Online Supplemental Appendix to

Online Shopping Intermediaries:

The Strategic Design of Search Environments

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In this online supplemental appendix, we provide arguments for two claims made in the main article.

SA.1. Evaluation Costs Quadratic in Breadth

The consumer's evaluation cost function in section 2 specified that search costs are linear in the breadth b but quadratic in depth d. We can show that the main result in Proposition 1 can continue to hold even for a search cost specification that is quadratic in both dimensions.

Proposition SA.1 *Suppose the consumer's evaluations cost is* $f(b,d;s) = \tau\left(\frac{b}{n}\right)^2 d^2$. *Let* $\tau > \mu(n\backslash e)^2$. *If* $n > e$ *then* $s^* = 1 - \frac{\mu}{2\tau}\left(\frac{n}{e}\right)^2 < 1$.

Proof: Let $s \in [0,1]$ be an arbitrary search environment and the consumer's evaluation objective be a modified version of (B1). Then the consumer's optimal evaluation plan is

$$\hat{b}(s) = \begin{cases} e & \text{if } s < \tilde{s} \\ \sqrt{\frac{\mu n^2}{2(1-s)\tau}} & \text{if } s \geq \tilde{s} \end{cases} \quad \text{and} \quad \hat{d}(s) = \begin{cases} \frac{\mu n^2}{2(1-s)\tau e^2} & \text{if } s < \tilde{s} \\ 1 & \text{if } s \geq \tilde{s} \end{cases}$$

where $\tilde{s} = 1 - \frac{\mu n^2}{2\tau e^2}$. Equilibrium prices $\hat{p}(s) = \frac{\mu \hat{d}(s)}{1-\frac{1}{\hat{b}(s)}} \left(= \frac{\mu}{1-\sqrt{\frac{2(1-s)\tau}{\mu n^2}}}\right)$ are clearly decreasing in $s > \tilde{s}$ and therefore s^* cannot exceed $\tilde{s}$. When $s < \tilde{s}$, equilibrium prices $\hat{p}(s) = \frac{\mu \hat{d}(s)}{1-\frac{1}{\hat{b}(s)}} \left(= \frac{\frac{(\mu n)^2}{2(1-s)\tau e^2}}{1-\frac{1}{e}}\right)$ are increasing in s. This implies that the optimal level of search aids $s^* = \tilde{s}$. ■

SA.2. Competing Intermediaries

The two intermediaries are located at the two ends of a Hotelling line. We show that the main result in Proposition 1 remains when the transportation cost (t) is sufficiently large. However, when the transportation cost is small, we show that it is optimal for the intermediary to provide full aids ($s^* = 1$).

Proposition SA. 2

(i) *When intermediaries are relatively differentiated* ($t \geq \bar{t} \equiv \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu$)*, the equilibrium outcomes are the same as in Proposition 1.*

(ii) *Otherwise, in equilibrium, the intermediary minimizes search costs in the search environment* ($s^* = 1$). *The symmetric equilibrium price is* $\mu\left(\frac{n}{n-1}\right)$.

Proof: We claim that there is a symmetric equilibrium in which both intermediaries set

$$s_j^* = \begin{cases} 1 - \frac{\mu}{\tau}e^{-2} & t \geq \bar{t} \\ 1 & 0 \leq t < \bar{t}, \end{cases}$$

where $\bar{t} \equiv \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu$.

To prove this claim we demonstrate directly that one intermediary, intermediary 1, cannot be more profitable by deviating from s_1^* given that the other intermediary, intermediary 2, chooses s_2^*.

