Online Appendix for "Customer Recognition in Experience versus Inspection Good Markets"

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This technical online Appendix contains the proofs of Proposition 2 and Lemma TA5. Lemmas TA1-TA4 below are pivotal to the proof of Proposition 2.

Lemma TA1. *Suppose* $0 \leq \lambda \leq 1 - \frac{\sqrt{5}}{5} \approx 0.553$ *and* $\frac{\lambda(1-\lambda)}{2-3\lambda} \leq \frac{t}{H-L} \leq \frac{(1-\lambda)(6-5\lambda)}{2}$. *There is a SPE where each firm charges* $a^D = \frac{2(2-\lambda^2)t}{3(1-\lambda)} + \frac{2\lambda(H-L)}{3}$ *in period 1 and* $b^S = \frac{1}{3}\left[\frac{t}{1-\lambda} + \lambda(H-L)\right]$ *and* $b^R = \frac{1}{3}\left[\frac{2t}{1-\lambda} - \lambda(H-L)\right]$ *to new and repeat customers, respectively, in period 2. In equilibrium, only some of the consumers realizing a low value from the first purchase will switch brand, and those realizing a high value never switch in period 2.*

Proof. We seek an equilibrium where only some of the consumers realizing a low value switch brands. We first look at the period-2 game. Without loss of generality, suppose consumers in $[0, \theta^D]$ have purchased product 0 and consumers in $(\theta^D, 1]$ have purchased product 1 in period 1. Let b_i^S and b_i^R denote firm i's ($i = 0, 1$) prices for switchers and repeat buyers, respectively. Suppose a consumer x buying product 0 in the first period has realized a low value. In period 2, she will repeat buy product 0 if $L - b_0^R - tx \geq \overline{v} - b_1^S - t(1-x)$ or equivalently $x \le x_0^D$, where

$$x_0^D = \frac{b_1^S - b_0^R + t - \lambda(H-L)}{2t}.$$

Otherwise, she will switch to product 1. Suppose a consumer x buying product 1 in the first period has realized a low value. In period 2, she will repeat buy product 1 if $L - b_1^R - t(1-x) \ge \overline{v} - b_0^S - tx$ or equivalently $x \ge x_1^D$, where

$$x_1^D = \frac{b_1^R - b_0^S + t + \lambda(H-L)}{2t}.$$

She will switch to product 0 otherwise.

The firms' period-2 profit functions are

$$R_0^D = [\lambda\theta^D + (1-\lambda)x_0^D]b_0^R + (1-\lambda)(x_1^D - \theta^D)b_0^S$$

and

$$R_1^D = [\lambda(1-\theta^D) + (1-\lambda)(1-x_1^D)]b_1^R + (1-\lambda)(\theta^D - x_0^D)b_1^S.$$

In the expression of R_0^D, $\lambda\theta^D$ is the consumers who purchased product 0 previously and realized a high value, and $(1-\lambda)x_0^D$ captures the consumers who purchased product 0 previously, realized a low value but are located sufficiently close to product 0. Both groups will repeat purchase product 0. Those who purchased product 1, realized a low value and are located sufficiently distant from product 1 (those in (θ^D, x_1^D)) will now switch to product 0. The number of such switchers is $(1-\lambda)(x_1^D - \theta^D)$. Each term in R_1^D has an analogous interpretation.

For $i = 0, 1$, we can verify that $\frac{\partial^2 R_i}{\partial (b_i^R)^2} = \frac{\partial^2 R_i}{\partial (b_i^S)^2} = \frac{-(1-\lambda)}{t} < 0$ and $\frac{\partial^2 R_i}{\partial b_i^R \partial b_i^S} = 0$. The Hessian matrix of each firm's period-2 objective is thus negative definite. The period-2 equilibrium readily follows from the first-order conditions: $b_0^S = \frac{1}{3}\left[\frac{2(1-(2-\lambda)\theta^D)t}{1-\lambda} + t + \lambda(H-L)\right]$, $b_0^R = \frac{1}{3}\left[\frac{2(1+\lambda)\theta^D t}{1-\lambda} + t - \lambda(H-L)\right]$, $b_1^S = \frac{1}{3}\left[\frac{2(2-\lambda)\theta^D t}{1-\lambda} - t + \lambda(H-L)\right]$, and $b_1^R = \frac{1}{3}\left[\frac{2(2-(1+\lambda)\theta^D)t}{1-\lambda} - t - \lambda(H-L)\right]$

Substituting these prices into R_0^D and R_1^D above and rearranging terms, we have firm i's $(i=0,1)$ period-2 equilibrium profits $R_i^D = \frac{(1-\lambda)\left[(b_i^R)^2+(b_i^S)^2\right]}{2t}$.

We now turn to period 1. Observing the period-1 prices a_0 and a_1, each consumer decides which product to buy. Her objective is to maximize expected total surplus over both periods, anticipating the period-2 price equilibrium. To derive the period-1 consumer choice rule, it suffices to identify the consumer indifferent between products 0 and 1 in period 1, θ^D. From the discussions above, we see that if consumer θ^D purchases product i $(i=0,1)$ in period 1 and realizes a low value, she will switch to the other product in period 2. Her expected total surplus of buying product 0 in period 1 is $V_0(\theta^D) = \left(\overline{v} - a_0 - t\theta^D\right) + \lambda\left(H - b_0^R - t\theta^D\right) + (1-\lambda)\left[\overline{v} - b_1^S - t(1-\theta^D)\right]$, and that of buying product 1 in period 1 is $V_1(\theta^D) = \left[\overline{v} - a_1 - t(1-\theta^D)\right] + \lambda\left[H - b_1^R - t(1-\theta^D)\right] + (1-\lambda)\left[\overline{v} - b_0^S - t\theta^D\right]$. Setting $V_0(\theta^D) = V_1(\theta^D)$ and substituting in b_0^S, b_0^R, b_1^S, and b_1^R yield

$$\theta^D = \frac{1}{2} - \frac{3(1-\lambda)(a_0 - a_1)}{4(2+\lambda-\lambda^2)t}.$$

