<u>Online Appendix for "The Demand Effects of Joint Product Advertising in Online Videos"</u>

Appendix A: Validating Integrity of Randomization

If videos are randomly assigned, we expect that: (1) the allocation of video in the full sample is proportionately transmitted to the subcategories of products (tops, dresses, bottoms, and auxiliary products) and (2) the mean characteristics of the treated and untreated products are statistically similar.

Table A1: Proportional transmission of treatment in product subcategories

Analysis	No. of treated products	Product subcategory	Percentage of treated products	95% CI for percentage treated products
Focal product analysis [297 Principal Products]	Yes -58 (19.5%)	Top (194)	40/194 (20.6%)	[13.9% - 25.1%]
		Bottom (15)	2/15 (13.3%)	[(-0.5%) - 39.6%]
	No - 239	Dress (88)	16/88 (18.2%)	[11.2% - 27.8%]
Coordinating product analysis [196 Products]	Yes - 62 (31.6%)	Top (43)	12/43 (27.9%)	[17.73% - 45.53%]
		Bottom (13)	6/13 (46.2%)	[6.35% - 56.91%]
	No - 134	Dress (20)	4/20 (20%)	[11.25% - 52.01%]
		Aux. Prod. (120)	40/120 (33.3%)	[23.31% - 39.95%]

If *p* denotes the proportion of video treated products in the full sample and *n* denotes the sample size of a subcategory, then the 95 percent CI for percentage of treated products in a subcategory = p +/- 1.96 x SQRT [p x (1-p)/n].[1] For example, for tops with n=194 and p=0.195, the 95% CI is [13.9% - 25.1%]. Table A1 reports the computed 95 percent CI for all product subcategories in the focal and coordinating product analyses, which shows that the observed percentage of treated products for each subcategory falls within its corresponding 95 percent CI indicating proportional transmission of video treatment from the full sample to different subcategories of products

Table A2: Proportional transmission of treatment for products with/without coordinating products

297 Principal Products	No. of products having coordinating products	No. of treated products	Percentage of products with/without coordinating products	95% CI for percentage of products with /without coordinating products
	Yes - 264/297 (88.9%)	Yes - 58/297	54/58 (93.1%)	[80.8% - 96.9%]
	No - 33/297	No - 239/297	210/239 (87.9%)	[84.8% - 92.9%]

[1] For using normal approximation of the error term of binomial distribution, we checked that n p>5 and n (1-p)>5 in each case.

Similarly, in Table A2, we found that the observed percentages of treated and control products having coordinating products are within the computed 95 percent CI, which shows the proportional transmission of treatment in products with and without coordinating products. Next, we found that the mean prices for the treated and control products are statistically similar for the sample of products in the focal and coordinating product analyses. Table A3 reports these results.

Table A3: Differences in prices of treated and control products

Analysis	Product type	Number	Mean	St. Dev.	Min	Max	*t-value* (Critical *t-value*)*
Focal product	Treated	58	20.39	4.74	7.9	34.9	-0.45 (12.70)
	Control	239	20.71	5.25	12.9	39.9	
Coordinating product	Treated	62	12.34	9.38	6	39.9	-0.02 (12.70)
	Control	134	12.37	7.84	6	29.9	

* The *t-values* are computed by Welch's t-test for unpaired groups with unequal sample sizes and variances. The critical *t-value* is read from the t-distribution table corresponding to degrees of freedom and two sided 95 % confidence interval.

Appendix B: Testing Ignorability of Treatment Assumption

The ignorability of treatment assumption requires that the treatment is not systematically assigned. Although random assignment of product videos in our case should satisfy this condition, nonetheless, we further check that the sales of treated and control products are statistically similar without videos. We consider three groups of treatment focal products: 1st treatment group →19 products for which videos were introduced between week 6-week 16; 2nd treatment group → 32 products for which videos were introduced between week 11-week 21; and 3rd treatment group →7 products for which videos were introduced between week 19-week 27. The control products in a week are those products that do not have their videos switched on in that week. For example, in week 10, all products in 1st treatment group are treatment products and all products that are left out of video assignment as well as the 2nd and 3rd treatment group products, as their videos are not switched on in this duration, are the control products. On similar lines, we consider three groups of treatment products in the coordinating product analysis. For example, 1st treatment group of coordinating products are the ones associated with the 1st treatment group of focal products. Table B1 reports the summary statistics for the three groups of treatment products in pre-video, video switch-on, and switch-off periods with their corresponding control products.

