

# **Online Appendix to: Brand Performance Volatility from Marketing Spending**

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The Appendix includes several additional proofs and tests to augment the main results of the paper. These include an illustrative example of financial cost of spending volatility in Section A1, proofs of propositions of brand performance volatility effects in Section A2, proofs of brand performance volatility effects under rational firm behavior in Section A3, proofs of theorems on optimal expenditure behavior in Section A4, and proofs for dynamic model extensions in Section A5. We also present, derivation of estimation equations in Section A6, and alternative model estimation results in Section A7.

## A1. Illustrative Example of Financial Cost of Spending Volatility

This section illustrates the additional financial burden that may arise from a volatile advertising strategy. Consider a brand manager with a budget of \$400,000 to be spent over the fiscal year (see Table A1). Under even spending we assume s/he invests \$100,000 in marketing activities every quarter. Under the alternative volatile strategy, we assume \$200,000 is spent in the first and third quarters, and zero is spent in the remaining two. Following the market response literature, we incorporate a carryover effect of marketing, i.e. sales do not drop immediately to their base level when expenditures are reduced to zero.

The upper panel of Table A1 shows the statement of cash flows associated with the two spending strategies. Column 3 and 4 present the incremental revenues, net of costs of goods sold, which accrue from marketing expenditures.<sup>1</sup> Consistent with the idea of the differential stimulus effect (Hanssens and Levien 1983; Simon 1982), incremental revenues are higher under alternating spending levels. The last two columns show the incremental cash flows (net revenues minus marketing expenditures). We assume that volatile spending generates cash flows that are 5% higher than those from even spending.<sup>2</sup>

Normally, the comparison of the two alternative spending schedules would stop at this point. However, the alternatives involve quite different levels of volatility of incremental revenues and cash flows, as shown in the last two rows of the upper panel. Volatility can be expressed as the range of monetary quantities or their standard deviation. By definition, it is zero for even spending but reaches a remarkable level under volatile spending. The financial side effect associated with volatile spending is demonstrated in the lower panel of Table A1.

Columns 1 and 2 show how cash flows accumulate over time until they reach their year-end total of \$80,000 and \$84,000, respectively. The accrual of cash flows is booked at the end of a quarter, for example, \$20,000 is booked under even spending at the end of the first quarter. In order to realize these cash flows, however, capital must be provided at the beginning of the quarter. Columns 3 and 4 list the required level of cash holdings. It equals the size of marketing expenditures in the first quarter but decreases in subsequent quarters due to the incremental cash flows generated by marketing in previous periods. These required cash holdings are not costless, as investors expect their invested capital to generate at least a certain rate of return. Since the volatile spending pattern creates negative cash flows in some quarters, more capital is locked up over time. Compared with even spending, these higher capital needs incur additional financing costs as shown in the last two columns. Assuming annual capital cost of 15% or 3.8% per quarter, these financing costs make the volatile spending a less attractive policy. Cash

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<sup>1</sup> The figures are generated by a typical market response function that accounts for carryover and differential stimulus effects. Marketing spending elasticity under even spending is assumed to be .30.

<sup>2</sup> Dubé, Hitsch, and Manchanda (2005) and Mahajan and Muller (1986) report gains in cash profits due to volatile spending are between 1% and 5%.

flows net of financing cost now amount to \$70,000 for even spending, but only \$68,000 for the volatile spending strategy.

**Table A1**  
**An Example of the Financial Costs of Spending Volatility**

| <i>Statement of cash flows (quarterly) in Thousand US\$</i> |                        |                   |   |                   |  |                   |
|---|------------------------|-------------------|---|-------------------|--|-------------------|
|   | Marketing expenditures |                   | Incremental revenues net of cost of goods due to marketing expenditures |                   | Incremental cash flows due to marketing expenditures |                   |
|   | Even spending          | Volatile spending | Even spending   | Volatile spending | Even spending  | Volatile spending |
| Quarter 1   | 100                    | 200               | 120   | 150               | 20   | -50               |
| Quarter 2   | 100                    | 0                 | 120   | 92                | 20   | 92                |
| Quarter 3   | 100                    | 200               | 120   | 150               | 20   | -50               |
| Quarter 4   | 100                    | 0                 | 120   | 92                | 20   | 92                |
| Total   | 400                    | 400               | 480   | 484               | 80   | 84                |
| Std. dev.   | 0                      | 100               | 0   | 29                | 0  | 71                |
| Range <sup>1)</sup>   | 0                      | 200               | 0   | 58                | 0  | 142               |

| <i>Cash balance sheet (quarterly) in Thousand US\$</i> |  |                   |                        |                   |   |                   |
|--|--|-------------------|------------------------|-------------------|---|-------------------|
|  | Cumulated cash flows due to marketing expenditures |                   | Required cash holdings |                   | Financing costs for capital lockup (3.8% per quarter) |                   |
|  | Even spending                                      | Volatile spending | Even spending          | Volatile spending | Even spending   | Volatile spending |
| Quarter 1  | 20   | -50               | 100                    | 200               | 4   | 8                 |
| Quarter 2  | 40   | 42                | 80                     | 50                | 3   | 2                 |
| Quarter 3  | 60   | -8                | 60                     | 158               | 2   | 6                 |
| Quarter 4  | 80   | 84                | 40                     | 8                 | 1   | 0                 |
| Total  |  |                   | 280                    | 416               | 10  | 16                |

<sup>1)</sup> Range = Maximum expenditure – Minimum expenditure

## A2. Proofs of Propositions of Brand Performance Volatility Effects

PROOF OF PROPOSITION 1A. We need to show that  $\frac{\partial \text{Var}[RV(MKT, CMKT)]}{\partial \text{Var}(MKT)} > 0$ . Taking the first derivative of Equation (4) w.r.t.  $\text{Var}(MKT)$  and setting  $> 0$  gives,

$$P^2 [Q'(\mu)]^2 + P^2 \rho Q'(\mu) Q'_c(\mu_c) \left[ \frac{\text{Var}(CMKT)}{\text{Var}(MKT)} \right]^{\frac{1}{2}} > 0, \quad (18)$$

which we divide by  $P^2 Q'(\mu)$  and rearrange to

$$Q'(\mu) \sqrt{\text{Var}(MKT)} > -\rho Q'_c(\mu_c) \sqrt{\text{Var}(CMKT)}. \quad (19)$$

Recall our expressions for elasticities,  $\varepsilon = Q' \cdot \mu / Q$  and  $\varepsilon_c = Q'_c \cdot \mu_c / Q_c$ , and coefficients of variation,  $CV = SD(MKT) / \mu$  and  $CV_c = SD(CMKT) / \mu_c$ . Dividing (19) by  $Q$  and expanding the l.h.s. with  $\mu$  and the r.h.s. with  $\mu_c$  produces

$$\varepsilon CV > -\rho \varepsilon_c CV_c,$$

which is equivalent to the condition in proposition 1A.  $\square$

PROOF OF PROPOSITION 1B. We need to show that  $\frac{\partial \text{Var}[RV(MKT, CMKT)]}{\partial \mu} < 0$ .

