_s - 6\lambda_ft_st_f + 6t_st_f\lambda_s)}{(5\lambda_f\lambda_s\beta - 2\beta\lambda_f^2 + 36t_st_f - 2\lambda_s^2\beta)} \tag{A139}$$

Note that the denominator of both β_{10} and β_{11} are positive by the assumptions that $\lambda_i \leq 1$, $t_i \geq 1$ and $\beta\lambda_i \leq t_i$. Therefore, the signs of these terms depend on the signs of the numerator.

We will establish the proposition via a series of claims.

Claim 4.1 *The numerator of β_{11} is convex in λ_s. Furthermore, it is negative at $\lambda_s = 0$ and increasing at $\lambda_s = 0$.*

Proof: Define numerator of β_{11}as $N(\beta_{11})$ We have:

$$\frac{\partial^2 N(\beta_{11})}{\partial^2\lambda_s^2} = 2\beta^2\lambda_f > 0. \tag{A140}$$

Also:

$$N(\beta_{11})|_{\lambda_s=0} = -6\beta\lambda_ft_st_f < 0 \tag{A141}$$

Also:

$$\left.\frac{\partial N(\beta_{11})}{\partial\lambda_s}\right|_{\lambda_s=0} = \lambda_f\left[\beta(t_f - \lambda_f) + t_s\right] + 6t_ft_s > 0 \tag{A142}$$

□

Define λ_s^{**} such that $\text{б}_{11}(\lambda_s^{**}) = 0$. We have:

Claim 4.2 *б_{11} is positive for $\lambda_s > \lambda_s^{**}(\lambda_f)$ and negative otherwise. Furthermore, $\lambda_s^{**}(\lambda_f) < \lambda_f$.*

Proof: Immediately follows from Claim 4.1. To see the second part note that:

$$N(\text{б}_{11})|_{\lambda_s=\lambda_f} = \beta\lambda_f^2(\beta t_f + t_s) > 0. \tag{A143}$$

Since $N(\text{б}_{11})$ is increasing and convex in λ_s, the result follows. □

Claim 4.3 *$\lambda_s^{**}(\lambda_f)$ is increasing in λ_f.*

Proof: Note that λ_s^{**} is defined by the equation $N(\text{б}_{11}) = 0$. Using the implicit function theorem, we have:

$$\frac{\partial\lambda_s^{**}}{\partial\lambda_f} = -\frac{\frac{\partial(N(\text{б}_{11})}{\partial\lambda_f}}{\frac{\partial N(\text{б}_{11})}{\partial\lambda_s}} \tag{A144}$$

which reduces to:

$$\frac{\partial\lambda_s^{**}}{\partial\lambda_f} = \frac{2\beta\lambda_f\lambda_s + t_f(t_s - \beta\lambda_s) + (t_st_f - \lambda_s^2\beta) + t_s(4t_f - \lambda_s)}{-\beta\lambda_f^2 + t_f\beta\lambda_f + 2\lambda_f\lambda_s\beta + \lambda_f t_s + 6t_st_f} > 0 \tag{A145}$$

□

Claim 4.4 *If $\lambda_s > \lambda_f$ then it is an equilibrium for both firms to use umbrella branding. Furthermore, when $\lambda_s \in (\lambda_s^{**}, \lambda_f)$, firms face a prisoner's dilemma and use umbrella branding even though they would be better off committing to individual branding.*

Proof: Note that from Claim 4.2 we know that if $\lambda_s > \lambda_s^{**}$, then $\text{б}_{11} > 0$. This implies that in this case both using umbrella branding is an equilibrium. Also, note that from the second part of Claim 4.2, it follows that if $\lambda_s > \lambda_f$ then umbrella branding is an equilibrium strategy, which proves part a. However, since $\lambda_s^{**}(\lambda_f) < \lambda_f$, it follows that for $\lambda_s \in (\lambda_s^{**}, \lambda_f)$ it is still an equilibrium for both firms to offer umbrella branding. However, we know that both firms would be better off committing to individual branding when $\lambda_s < \lambda_f$. This implies that firms face a prisoner's dilemma in the region $\lambda_s \in (\lambda_s^{**}, \lambda_f)$. This proves part a and the first part of part b of the Proposition. The last portion of the statement in part b is proved in Claim 4.3.□