Table B1: Treatment group-wise summary statistics

Weekly sales in numbers	Focal Product				Coordinating Product			
	No. of product-weeks	Mean	Std. Dev.	*t-value* (critical *t-value*)*	No. of product-weeks	Mean	Std. Dev.	*t-value* (critical *t-value*)
Pre-video period (Wk 1- Wk 5)								
1st Treat Gr.	75	133.8	162.3	0.95 (1.99)	140	93.92	209.72	1.37 (1.98)
Control Gr.	704	115.64	103.28		681	69.23	76.93	
Video Switch-on period (Wk 6- Wk 16)								
1st Treat Gr.	250	159.84	256.93	2.41 (1.97)	320	119.11	266.57	3.9 (1.96)
Control Gr.	2439	120.14	127.8		1564	60.24	97.83	
Video Switch-off period (Wk 17- Wk 28)								
1st Treat Gr.	134	111.04	230.84	0.69 (1.97)	210	64.11	157.17	1.13 (1.97)
Control Gr.	2890	97.27	94.67		1393	51.68	66.79	
Pre-video period (Wk 1- Wk 10)								
2nd Treat Gr.	243	119.9	119.93	-0.15 (1.96)	208	69.10	79.19	-0.89 (1.96)
Control Gr.	2177	121.16	127.16		1604	74.57	109.60	
Video Switch-on period (Wk 11- Wk 21)								
2nd Treat Gr.	316	111.92	119.05	2.42 (1.96)	329	77.64	66.20	2.28 (1.96)
Control Gr.	2105	94.67	116.81		1158	66.90	101.35	
Video Switch-off period (Wk 22- Wk 28)								
2nd Treat Gr.	200	100.36	101.47	1.43 (1.97)	125	41.27	38.03	-0.35 (1.96)
Control Gr.	1463	89.55	93.44		769	42.82	79.56	
Pre-video period (Wk 1- Wk 18)								
3rd Treat Gr.	93	126.77	150.92	0.7 (1.99)	55	56.37	50.24	-2.2 (1.99)
Control Gr.	3767	115.7	123.7		2575	71.94	104.93	
Video Switch-on period (Wk 19- Wk 27)								
3rd Treat Gr.	74	123.53	96.69	3.11 (1.99)	138	58.09	23.41	0.73 (1.96)
Control Gr.	2088	87.84	102.51		1146	55.68	88.59	
Video Switch-off period (Wk 28)								
3rd Treat Gr.	7	69.14	50.01	0.52 (2.44)	3	36.33	9.29	-0.5 (2.57)
Control Gr.	233	59.21	47.28		132	39.74	47.57	

* The *t-values* are computed by Welch's t-test for unpaired groups with unequal sample sizes and variances. The critical *t-value* is read from the t-distribution table corresponding to the degrees of freedom and two sided 95 % confidence interval.

In Table B1, we also report the *t-value* and the critical *t-value* for the difference in means of treatment and control products in each period. A *t-value* higher than critical *t-value* for a pair of treatment and control products indicates statistically different mean weekly sales for the two. For both focal and coordinating products, we mostly find statistically similar mean weekly sales for the treatment and control products in pre-video and video switch-off periods indicating the similarity of the two groups in absence of video treatment. But we find a significantly higher sales for the treatment products during their video switch-on period as compared to the control products.

We examined the differential trends in weekly sales for the three groups of treatment products during the period when their videos are switched-on from the average seasonality for control products with the following specification

$$Sales_{it} = \alpha_i + \alpha_t + Treat1 \times \alpha_t + Treat2 \times \alpha_t + Treat3 \times \alpha_t + \boldsymbol{\beta} \times \boldsymbol{X_{it}} + \varepsilon_{it}, \qquad \text{------ (B1)}$$

where, *Treat1*, *Treat2*, and *Treat3* are, respectively, the indicator variables for the 1st, 2nd, and 3rd treatment groups. All other terms have the same meaning as in our specification (1). The 28 weekly coefficients for $Treat1 \times \alpha_t$, $Treat2 \times \alpha_t$, and $Treat3 \times \alpha_t$ capture the deviations of weekly sales of the 1st, 2nd, and 3rd treatment group products from the average seasonality in weekly sales (captured by week fixed effect α_t), respectively.