Taking the first derivative of Equation (4) w.r.t.  $\mu$  and setting  $< 0$  gives,

$$2P^2 Q''(\mu) Q'(\mu) \text{Var}(MKT) + 2P^2 \rho Q''(\mu) Q'_c(\mu_c) [\text{Var}(MKT) \text{Var}(CMKT)]^{\frac{1}{2}} < 0. \quad (20)$$

Note that  $Q''(\mu) < 0$ . Dividing (20) by  $2P^2 Q''(\mu) \text{Var}(MKT)$  and rearranging the result gives

$$Q'(\mu) \sqrt{\text{Var}(MKT)} > -\rho Q'_c(\mu_c) \sqrt{\text{Var}(CMKT)}, \quad (21)$$

which is equivalent to expression (19). As done before, we can transform this expression to

$$\varepsilon CV > -\rho \varepsilon_c CV_c,$$

which is equivalent to the condition in proposition 1B.  $\square$

PROOF OF PROPOSITION 1C. We need to show that  $\frac{\partial \text{Var}[RV(MKT, CMKT)]}{\partial Q'(\mu)} > 0$ .

Taking the first derivative of Equation (4) w.r.t.  $Q'(\mu)$  and setting  $> 0$  gives,

$$2P^2 Q'(\mu) \text{Var}(MKT) + 2P^2 \rho Q'_c(\mu_c) [\text{Var}(MKT) \text{Var}(CMKT)]^{\frac{1}{2}} > 0, \quad (22)$$

which we divide by  $P^2 Q'(\mu) [\text{Var}(MKT)]^{1/2}$  and rearrange to

$$Q'(\mu) \sqrt{\text{Var}(MKT)} > -\rho Q'_c(\mu_c) \sqrt{\text{Var}(CMKT)}, \quad (23)$$

which is equivalent to expression (19). As done before, we can transform this expression to

$$\varepsilon CV > -\rho \varepsilon_c CV_c,$$

which is equivalent to the condition in proposition 1C.  $\square$

PROOF OF PROPOSITION 2A. We need to show that  $\frac{\partial \text{Var}[CF(MKT, CMKT)]}{\partial \text{Var}(MKT)} > 0$ . Taking the

first derivative of Equation (5) w.r.t.  $\text{Var}(MKT)$  and setting  $> 0$  gives,

$$\left[ (P-C) Q'(\mu) - 1 \right]^2 + (P-C)^2 \rho Q'(\mu) Q'_c(\mu_c) \left[ \frac{\text{Var}(CMKT)}{\text{Var}(MKT)} \right]^{\frac{1}{2}} > 0. \quad (24)$$

Selectively, we expand terms of (24) with  $Q$ ,  $\mu$ , and  $\mu_c$ , respectively, to obtain

$$\left[ \frac{\varepsilon(P-C)Q - \mu}{\mu^2} \right]^2 > -(P-C)^2 \rho \varepsilon \varepsilon_c \frac{Q^2}{\mu \mu_c} \left[ \frac{\text{Var}(CMKT)}{\text{Var}(MKT)} \right]^{\frac{1}{2}}. \quad (25)$$

Dividing this expression by  $\frac{\varepsilon^2(P-C)^2 Q^2}{\mu^2}$  gives,

$$\left[ \frac{\varepsilon(P-C)Q - \mu}{\varepsilon^2(P-C)^2 Q^2} \right]^2 > -\rho \frac{\varepsilon_c}{\varepsilon} \frac{\mu}{\mu_c} \left[ \frac{\text{Var}(CMKT)}{\text{Var}(MKT)} \right]^{\frac{1}{2}}. \quad (26)$$

Substituting for the near-optimal expenditure level  $\tilde{\mu}$  according to Equation (6) and the coefficients of variation,  $CV$  and  $CV_c$ , we can write for (26)

$$\left( \frac{\tilde{\mu} - \mu}{\tilde{\mu}} \right)^2 > -\rho \frac{\varepsilon_c CV_c}{\varepsilon CV},$$

which is equivalent to the condition in proposition 2A.  $\square$

PROOF OF COROLLARY 1. We need to show that  $\left( \frac{\tilde{\mu} - \mu}{\tilde{\mu}} \right)^2 < k$ , where  $k = -\rho \frac{\varepsilon_c CV_c}{\varepsilon CV}$ . From  $\varepsilon_c$

$< 0$  and  $\rho, \varepsilon, CV, CV_c > 0$ , it follows  $k > 0$ . Assume expenditures are set at the optimal level, i.e.  $\mu = \mu^*$ .

Since  $\mu^* = \varepsilon^*(P-C)Q^*$ , it follows with (6) that  $\tilde{\mu} = \mu^* = \varepsilon^*(P-C)Q^*$ . Let  $\kappa \in \mathbb{R}$  be an arbitrary constant that measures how close actual expenditures are to the optimal level:  $\mu = |\mu^* - \kappa|$ . For

$\kappa \rightarrow 0$ ,  $\mu \rightarrow \mu^*$  and  $\tilde{\mu} \rightarrow \mu^*$ . As a consequence,  $|\tilde{\mu} - \mu| \rightarrow 0$  and therefore,  $\left(\frac{\tilde{\mu} - \mu}{\tilde{\mu}}\right)^2 \Big|_{\kappa \rightarrow 0} = 0$ . Hence,

there is a  $\kappa$  small enough to satisfy  $\left(\frac{\tilde{\mu} - \mu}{\tilde{\mu}}\right)^2 < k$ .  $\square$

PROOF OF PROPOSITION 2B. This proposition implies that the first derivative of (5) has a root.

Hence,

$$\begin{aligned} \frac{\partial \text{Var}[CF(MKT, CMKT)]}{\partial \mu} &= 0 \\ &= 2(P-C)Q''(\mu)[(P-C)Q'(\mu)-1]\text{Var}(MKT) \\ &\quad + 2(P-C)^2 \rho Q''(\mu)Q'_c(\mu_c)[\text{Var}(MKT)\text{Var}(CMKT)]^{\frac{1}{2}}. \end{aligned} \quad (27)$$

Dividing (27) by  $Q'' < 0$ , expanding  $Q'$  and  $Q'_c$  with  $Q$ ,  $\mu$ , and  $\mu_c$ , respectively, and substituting terms for elasticities, coefficients of variation and  $\tilde{\mu}$  from (6), we solve for the root

$$\mu_0 = \tilde{\mu} \left( \frac{\varepsilon CV + \rho \varepsilon_c CV_c}{\varepsilon CV} \right). \quad (28)$$

Because  $\varepsilon CV > -\rho \varepsilon_c CV_c$ , the root is defined for positive mean expenditure levels. For a

U-shaped relation, we must show that  $\partial \text{Var}[CF(MKT, CMKT)]/\partial \mu|_{\mu < \mu_0} < 0$  and