Claim 4.5 *The numerator of б_{10} is convex in λ_s. Furthermore, it is negative at $\lambda_s = 0$ and increasing at $\lambda_s = 0$.*

Proof: Define numerator of б_{10} as $N(\text{б}_{10})$ We have:

$$\frac{\partial^2 N(\text{б}_{10})}{\partial^2\lambda_s^2} = 4\beta^2(\lambda_f + \lambda_s) > 0. \tag{A146}$$

Also:

$$N(\text{б}_{10})|_{\lambda_s=0} = 2t_s\lambda_f\beta\left[\lambda_f - 3t_s\right] < 0 \tag{A147}$$

Also:

$$\left.\frac{\partial N(\text{б}_{10})}{\partial\lambda_s}\right|_{\lambda_s=0} = \lambda_f\left[\beta(t_f - \lambda_f) + (t_s - \beta\lambda_f)\right] + 6t_ft_s > 0 \tag{A148}$$

□

Define λ_s^* such that $\text{б}_{10}(\lambda_s^*) = 0$.

Claim 4.6 *б_{10} is positive for $\lambda_s > \lambda_s^*(\lambda_f)$ and negative otherwise. Furthermore, $\lambda_s^*(\lambda_f) < \lambda_f$.*

Proof: The first part follows from Claim 4.5. To see the second part note that:

$$N(\text{б}_{10})|_{\lambda_s=\lambda_f} = 3\beta\lambda_f^2(\beta t_f + t_s) > 0. \tag{A149}$$

Since $N(\text{б}_{10})$ is increasing and convex in λ_s, the result follows. □

Claim 4.7 *$\lambda_s^*(\lambda_f)$ is increasing in λ_f.*

Proof: Note that λ_s^* is defined by the equation $N(\mathscr{b}_{10}) = 0$. Using the implicit function theorem, we have:

$$\frac{\partial \lambda_s^*}{\partial \lambda_f} = -\frac{\frac{\partial (N(\mathscr{b}_{10})}{\partial \lambda_f}}{\frac{\partial N(\mathscr{b}_{10})}{\partial \lambda_s}} \tag{A150}$$

which reduces to:

$$\frac{\partial \lambda_s^*}{\partial \lambda_f} = \frac{2\beta\lambda_s(2\lambda_f - \lambda_s) + 4t_s(t_f - \lambda_f) + t_f(t_s - \lambda_s\beta) + t_s(t_f - \lambda_s)}{2(3t_s t_f - \beta\lambda_f^2) + 4\lambda_f\lambda_s\beta + t_f\beta\lambda_f + 4\lambda_s\beta t_f + \lambda_f t_s} > 0 \tag{A151}$$

where the inequality follows since $\lambda_i \leq 1$, $t_i \geq 1$ and $\beta\lambda_i < t_i$ and the fact that $\lambda_s^* < \lambda_f$. □

Claim 4.8 *If $\lambda_s < \lambda_s^*(\lambda_f)$ then it is an equilibrium for both firms to choose individual branding.*

Proof: Under the conditions specified in the claim, it follows that $\mathscr{b}_{10} > 0$. This implies that firms would not like to deviate from individual branding and it is therefore an equilibrium. This proves part d. □

Claim 4.9 $\lambda_s^* < \lambda_s^{**}$.

Proof: We have:

$$N(\mathscr{b}_{10}) - N(\mathscr{b}_{11}) = \beta\left[\lambda_f\beta\lambda_s^2 + 2t_f\lambda_s^2\beta + (2t_s - \beta\lambda_s)\lambda_f^2\right] > 0 \tag{A152}$$

which completes the proof. □

Claim 4.10 *If $\lambda_s \in (\lambda_s^*, \lambda_s^{**})$ then one firm uses individual branding and the other one uses umbrella branding.*

Proof: This follows since in this range $\mathscr{b}_{10} > 0$ and $\mathscr{b}_{11} < 0$ and therefore the symmetric solution cannot be an equilibrium and the asymmetric equilibrium exists. This proves part c.□