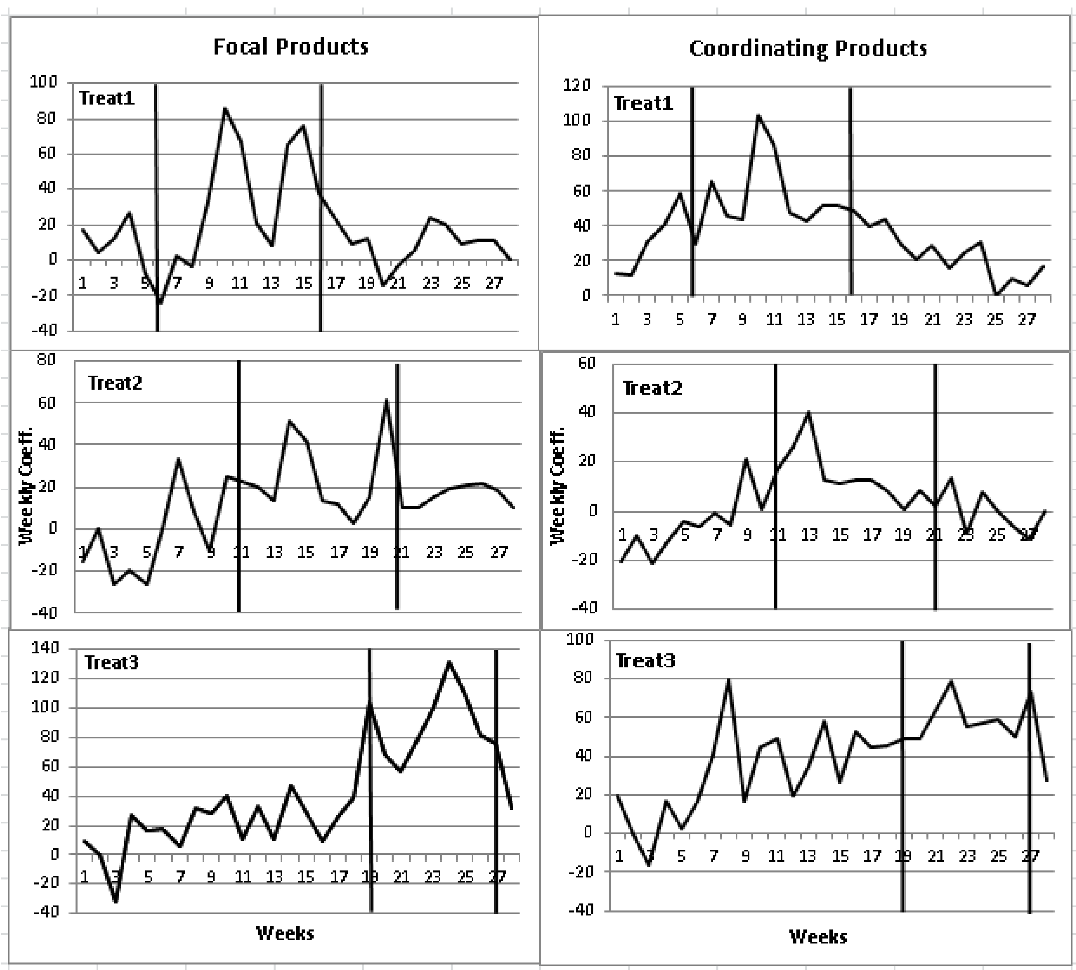

Figure B: Weekly sales trends for different groups of treatment products

We plot the weekly coefficients for the three treatment groups for focal products in the left pane of Figure B. Similarly, we estimated the weekly coefficients for the three treatment groups for coordinating products and plot them in the right pane of Figure B. In Figure B, we use two vertical lines to demarcate the period when the focal products' videos were switched on. For all treatment groups for both focal and coordinating products, we find a higher average weekly coefficient values in the video treatment period as compared to the period without treatment. We also find that the values of weekly coefficients decrease after the videos are switched off for all treatment groups.