$\partial \text{Var}[CF(MKT, CMKT)]/\partial \mu|_{\mu > \mu_0} > 0$ . Rewriting Equation (27) by substituting terms for

elasticities, coefficients of variation and  $\tilde{\mu}$ , these inequalities imply  $\tilde{\mu} \left( \frac{\varepsilon CV + \rho \varepsilon_c CV_c}{\varepsilon CV} \right) - \mu > 0$  and

$\tilde{\mu} \left( \frac{\varepsilon CV + \rho \varepsilon_c CV_c}{\varepsilon CV} \right) - \mu < 0$ , respectively. Let  $\mu = z\mu_0$ , with  $z > 0$ . Note that  $\mu < \mu_0$  for  $z < 1$  and

$\mu > \mu_0$  for  $z > 1$ . Substituting  $\mu_0$  for (28), we easily verify that  $\partial \text{Var}[CF(MKT)]/\partial \mu|_{z < 1} < 0$  and

$\partial \text{Var}[CF(MKT)]/\partial \mu|_{z > 1} > 0$ .  $\square$

PROOF OF COROLLARY 2. Set  $\mu = \mu^*$ , which implies with (6) that  $\tilde{\mu} = \mu^*$ . Because  $Q'_c < 0$  and  $\rho > 0$ ,  $(\varepsilon CV + \rho \varepsilon_c CV_c) / \varepsilon CV < 1$ . Then, result (28) implies that  $\mu_0 < \mu^*$ .  $\square$

PROOF OF PROPOSITION 2C. The derivative of (5) w.r.t.  $Q'(\mu)$  is given by

$$\begin{aligned} \frac{\partial \text{Var}[CF(MKT, CMKT)]}{\partial Q'(\mu)} &= 2(P-C)[(P-C)Q'(\mu)-1]\text{Var}(MKT) \\ &+ 2(P-C)^2 \rho Q'_c(\mu_c) [\text{Var}(MKT)\text{Var}(CMKT)]^{\frac{1}{2}}. \end{aligned} \quad (29)$$

Note that Equation (27), the derivative of (5) w.r.t  $\mu$ , equals Equation (29) scaled by  $Q''(\mu)$ . Hence, both derivatives have the same root, as given by Equation (28). However, because  $Q'' < 0$ , the inequality conditions for  $\partial \text{Var}[CF(MKT, CMKT)] / \partial Q'(\mu)$  at  $\mu < \mu_0$  and  $\mu > \mu_0$ , respectively, are reversed.

Following the chain of proof for proposition 2B, we can prove that these inequalities hold. Specifically,

$$\partial \text{Var}[CF(MKT, CMKT)] / \partial Q'(\mu) \Big|_{\mu < \mu_0} > 0 \Rightarrow \mu < \tilde{\mu} \left( \frac{\varepsilon CV + \rho \varepsilon_c CV_c}{\varepsilon CV} \right), \text{ with } \varepsilon CV > -\rho \varepsilon_c CV_c$$

and

$$\partial \text{Var}[CF(MKT, CMKT)] / \partial Q'(\mu) \Big|_{\mu > \mu_0} < 0 \Rightarrow \mu > \tilde{\mu} \left( \frac{\varepsilon CV + \rho \varepsilon_c CV_c}{\varepsilon CV} \right),$$

which is equivalent to the conditions in proposition 2C.  $\square$

PROOF OF PROPOSITION 3A. We need to show that  $\frac{\partial \text{Var}[RV(MKT, CMKT)]}{\partial \text{Var}(CMKT)} > 0$ . Taking the

first derivative of Equation (4) w.r.t.  $\text{Var}(CMKT)$  and setting  $> 0$  gives,

$$P^2 [Q'_c(\mu_c)]^2 + P^2 \rho Q'(\mu) Q'_c(\mu_c) \left[ \frac{\text{Var}(MKT)}{\text{Var}(CMKT)} \right]^{\frac{1}{2}} > 0,$$

which we divide by  $P^2$  to obtain

$$[Q'_c(\mu_c)]^2 + \rho Q'(\mu) Q'_c(\mu_c) \left[ \frac{\text{Var}(MKT)}{\text{Var}(CMKT)} \right]^{\frac{1}{2}} > 0. \quad (30)$$

Consider first a substitutive cross-effect, i.e.  $Q'_c(\mu_c) < 0$  and therefore  $\varepsilon_c < 0$ . Dividing (30) by  $Q'_c(\mu_c)$  and rearranging the result gives,

$$Q'_c(\mu_c)\sqrt{\text{Var}(CMKT)} < -\rho Q'(\mu)\sqrt{\text{Var}(MKT)}. \quad (31)$$

Dividing (31) by  $Q$  and expanding the l.h.s. with  $\mu_c$  and the r.h.s.  $\mu$ , we can rewrite and simplify the inequality to

$$\varepsilon_c CV_c < -\rho\varepsilon CV, \quad ,$$

which proves condition (7a) for proposition 3A.

Consider now a market-expanding cross-effect, i.e.  $Q'_c(\mu_c) > 0$  and therefore  $\varepsilon_c > 0$ . Divide (30) by  $Q'_c(\mu_c)$  and rearrange the result to

$$Q'_c(\mu_c)\sqrt{\text{Var}(CMKT)} > -\rho Q'(\mu)\sqrt{\text{Var}(MKT)}. \quad (32)$$

Again, we can rewrite this expression in terms of elasticities and coefficients of variation

$$\varepsilon_c CV_c > -\rho\varepsilon CV, \quad ,$$

which proves condition (7b) for proposition 3A.  $\square$

PROOF OF PROPOSITION 3B. We need to show that  $\frac{\partial \text{Var}[CF(MKT, CMKT)]}{\partial \text{Var}(CMKT)} > 0$ . Taking the first derivative of Equation (5) w.r.t.  $\text{Var}(CMKT)$  and setting  $> 0$  gives,

$$(P-C)^2 [Q'_c(\mu_c)]^2 + (P-C)^2 \rho Q'(\mu) Q'_c(\mu_c) \left[ \frac{\text{Var}(MKT)}{\text{Var}(CMKT)} \right]^{\frac{1}{2}} > 0,$$

which we divide by  $(P-C)^2$  to obtain

$$[Q'_c(\mu_c)]^2 + \rho Q'(\mu) Q'_c(\mu_c) \left[ \frac{\text{Var}(MKT)}{\text{Var}(CMKT)} \right]^{\frac{1}{2}} > 0. \quad (33)$$

Note that this inequality is identical to inequality (30). With inequalities (31) and (32), we already showed that the conditions (7a) and (7b) satisfy inequality (33).  $\square$

PROOF OF PROPOSITION 4A. The first derivative of Equation (4) w.r.t.  $\rho$  is given by

$$\frac{\partial \text{Var}[RV(MKT, CMKT)]}{\partial \rho} = 2P^2 Q'(\mu) Q'_c(\mu_c) [\text{Var}(MKT)\text{Var}(CMKT)]^{\frac{1}{2}} \quad (34)$$