Note that if $\lambda_s < \lambda_s^*$ then $\mathscr{b}_{10} < 0$ and therefore both firms using an individual branding strategy is an equilibrium strategy. Also, note that if $\mathscr{b}_{10} < 0$ and $\mathscr{b}_{11} > 0$ then there could be multiple equilibria with both the symmetric equilibrium being valid. However, $\mathscr{b}_{11} > 0$ only if $\lambda_s > \lambda_s^{**}$. Since from Claim 4.9, we know that $\lambda_s^* < \lambda_s^{**}$, it follows that if $\mathscr{b}_{11} > 0$ then $\mathscr{b}_{10} > 0$ and in this case both firms offering umbrella branding is the unique equilibrium. Furthermore, since the case $\mathscr{b}_{10} < 0$ and $\mathscr{b}_{11} > 0$ can be ruled out, we can rule out the case in which both symmetric equilibrium are valid. This completes the proof. □

Proof of Proposition 5

First, note that the follower who buys the low-quality product and is indifferent between firm 1 and 2 is indexed by θ_{fl}, which is given by:

$$\theta_{fl} = \frac{p_{2L} - p_{1L} + t_f}{2t_f} \tag{A153}$$

Now consider the followers with high follower effect and who buy the high-quality product. The consumer who is indifferent between buying firm 1 and 2's product is indexed by θ_{fh} which is given by:

$$\theta_{fh} = \frac{p_{2H} - p_{1H} - (\lambda_f + \Lambda)(1 - 2x_1^e) + t_f}{2t_f} \tag{A154}$$

Finally, the snob who is indifferent between the two firms is indexed by θ_s which is:

$$\theta_s = \frac{p_{2H} - p_{1H} + \alpha\beta\lambda_s(1 - 2y_1^e) + t_s}{2t_s} \tag{A155}$$

The rational expectation conditions imply that:

$$\Omega_1(x_1^e) = F(\theta_s) - x_1^e = 0 \quad \text{(A156)}$$

$$\Omega_2(y_1^e) = F(\theta_{fh}) - y_1^e = 0 \quad \text{(A157)}$$

Note that the expectations x_1^e and y_1^e depend on the prices that the firms charge. Using the implicit function theorem, it is easy to show that:

$$\frac{\partial x_1^e}{\partial p_{1H}} = \frac{f(\theta_s)t_f\left[-t_f + \alpha\beta\lambda_s f(\theta_{fh})\right]}{2\left[t_s t_f + f(\theta_s)f(\theta_{fh})\lambda_s\alpha\beta(\lambda_f + \Lambda)\right]} \quad \text{(A158)}$$

$$\frac{\partial y_1^e}{\partial p_{1H}} = \frac{-f(\theta_{fh})\left[t_s + (\lambda_s + \Lambda)f(\theta_s)\right]}{2\left[t_s t_f + f(\theta_s)f(\theta_{fh})\lambda_s\alpha\beta(\lambda_f + \Lambda)\right]} \quad \text{(A159)}$$

The profit function for firm 1 is given by:

$$\Pi_1^I = \left[F(\theta_s) + \alpha\beta F(\theta_{fh})\right](p_{1H} - c) + (1-\alpha)\beta F(\theta_{fl})p_{1L} \quad \text{(A160)}$$

The first order conditions imply that:

$$0 = \left[F(\theta_s) + \alpha\beta F(\theta_{fh})\right] + (p_{1H} - c)\left[f(\theta_s)\cdot\frac{\partial\theta_s}{p_{1H}} + \alpha\beta\cdot\frac{\partial\theta_{fh}}{\partial p_{1H}}\right] \quad \text{(A161)}$$

$$0 = (1-\alpha)\beta\left[F(\theta_{fl}) - \frac{p_{1L}f(\theta_{fl})}{2t_f}\right] \quad \text{(A162)}$$

Using symmetry and equations (A158) and (A159) we get:

$$p_{1H}^* = c + \frac{(1+\alpha\beta)\left(\alpha\beta\lambda_s(\lambda_f + \Lambda)f^2(\frac{1}{2}) + t_s t_f\right)}{f(\frac{1}{2})\left(t_f + \alpha\beta(t_s + \lambda_f + \Lambda - \lambda_s)\right)} \quad \text{(A163)}$$

$$p_{1L}^* = \frac{t_f}{f(\frac{1}{2})} \quad \text{(A164)}$$

To determine the impact of λ_f, we now impose the condition that the value distributions are uniform and $\beta = 1$. Using this we can obtain the profits under individual branding. Since the profits from umbrella branding remains the same as in the base case, we can then calculate the profitability of using umbrella branding strategy. After simplification this reduces to:

$$\Pi_1^U - \Pi_1^I = \frac{\alpha(t_s - t_f)^2 - \lambda_s(t_f\alpha^2 + t_f + \lambda_s\alpha) + D1}{2\left(t_f + \alpha t_s\alpha(\lambda_f + \Lambda - \lambda_s)\right)} \quad \text{(A165)}$$

where:

$$D1 = (\lambda_f + \Lambda)(-\alpha^3\lambda_s - 2\alpha^2\lambda_s + t_f\alpha^2) + (t_s - \lambda_f)\Lambda\alpha + (\lambda_s\alpha - t_f)\lambda_f - (\lambda_s^2 + \lambda_f^2)\alpha \quad \text{(A166)}$$

Since the denominator of (A165) is positive, it is profitable to use umbrella branding whenever the numerator is positive. To evaluate the impact of λ_f, note that the numerator is decreasing in λ_f, since it is given by:

$$\frac{\partial\text{Numerator}}{\partial\lambda_f} = -(1-\alpha)^2(t_f - \alpha\lambda_s) - 2\alpha^2\lambda_s - \alpha\Lambda - 2\alpha\lambda_f < 0 \quad \text{(A167)}$$

which proves the first part of the proposition.

To see the second part of the proposition, note that the critical λ_f^* is implicitly defined by the equation:

$$\zeta(\lambda_f^*) = \alpha(t_s - t_f)^2 - \lambda_s(t_f\alpha^2 + t_f + \lambda_s\alpha) + D1(\lambda_f^*) = 0 \quad \text{(A168)}$$

Using the implicit function theorem, we have:

$$\frac{d\lambda_f^*}{d\alpha} = -\frac{\frac{\partial\zeta}{\partial\alpha}}{\frac{\partial\zeta}{\partial\lambda_f}} \quad \text{(A169)}$$

Since the denominator is negative, we only need to look at $\frac{\partial\zeta}{\partial\alpha}$. For small λ_s this reduces to:

$$\frac{\partial\zeta}{\partial\alpha} = (t_s - t_f)^2 - \lambda_f(\lambda_f + \Lambda) + 2\alpha t_f(\lambda_f + \Lambda) > (t_s - t_f)(t_s - t_f - \lambda_f) + 2\alpha t_f(\lambda_f + \Lambda) > 0 \quad \text{(A170)}$$

where the last two inequalities follow since $t_s - t_f > \lambda_f + \Lambda$.

Similarly, the sign of $\frac{\partial \lambda_f^*}{\partial t_f}$ is the same as the the sign of $\frac{\partial \zeta}{\partial t_f}$. We have:

$$\frac{\partial \zeta}{\partial t_f} = \alpha\left(\alpha(\lambda_f + \Lambda) - 2(t_s - t_f)\right) < 0 \tag{A171}$$

This completes the proof. □

Case when λ_s is large.

In this case, we set the parameters to be $t_s = 2, t_f = 1, \lambda_f = 0, \lambda_s = 1$. Then, we have:

$$\frac{\partial \zeta}{\partial \alpha} = -2\alpha - \Lambda(3\alpha^2 + 2\alpha - 2) \tag{A172}$$

which is positive for small α. However, if $\alpha > \frac{\sqrt{7}-1}{3}$ then $(3\alpha^2 + 2\alpha - 2) > 0$ and individual branding becomes more attractive as α increases.

Case when some snobs move down.

The analysis for the individual branding remains the same as in Proposition 5. However, under umbrella branding now some snobs move to the low-end market. The analysis proceeds as before. For small λ_s and λ_f, we find that:

$$\Pi_1^U = \frac{t_s\left((t_s - \alpha\Lambda)(1-\alpha) + (3\alpha+1)t_f\right)}{2\left(\alpha(t_f - \Lambda) + t_s\right)} \tag{A173}$$

Using the profits under individual branding case derived in Proposition 5, we find that:

$$\Pi_1^U - \Pi_1^I = \frac{-\mathcal{A}_1}{(\alpha(t_f - \Lambda) + t_s)(t_f + t_s\alpha + \alpha\Lambda)} \tag{A174}$$

where:

$$\mathcal{A}_1 = \alpha\left[-\alpha^2(1-\alpha)\Lambda^2(t_s - t_f) + \alpha\Lambda(-2t_s^2\alpha + t_s^2 + \alpha^2 t_f^2 + 8\alpha t_s t_f + t_f^2 - 2t_f^2\alpha + \alpha^2 t_s^2) + \alpha(1-\alpha)(t_s - t_f)^3\right] \tag{A175}$$