(i) Suppose $t \geq \bar{t}$ and consider any deviation $s_1 \neq s_1^* = s_2^* = 1 - \frac{\mu}{\tau}e^{-2}$, with the corresponding profits denoted by $\tilde{\pi}_1(s_1)$. Any deviation $s_1 \in \left[0, 1 - \frac{\mu}{\tau}e^{-2}\right)$ leads to profits

$$\tilde{\pi}_1(s_1) = \frac{1}{2t}\left(\frac{1}{e^2-1}\right)\frac{\rho\mu^2}{(1-s_1)\tau}\left[t - \frac{1}{e^2(e^2-1)}\frac{\mu^2}{(1-s_1)\tau} + \frac{\mu}{e^2-1}\right],$$

which we show is increasing on this interval. Specifically, $\frac{\partial\tilde{\pi}_1}{\partial s_1} > 0$ as long as

$$t > \left[\frac{2\mu}{e^2(e^2-1)(1-s_1)\tau} - \frac{1}{e^2-1}\right]\mu.$$

for all s_1. We have,

$$t \geq \bar{t} = \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu > \frac{\mu}{e^2-1} > \left[\frac{2\mu}{e^2(e^2-1)(1-s_1)\tau} - \frac{1}{e^2-1}\right]\mu.$$

where the first inequality holds by assumption, the second since $n > e^2$, and the third for $s_1 \in \left[0, 1 - \frac{\mu}{\tau}e^{-2}\right)$. Therefore, any deviation $s_1 < s_1^* = 1 - \frac{\mu}{\tau}e^{-2}$ is not profitable.

Any deviation $s_1 \in \left(1 - \frac{\mu}{\tau}e^{-2}, 1 - \frac{\mu}{n\tau}\right)$ leads to

$$\tilde{\pi}_1(s_1) = \left(\frac{1}{2t}\right)\frac{\rho\mu^2}{\mu-(1-s_1)\tau}\left\{t - \frac{\mu^2}{\mu-(1-s_1)\tau} + \mu\ln\left[\frac{\mu}{(1-s_1)\tau}\right] - \mu - \left[-\frac{\mu}{1-\frac{1}{e^2}} + \mu\right]\right\}.$$

This deviation is not profitable if $\frac{\partial \tilde{\pi}_1}{\partial s_1} < 0$, which requires

$$t > \mu f(s_1) \equiv \left\{\frac{\mu}{(1-s_1)\tau} + \frac{2\mu}{\mu-(1-s_1)\tau} - \ln\left[\frac{\mu}{(1-s_1)\tau}\right] - \frac{1}{e^2-1}\right\}\mu.$$

Since $t > \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu > (e^2 + \frac{1}{e^2-1})\mu = \mu f\left(1 - \frac{\mu}{\tau}e^{-2}\right)$, profits are decreasing near (and to the right of) s_1^*. Note that the function $f(s_1)$ is strictly increasing in s_1. Therefore, the condition $\pi_1^* > \tilde{\pi}(s_1)$ at $s_1 = 1 - \frac{\mu}{n\tau}$, the right endpoint of the interval, is sufficient for $\pi_1^* > \tilde{\pi}(s_1)$ at any $s_1 \in \left(1 - \frac{\mu}{\tau}e^{-2}, 1 - \frac{\mu}{n\tau}\right)$. This condition is

$$\pi_1^* = \frac{\rho\mu}{2}\left(\frac{e^2}{e^2-1}\right) > \left(\frac{\rho\mu}{2t}\right)\left(\frac{n}{n-1}\right)\left[t + \mu\ln(n) - \frac{n}{n-1}\mu + \left(\frac{e^2}{e^2-1}\right)\mu - 2\mu\right] = \tilde{\pi}_1\left(1 - \frac{\mu}{n\tau}\right),$$

which holds by our assumption $t > \bar{t} = \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu$.

Any deviation $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right]$ leads to a profit of

$$\tilde{\pi}_1(s_1) = \left(\frac{\rho\mu}{2t}\right)\left(\frac{n}{n-1}\right)\left[t + \mu\ln(n) - \frac{n}{n-1}\mu - (1-s_1)n\tau + \frac{\mu}{e^2-1}\right],$$

which is increasing in s_1 and therefore bounded above by $\tilde{\pi}(s_1 = 1)$. The condition $t > \bar{t} = \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu$ directly implies that

$$\pi_1^* = \frac{\rho\mu}{2}\left(\frac{e^2}{e^2-1}\right) > \left(\frac{\rho\mu}{2t}\right)\left(\frac{n}{n-1}\right)\left[t + \mu\ln(n) - \frac{n}{n-1}\mu + \frac{\mu}{e^2-1}\right] = \tilde{\pi}_1(1) \geq \tilde{\pi}_1(s_1),$$

for all $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right]$.