Again, we can rewrite this expression in terms of elasticities and coefficients of variation

$$\frac{\partial \text{Var} [RV(MKT, CMKT)]}{\partial \rho} = 2(PQ)^2 \varepsilon CV \varepsilon_c CV_c. \quad (35)$$

Since  $P$ ,  $Q$ ,  $\varepsilon$ ,  $CV$ , and  $CV_c$  are always strictly positive, it is easy to show that

$$\frac{\partial \text{Var} [RV(MKT, CMKT)]}{\partial \rho} > 0 \quad \text{iff } \varepsilon_c > 0$$

and

$$\frac{\partial \text{Var} [RV(MKT, CMKT)]}{\partial \rho} < 0 \quad \text{iff } \varepsilon_c < 0. \quad \square$$

PROOF OF PROPOSITION 4B. The first derivative of Equation (5) w.r.t.  $\rho$  is given by

$$\frac{\partial \text{Var} [CF(MKT, CMKT)]}{\partial \rho} = 2(P-C)^2 Q'(\mu) Q'_c(\mu_c) [\text{Var}(MKT) \text{Var}(CMKT)]^{\frac{1}{2}},$$

which can be rewritten as

$$\frac{\partial \text{Var} [CF(MKT, CMKT)]}{\partial \rho} = 2(P-Q)^2 \varepsilon CV \varepsilon_c CV_c. \quad (36)$$

Note that  $(P-C)^2$  is always strictly positive. Following the chain of proof for proposition 4A, it is easy to show that proposition 4B also holds.  $\square$

### A3. Proofs of Brand Performance Volatility Effects under Rational Firm Behavior

Table 2 summarizes the brand performance volatility effects that follow from our propositions if we assume that firms follow a rational, competitive Nash behavior. Note that Theorem 2 requires

$\partial \sigma_{RV}^2 / \partial \sigma_{MKT}^2 > 0$  for any optimal, volatile marketing policy. Consistent with proposition 1A, this

implies that  $\varepsilon CV > -\rho \varepsilon_c CV_c$  always holds. In the following, we frequently refer to this result to prove the statements in Table 2.

#### Impact of Own Marketing on Performance Volatility

PROOF OF EFFECTS ON VARIANCE OF REVENUES. Since  $\varepsilon CV > -\rho \varepsilon_c CV_c$  the statements in Table 2 directly follow from propositions 1A, 1B, and 1C.  $\square$

PROOF OF THE EFFECT OF EXPENDITURE VARIANCE ON VARIANCE OF CASH FLOWS. From

$\varepsilon CV > -\rho \varepsilon_c CV_c$ , it follows  $-\rho \frac{\varepsilon_c CV_c}{\varepsilon CV} < 1$ . Proposition 2A states that cash-flow variance increases

with expenditure variance if  $\left(\frac{\tilde{\mu} - \mu}{\tilde{\mu}}\right)^2 > -\rho \frac{\varepsilon_c CV_c}{\varepsilon CV}$ . Hence, a higher variance of own marketing

expenditures always increases the variance of cash flows if  $\left(\frac{\tilde{\mu} - \mu}{\tilde{\mu}}\right)^2 \geq 1$ . However, there may exist a

budget that satisfies  $0 \leq \left(\frac{\tilde{\mu} - \mu}{\tilde{\mu}}\right)^2 \leq 1$ , which proves the result in Table 2. Since the result in Table 2 is

fully consistent with proposition 2A, the proof for corollary 1 given above applies here, too. Thus, corollary 1 holds under the assumption of rational firm behavior.  $\square$

PROOF OF THE EFFECT OF LEVEL OF EXPENDITURES ON VARIANCE OF CASH FLOWS. Since

$\varepsilon CV > -\rho \varepsilon_c CV_c$  the statement in Table 2 directly follows from proposition 2B. For this reason, the proof of corollary 2 also applies here. Thus, corollary 2 holds under the assumption of rational firm behavior.  $\square$

PROOF OF THE EFFECT OF RESPONSIVENESS ON VARIANCE OF CASH FLOWS. Since

$\varepsilon CV > -\rho \varepsilon_c CV_c$  the statement in Table 2 directly follows from proposition 2C.  $\square$

### **Impact of Competitive Marketing on Performance Volatility**

PROOF OF THE EFFECT OF COMPETITIVE EXPENDITURE VARIANCE ON VARIANCE OF REVENUES AND CASH FLOWS. Consider first the case of substitutive cross-effects, i.e.  $\varepsilon_c < 0$ . To demonstrate a positive effect of competitive expenditure variance on variance of revenues and cash flows, we need to show that the following inequalities are both satisfied:

$$\varepsilon_c CV_c < -\rho \varepsilon CV \quad (\text{see condition 7a}) \quad (37)$$

and

$$\varepsilon CV > -\rho \varepsilon_c CV_c. \quad (38)$$

Because  $\varepsilon$ ,  $CV$ , and  $CV_c > 0$ , these inequalities are always satisfied for  $\rho \leq 0$ . For  $\rho > 0$ , we rearrange (37) to  $-\frac{1}{\rho} \varepsilon_c CV_c > \varepsilon CV$ . Then

$$-\frac{1}{\rho} \varepsilon_c CV_c > -\rho \varepsilon_c CV_c \quad (39)$$

must hold to satisfy (38). Recall that  $\varepsilon_c < 0$  and  $\rho > 0$ . (39) can be simplified to  $-1/\rho < -\rho$  and therefore  $\rho^2 < 1$ , which is always satisfied because  $0 < \rho \leq 1$ . Hence, a positive effect is consistent with  $\rho > 0$ . However, we can easily choose reasonable parameter values that are also consistent with a negative effect. Here, inequalities

$$\varepsilon_c CV_c > -\rho \varepsilon CV \quad (40)$$

and (38) must both be satisfied. Assume  $\varepsilon = .19$ ,  $\varepsilon_c = -.05$ ,  $\rho = .35$ ,  $CV = .365$ ,  $CV_c = .303$  and insert these parameter values into (40) and (38) to demonstrate that the inequalities are satisfied.  $\square$

Consider now the case of market-expanding cross-effects, i.e.  $\varepsilon_c > 0$ . For a positive effect of competitive expenditure variance on variance of revenues and cash flows, inequality (38) together with

$$\varepsilon_c CV_c > -\rho \varepsilon CV \quad (\text{see condition 7b}) \quad (41)$$

must be satisfied. Again, it is straightforward to show that these inequalities are always satisfied for

$\rho \geq 0$  because  $\varepsilon$ ,  $CV$ , and  $CV_c > 0$ . For  $\rho < 0$ , we rearrange (41) to  $-\frac{1}{\rho} \varepsilon_c CV_c < \varepsilon CV$ . From this, we

conclude that

$$-\frac{1}{\rho} \varepsilon_c CV_c > -\rho \varepsilon_c CV_c \quad (42)$$

satisfies (38). Recall that  $\varepsilon_c > 0$  and  $\rho < 0$ . (42) can be simplified to  $-1/\rho > -\rho$  and therefore  $|\rho|^2 < 1$ ,

which is always satisfied because  $-1 \leq \rho < 0$ . Hence, a positive effect is consistent with

$\rho < 0$ . But again, we can easily choose reasonable parameter values that are consistent with a negative effect. For example, the parameter values  $\varepsilon = .19$ ,  $\varepsilon_c = .03$ ,  $\rho = -.35$ ,  $CV = .365$ ,  $CV_c = .303$  satisfy both (38) and the reverse of inequality (41).  $\square$

PROOF OF THE EFFECT OF EXPENDITURE CORRELATION ON VARIANCE OF REVENUES AND CASH FLOWS.