In period 1, firms 0 and 1 choose prices a_0 and a_1 to maximize their total expected profits $\pi_0^D = \theta^D a_0 + R_0^D$ and $\pi_1^D = (1-\theta^D)a_1 + R_1^D$, respectively. It is straightforward to verify that for $i=0,1$, $\frac{d^2\pi_i}{da_i^2} = \frac{-(1-\lambda)[7+8\lambda(1-\lambda)]}{4(2+\lambda-\lambda^2)t} < 0$, i.e., π_i is strictly concave in a_i. The firms' FOCs lead to a candidate SPE: $a_0 = a_1 = a^D$, where

$$a^D = \frac{2(2-\lambda^2)t}{3(1-\lambda)} + \frac{2\lambda(H-L)}{3},$$

and $b_0^S = b_1^S = b^S$ and $b_0^R = b_1^R = b^R$, where

$$b^S = \frac{1}{3}\left[\frac{t}{1-\lambda} + \lambda(H-L)\right] \text{ and } b^R = \frac{1}{3}\left[\frac{2t}{1-\lambda} - \lambda(H-L)\right].$$

Each firm makes total profits $\pi^D = \frac{(2-\lambda^2)t}{3(1-\lambda)} + \frac{\lambda(H-L)}{3} + R^D$, where R^D is its period-2 profits

$$R^D = \frac{1}{18(1-\lambda)t}\left\{[2t - \lambda(1-\lambda)(H-L)]^2 + [t + \lambda(1-\lambda)(H-L)]^2\right\}.$$

We can check that $x_0^D < \frac{1}{2} < x_1^D$ always holds, that $x_0^D \geq 0 \Leftrightarrow x_1^D \leq 1 \Leftrightarrow \frac{t}{H-L} \geq \frac{\lambda(1-\lambda)}{2-3\lambda}$, that $H - b^R \geq \overline{v} - b^S \Leftrightarrow \frac{t}{H-L} \leq (1-\lambda)(3-\lambda)$ (i.e., those realizing a high value do not switch). For $\frac{\lambda(1-\lambda)}{2-3\lambda} < (1-\lambda)(3-\lambda)$, we need $\lambda \leq 2 - \sqrt{2}$.

Next, we still need to ensure that neither firm has an incentive to unilaterally deviate from the period-2 equilibrium above. Due to symmetry, it suffices to rule out firm 0's such incentive. We see from above that each firm's period-2 price for switchers (b^S) is independent of that for repeat buyers (b^R), as the two prices serve separate market segments. Below, we derive the conditions for firm 0 not to deviate from either price.

First, suppose firm 0 wishes to deviate to a different price for repeat customers, b_0^{RD}. From the above deduction, it is clear that firm 0 will not deviate to any b_0^{RD} that attracts all of its own H customers. Therefore, suppose b_0^{RD} is high enough such that firm 0 loses some of its H customers to firm 1, i.e., $H - \frac{t}{2} - b_0^{RD} < \lambda H + (1-\lambda)L - \frac{t}{2} - b^S \Leftrightarrow b_0^{RD} - b^S > (1-\lambda)(H-L)$. Let y_0 denote firm 0's H customer who is indifferent to repeat buying product 0 and switching to product 1, i.e., $H - ty_0 - b_0^{RD} = \lambda H + (1-\lambda)L - t(1-y_0) - b^S$, which leads to $y_0 = \frac{b^S - b_0^{RD} + t + (1-\lambda)(H-L)}{2t}$. Firm 0's period-2 demand from repeat buyers is then $D_0^{RD} = \lambda y_0 + (1-\lambda)x_0 = \frac{b^S - b_0^{RD} + t}{2t}$, where x_0 is as given above. By deviating, firm 0 wishes to maximize $b_0^{RD} D_0^{RD}$, which leads to the optimal deviating price, $b_0^{RD} = \frac{b^S + t}{2}$. When $\lambda \leq 2 - \sqrt{2}$, we have that $b_0^{RD} - b^S > (1-\lambda)(H-L) \Leftrightarrow \frac{t}{H-L} > \frac{(1-\lambda)(6-5\lambda)}{2-3\lambda}$ and that $\frac{(1-\lambda)(6-5\lambda)}{2-3\lambda} > (1-\lambda)(3-\lambda) \Leftrightarrow \lambda < 2$, which always holds. Therefore, when $\frac{t}{H-L} \leq (1-\lambda)(3-\lambda)$, firm 0 will not unilaterally deviate from b^R.