Note that the denominator of (A174) is positive. Therefore, umbrella branding is more attractive if $\mathcal{A}_1$ is negative and individual branding is more attractive if $\mathcal{A}_1$ is positive. To see the impact of α we have:

$$\frac{\partial \mathcal{A}_1}{\partial \alpha} = -\alpha(t_s - t_f)(2-3\alpha)\Lambda^2 + (16\alpha t_s t_f + t_f^2 - 4t_s^2\alpha + t_s^2 - 4t_f^2\alpha + 3\alpha^2 t_s^2 + 3\alpha^2 t_f^2)\Lambda + (t_s - t_f)^3(1-2\alpha) \tag{A176}$$

For small α this reduces to:

$$\frac{\partial \mathcal{A}_1}{\partial \alpha} = (t_s - t_f)^3 + (t_s^2 + t_f^2)\Lambda > 0 \tag{A177}$$

which is positive.

Proof of Proposition 6

First, consider the case when both firms use individual branding. Note that in this case, we have two markets each of which consists of both snobs and followers. The utility that a follower in segment 1 at θ_{f1} derives from buying from firm 1 is:

$$U_1^f = r - p_{11} - t\theta_{f1} + \lambda_f \frac{x_s^e}{2} \tag{A178}$$

where p_{11} is firm 1's price in the first market and x_s^e is the market share for firm 1 among snobs in market 1. Note that we divide x_s^e by two since the total number of snobs in the first market is $\frac{1}{2}$. The corresponding utility for the follower from purchasing brand 1 is:

$$U_2^f = r - p_{21} - t(1 - \theta_{f1}) + \lambda_f \frac{(1 - x_s^e)}{2} \tag{A179}$$

The indifferent follower is given by:

$$\theta_{f1} = \frac{2(p_{21} - p_{11}) + 2t + \lambda_f(2x_s^e - 1)}{4t} \tag{A180}$$

Similarly, the indifferent snob in market 1, indexed by θ_{s1}, is given by:

$$\theta_{s1} = \frac{2(p_{21} - p_{11}) + 2t - \lambda_s(2x_f^e - 1)}{4t} \tag{A181}$$

Rational expectations imply that:

$$\theta_{s1} = x_s^e \tag{A182}$$

$$\theta_{f1} = x_f^e \tag{A183}$$

This implies that the demand function is:

$$x_s = \frac{(4t + 2\lambda_f)(p_{21} - p_{11}) + 4t^2 + \lambda_s\lambda_f}{2(4t^2 + \lambda_s\lambda_f)} \tag{A184}$$

$$x_f = \frac{(4t - 2\lambda_s)(p_{21} - p_{11}) + 4t^2 + \lambda_s\lambda_f}{2(4t^2 + \lambda_s\lambda_f)} \tag{A185}$$

Since the two markets are symmetric, the demand function for the other segments are similar. The other segment's demand are denoted by y_s and y_f and their prices by p_{12} and p_{22}. The profit function for firm 1 is therefore:

$$\Pi_1(0,0) = (x_s + x_f)p_{11} + (y_s + y_f)p_{12} \tag{A186}$$

Using the first order conditions we get the equilibrium prices are

$$p_{11}^* = \frac{4t^2 + \lambda_s\lambda_f}{4t + \lambda_f - \lambda_s} \tag{A187}$$

Denote this price as $p^I(\lambda_s, \lambda_f)$. We note that:

$$\frac{\partial p^I(\lambda_s, \lambda_f)}{\partial \lambda_s} = \frac{(2t + \lambda_f)^2}{(4t + \lambda_f - \lambda_s)^2} > 0 \tag{A188}$$

Furthermore:

$$\frac{\partial^2 p^I(\lambda_s, \lambda_f)}{\partial \lambda_s^2} = \frac{2(2t + \lambda_f)^2}{(4t + \lambda_f - \lambda_s)^3} > 0 \tag{A189}$$

Also:

$$\frac{\partial^2 p^I(\lambda_s, \lambda_f)}{\partial \lambda_s \lambda_f} = \frac{2(2t - \lambda_s)(2t + \lambda_f)}{(4t + \lambda_f - \lambda_s)^3} > 0 \tag{A190}$$