The relationship between expenditure correlation and variance of revenues and cash flows depends on the sign of the cross-effect. The results in Table 2 are fully consistent with the results in Table 1 that are derived from propositions 4A and 4B. We have shown above that these propositions hold.  $\square$

#### A4. Proofs of Theorems on Optimal Expenditure Behavior

##### Proof of Theorem 1

Expression (8) states the maximization problem that we want to solve. Define  $m = (P - C)/P$  that measures the profit margin in percent. We can then write for the cash flows

$$CF = m \cdot PQ - MKT \quad (43)$$

and derive their variance

$$\sigma_{CF}^2 = m^2 P^2 \sigma_Q^2 + \sigma_{MKT}^2 - 2mP \text{Cov}(Q, MKT). \quad (44)$$

Note that  $\text{Cov}(Q, MKT) = \rho_{Q, MKT} \sqrt{\sigma_Q^2 \sigma_{MKT}^2}$ , where  $\rho_{Q, MKT}$  measures the correlation between unit sales and marketing expenditures. Using this relation and  $\sigma_{RV}^2 = P^2 \sigma_Q^2$ , we can write for (44)

$$\sigma_{CF}^2 = m^2 \sigma_{RV}^2 + \sigma_{MKT}^2 - 2mP \rho_{Q, MKT} \sqrt{\sigma_Q^2 \sigma_{MKT}^2}. \quad (45)$$

Insert this expression into the profit function (8) to obtain:

$$\max_{\mu, \sigma_{MKT}^2} \Pi = (P - C)Q(\mu, \sigma_{MKT}^2, CMKT) - \mu - (w + rm^2) \sigma_{RV}^2 - r \left( \sigma_{MKT}^2 + 2rm \rho_{Q, MKT} \sqrt{\sigma_Q^2 \sigma_{MKT}^2} \right) - f. \quad (46)$$

For this maximization problem, we derive the first-order conditions (9a) and (9b) that each competitor has to satisfy under Nash competition.

Assuming  $r = 0$ , the first-order conditions reduce to

$$\frac{\partial \Pi}{\partial \mu} = (P - C) \frac{\partial Q}{\partial \mu} - 1 - w \frac{\partial \sigma_{RV}^2}{\partial \mu} = 0, \quad (47a)$$

$$\frac{\partial \Pi}{\partial \sigma_{MKT}^2} = (P - C) \frac{\partial Q}{\partial \sigma_{MKT}^2} - w \frac{\partial \sigma_{RV}^2}{\partial \sigma_{MKT}^2} = 0. \quad (47b)$$

Multiply (47a) with  $\mu$  and expand terms with  $Q$  and  $\sigma_{RV}^2$ , respectively, to obtain

$$\frac{\partial \Pi}{\partial \mu} = (P - C) \varepsilon_{Q, \mu} Q - \mu - w \varepsilon_{\sigma_{RV}^2, \mu} \sigma_{RV}^2 = 0, \quad (48a)$$

where  $\varepsilon_{Q,\mu} = \frac{\partial Q}{\partial \mu} \frac{\mu}{Q}$  and  $\varepsilon_{\sigma_{RV}^2, \mu} = \frac{\partial \sigma_{RV}^2}{\partial \mu} \frac{\mu}{\sigma_{RV}^2}$ . Similarly, we multiply (47b) with  $\sigma_{MKT}^2$  and expand terms with  $Q$  and  $\sigma_{RV}^2$ , respectively, and obtain

$$\frac{\partial \Pi}{\partial \sigma_{MKT}^2} = (P-C) \varepsilon_{Q, \sigma_{MKT}^2} Q - w \varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2} \sigma_{RV}^2 = 0, \quad (48b)$$

where  $\varepsilon_{Q, \sigma_{MKT}^2} = \frac{\partial Q}{\partial \sigma_{MKT}^2} \frac{\sigma_{MKT}^2}{Q}$  and  $\varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2} = \frac{\partial \sigma_{RV}^2}{\partial \sigma_{MKT}^2} \frac{\sigma_{MKT}^2}{\sigma_{RV}^2}$ .

Setting (48a) = (48b), we can solve for the optimal expenditure level

$$\mu^* = (P-C) Q^* \varepsilon_{Q, \mu}^* - \frac{\varepsilon_{\sigma_{RV}^2, \mu}^* (P-C) Q^* \varepsilon_{Q, \sigma_{MKT}^2}^*}{\varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2}^*}, \quad (49)$$

where the asterisk means that variables are at their optimum. Rearranging terms shows that (49) is equivalent to Theorem 1.  $\square$

## Proof of Theorem 2

Solve expression (48a) for the revenue volatility

$$\sigma_{RV}^2 = \frac{(P-C) \varepsilon_{Q, \mu} - \mu}{w \varepsilon_{\sigma_{RV}^2, \mu}} \quad (50)$$

and substitute for the optimal expenditure level (49) to obtain the revenue volatility at optimum

$$\left(\sigma_{RV}^2\right)^* = \frac{(P-C) Q^* \varepsilon_{Q, \sigma_{MKT}^2}^*}{w \varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2}^*}. \quad (51)$$

Substitute  $\varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2}$  for  $\frac{\partial \sigma_{RV}^2}{\partial \sigma_{MKT}^2} \frac{\sigma_{MKT}^2}{\sigma_{RV}^2}$  in (51) and solve for the optimal variance of marketing expenditures

$$\left(\sigma_{MKT}^2\right)^* = \frac{(P-C) Q^*}{w} \left( \frac{\varepsilon_{Q, \sigma_{MKT}^2}^*}{\partial \sigma_{RV}^2 / \partial \sigma_{MKT}^2} \right). \quad (52)$$

Since variances are always positive,  $\partial \sigma_{RV}^2 / \partial \sigma_{MKT}^2 > 0$  must hold for (52), which is consistent with Theorem 2.  $\square$

## A5. Proofs for Dynamic Model Extensions

### Proofs of Propositions of Brand Performance Volatility Effects

Note that the structure of the general response function (1) and thus the variance functions of revenues and cash flows does not change if we substitute own and competitive marketing expenditures,  $MKT$  and  $CMKT$ , for their stock variables  $S$  and  $S_C$ , respectively. As a result, all brand performance volatility effects for the dynamic model are identical to their counterparts that we derived and proved for the static model. The only difference is that the variance, the mean, and the responsiveness are expressed in terms of marketing stocks instead of marketing expenditures.