Second, suppose firm 0 wishes to deviate to a different price for switchers, b_0^{SD}. From the above deduction, we see that firm 0 will not deviate to any b_0^{SD} that does not attract any of firm 1's H customers. Therefore, suppose b_0^{SD} is low enough such that it also attracts some of firm 1's H customers, i.e., $\lambda H + (1-\lambda)L - \frac{t}{2} - b_0^{SD} > H - \frac{t}{2} - b^R \Leftrightarrow b_0^{SD} - b^R \leq -(1-\lambda)(H-L)$. Let y_1 denote firm 1's H customer who is indifferent to repeat buying product 1 and switching to product 0, i.e., $H - t(1-y_1) - b^R = \lambda H + (1-\lambda)L - ty_1 - b_0^{SD}$, which leads to $y_1 = \frac{b^R - b_0^{SD} + t - (1-\lambda)(H-L)}{2t}$. Firm 0's period-2 demand from switchers is $D_0^{SD} = \lambda(y_1 - \frac{1}{2}) + (1-\lambda)(x_1 - \frac{1}{2}) = \frac{b^R - b_0^{SD} + t}{2t} - \frac{1}{2}$, where x_1 is as given above. Firm 0 wishes to maximize $b_0^{SD} D_0^{SD}$, which leads to the optimal deviating price, $b_0^{SD} = \frac{b^R}{2}$. When $\lambda \leq 2 - \sqrt{2}$, we have that $b_0^{SD} - b^R \leq -(1-\lambda)(H-L) \Leftrightarrow \frac{t}{H-L} \geq \frac{(1-\lambda)(6-5\lambda)}{2}$ and that $\frac{(1-\lambda)(6-5\lambda)}{2} < (1-\lambda)(3-\lambda)$ always holds. Therefore, when $\frac{\lambda(1-\lambda)}{2-3\lambda} \leq \frac{t}{H-L} \leq \frac{(1-\lambda)(6-5\lambda)}{2}$, firm 0 will not unilaterally deviate from b^S. Note that $\frac{\lambda(1-\lambda)}{2-3\lambda} \leq \frac{(1-\lambda)(6-5\lambda)}{2} \Leftrightarrow 5\lambda^2 - 10\lambda + 4 > 0$, which holds when $\lambda \leq 1 - \frac{\sqrt{5}}{5}$. Q.E.D.

Lemma TA2. *When $\frac{t}{H-L} > \frac{3(2-\lambda)}{2}$, there is a SPE where each firm charges $a^D = \frac{4t}{3}$ in period 1 and $b^R = \frac{2t}{3}$ and $b^S = \frac{t}{3}$ to repeat customers and switchers, respectively, in period 2, making total profits of $\pi^D = \frac{17t}{18}$. In equilibrium, some of the consumers realizing a low or high value switch brands in period 2.*

Proof. We now seek an equilibrium where some of the consumers realizing a low or high value switch brands in the second period. We start with analyzing the period-2 competition. Again, without loss suppose consumers in $[0, \theta^D]$ have purchased product 0 and consumers in $(\theta^D, 1]$ have purchased product 1 in period 1. Consider consumer x who has purchased product 0 previously and realized value L. In period 2, she will repeat purchase product 0 if and only if $x \leq x_0^D$. Similarly, if consumer x has purchased product 1 and realized value L in period 1, in period 2 she will repeat purchase product 1 if and only if $x \geq x_1^D$. Here x_0^D and x_1^D are as given in the proof of Lemma TA1 above.

If consumer x has purchased product 0 and realized value H, in period 2 she will repeat purchase product 0 if $H - b_0^R - tx \geq \overline{v} - b_1^S - t(1-x)$ or equivalently $x \leq y_0^D$, where

$$y_0^D = \frac{b_1^S - b_0^R + t + (1-\lambda)(H-L)}{2t}.$$

Otherwise, she will switch to product 1. If consumer x has purchased product 1 and realized value H, in period 2 she will repeat purchase product 1 if $H - b_1^R - t(1-x) \geq \overline{v} - b_0^S - tx$ or equivalently $x \geq y_1^D$, where

$$y_1^D = \frac{b_1^R - b_0^S + t - (1-\lambda)(H-L)}{2t}.$$

She will switch to product 0 otherwise.

Therefore, firms 0 and 1's period-2 profit functions are

$$R_0^D = [\lambda y_0^D + (1-\lambda)x_0^D]b_0^R + \left[\lambda(y_1^D - \theta^D) + (1-\lambda)(x_1^D - \theta^D)\right] b_0^S$$

and

$$R_1^D = [\lambda(1-y_1^D) + (1-\lambda)(1-x_1^D)]b_1^R + \left[\lambda(\theta^D - y_0^D) + (1-\lambda)(\theta^D - x_0^D)\right] b_1^S.$$

Since $\frac{\partial^2 R_i}{\partial (b_i^R)^2} = \frac{\partial^2 R_i}{\partial (b_i^S)^2} = -\frac{1}{t}$ and $\frac{\partial^2 R_i}{\partial b_i^R \partial b_i^S} = 0$, each firm i's period-2 Hessian matrix is negative definite. The unique period-2 price equilibrium then follows from the FOCs: $b_0^S = t - \frac{4t\theta^D}{3}$, $b_0^R = \frac{t+2t\theta^D}{3}$, $b_1^S = \frac{-t+4t\theta^D}{3}$, and $b_1^R = t - \frac{2t\theta^D}{3}$. We then obtain the period-2 profits: $R_0^D = \frac{(b_0^R)^2+(b_0^S)^2}{2t}$ and $R_1^D = \frac{(b_1^R)^2+(b_1^S)^2}{2t}$.