Also, note that:

$$\frac{\partial p^I(\lambda_s, \lambda_f)}{\partial \lambda_f} = \frac{-(2t - \lambda_s)^2}{(4t + \lambda_f - \lambda_s)^2} < 0 \tag{A191}$$

The equilibrium profits are:

$$\Pi_1^*(0,0) = \frac{4t^2 + \lambda_s\lambda_f}{4t + \lambda_f - \lambda_s} \tag{A192}$$

Now, let us consider the case when both firms use umbrella branding. The utility that a follower in segment 1 at θ_{f1} derives from buying from firm 1 is:

$$U_1^f = r - p_{11} - t\theta_{f1} + \lambda_f\left(\frac{x_s^e + y_s^e}{2}\right) \tag{A193}$$

where p_{11} is firm 1's price in the first market and x_s^e is the market share for firm 1 among snobs in market 1 and y_s^e is the market share for firm 1 among snobs in market 2. The corresponding utility for the follower from purchasing brand 1 is:

$$U_2^f = r - p_{21} - t(1 - \theta_{f1}) + \lambda_f\frac{(2 - x_s^e - ys^e)}{2} \tag{A194}$$

The indifferent follower is given by:

$$\theta_{f1} = \frac{(p_{21} - p_{11}) + t + \lambda_f(x_s^e + y_s^e)}{2t} \tag{A195}$$

Similarly, the indifferent snob in market 1 is indexed by θ_{s1} and is given by:

$$\theta_{s1} = \frac{(p_{21} - p_{11}) + t - \lambda_s(x_f^e + y_f^e)}{2t} \tag{A196}$$

For segment 2 we have analogously:

$$\theta_{f2} = \frac{(p_{22} - p_{12}) + t + \lambda_f(x_s^e + y_s^e)}{2t} \tag{A197}$$

$$\theta_{s2} = \frac{(p_{22} - p_{12}) + t - \lambda_s(x_f^e + y_f^e)}{2t} \tag{A198}$$

Rational expectations imply that:

$$\theta_{s1} = x_s^e \tag{A199}$$

$$\theta_{f1} = x_f^e \tag{A200}$$

$$\theta_{s2} = y_s^e \tag{A201}$$

$$\theta_{f2} = y_f^e \tag{A202}$$

This implies that the demand function is:

$$x_s = \frac{(2t^2 + \lambda_s\lambda_f - t\lambda_s)(p_{21} - p_{11}) + \lambda_s(t + \lambda_f)(p_{12} - p_{22}) + 2t^3 + 2\lambda_s\lambda_f t}{4t(t^2 + \lambda_s\lambda_f)} \tag{A203}$$

$$x_f = \frac{(2t^2 + \lambda_s\lambda_f + t\lambda_f)(p_{21} - p_{11}) + \lambda_f(t - \lambda_f)(p_{12} - p_{22}) + 2t^3 + 2\lambda_s\lambda_f t}{4t(t^2 + \lambda_s\lambda_f)} \tag{A204}$$

Since the two markets are symmetric, the demand function for the other segments are similar; denote the corresponding demands by y_s and y_f and the prices by p_{12} and p_{22}. The profit function for firm 1 is therefore:

$$\Pi_1(1,1) = (x_s + x_f)p_{11} + (y_s + y_f)p_{12} \tag{A205}$$

Using the first-order conditions, we find that the equilibrium price is:

$$p_{11}^* = \frac{2(t^2 + \lambda_s\lambda_f)}{2t + \lambda_f - \lambda_s} \tag{A206}$$

Denote the price under umbrella branding by $p^U(\lambda_s, \lambda_f)$. This is given by(A206). Comparing and (A187), we see that:

$$p^I(\lambda_s, \lambda_f) = p^U\left(\frac{\lambda_s}{2}, \frac{\lambda_f}{2}\right) \tag{A207}$$

This implies that in equilibrium the profitability of using umbrella branding versus individual branding can be expressed as:

$$\Delta = p^U(\lambda_s, \lambda_f) - p^U\left(\frac{\lambda_s}{2}, \frac{\lambda_f}{2}\right) \tag{A208}$$