Table B2: Estimates with differential sales trends for the treated and control products

Dependent Variable (Weekly sales in numbers)	Coefficient Estimates (Robust Cluster Corrected Std. Errors)			
	Focal products analysis		Coordinating product analysis	
Product video	14.01^{**} (6.42)	11.04^{**} (5.01)	10.44^{**} (5.21)	7.45^{**} (3.70)
Treat 1. Linear time trends	1.52 (1.07)	3.56 (4.39)	-1.54 (1.09)	2.21 (2.55)
Treat 2. Linear time trends	1.13 (0.72)	2.30 (4.65)	0.25 (0.46)	2.75 (2.54)
Treat 3. Linear time trends	2.82^{*} (1.66)	16.59 (12.02)	1.31^{*} (0.78)	5.49^{**} (2.45)
Treat 1. Square time trends		-0.07 (0.15)		-0.13 (0.11)
Treat 2. Square time trends		-0.04 (0.14)		-0.08 (0.08)
Treat 3. Square time trends		-0.39 (0.34)		-0.13 (0.09)
No. of product-weeks (No. of products)	6828 (297)	6828 (297)	4708 (196)	4708 (196)

***, **, * = statistically significant at α = 0.01, α = 0.05, and α = 0.10 levels (two-sided test), respectively. The standard errors cluster corrected at the product level.

Next, we checked that the estimated treatment effect in our difference-in-difference specification is not due to pre-existing differential sales trends in the treatment and control products with the following specifications (B2) for the focal product analysis.

$$Sales_{it} = \alpha_i + \alpha_t + \delta_1 \, Vidwk_{it} + \delta_2 \, Treat1 \times t + \delta_3 \, Treat2 \times t + \delta_4 \, Treat3 \times t + \delta_5 \, Treat1 \times t^2 + \delta_6 \, Treat2 \times t^2 + \delta_7 \, Treat3 \times t^2 + \boldsymbol{\beta} \boldsymbol{X_{it}} + \varepsilon_{it}, \qquad \text{----- (B2)}$$

where, t= 1,2,3, …. 28 and α_t denote weeks the time fixed-effects, respectively. In this specification, coefficients δ_2, δ_3, and δ_4 account for the differential linear time trends and δ_5, δ_6, and δ_7 capture the differential quadratic time trends for the three groups of treated products, respectively. Thus coefficient δ_1 in this specification estimates the net gain in sales of the treated products due to the introduction of videos after controlling for the differential time trends in sales for the treated and control products. We ran similar specification for the coordinating product analysis and report the coefficient estimates for the two analyses in Table B2. We found qualitatively similar treatment effect coefficients indicating the robustness of our results.

Appendix C: Effect of Stocked-out Products

If the random assignment of videos in the full sample is proportionately transmitted to the products that are stocked out and products that are not stocked out, it indicates that the probability of stock out is not systematically different for the treated and control products. Therefore, we performed a similar integrity check on randomization as in Appendix A, and found that the observed proportions of stocked-out treated and control products fall within their computed 95 percent CI indicating equal probability of stock out for the treated and control products.

Table C1: Stock-out probabilities

Analysis	No. of stocked-out products	No. of treated products	Percentage of stocked-out products	95 percent CI for percentage of stocked-out products
Focal products analysis	Yes 53/297 (17.9%)	Yes 58/297	11/58 (18.9%)	[7.9% - 27.7%]
	No 244/297	No 239/297	42/239 (17.6%)	[13.0% - 22.7%]
Coordinating products analysis	Yes 25/196 (12.8%)	Yes 62/196	5/62 (8.1%)	[4.4% - 21.1%]
	No 171/196	No 134/196	20/134 (14.9%)	[7.1% - 18.4%]

We further conducted focal product analysis for only those products that were not stocked-out, and found qualitatively similar treatment effect estimates as shown in Table C2. This further reassures us that our results are not due to systematic differences in attrition rates of treated and control products.