We now turn to period 1. For given period-1 prices a_0 and a_1, we first characterize the consumer indifferent to products 0 and 1, θ^D. Notice, here the consumers in $[y_0^D, y_1^D]$ always switch brands, even after realizing a high value from their first purchase. Therefore, consumer θ^D's total expected surplus of buying product 0 in period 1 is $V_0(\theta^D) = \left(\overline{v} - a_0 - t\theta^D\right) + \left[\overline{v} - b_1^S - t(1-\theta^D)\right]$, and that of buying product 1 in period 1 is $V_1(\theta^D) = \left[\overline{v} - a_1 - t(1-\theta^D)\right] + (\overline{v} - b_0^S - t\theta^D)$. Setting $V_0(\theta^D) = V_1(\theta^D)$ and substituting in b_0^S and b_1^S yield $\theta^D = \frac{1}{2} - \frac{3(a_0-a_1)}{8t}$. In period 1, firms 0 and 1 set prices a_0 and a_1 to maximize their total expected profits $\pi_0^D = \theta^D a_0 + R_0^D$ and $\pi_1^D = (1-\theta^D)a_1 + R_1^D$, respectively.

Since $\frac{d^2\pi_i^D}{da_i^2} = \frac{-7}{16t} < 0$ for $i = 0, 1$, a candidate SPE then follows from the FOCs: $a_0 = a_1 =$

a^D, where $a^D = \frac{4t}{3}$. We can further verify that in period 2 each firm charges price $b^S = \frac{t}{3}$ to switchers and $b^R = \frac{2t}{3}$ to repeat customers. Each firm makes total profits $\pi^D = \frac{17t}{18}$. In equilibrium, $x_0^D = \frac{1}{3} - \frac{\lambda(H-L)}{2t}$, $x_1^D = \frac{2}{3} + \frac{\lambda(H-L)}{2t}$, $y_0^D = \frac{1}{3} + \frac{(1-\lambda)(H-L)}{2t}$, $y_1^D = \frac{2}{3} - \frac{(1-\lambda)(H-L)}{2t}$. Note that $0 < x_0^D < \frac{1}{2} < x_1^D < 1 \Leftrightarrow \frac{t}{H-L} > \frac{3\lambda}{2}$ and that $0 < y_0^D < \frac{1}{2} < y_1^D < 1 \Leftrightarrow \frac{t}{H-L} > 3(1-\lambda)$.

Next, we need to ensure that neither firm has an incentive to unilaterally deviate from the period-2 equilibrium above. Due to symmetry, we focus on firm 0. We see from above that each firm's period-2 prices for switchers and for repeat buyers serve separate market segments. Below, we derive the conditions for firm 0 not to deviate from either price.

First, suppose firm 0 wishes to deviate to a different price for repeat customers, b_0^{RD}. From the above deduction, it is clear that firm 0 will not deviate to any b_0^{RD} that induces some of its H customers to switch to firm 1. Therefore, suppose b_0^{RD} is low enough such that it retains all of its H customers, i.e., $H - \frac{t}{2} - b_0^{RD} > \lambda H + (1-\lambda)L - \frac{t}{2} - b^S \Leftrightarrow b_0^{RD} - b^S < (1-\lambda)(H-L)$. Firm 0's demand from its own repeat customers is $D_0^{RD} = \frac{\lambda}{2} + \frac{(1-\lambda)\left[b^S - b_0^{RD} + t - \lambda(H-L)\right]}{2t}$. Maximizing $b_0^{RD} D_0^{RD}$ leads to the optimal deviating price $b_0^{RD} = \frac{b^S - \lambda(H-L)}{2} + \frac{t}{2(1-\lambda)}$. However, we have $b_0^{RD} - b^S < (1-\lambda)(H-L) \Leftrightarrow \frac{t}{H-L} < \frac{3(1-\lambda)(2-\lambda)}{2+\lambda}$, which never holds when $\frac{t}{H-L} > \max\{\frac{3\lambda}{2}, 3(1-\lambda)\}$. Therefore, when $\frac{t}{H-L} > \max\{\frac{3\lambda}{2}, 3(1-\lambda)\}$, firm 0 will not unilaterally deviate to a different price for repeat customers.

Second, suppose firm 0 wishes to deviate to a different price for switchers, b_0^{SD}. From the above deduction, we see that firm 0 will not deviate to any b_0^{SD} that attracts any of firm 1's H customers. Therefore, suppose b_0^{SD} is high enough such that it attracts only some of firm 1's L customers, i.e., $\lambda H + (1-\lambda)L - \frac{t}{2} - b_0^{SD} < H - \frac{t}{2} - b^R \Leftrightarrow b_0^{SD} > b^R - (1-\lambda)(H-L)$. Firm 0's period-2 demand from switchers is then $D_0^{SD} = (1-\lambda)\left(x_1 - \frac{1}{2}\right) = (1-\lambda)\left(\frac{b^R - b_0^{SD} + t + \lambda(H-L)}{2t} - \frac{1}{2}\right)$. Maximizing $b_0^{SD} D_0^{SD}$ leads to the optimal deviating price, $b_0^{SD} = \frac{b^R + \lambda(H-L)}{2}$. We can check that $b_0^{SD} > b^R - (1-\lambda)(H-L) \Leftrightarrow \frac{t}{H-L} < \frac{3(2-\lambda)}{2}$. Note that $\frac{3(2-\lambda)}{2} > \max\{\frac{3\lambda}{2}, 3(1-\lambda)\}$. Therefore, when $\frac{t}{H-L} > \frac{3(2-\lambda)}{2}$, firm 0 will not unilaterally deviate to a different price for switchers. Q.E.D.