To prove that all propositions and corollaries also hold with respect to marketing expenditures, it is sufficient to show that the variance, the mean, and the responsiveness of expenditures differ from their stock equivalents only by a scaling factor. All results hold for competitive marketing stock and expenditures, as well.

MEAN. Taking the expectation of the marketing stock equation (10) gives

$$E(S_t) = \lambda E(S_{t-1}) + E(MKT_t). \quad (53)$$

Since  $\lambda < 1$ , the mean  $E(S_t)$  is identical for all values of  $t$ . The above expression can be rearranged to

$$E(S) = \frac{\mu}{1-\lambda}, \quad (54)$$

where  $\mu$  denotes the mean of own marketing expenditures. This result shows that the mean of own marketing stock is identical with the mean of its expenditures up to the scale  $1/1-\lambda$ .  $\square$

VARIANCE. We arrange (10) to

$$S_t - \lambda S_{t-1} = MKT_t$$

and take the variance of both sides

$$Var(S_t) + \lambda^2 Var(S_{t-1}) - 2\lambda \rho_{ARI} SD(S_t) SD(S_{t-1}) = Var(MKT_t), \quad (55)$$

where  $\rho_{ARI}$  measures the autocorrelation of the stock and all other terms are defined as earlier. Since  $SD(S_t) - SD(S_{t-1}) \rightarrow 0$  for longer time-series of  $S$ , we can use  $SD(S_t) \cong SD(S_{t-1})$  and simplify (55) to

$$Var(S) = \frac{1}{1 + \lambda^2 - 2\lambda \rho_{AR(1)}} Var(MKT). \quad (56)$$

The denominator of (56) is a constant and always greater than zero. To see that note that the following inequality must be met

$$\frac{1 + \lambda^2}{2\lambda} > \rho_{ARI}.$$

Since  $\rho_{ARI}$  cannot take on values greater than 1 we can rearrange this inequality to  $(1 - \lambda^2) > 0$ , which is always true.  $\square$

RESPONSIVENESS. From the econometric literature on distributed-lag models (Hanssens, Parsons, and Schultz 2001), it is well established that short-term and long-term sales effects relate to each other as follows

$$Q'_s(\cdot) = \frac{Q'(\cdot)}{1 - \lambda}. \quad (57)$$

Hence, sales responsiveness w.r.t. stock is identical to the sales responsiveness w.r.t. expenditures up to the scale of  $1/1 - \lambda$ .  $\square$

In addition, the conditions that are part of the propositions and summarized in Table 1 and 2 only change by a scaling factor. They can be rewritten by simply substituting

$$\begin{aligned} \varepsilon CV \text{ for } \varepsilon CV \frac{1}{\sqrt{1 + \lambda^2 - 2\lambda\rho_{ARI,S}}}, \\ \varepsilon_c CV_c \text{ for } \varepsilon_c CV_c \frac{1}{\sqrt{1 + \lambda_c^2 - 2\lambda_c\rho_{ARI,S_c}}}, \end{aligned}$$

and

$$\rho \text{ for } \rho_{S,S_c},$$

where  $\lambda_c$  denotes the carryover coefficient of competitive marketing expenditures and  $\rho_{S,S_c}$  measures the correlation between own marketing stock and competitive marketing stock.  $\square$

### Proofs of Theorems on Optimal Expenditure Behavior

EXTENSION OF THEOREM 1. The firm faces the following dynamic profit maximization problem

$$\max_{MKT, \sigma_{MKT}^2} \int_0^{\infty} e^{-dt} \left[ (P - C)Q(S, S_c, \sigma_{MKT}^2) - MKT - w\sigma_{RV}^2 - r\sigma_{CF}^2 - f \right] dt \quad (58)$$

subject to

$$\frac{dS}{dt} = -\phi S + MKT, \quad (59)$$

with

$$MKT \geq 0, \quad \sigma_{MKT}^2 \geq 0, \quad S \geq 0, \quad \text{and } S(0) = S_0. \quad (60)$$

Equation (59) describes the dynamic process for the evolution of the marketing stock (Nerlove and Arrow 1962), where  $\phi$  is the depreciation rate of the stock. Note that time enters the objective

function (58) explicitly through the discount term  $e^{-dt}$ , with discount rate  $d$  and  $0 < d < \infty$ . Since the discount rate is a constant the problem is quasiautonomous. Given the infinite time horizon, the solution to  $S(t)$  is likely to tend to a steady state in the long run (Kamien and Schwartz 1991, 95f).

Using (59) and assuming again  $r = 0$ , we can restate the maximization problem as follows

$$\max \int_0^{\infty} e^{-dt} \left[ (P-C)Q(S, S_C, \sigma_{MKT}^2) - \frac{dS}{dt} - \phi S - w\sigma_{RV}^2 - f \right] dt. \quad (61)$$

The first-order conditions are given by the Euler equation

$$\frac{\partial F}{\partial S} \left[ t, S^*(t), \frac{dS^*(t)}{dt}, (\sigma_{MKT}^2)^* \right] - \frac{d}{dt} \left[ \frac{\partial}{\partial (dS/dt)} F \left( t, S^*(t), \frac{dS^*(t)}{dt}, (\sigma_{MKT}^2)^* \right) \right] = 0 \quad (62a)$$

$$\frac{\partial F}{\partial \sigma_{MKT}^2} = 0, \quad (62b)$$

where  $F \left( t, S^*(t), \frac{dS^*(t)}{dt}, (\sigma_{MKT}^2)^* \right)$  is the integrand and the asterisk indicates that variable values are at their optimum. Under Nash competition, each competitor has to satisfy these first-order conditions. The needed derivatives are given by

$$\frac{\partial F}{\partial S} = e^{-dt} \left[ (P-C) \frac{\partial Q}{\partial S} - \phi - w \frac{\partial \sigma_{RV}^2}{\partial S} \right], \quad (63a)$$

$$\frac{\partial F}{\partial (dS/dt)} = -e^{-dt}, \quad (63b)$$

$$\frac{d}{dt} \frac{\partial F}{\partial (dS/dt)} = d e^{-dt}. \quad (63c)$$

By using (63a) and (63c), we can write for (62a) and (62b)

$$(P-C) \frac{\partial Q}{\partial S} - \phi - w \frac{\partial \sigma_{RV}^2}{\partial S} - d = 0, \quad (64a)$$

$$(P-C) \frac{\partial Q}{\partial \sigma_{MKT}^2} - w \frac{\partial \sigma_{RV}^2}{\partial \sigma_{MKT}^2} = 0. \quad (64b)$$

Consider first expenditure volatility, which appears only in the integrand, and first-order condition (64b). Multiply the expression with  $\sigma_{MKT}^2$  and expand the first term with  $Q$  to obtain

$$(P-C) Q \varepsilon_{Q, \sigma_{MKT}^2} - w \frac{\partial \sigma_{RV}^2}{\partial \sigma_{MKT}^2} (\sigma_{MKT}^2) = 0.$$