Table C2: Estimates for only stocked-out products

Dependent Variable (Weekly sales in numbers)	Coefficient Estimates (Robust Cluster Corrected Std. Errors)	
	Focal products analysis for full sample of products	Focal product analysis for products that are not stocked-out
Product video	16.70** (8.02)	12.27** (6.05)
Catalogue	103.80** (48.95)	106.44** (51.08)
Website home page	61.08*** (15.53)	55.19*** (17.06)
Category front page	29.44 (20.90)	18.51 (20.31)
Price markdown	76.62*** (8.75)	83.43*** (11.03)
Email promotion	68.91* (41.75)	70.06 (45.59)
No. of product-weeks (No. of products)	6828 (297)	5944 (244)
R Square	0.56	0.48

***, **, * = statistically significant at $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.10$ levels (two-sided test), respectively.
The standard errors are cluster corrected at the product level.

Appendix D: Effect of Promotional Control Variables

The retailer may systematically promote products with video to derive maximum benefit, which would make the promotion related control variables endogenous in our specifications. We ran specifications (D1) to check whether the retailer offered statistically different promotions to the treated products during their video switch-on period

$$Prom_{it} = \alpha_i + \alpha_t + \beta\, Vidwk_{it} + \varepsilon_{it}, \qquad \text{------- (D1)}$$

where, $Prom_{it}$ denotes the number of promotions offered on product i in week t. We used the number of non-price and price promotions separately in specification (D1). The number of non-price promotions for product i in week t is the sum of indicator variables $Homepgwk_{it}$, $Catpgwk_{it}$, $Catwk_{it}$, and $Emailwk_{it}$ as described in specification (1). The number of price promotion is one if the product i gets a price markdown in week t, and 0 otherwise. All other variables have the same meaning as in specification (1). A positive and significant estimate of β would indicate a higher number of promotions for the treated products in the video switch-on period. We found insignificant estimates of β for both non-price promotions [-0.028 (Std. Err. 0.022) and 0.002 (Std. Err. 0.02)] and price promotions [0.029 (Std. Err. 0.026) and -0.007 (Std. Err. 0.01)] for the focal and coordinating products, respectively. Thus we fail to find evidence for statistically different promotions for the treated products during their video switch-on period.

To further show that our estimated treatment effect is not merely due to the endogenous promotion variables, we estimated the treatment effect on a sample of only those treated and control products that were not exposed to any promotions. But before that, we verify the random assignment of videos in the sample of products that received no promotions. The results in Table D1 show that the proportion of video allocation in the full sample of non-promoted products and in the various subcategories of products in this sample are statistically similar.

Table D1: Randomization check

Analysis	Video assignment	Product subcategory	Percentage of treated products	95% CI for percentage of treated products
Focal product analysis [152 Principal Products]	Yes -22 (14.5%)	Top (99)	13/99 (13.1%)	[7.5% - 21.4%]
		Bottom (10)	1/10 (10.0%)	[(-7.3%) - 36.3%]
	No - 130	Dress (43)	8/43 (18.6%)	[3.9% - 24.9%]
Coordinating product analysis [66 Products]	Yes - 21 (31.8%)	Top (21)	3/21 (14.3%)	[11.9% - 51.7%]
		Bottom (11)	5/11 (45.5%)	[4.3% - 59.3%]
	No - 45	Dress (8)	0/8 (0%)	[-0.4% - 64.1%]
		Aux. Prod. (26)	13/26 (50%)	[13.9% - 49.7%]

We then ran specifications (1) and (2) on the sample of products that received no promotions and report the coefficient estimates in Table D2. Table D2 shows qualitatively similar video coefficients (magnitude and statistical significance) for the sample of products that received no promotions and the full sample of products.

Table D2: Parameter estimates for products that received no promotion

Dependent Variable (Weekly sales in numbers)	Coefficient Estimates (Robust Cluster Corrected Std. Errors)			
	Focal product analysis		Coordinating product analysis	
	All products	Products without promotion	All products	Products without promotion
Product video	16.70^{**} (8.02)	18.54^{**} (8.34)	12.34^{**} (5.61)	13.12^{**} (6.47)
No. of Product-weeks (No. of products)	6828 (297)	3480 (152)	4708 (196)	1498 (66)

$^{***}, ^{**}, ^{*}$ = statistically significant at $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.10$ levels (two-sided test), respectively.