This can be equivalently expressed as:

$$\Delta = \int_{\frac{\lambda_f}{2}}^{\lambda_f} \frac{\partial p^U(\lambda_s, y)}{\partial y} dy + \int_{\frac{\lambda_s}{2}}^{\lambda_s} \frac{\partial p^U(x, \frac{\lambda_f}{2})}{\partial x} dx \tag{A209}$$

Note that:

$$\frac{\partial \Delta}{\partial \lambda_s} = \int_{\frac{\lambda_f}{2}}^{\lambda_f} \frac{\partial^2 p^U(\lambda_s, y)}{\partial \lambda_s \partial y} dy + \left[\frac{\partial}{\partial \lambda_s} p^U\left(\lambda_s, \frac{\lambda_f}{2}\right) - \frac{1}{2} \cdot \frac{\partial}{\partial \lambda_s} p^U\left(\frac{\lambda_s}{2}, \frac{\lambda_f}{2}\right)\right] \tag{A210}$$

Since equilibrium prices are increasing and convex in λ_s, the second term in brackets in (A210) is positive. Furthermore, the cross derivatives are also positive, which implies that the first term is positive. This implies that umbrella branding becomes more attractive as λ_s increases. To see that a critical λ_s^* exists, we can solve directly for Δ which is:

$$\Delta = \frac{(2t^2 - \lambda_s\lambda_f)(\lambda_s - \lambda_f) + 6\lambda_s\lambda_f t}{(2t + \lambda_f - \lambda_s)(4t + \lambda_f - \lambda_s)} \tag{A211}$$

If $\lambda_s \to 0$ then $\Delta < 0$ and if $\lambda_s \to 1$ then $\Delta > 0$, which completes the proof. □

Proof of Proposition 7

First, consider the case when the firms are extending the brand names to the follower-market. In this case, $\delta > 0$. If a consumer located at θ in the follower market buys the product from firm 1, then her (indirect) utility is given by:

$$U_f^1 = r_{1f} - t_f\theta - p_{1L} \tag{A212}$$

where:

$$r_{1f} = \begin{cases} r_f + \delta & \text{if firm 1 uses umbrella branding.} \\ 0 & \text{otherwise.} \end{cases} \tag{A213}$$

Similarly, we can define the utility from buying firm 2's product. The indifferent consumer θ_f is given by:

$$\theta_f = \frac{r_{1f} - r_{2f} + p_{2L} - p_{1L} + t_f}{2t_f} \tag{A214}$$

The profits in the follower segment for firm 1 is therefore:

$$\Pi_{1f} = p_{1L}F(\theta_f) \tag{A215}$$

Define $\Delta_1 = r_{1f} - r_{2f}$. Before considering the impact of branding strategy on profits in this case, note that the profits from the snob segment do not depend on the branding strategy. Therefore, we can focus on the profits from the follower segment. Note that the impact of the branding strategy in this case is captured in the term Δ_1.

The first-order conditions for the equilibrium prices are:

$$0 = \frac{-p_{1L}f(\theta_f)}{2t_f} + F(\theta_f) \tag{A216}$$

$$0 = \frac{-p_{2L}f(\theta_f)}{2t_f} + 1 - F(\theta_f) \tag{A217}$$

The first-order condition implies that:

$$\Pi_{1f}^* = \frac{2t_f F^2(\theta_f^*)}{f(\theta_f^*)} \tag{A218}$$

which is increasing in θ_f^* because of the log-concavity of $F(\cdot)$. Therefore, it is sufficient to show that θ_f^* is increasing in Δ_1. Using (A216), (A217) and (A214), we obtain the equation which defines θ_f^*. Define:

$$\Omega(x) \equiv \frac{1 - 2F(x)}{f(x)} + \left[\frac{1}{2} + \frac{\Delta_1}{2t_f}\right] - x \tag{A219}$$

Using log-concavity, $\Omega'(x) < 0$. Also, θ_f^* is defined by $\Omega(\theta_f^*) = 0$. Note that $\Omega(0) > 0$ and for interior solution, we must have the case that $\Omega(1) > 0$. Therefore, the sign of $\frac{\partial \theta_f^*}{\partial \Delta_1}$ is the same as the sign of $\frac{\partial \Omega}{\partial \Delta_1}$, which is positive. Consequently, as Δ_1 increases, the firm's profit increases. Therefore, when firms move from the snob market to the follower market, they both have incentives to use umbrella branding. Using similar arguments, we can show that when firms move from the follower market to the snob market, they will use individual branding. □