Solve the equation for  $(\sigma_{MKT}^2)^*$  to obtain

$$(\sigma_{MKT}^2)^* = \frac{(P-C)Q^*}{w} \left( \frac{\varepsilon_{Q,\sigma_{MKT}^2}^*}{\partial \sigma_{RV}^2 / \partial \sigma_{MKT}^2} \right), \quad (65)$$

with  $\partial \sigma_{RV}^2 / \partial \sigma_{MKT}^2 > 0$ . Solution (65) is consistent with Theorem 2.  $\square$

THEOREM 1. Multiply (64a) with  $S$  and expand terms with  $Q$  and  $\sigma_{RV}^2$ , respectively, to obtain

$$(P-C)Q \varepsilon_{Q,S} - w \varepsilon_{\sigma_{RV}^2, S} \sigma_{RV}^2 - (\phi + d)S = 0, \quad (66)$$

where  $\varepsilon_{Q,S} = \frac{\partial Q}{\partial S} \frac{S}{Q}$  and  $\varepsilon_{\sigma_{RV}^2, S} = \frac{\partial \sigma_{RV}^2}{\partial S} \frac{S}{\sigma_{RV}^2}$ . Similarly, multiply (64b) with  $\sigma_{MKT}^2$  and expand terms

with  $Q$  and  $\sigma_{RV}^2$ , respectively, and obtain

$$(P-C)Q \varepsilon_{Q,\sigma_{MKT}^2} - w \varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2} \sigma_{RV}^2 = 0, \quad (67)$$

where  $\varepsilon_{Q,\sigma_{MKT}^2} = \frac{\partial Q}{\partial \sigma_{MKT}^2} \frac{\sigma_{MKT}^2}{Q}$  and  $\varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2} = \frac{\partial \sigma_{RV}^2}{\partial \sigma_{MKT}^2} \frac{\sigma_{MKT}^2}{\sigma_{RV}^2}$ .

Rearrange (66) and (67) to

$$\frac{(P-C)Q \varepsilon_{Q,S} - (\phi + d)S}{\varepsilon_{\sigma_{RV}^2, S}} = w \sigma_{RV}^2, \quad (68)$$

$$\frac{(P-C)Q \varepsilon_{Q,\sigma_{MKT}^2}}{\varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2}} = w \sigma_{RV}^2. \quad (69)$$

Setting (68) = (69), we can solve for the optimal stock

$$S^* = \frac{(P-C)Q^* \varepsilon_{Q,S}^*}{\phi + d} - \frac{\varepsilon_{\sigma_{RV}^2, S}^* (P-C)Q^* \varepsilon_{Q,\sigma_{MKT}^2}^*}{(\phi + d) \varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2}^*}. \quad (70)$$

The solution establishes a global maximum because the integrand  $F\left(t, S^*(t), dS^*(t)/dt, (\sigma_{MKT}^2)^*\right)$  is concave in  $S^*(t)$  and  $dS^*(t)/dt$ . In addition, we do not need a necessary transversality condition in infinite horizon problems (Kamien and Schwartz 1991, 95f).

Given  $\phi S = MKT - dS/dt$ , we multiply (70) with  $\phi$  to obtain

$$MKT^* = \frac{(P-C)Q^* \varepsilon_{Q,MKT}^*}{\phi + d} - \frac{\varepsilon_{\sigma_{RV}^2, MKT}^* (P-C)Q^* \varepsilon_{Q, \sigma_{MKT}^2}^*}{(\phi + d) \varepsilon_{\sigma_{RV}^2, \sigma_{MKT}^2}^*} + \frac{dS^*}{dt}, \quad (71)$$

where  $\varepsilon_{Q, MKT}^2 = \phi \varepsilon_{Q, S}^2$  and  $\varepsilon_{\sigma_{RV}^2, MKT} = \phi \varepsilon_{\sigma_{RV}^2, S}$ . Since the optimal solution is a steady-state solution in an autonomous problem with infinite horizon,  $dS^*/dt$  measures the deviations of optimal marketing expenditures around their long-term equilibrium level, i.e. around the optimal mean expenditure level  $\mu_{long-term}^*$ . The optimal variance of marketing expenditures and thus  $dS^*/dt$  is given by expression (65).

Since  $dS^*/dt = 0$  for the optimal mean expenditure level,  $\mu_{long-term}^*$ , expression (71) reduces to the extended form of Theorem 1 in (12).  $\square$

## A6. Derivation of Estimation Equations

In this section, we derive the estimation equations (16) and (17) from equations (14) and (15). We start with a log transformation of equations (14) and (15),

$$\ln V(REV) = \ln \gamma_0 + \gamma_1 \ln V(MKT) + \gamma_2 \ln A(MKT) + \gamma_3 \ln V(CMKT) + \gamma_4 CORR + \gamma_5 RESP + \mathbf{X}\gamma + v, \quad (72)$$

$$\ln V(CF) = \ln \delta_0 + \delta_1 \ln V(REV) + \delta_2 \ln V(MKT) + \delta_3 \ln A(MKT) + \delta_4 A(MKT) + v. \quad (73)$$

Then we take the total differentials to obtain,

$$\begin{aligned} \frac{1}{V(REV)} dV(REV) &= \frac{\gamma_1}{V(MKT)} dV(MKT) + \frac{\gamma_2}{A(MKT)} dA(MKT) \\ &+ \frac{\gamma_3}{V(CMKT)} dV(CMKT) + \gamma_4 dCORR + \gamma_5 dRESP + d\mathbf{X}\gamma + dv, \end{aligned} \quad (74)$$

$$\begin{aligned} \frac{1}{V(CF)} dV(CF) &= \frac{\delta_1}{V(REV)} dV(REV) + \frac{\delta_2}{V(MKT)} dV(MKT) + \frac{\delta_3}{A(MKT)} dA(MKT) \\ &+ \delta_4 dA(MKT) + dv, \end{aligned} \quad (75)$$

which define the structural form of our estimation equations (16) and (17).

## **A7. Alternative Model Estimation Results**

In this section, we provide a summary of results of alternative brand sales model estimations and the correlation matrix of variables.

Note that the estimation of our suggested brand sales model (Equation 13) is based on a fixed-effects model with brand-specific effects for own and competitive marketing expenditures. Estimation of the alternative partial adjustment and Koyck specifications requires transforming the data into first differences. These models include the lagged dependent variable among predictor variables whose effect can only be consistently estimated in a first-difference panel data model (Greene 2006). Since the lagged difference of sales is still correlated with the differenced error term it needs to be instrumented. Values for sales lagged by two or three periods, respectively, provide valid instruments (Greene 2006). However, they further reduce the sample size and may turn out to be weak instruments and. Finally, we note that because of the inherent differences between model specifications comparisons of model fit in terms of Pseudo  $R^2$  (for first differences of the dependent variable) are limited. Note also that positive cross-effects for competitive marketing expenditures are not uncommon for pharmaceutical markets (see for example, Fischer and Albers 2010).