Next, in Table D3, we compared the treatment effect estimates from specifications (1) and (2) with and without the promotional control variables. We find that in all cases, the point estimates of the treatment coefficient are similar (in magnitude as well as significance), which assures us that our treatment estimates are robust to the endogeneity of promotional control variables.

Table D3: Treatment effect of product videos with and without control variable

Dependent Variable (Weekly sales in numbers)		Product Video Coefficient Estimates (Robust Cluster Corrected Std. Errors)		
		(A)	(B)	(C)
Focal product analysis	With control variables	16.70** (8.02)	23.55** (9.81)	17.47 (10.11)
	Without control variables	16.82** (8.39)	22.54** (10.24)	18.82 (13.48)
Coordinating product analysis	With control variables	12.34** (5.61)	10.41** (4.92)	11.58* (6.12)
	Without control variables	11.48** (5.25)	9.54** (4.46)	11.97* (6.89)

***, **, * = statistically significant at α = 0.01, α = 0.05, and α = 0.10 levels (two-sided test), respectively. Columns A, B, and C, respectively, display the coefficient estimates for the full dataset, pre-treatment and switch-on-period data, and treatment switch-on and switch-off-period data only. The standard errors are cluster corrected at the product level

Appendix E: Substitution Effect of Product Video

The customers may simply shift their demands of control products to the treated products after watching their videos. If this is the case, then introduction of product videos does not result in an overall increase in product demands and hence not beneficial for the retailer. We ran the following interrupted time series specification (E1) on 239 control products in our focal product analysis to check for this possibility

$$Sales_{it} = \alpha_i + \alpha_t + \delta_1 \, t + \delta_2 \, Wkvid_t + \delta_3 \, t \times Wkvid_t + \boldsymbol{\beta} \, \boldsymbol{X_{it}} + \varepsilon_{it} \quad , \qquad \text{-- (E1)}$$

where, t = {1, 2, 3, …28} denotes the weeks; *Wkvid* is an indicator variable equal to one during week 6-week 21 when the majority of videos are introduced on the retailer's website, and zero otherwise; and the remaining terms have same meaning as in specification (1). In this specification, δ_1 captures the average sales trend over the entire period, δ_2 captures the change in level of sales during the video switch-on weeks, and δ_3 captures the incremental change in sales trend (slope) during the video switch-on weeks. We additionally control for the seasonality in product demands through time fixed-effects.[2]

[2] Besides one week indicator variable dropped out of the total 28 weeks, three additional weeks' fixed-effect indicators in specifications (E1) are dropped due to multicollinearity → two between week 6 – week 21 due to multicollinearity with variable *Wkvid* and *Wkvid* × *t*, and one in the remaining weeks for multicollinearity with variable *t*.

Table E1: Evidence of no substitution effect of product video

Dependent Variable (Weekly sales in numbers)	Coefficient Estimates (Robust Cluster Corrected Std. Errors)			
	Focal product analysis		Coordinating product analysis	
Weeks with video (*Wkvid*)	73.82^{***} (11.78)	65.39^{***} (11.09)	86.99^{***} (9.41)	52.84^{***} (10.01)
Linear time trend (t)		-3.37^{***} (0.31)		-3.41^{***} (0.42)
$Wkvid \times t$		-1.77^{***} (0.51)		-2.27^{***} (0.51)
No. of product-weeks (No. of products)	5436 (239)	5436 (239)	3180 (134)	3180 (134)

***, **, * = statistically significant at α = 0.01, α = 0.05, and α = 0.10 levels (two-sided test), respectively. Standard errors are cluster corrected at the product level

In Table E1, we find a significant and positive value for δ_2, which indicates a higher level of sales of control products during the period when videos for the majority of treated products were switched on. A negative and significant estimate for δ_1 indicates that the control products have an average declining sales trend. A negative and significant estimate for δ_3 indicates a higher rate of sales decline during the video switch-on weeks. But we find that the higher gain in sales level due to δ_2 more than compensates for the loss due to a higher declining rate of sales during video switch-on period. We observe similar results in the coordinating product analysis. Thus, the estimated increase in sales of treated products are not at the expense of loss in sales of control products.