Lemma TA3. *When* $\lambda > \frac{\sqrt{5}-1}{2} \approx 0.618$ *and* $\frac{\lambda[(1+2\lambda)\sqrt{1-\lambda} + 2(1-\lambda)\sqrt{\lambda}]}{2(2-\lambda)\sqrt{\lambda} - (1+\lambda)\sqrt{1-\lambda}} \leq \frac{t}{H-L} \leq \frac{\lambda(2+\lambda)}{1+\lambda}$, *there is a SPE where each firm charges price* $a^D = \frac{2}{3\lambda}[(3-\lambda)t - \lambda(1-\lambda)(H-L)]$ *in period 1 and* $b^R = \frac{(1+\lambda)t+\lambda(1-\lambda)(H-L)}{3\lambda}$ *to repeat customers and* $b^S = \frac{(2-\lambda)t-\lambda(1-\lambda)(H-L)}{3\lambda}$ *to switchers in period 2. In equilibrium, all consumers realizing a low value (L) and some of the consumers realizing a high value (H) switch brands.*

Proof. We seek an equilibrium where all consumers realizing a low value switch brands and where some of the consumers realizing a high value switch brands. Because the deduction of the current equilibrium closely parallels that of Lemma TA2, here we will be brief.

We first derive the period-2 price equilibrium. Suppose consumers in $[0, \theta^D]$ have purchased product 0 and those in $[\theta^D, 1]$ have purchased product 1 in period 1. Then, firms 0 and 1's period-2 profit functions are

$$R_0^D = \lambda y_0^D b_0^R + \left[\lambda(y_1^D - \theta^D) + (1-\lambda)(1-\theta^D)\right] b_0^S$$

and

$$R_1^D = \lambda(1-y_1^D)b_1^R + \left[\lambda(\theta^D - y_0^D) + (1-\lambda)\theta^D\right] b_1^S,$$

where y_0^D and y_1^D are as given in the proof of Lemma TA2 above.

For $i = 0, 1$, we have $\frac{\partial^2 R_i}{\partial (b_i^R)^2} = \frac{\partial^2 R_i}{\partial (b_i^S)^2} = \frac{-\lambda}{t}$ and $\frac{\partial^2 R_i}{\partial b_i^R \partial b_i^S} = 0$. The second-order conditions are thus satisfied. From the FOCs, we obtain a unique period-2 price equilibrium: $b_0^R = \frac{(2\theta+\lambda)t+\lambda(1-\lambda)(H-L)}{3\lambda}$, $b_0^S = \frac{(4-\lambda-4\theta)t-\lambda(1-\lambda)(H-L)}{3\lambda}$, $b_1^R = \frac{(2+\lambda-2\theta)t+\lambda(1-\lambda)(H-L)}{3\lambda}$, and $b_1^S = \frac{(4\theta-\lambda)t-\lambda(1-\lambda)(H-L)}{3\lambda}$. Each firm i's period-2 profits are $R_i = \frac{\lambda}{2t}\left[\left(b_i^R\right)^2 + \left(b_i^S\right)^2\right]$.

We now turn to period 1. Given period-1 prices a_0 and a_1, we can easily check that the consumer indifferent to products 0 and 1 is $\theta^D = \frac{1}{2} - \frac{3\lambda(a_0-a_1)}{8t}$. Firms 0 and 1's period-1 objectives are $\pi_0^D = \theta^D a_0 + R_0^D$ and $\pi_1^D = (1-\theta^D)a_1 + R_1^D$, respectively. Since $\frac{d^2\pi_i^D}{da_i^2} = \frac{-7}{16t} < 0$ for $i = 0, 1$, the second-order conditions are satisfied. The SPE then follows from the FOCs: $a_0 = a_1 = a^D$, where $a^D = \frac{2}{3\lambda}[(3-\lambda)t - \lambda(1-\lambda)(H-L)]$. In period 2, each firm charges $b^R = \frac{(1+\lambda)t+\lambda(1-\lambda)(H-L)}{3\lambda}$ to repeat customers and $b^S = \frac{(2-\lambda)t-\lambda(1-\lambda)(H-L)}{3\lambda}$ to switchers. We have $y_0^D = \frac{(1+\lambda)t+\lambda(1-\lambda)(H-L)}{6\lambda t}$ and $y_1^D = \frac{(-1+5\lambda)t-\lambda(1-\lambda)(H-L)}{6\lambda t}$. We can easily check that $y_0^D > 0$ and $y_1^D < 1$ always hold, and that $y_0^D \leq \frac{1}{2} \leq y_1^D \Leftrightarrow \frac{t}{H-L} \geq \frac{\lambda(1-\lambda)}{2\lambda-1}$. When $L - b^R \leq \overline{v} - t - b^S \Leftrightarrow \frac{t}{H-L} \leq \frac{\lambda(2+\lambda)}{1+\lambda}$, those realizing a low value always switch. We also have $\frac{\lambda(1-\lambda)}{2\lambda-1} \leq \frac{\lambda(2+\lambda)}{1+\lambda} \Leftrightarrow \lambda \geq \frac{\sqrt{5}-1}{2}$.

Next, we need to ensure that neither firm has an incentive to unilaterally deviate from the period-2 equilibrium above. Due to symmetry, we focus on firm 0. We see from above that each firm's period-2 prices for switchers (b^S) and for repeat buyers (b^R) serve separate market segments. Below, we derive the conditions for firm 0 not to deviate from either price.