(ii) Now suppose $t \leq \bar{t} = \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu$. We first consider any deviation $s_1 \in \left[0, 1 - \frac{\mu}{\tau}e^{-2}\right)$. This leads to a deviation profit of

$$\tilde{\pi}_1(s_1) = \frac{\rho\mu^2}{2t\tau}\left[\frac{1}{(1-s_1)(e^2-1)}\right]\left\{t - \frac{\mu^2}{\tau e^2}\left[\frac{1}{(1-s_1)(e^2-1)}\right] + \mu\left[\frac{n}{n-1} - \ln(n)\right]\right\}.$$

Characterizing the shape of this deviation profit function depends on the level of t. We argue that $\tilde{\pi}_1(s_1) \leq \pi_1^*$ for different three levels of t.

For $0 < t < \left[\ln(n) - \frac{n}{n-1}\right]\mu$, the expression for the demand at intermediary 1,

$$\tilde{D}_1 = \frac{1}{2t}\left\{t - \frac{\mu^2}{\tau e^2}\frac{1}{(1-s_1)(e^2-1)} + \mu\left[\frac{n}{n-1} - \ln(n)\right]\right\} < 0.$$

So any deviation under this condition is not profitable.

For $\left[\ln(n) - \frac{n}{n-1}\right]\mu \leq t \leq \left[\ln(n) - \frac{n}{n-1} + \frac{2}{e^2-1}\right]\mu$, the derivative $\partial\tilde{\pi}_1/\partial s_1$ has the following property:

$$\left.\frac{\partial\tilde{\pi}_1}{\partial s_1}\right|_{s_1=1-\frac{\mu}{\tau}e^{-2}} < 0 < \left.\frac{\partial\tilde{\pi}_1}{\partial s_1}\right|_{s_1=0}.$$

Since the derivative is continuous, it means that any maximizer, $\hat{s}_1$, of $\tilde{\pi}_1(s_1)$ in $\left[0,1-\frac{\mu}{\tau}\mathrm{e}^{-2}\right)$ must solve $\frac{\partial\tilde{\pi}_1}{\partial s_1}=0$. This solution is expressed $\hat{s}_1 = 1-\frac{2\mu}{\tau}\left\{e^2(e^2-1)\left[\frac{t}{\mu}-\ln(n)+\frac{n}{n-1}\right]\right\}^{-1}$ and leads to profits

$$\tilde{\pi}_1(\hat{s}_1) = \left(\frac{e^2\rho\mu}{8}\right)\left[\frac{t}{\mu}-\ln(n)+\frac{n}{n-1}\right]\left\{1-\frac{\mu}{t}\left[\ln(n)-\frac{n}{n-1}\right]\right\}.$$

Under the condition that $t \le \left[\ln(n)-\frac{n}{n-1}+\frac{2}{e^2-1}\right]\mu$,

$$\tilde{\pi}_1(\hat{s}_1) \le \frac{1}{4}\left(\frac{e^2\rho\mu}{e^2-1}\right)\left(\frac{\frac{2}{e^2-1}}{\ln(n)-\frac{n}{n-1}+\frac{2}{e^2-1}}\right) < \frac{\rho\mu}{2(1-1/n)},$$

where the last term is the profit π_1^* the intermediary earns by sticking to $s_1^*=1$. Hence, no deviation $s_1 \in \left[0,1-\frac{\mu}{\tau}e^{-2}\right)$ is profitable.

For $[\ln(n)-\frac{n}{n-1}+\frac{2}{e^2-1}]\mu < t \le \bar{t} = \frac{n(e^2-1)}{n-e^2}\left[\ln(n)-\frac{n}{n-1}+\frac{1}{e^2-1}\right]\mu$, we have $\frac{\partial\tilde{\pi}_1}{\partial s_1} > 0$ for all $s_1 \in \left[0,1-\frac{\mu}{\tau}\mathrm{e}^{-2}\right)$. Thus,

$$\tilde{\pi}_1(s_1) \le \tilde{\pi}_1\left(1-\frac{\mu}{\tau}e^{-2}\right) = \frac{\rho\mu}{2t}\left(\frac{e^2}{e^2-1}\right)\left\{t-\mu\left[\frac{1}{e^2-1}-\frac{n}{n-1}+\ln(n)\right]\right\} \le \pi_1^*,$$

for all $s_1 \in \left[0,1-\frac{\mu}{\tau}e^{-2}\right)$. Hence, for any $t \le \bar{t} = \frac{n(e^2-1)}{n-e^2}\left[\ln(n)-\frac{n}{n-1}+\frac{1}{e^2-1}\right]\mu$, there is no profitable deviation for any $s_1 \in \left[0,1-\frac{\mu}{\tau}e^{-2}\right)$.