Appendix F: Computation of Omitted Variable Bias

Any time varying unobserved shock to the demand of one product can simultaneously affect the demand of its complementary product, thereby making the sum of sales of associated focal products endogenous in specification (3). This could lead to biased estimate of coefficient δ_2, and possibly biased estimates for other coefficients in specification (3). In the following, we derive the expression for omitted variable bias for coefficients of specification (3) and show that as long as videos are randomly assigned, the coefficient associated with video variables remains unbiased.

For simplicity, we use the following specification for specification (3)

$$S_c = \delta_1 V + \delta_2 S_f + \delta_3 V S_f + \varepsilon, \qquad \text{------------------ (F1)}$$

where, S_c and S_f, respectively, denote the coordinating and focal products sales, and V denotes the video variable. Let's say an omitted variable Z in the error term of (F1) is correlated with S_f. So the full specification that would give unbiased coefficient estimates is

$$S_c = \delta_1 V + \delta_2 S_f + \delta_3 V S_f + \delta_z Z + \mathrm{u} \qquad \text{-------------------- (F2)}$$

From the partialling out explanation of multiple regression, the biased coefficient of endogenous variable S_f in specification (F1) can be obtained as

$$\widehat{\delta_2} = \frac{Cov\,(S_c,\ \gamma)}{Var\,(\gamma)}, \qquad \text{------------------- (F3)}$$

where, γ is the OLS error from regressing S_f on V and VS_f. Substituting for S_c in (F3) from (F2), we get

$$\widehat{\delta_2} = \frac{Cov\,(\delta_1 V + \delta_2 S_f + \delta_3 V S_f + \delta_z Z + \mathrm{u},\ \gamma)}{Var\,(\gamma)}$$

$$\widehat{\delta_2} = \delta_1 \frac{Cov\,(V,\ \gamma)}{Var\,(\gamma)} + \delta_2 \frac{Cov\,(S_f,\ \gamma)}{Var\,(\gamma)} + \delta_3 \frac{Cov\,(V S_f,\ \gamma)}{Var\,(\gamma)} + \delta_z \frac{Cov\,(\mathrm{Z},\ \gamma)}{Var\,(\gamma)} + \frac{Cov\,(\mathrm{u},\ \gamma)}{Var\,(\gamma)}$$

The OLS error γ (satisfying OLS assumptions) is uncorrelated with variables V and VS_f, i.e., Cov (V, γ) = Cov (VS_f, γ) = 0. Also it can be easily shown that Cov (S_f, γ) = Var (γ). The error u from full specification (F2) is uncorrelated to the endogenous variable S_f and thus uncorrelated to γ, i.e. Cov (u, γ) =0. Substituting these values, the above equation simplifies to

$$\widehat{\delta_2} = \delta_2 + \delta_z \frac{Cov\,(\mathrm{Z},\ \gamma)}{Var\,(\gamma)} \qquad \text{------------------ (F4)}$$

Bias in δ_2 – The omitted variable Z is correlated to S_f and thus it is correlated with γ, error from regressing S_f on variables V and VS_f. Therefore, $Cov\,(\mathrm{Z}, \gamma) \neq 0$ and hence $\widehat{\delta_2}$, coefficient of S_f estimated from specification (F1), will be biased i.e. different from its unbiased value δ_2.

Bias in δ_1 – For estimating bias in δ_1, γ is the error from regressing V on variables S_f and VS_f. Due to random video assignment, the omitted variable Z is uncorrelated with V and thus it is uncorrelated with γ, i.e. Cov (Z, γ) =0. Therefore, $\widehat{\delta_1}$ – coefficient of V estimated from specification (F1) – will be equal to its unbiased value δ_1.