First, suppose firm 0 wishes to deviate to a different price for repeat customers, b_0^{RD}. Again, firm 0 will not deviate to any b_0^{RD} that induces some of its H customers to switch to firm 1. Therefore, suppose b_0^{RD} is low enough such that it retains all of its H customers, i.e., $b_0^{RD} - b^S < (1-\lambda)(H-L)$. We can easily derive the optimal deviating price, $b_0^{RD} = \frac{b^S - \lambda(H-L)}{2} + \frac{t}{2(1-\lambda)}$. When $\lambda > \frac{\sqrt{5}-1}{2}$, we have $\lambda^2 - 6\lambda + 2 < 0$ and $b_0^{RD} - b^S < (1-\lambda)(H-L) \Leftrightarrow \frac{t}{H-L} < \frac{\lambda(1-\lambda)(5-2\lambda)}{-(\lambda^2-6\lambda+2)}$. When $\lambda > \frac{\sqrt{5}-1}{2}$ and $\frac{t}{H-L} > \frac{\lambda(1-\lambda)}{2\lambda-1}$, the last inequality never holds since $\frac{\lambda(1-\lambda)(5-2\lambda)}{-(\lambda^2-6\lambda+2)} < \frac{\lambda(1-\lambda)}{2\lambda-1} \Leftrightarrow (1-\lambda)^2 > 0$. Therefore, firm 0 will not unilaterally deviate from b^R when $\lambda > \frac{\sqrt{5}-1}{2}$ and when $\frac{\lambda(1-\lambda)}{2\lambda-1} \leq \frac{t}{H-L} \leq \frac{\lambda(2+\lambda)}{1+\lambda}$.

Second, suppose firm 0 wishes to deviate to a different price for switchers, b_0^{SD}. Again, suppose b_0^{SD} is high enough such that it attracts only some of firm 1's L customers, i.e., $b_0^{SD} > b^R - (1-\lambda)(H-L)$. Firm 0's period-2 demand from switchers is then $D_0^{SD} = (1-\lambda)\left(\frac{b^R - b_0^{SD} + t + \lambda(H-L)}{2t} - \frac{1}{2}\right)$. Maximizing $b_0^{SD} D_0^{SD}$ leads to the optimal deviating price, $b_0^{SD} = \frac{b^R + \lambda(H-L)}{2}$, and the associated profit from switchers, $\frac{(1-\lambda)[b^R + \lambda(H-L)]^2}{8t}$. We can check that $b_0^{SD} > b^R - (1-\lambda)(H-L) \Leftrightarrow \frac{t}{H-L} < \frac{\lambda(5-2\lambda)}{1+\lambda}$, which always holds when $\frac{\lambda(1-\lambda)}{2\lambda-1} \leq \frac{t}{H-L} \leq \frac{\lambda(2+\lambda)}{1+\lambda}$, since $\frac{\lambda(2+\lambda)}{1+\lambda} < \frac{\lambda(5-2\lambda)}{1+\lambda}$. From the above deduction process, firm 0's profit from switchers in the candidate equilibrium is $\frac{\lambda(b^S)^2}{2t}$. When $\lambda > \frac{\sqrt{5}-1}{2}$, we have that $2(2-\lambda)\sqrt{\lambda} - (1+\lambda)\sqrt{1-\lambda} > 0$ and that $\frac{(1-\lambda)[b^R + \lambda(H-L)]^2}{8t} < \frac{\lambda(b^S)^2}{2t}$ is equivalent to $\frac{t}{H-L} > \frac{\lambda[(1+2\lambda)\sqrt{1-\lambda} + 2(1-\lambda)\sqrt{\lambda}]}{2(2-\lambda)\sqrt{\lambda} - (1+\lambda)\sqrt{1-\lambda}}$. When $\lambda > \frac{\sqrt{5}-1}{2}$, we can further verify that $\frac{\lambda(1-\lambda)}{2\lambda-1} < \frac{\lambda[(1+2\lambda)\sqrt{1-\lambda} + 2(1-\lambda)\sqrt{\lambda}]}{2(2-\lambda)\sqrt{\lambda} - (1+\lambda)\sqrt{1-\lambda}}$ is equivalent to $2(1-\lambda)\sqrt{1-\lambda} < \lambda\sqrt{\lambda}$, which always holds. Therefore, when $\frac{\lambda[(1+2\lambda)\sqrt{1-\lambda} + 2(1-\lambda)\sqrt{\lambda}]}{2(2-\lambda)\sqrt{\lambda} - (1+\lambda)\sqrt{1-\lambda}} \leq \frac{t}{H-L} \leq \frac{\lambda(2+\lambda)}{1+\lambda}$, firm 0 will not unilaterally deviate from b^S. Q.E.D.

Lemma TA4. *(1). There is no pure-strategy SPE where all consumers realizing a low value switch and no consumer realizing a high value switches in period 2. (2). There is no pure-strategy SPE where all consumers switch brands in period 2.*

Proof. (1). Suppose there is a pure-strategy SPE where all consumers realizing L switch brands and no consumer realizing H switches brands in period 2. Suppose that in equilibrium consumers in $[0, \theta]$ purchase product 0 and those in $[\theta, 1]$ purchase product 1 in period 1, and that firm i ($i = 0, 1$) charges b_i^R and b_i^S to repeat customers and switchers, respectively, in period 2. The firms' period-2 profits are then $R_0 = \lambda\theta b_0^R + (1-\lambda)(1-\theta)b_0^S$ and $R_1 = \lambda(1-\theta)b_1^R + (1-\lambda)\theta b_1^S$, respectively. After realizing L, consumer 0 switches to product 1 iff $L - b_0^R \leq \overline{v} - b_1^S - t$. After purchasing product 0 and realizing H, consumer θ will repeat buy product 0 iff $H - b_0^R - t\theta \geq \overline{v} - b_1^S - t(1-\theta)$. We therefore have $t - \lambda(H-L) \leq b_0^R - b_1^S \leq (1-\lambda)(H-L) + t(1-2\theta)$. However, if $b_0^R - b_1^S > t - \lambda(H-L)$, firm 1 benefits from slightly raising b_1^S. If $b_0^R - b_1^S = t - \lambda(H-L)$, firm 0 benefits from slightly raising b_0^R. A contradiction.