**TABLE A2**  
**Overview of Estimation Results of Alternative Brand Sales Model Specifications**

| <i>France</i>  | <i>Gastro-intestinal categories</i> |                |                |                |                | <i>Cardio-vascular categories</i> |                |                |                |                |
|--|-------------------------------------|----------------|----------------|----------------|----------------|-----------------------------------|----------------|----------------|----------------|----------------|
|  | Eq. 13                              | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      | Eq. 13                            | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      |
| Average effect of own marketing expenditures         | .14                                 | .07            | .13            | .20            | .16            | .02                               | .04            | .01            | .03            | .01            |
| Average effect of competitive marketing expenditures | .11                                 | .06            | .04            | .05            | .05            | -.04                              | .03            | .02            | -.02           | -.03           |
| Effect of lagged sales                               | n.a.                                | .66*<br>(.31)  | .68*<br>(.31)  | .32**<br>(.11) | .35**<br>(.10) | n.a.                              | .59**<br>(.10) | .59**<br>(.10) | .49**<br>(.06) | .45**<br>(.06) |
| Pseudo-R <sup>2</sup>                                | .87                                 | .39            | .39            | .37            | .39            | .90                               | .33            | .34            | .47            | .46            |
| <i>Germany</i>                                       | <i>Gastro-intestinal categories</i> |                |                |                |                | <i>Cardio-vascular categories</i> |                |                |                |                |
|  | Eq. 13                              | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      | Eq. 13                            | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      |
| Average effect of own marketing expenditures         | .16                                 | .15            | .06            | .17            | .16            | .00                               | -.01           | -.01           | .00            | .02            |
| Average effect of competitive marketing expenditures | -.01                                | .03            | .01            | .02            | .02            | .02                               | .03            | .02            | .00            | .04            |
| Effect of lagged sales                               | n.a.                                | .69**<br>(.19) | .74**<br>(.19) | .34*<br>(.15)  | .39**<br>(.14) | n.a.                              | .65**<br>(.11) | .65**<br>(.10) | .50**<br>(.06) | -.29<br>(.51)  |
| Pseudo-R <sup>2</sup>                                | .45                                 | .08            | .03            | .45            | .20            | .91                               | .14            | .15            | .48            | .45            |

Table A2 (continued)

| <i>Italy</i>   | <i>Gastro-intestinal categories</i> |                |                |                |                | <i>Cardio-vascular categories</i> |                |                |                |                |
|--|-------------------------------------|----------------|----------------|----------------|----------------|-----------------------------------|----------------|----------------|----------------|----------------|
|  | Eq. 13                              | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      | Eq. 13                            | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      |
| Average effect of own marketing expenditures         | .03                                 | .02            | .08            | .01            | .00            | .08                               | .16            | .12            | .12            | .13            |
| Average effect of competitive marketing expenditures | -.01                                | -.09           | -.06           | -.07           | -.06           | -.02                              | .08            | .09            | .03            | .04            |
| Effect of lagged sales                               | n.a.                                | .12<br>(.42)   | .20<br>(.28)   | .04<br>(.33)   | .17<br>(.19)   | n.a.                              | .10<br>(.31)   | .11<br>(.31)   | -.13<br>(.39)  | .26**<br>(.05) |
| Pseudo-R <sup>2</sup>                                | .60                                 | .22            | .37            | .32            | .31            | .93                               | .15            | .15            | .27            | .24            |
| <i>UK</i>  | <i>Gastro-intestinal categories</i> |                |                |                |                | <i>Cardio-vascular categories</i> |                |                |                |                |
|  | Eq. 13                              | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      | Eq. 13                            | Partial (a)    | Partial (b)    | Koyck (a)      | Koyck (b)      |
| Average effect of own marketing expenditures         | .03                                 | .01            | -.05           | .03            | .00            | .00                               | .03            | .04            | .02            | .02            |
| Average effect of competitive marketing expenditures | -.01                                | .03            | .03            | -.02           | -.01           | .00                               | .01            | -.03           | .00            | -.01           |
| Effect of lagged sales                               | n.a.                                | .85**<br>(.18) | .85**<br>(.17) | .52**<br>(.10) | .51**<br>(.11) | n.a.                              | .43**<br>(.08) | .43**<br>(.09) | .52**<br>(.06) | .52**<br>(.04) |
| Pseudo-R <sup>2</sup>                                | .62                                 | .20            | .29            | .69            | .68            | .89                               | .60            | .60            | .60            | .64            |

Notes: Standard errors in parentheses. Two-sided t-test applies. \*\* p < .01; \* p < .05. Pseudo R<sup>2</sup> is the squared correlation between predicted and actual values of the dependent variable. Standard R<sup>2</sup> is not defined for GLS and IV estimation approaches, which were used for estimating the models.

The estimates of effects for own and competitive marketing expenditures represent average total effects across brands. Therefore, standard errors are not presented here.

*Model specifications:*

|             |   |
|-------------|---|
| Eq. 13      | = Brand sales model with serial error correlation structure as of Equation (13)                                       |
| Partial (a) | = Partial adjustment model with current + lagged own and competitive marketing effects (no serial error correlation). |
| Partial (b) | = Partial adjustment model with only current own and competitive marketing effects (no serial error correlation).     |
| Koyck (a)   | = Koyck model with current + lagged own and competitive marketing effects (serial error correlation).                 |
| Koyck (b)   | = Koyck model with only current own and competitive marketing effects (serial error correlation).                     |

**TABLE A3**  
**Correlation Matrix (Moving Variances and Moving Averages based on 8 Quarter Window)**

|  | Variance of Revenues   | Variance of Cash Flows | Variance of marketing expenditures | Level of marketing expenditures | Marketing responsiveness | Variance of competitive marketing expenditures | Correlation between own and competitive marketing expenditures |
|--|------------------------|------------------------|------------------------------------|---------------------------------|--------------------------|--|--|
| Variance of Revenues   | 1                      |                        |                                    |                                 |                          |  |  |
| Variance of Cash Flows   | .965***                | 1                      |                                    |                                 |                          |  |  |
| Variance of marketing expenditures                             | .238***                | .383***                | 1                                  |                                 |                          |  |  |
| Level of marketing expenditures <sup>1)</sup>                  | .344***, <sup>1)</sup> | .384***, <sup>1)</sup> | .603***, <sup>1)</sup>             | 1                               |                          |  |  |
| Marketing responsiveness                                       | .072***                | .062***                | .050**                             | .085***                         | 1                        |  |  |
| Variance of competitive marketing expenditures                 | -.045**                | -.03                   | .064***                            | .168***                         | -.139***                 | 1  |  |
| Correlation between own and competitive marketing expenditures | -.086***               | -.115***               | -.065***                           | -.051**                         | .130***                  | .014   | 1  |

Notes: \*\*\*  $p < .01$ ; \*\*  $p < .05$ ; \*  $p < .10$

<sup>1)</sup> Note that this variable is not corrected for brand size. Larger brands have higher expenditure levels that involve larger variance.

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