Bias in δ_3 – For estimating bias in δ_3, equation (F4) can be written as

$$\widehat{\delta_3} = \delta_3 + \delta_z \frac{Cov\,(Z\,,\ \gamma\,)}{Var\,(\gamma)},$$

where, $VS_f = \pi_1 V + \pi_2 S_f + \gamma$. Substituting the value of γ in the covariance term we get

$$\widehat{\delta_3} = \delta_3 + \delta_z \frac{Cov\,(Z,\ VS_f - \pi_1 V - \pi_2 S_f)}{Var\,(\gamma)}$$

$$\widehat{\delta_3} = \delta_3 + \frac{\delta_z}{Var\,(\gamma)}[Cov\,(Z, VS_f) - \pi_1\, Cov\,(Z, V) - \pi_2\, Cov\,(Z, S_f)] \quad \text{------------- (F5)}$$

Due to random video assignment, *Cov* (*V*, *Z*) = *Cov* (*V*, S_f) = *Cov* (*V*, ZS_f) = 0. Moreover, *Cov* (*V*, ZS_f) = E(VZS_f) – E(V) E(ZS_f) =0 ➔ E(VZS_f) = E(V) E(ZS_f). Therefore, the first covariance term in brackets in (F5) can be simplified as

$$Cov\,(Z, VS_f) = E(ZVS_f) - E(Z)E(VS_f) = E(V)E(ZS_f) - E(Z)E(V)E(S_f) = E(V)Cov\,(Z, S_f)$$

The coefficient π_2 can be obtained by regressing VS_f on S_f, i.e.

$$\pi_2 = \frac{Cov\,(VS_f,\ S_f)}{Var\,(\,S_f)} = \frac{E(S_f{}^2 V) - E(VS_f)E(S_f)}{Var\,(\,S_f)} = \frac{E(V)[E(S_f{}^2) - E(S_f)E(S_f)]}{Var\,(\,S_f)} = E(V)$$

Therefore, the first and third term inside the brackets in equation (F5) cancels out and the second term is equal to zero making the coefficient $\widehat{\delta_3}$ – estimated from specification (F1) – equal to its unbiased value δ_3. The intuition behind this result is that since *V* is randomly assigned, the correlation between VS_f and *Z* comes entirely from the correlation between S_f and *Z*. Once the correlation between *Z* and S_f is absorbed by the main S_f term, the interaction term VS_f becomes exogenous in specification (F1).

Simultaneity issue – The systems of equations in specification (3) also suffers from simultaneity issue because the dependent variable in one equation is part of an independent variable in the second equation, and the dependent variable in the second is part of an independent variable in the first. For instance, suppose a pair of pants as a coordinating product has three associated focal products (a top, a sandal, and a scarf), but the top, in turn, appears as a coordinating product for three focal products (the same pair of pants, a purse, and an earring). Let $S_{c(pant)}$, $S_{c(top)}$, $S_{f(pant)}$,and $S_{f(top)}$, respectively, denote the sales of the coordinating product pair of pants, coordinating product top, associated focal products including the pair of pants, and associated focal products including the top. Then specification (F1) for sales of the coordinating products, pair of pants and top, can be written as

$$S_{c(pant)} = \delta_1 V + \delta_2 S_{f(top)} + \delta_3 V S_{f(top)} + \varepsilon_1 \quad \text{-------------------- (F6)}$$

$$S_{c(top)} = \delta_1 V + \delta_2 S_{f(pant)} + \delta_3 V S_{f(pant)} + \varepsilon_2 \quad \text{--------------------- (F7)}$$

Suppose $S_{f(top)} = a \times S_{c(top)}$ and $S_{f(pant)} = b \times S_{c(pant)}$, where a and b are constants. Solving for $S_{f(top)}$, it can be shown that $S_{f(top)}$ is a linear function of ε_1 and hence endogenous in equation (F6). Similarly, $S_{f(pant)}$ is a linear function of ε_2 and therefore endogenous in (F7). Thus, simultaneity can be viewed as endogeneity due to omitted variable in error term that is correlated with the associated focal products sales. Therefore, as per the above exposition on omitted variable bias, such endogeneity of sales of associated focal products (S_f) due to simultaneity will lead to bias in coefficient (δ_2) but it would not bias our coefficients of interest (δ_1 and δ_3) as long as the videos are assigned randomly.

Endogenous promotion variables – The above exposition on omitted variable bias suggests that estimating our specifications with endogenous promotional variables would lead to biased coefficients for the promotional variables but unbiased coefficients for the video variable and its interaction with the promotional variables. For this reason, we find qualitatively similar estimates for coefficients of video variables in specifications (1), (2), & (3) with and without inclusion of the endogenous promotional variables. Similarly, the estimated coefficients for video variable and its interaction with endogenous promotional variables in specifications (4) and (5) would be unbiased.