(2). Suppose there is a pure-strategy SPE where all consumers switch brands in period 2. Suppose that in equilibrium consumers in $[0, \theta]$ ($[\theta, 1]$) purchase product 0 (1) in period 1. Then, the firms' period-2 profits are $R_0 = (1-\theta)b_0^S$ and $R_1 = \theta b_1^S$. After realizing H, consumer 0 switches to 1 iff $H - b_0^R \leq \overline{v} - b_1^S - t \Leftrightarrow b_0^R \geq b_1^S + t + (1-\lambda)(H-L)$. However, by unilaterally deviating to a price $b_0^R < b_1^S + t + (1-\lambda)(H-L)$ firm 0 can also sell to some of the consumers who realize H with its product while still selling to all consumers in $[\theta, 1]$ at price b_0^S. This strictly increases firm 0's period-2 profits. A contradiction. Q.E.D.

Proof of Proposition 2. We note that there are only five possible brand-switching patterns in period 2: (1) only some of the L consumers switch, (2) some of the L and H consumers switch, (3) some of the H consumers and all L consumers switch, (4) all L consumers switch but no H consumer switchers, and (5) all consumers (L and H) switch. Note that the conditions in Lemmas TA1-3 do not overlap. The statements of the Proposition then directly follow from Lemmas TA1-4 above. Q.E.D.

Lemma TA5. *Suppose ex ante consumers observe their values of both products. Without BPD, there is a unique symmetric SPE where each firm prices at t in both periods and makes total profits of t. With BPD, there is a unique symmetric SPE where each firm prices at $\frac{4t}{3}$ in period 1 and charges prices $\frac{2t}{3}$ to repeat customers and $\frac{t}{3}$ to switchers, making total profits of $\frac{17t}{18}$.*

Proof. (1). In the inspection good duopoly, ex ante consumers observe their values of both products. Without BPD each consumer's period-2 choice only depends on the firms' period-2 prices, and not on her period-1 purchase history. The equilibrium thus reduces to replications of the static equilibrium, which we now derive. Let p_0 and p_1 denote the prices of firms 0 and 1, respectively, in the static model. There are $\lambda^2 + (1-\lambda)^2$ consumers (called the Type-1 consumers) who value both goods equally (at L or H), $\lambda(1-\lambda)$ consumers who value good 0 at H and good 1 at L (called the Type-2 consumers), and $\lambda(1-\lambda)$ consumers who value good 0 at L and good 1 at H (the Type-3 consumers). Let θ_k $(k = 1, 2, 3)$ denote the Type-k consumer indifferent between the two goods, i.e., $v - t\theta_1 - p_0 = v - t(1-\theta_1) - p_1$, $H - t\theta_2 - p_0 = L - t(1-\theta_2) - p_1$, $L - t\theta_3 - p_0 = H - t(1-\theta_3) - p_1$. We then have $\theta_1 = \frac{p_1 - p_0 + t}{2t}$, $\theta_2 = \frac{p_1 - p_0 + t + (H-L)}{2t}$, and $\theta_3 = \frac{p_1 - p_0 + t - (H-L)}{2t}$. Firm 0's demand is

$$D_0 = [\lambda^2 + (1-\lambda)^2]\theta_1 + \lambda(1-\lambda)(\theta_2 + \theta_3) = \frac{p_1 - p_0 + t}{2t}.$$

Firms 0 and 1's per-period profit functions are $\frac{(p_1 - p_0 + t)p_0}{2t}$ and $\left[1 - \frac{p_1 - p_0 + t}{2t}\right] p_1$, respectively. The unique equilibrium is then $p_0 = p_1 = t$. Each firm thus makes total profits of t.

(2). Each firm i $(i = 0, 1)$ charges a single price a_i in period 1 due to the lack of consumer purchase history. In period 2, firm i charges prices b_i^R to its repeat buyers and b_i^S to switchers.

We start with analyzing the period-2 competition. It is simple to verify that in period 1, if a Type-k consumer x purchases product 0, then the Type-k consumers in $[0, x)$ will also purchase product 0. Similarly, if a Type-k consumer x purchases product 1, then the Type-k consumers in $(x, 1]$ will also purchase product 1. Therefore, without loss of generality let θ_k

$(k = 1, 2, 3)$ denote the Type-k consumer indifferent between the two goods in period 1.