Now consider deviations $s_1 \in \left(1-\frac{\mu}{\tau}e^{-2}, 1-\frac{\mu}{n\tau}\right)$ when $s_2^*=1$.

Intermediary 1's profit is given by

$$\tilde{\pi}_1(s_1) = \frac{\rho\mu^2}{\mu-(1-s_1)\tau}\left(\frac{1}{2t}\right)\left\{t+\mu\left[\ln\left(\frac{\mu}{(1-s_1)\tau}\right)-\frac{\mu}{\mu-(1-s_1)\tau}-1-\ln(n)+\frac{n}{n-1}\right]\right\}.$$

It can be shown that

$$\frac{\partial\pi_1}{\partial s_1} > 0 \iff t < \{\frac{\mu}{(1-s_1)\tau}+\frac{2\mu}{\mu-(1-s_1)\tau}-\ln\left[\frac{\mu}{(1-s_1)\tau}\right]-\frac{n}{n-1}+\ln(n)\}\mu \equiv \mu f(s_1).$$

where $f(s_1) > 0$ is increasing on $\left(1-\frac{\mu}{\tau}e^{-2}, 1-\frac{\mu}{n\tau}\right)$.

Suppose $0 \le t \le [e^2+\frac{2}{e^2-1}-\frac{n}{n-1}+\ln(n)]\mu = \mu f\left(1-\frac{\mu}{\tau}e^{-2}\right)$. Then $\frac{\partial\pi_1}{\partial s_1} > 0$ for all $s_1 \in \left(1-\frac{\mu}{\tau}e^{-2},\ 1-\frac{\mu}{n\tau}\right)$. Thus,

$$\tilde{\pi}_1(s_1) \le \tilde{\pi}_1\left(1-\frac{\mu}{n\tau}\right) = \frac{\rho\mu}{2}\left(\frac{n}{n-1}\right)\left(1-\frac{\mu}{t}\right)$$

for all $s_1 \in \left(1 - \frac{\mu}{\tau}e^{-2}, 1 - \frac{\mu}{n\tau}\right)$. However, by choosing $s_1^* = 1$, intermediary 1 earns $\pi_1^* = \frac{\rho\mu}{2(1-1/n)}$, which exceeds $\tilde{\pi}_1\left(1 - \frac{\mu}{n\tau}\right)$.

Suppose $[e^2 + \frac{2}{e^2-1} - \frac{n}{n-1} + \ln(n)]\mu < t < \bar{t} = \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]\mu$. Then $\frac{\partial \pi_1}{\partial s_1} < 0$ near $s_1 = 1 - \frac{\mu}{\tau}e^{-2}$. In this case, $\tilde{\pi}_1(s_1)$ is bounded by either $\tilde{\pi}_1\left(s_1 = 1 - \frac{\mu}{\tau}e^{-2}\right)$ or $\tilde{\pi}_1\left(s_1 = 1 - \frac{\mu}{n\tau}\right)$. We know from above that both of these values are exceeded by the profit π_1^*. Hence there is no profitable deviation $s_1 \in \left(1 - \frac{\mu}{\tau}e^{-2}, 1 - \frac{\mu}{n\tau}\right)$.

Finally consider any deviation $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right)$. This leads to profits given by

$$\tilde{\pi}_1(s_1) = \frac{\rho\mu}{2t}\left(\frac{n}{n-1}\right)[t - (1 - s_1)n\tau],$$

which is obviously increasing in s_1. Therefore, choosing $s_1^* = 1$ gives intermediary 1 more profit than any in $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right)$.■