We now identify each Type-k consumer's period-2 choice. If a Type-1 consumer x purchases product 0 in period 1, in period 2 she will repeat purchase 0 if and only if $v - tx - b_0^R \geq v - t(1-x) - b_1^S$ or $x \leq x_{1A} \equiv \frac{b_1^S - b_0^R + t}{2t}$. Here v equals either H or L. If a Type-1 consumer x purchases product 1 in period 1, she will repeat purchase 1 if and only if $v - t(1-x) - b_1^R \geq v - tx - b_0^S$ or $x \geq x_{1B} \equiv \frac{b_1^R - b_0^S + t}{2t}$. If a Type-2 consumer x purchases product 0 in period 1, she will repeat purchase 0 if and only if $H - tx - b_0^R \geq L - t(1-x) - b_1^S$ or $x \leq x_{2A} \equiv \frac{b_1^S - b_0^R + t + (H-L)}{2t}$. If a Type-2 consumer x purchases product 1 in period 1, she will repeat purchase 1 if and only if $L - t(1-x) - b_1^R \geq H - tx - b_0^S$ or $x \geq x_{2B} \equiv \frac{b_1^R - b_0^S + t + (H-L)}{2t}$. Similarly, if a Type-3 consumer x purchases product 0 in period 1, she will reeapt purchase 0 if and only if $L - tx - b_0^R \geq H - t(1-x) - b_1^S$ or $x \leq x_{3A} \equiv \frac{b_1^S - b_0^R + t - (H-L)}{2t}$. If a Type-3 consumer x purchases product 1 in period 1, she will repeat purchase 1 if and only if $H - t(1-x) - b_1^R \geq L - tx - b_0^S$ or $x \geq x_{3B} \equiv \frac{b_1^R - b_0^S + t - (H-L)}{2t}$.

In period 2, firm 0's demand from repeat customers is

$$D_0^R = [\lambda^2 + (1-\lambda)^2] x_{1A} + \lambda(1-\lambda)(x_{2A} + x_{3A}),$$

and its demand from switchers is

$$D_0^S = [\lambda^2 + (1-\lambda)^2](x_{1B} - \theta_1) + \lambda(1-\lambda)\left[(x_{2B} - \theta_2) + (x_{3B} - \theta_3)\right].$$

Firm 1's demand from repeat customers is

$$D_1^R = [\lambda^2 + (1-\lambda)^2](1 - x_{1B}) + \lambda(1-\lambda)\left[(1 - x_{2B}) + (1 - x_{3B})\right],$$

and its demand from switchers is

$$D_1^S = [\lambda^2 + (1-\lambda)^2](\theta_1 - x_{1A}) + \lambda(1-\lambda)\left[(\theta_2 - x_{2A}) + (\theta_3 - x_{3A})\right].$$

Each firm i's period-2 profit function is $R_i = D_i^R b_i^R + D_i^S b_i^S$. Let $\Phi \equiv [\lambda^2 + (1-\lambda)^2]\theta_1 + \lambda(1-\lambda)(\theta_2+\theta_3)$. The firms' period-2 first-order conditions are $\frac{\partial R_0}{\partial b_0^R} = \frac{b_1^S - 2b_0^R + t}{2t} = 0$, $\frac{\partial R_0}{\partial b_0^S} = \frac{b_1^R - 2b_0^S + t}{2t} - \Phi = 0$, $\frac{\partial R_1}{\partial b_1^R} = 1 - \frac{2b_1^R - b_0^S + t}{2t} = 0$, and $\frac{\partial R_1}{\partial b_1^S} = \Phi - \frac{2b_1^S - b_0^R + t}{2t} = 0$. We can check that $\frac{\partial R_0}{\partial b_0^R} = 0$ and $\frac{\partial R_1}{\partial b_1^S} = 0$ jointly lead to $b_0^R = \frac{(1+2\Phi)t}{3}$ and $b_1^S = \frac{(-1+4\Phi)t}{3}$, and that $\frac{\partial R_0}{\partial b_0^S} = 0$ and $\frac{\partial R_1}{\partial b_1^R} = 0$ jointly lead to $b_0^S = \frac{(3-4\Phi)t}{3}$ and $b_1^R = \frac{(3-2\Phi)t}{3}$. Straightforward but tedious algebra then verifies $R_i = \frac{1}{2t}\left[\left(b_i^R\right)^2 + \left(b_i^S\right)^2\right]$.

Next, we turn to period 1. Recall, by construction θ_k $(k = 1,2,3)$ represents the Type-k consumer indifferent to the two goods in period 1. For example, consider θ_2. Buying product 0 in period 1 yields her total surplus $V_0(\theta_2) = (H - a_0 - t\theta_2) + \left[L - b_1^S - t(1-\theta_2)\right]$. Buying product 1 yields her total surplus $V_1(\theta_2) = [L - a_1 - t(1-\theta_2)] + \left(H - b_0^S - t\theta_2\right)$. That $V_0(\theta_2) = V_1(\theta_2)$ leads to

$$b_0^S - b_1^S = a_0 - a_1.$$

We can easily check that this relation is also obtained by analyzing the period-1 choices of θ_1 and θ_3. Substituting b_0^S and b_1^S into the above equation and rearranging, we have

$$\Phi = \frac{1}{2} - \frac{3(a_0 - a_1)}{8t}.$$

Note that by construction Φ stands for firm 0's period-1 demand. Firms 0 and 1's period-1 objective functions are

$$\pi_0 = \left[\frac{1}{2} - \frac{3(a_0 - a_1)}{8t}\right] a_0 + R_0$$

and

$$\pi_1 = \left[\frac{1}{2} + \frac{3(a_0 - a_1)}{8t}\right] a_1 + R_1,$$

respectively. The firms' period-1 first-order conditions are $\frac{d\pi_0}{da_0} = \frac{1}{2} + \frac{-7a_0 + a_1}{16t} = 0$ and $\frac{d\pi_1}{da_1} = \frac{1}{2} + \frac{a_0 - 7a_1}{16t} = 0$. These two equations jointly lead to $a_0 = a_1 = \frac{4t}{3}$ and $\Phi = \frac{1}{2}$. We then readily obtain $b_0^R = b_1^R = \frac{2t}{3}$, $b_0^S = b_1^S = \frac{t}{3}$. Each firm makes total profits of $\frac{17t}{18}$. Q.E.D.