Online Appendix

Formal Description of the SUE Refinement

The SUE refinement constrains off-equilibrium beliefs in a candidate PBE using the on-equilibrium beliefs in an alternative PBE. If the candidate PBE does not survive such a belief restriction, then it is said to be *defeated* by the alternative PBE. A SUE is a PBE that is not defeated by any alternative PBE. Consider a candidate PBE in which the deal price d is not used in equilibrium. Suppose there exists an alternative PBE in which the deal price d is used in equilibrium and at least one of the merchant types that sets this deal price realizes higher profit than in the candidate PBE. Then, the refinement requires that the beliefs following the deal price d in the candidate PBE do not assign lower probability than the alternative PBE to the merchant types that are strictly better off in the alternative PBE. Formally, let T denote the set of merchant types that set the deal price d in the alternative PBE. Let $T_1 \subseteq T$ denote the set of types that realize strictly higher profit in the alternative PBE than in the candidate PBE. Let $\mu(t \mid d)$ denote the belief in the candidate equilibrium that the merchant's type is t at some information set following a deal price d. Let $\mu'(t \mid d)$ denote the corresponding belief in the alternative equilibrium. The candidate PBE is not defeated by the alternative PBE iff $\mu(t \mid d) \geq \mu'(t \mid d)$ for any $t \in T_1$.

Two implications follow from such a belief restriction. First, a candidate PBE will be defeated if the type H merchant is better off in the alternative PBE. This is because consumer beliefs at the deal price d must be at least as optimistic as in the alternative PBE, i.e., we require that $\mu(H \mid d) \geq \mu'(H \mid d)$. Given any beliefs that satisfy this restriction, consumer demand for the type H merchant at the deal price d cannot be lower than that in the alternative PBE, because consumers derive higher utility from the type H merchant and their decisions must be sequentially rational. It follows that it will be profitable for the type H merchant to deviate to $d_H = d$ in the candidate PBE, because it can earn at least as much profit as in the alternative PBE. Hence, the candidate PBE is defeated by the alternative PBE.

A second implication is that a candidate PBE will not be defeated if only the type L merchant is better off in the alternative PBE. In this case, we require that consumer beliefs are not more optimistic than in the alternative PBE, i.e., $\mu(L \mid d) \geq \mu'(L \mid d)$. But this allows for the beliefs to be pessimistic, i.e., $\mu(L \mid d) = 1$. Since to be a PBE, the candidate PBE must have survived under such pessimistic beliefs, it follows that it is not defeated by the alternative PBE.

It follows from these two implications that a PBE that yields the highest profit for the type H merchant: (a) will defeat any other PBE that yields lower profit for the type H merchant, and (b) is itself not defeated by any other PBE. Thus, only the PBE that yields the highest profit for the type H merchant can be a SUE.

Repeat Purchases

New consumers who buy the deal will make a repeat purchase at the regular price p if they find that the product meets their needs. We assume that future profit and future utility are not discounted. New consumers whose needs are met will buy again and obtain a surplus $r-p \geq 0$. The probability that a type t merchant's product meets a new consumer's need is α_t. Hence, a new consumer's utility from the deal is

$$\begin{aligned} u_{NC} &= \theta\left(r\alpha_H + (r-p)\,\alpha_H\right) + (1-\theta)\left(\alpha_L r + \alpha_L\,(r-p)\right) - d_t, \\ &= \left(\theta r\alpha_H + (1-\theta)\,r\alpha_L\right)\left(2-\frac{p}{r}\right) - d_t. \end{aligned} \tag{32}$$

Comparing equation (32) with equation (2), we observe that repeat purchases increase new consumers' willingness to pay for the deal by a factor $\left(2-\frac{p}{r}\right)$. It will be sufficient to consider repeat purchases by new consumers. Experienced consumers already know the merchant and whether its product can meet their needs. Hence, the deal will neither affect their repeat purchase decision nor the associated merchant revenue. As before, we normalize $r=1$. We assume $p > (2-p)\,\alpha_H$ such that it does not restrict the deal price at which new consumers can be acquired.

We then proceed as in the main model. We show that a separating equilibrium that resembles the symmetric information benchmark can be supported only if the website displays deal sales. Moreover, it can be supported for a larger range of parameters than in the main model. In the symmetric information benchmark, new consumers are willing to pay $(2-p)\,\alpha_t$ for a type t merchant (from equation (32)). Therefore, a type t merchant charges $d_t = (2-p)\,\alpha_t$, and all new consumers and matched consumers buy the deal. Offering a deal will be attractive for a type t merchant iff

$$\alpha_t\,(N+\alpha_t)\,(2-p) + pN\alpha_t > p\alpha_t, \tag{33}$$

Comparing equation (33) with equation (5), we note that offering a deal is more attractive because of repeat purchases. The website's expected profit is

$$\Pi^W_{\text{SI}} = (1-\lambda)\,(2-p)\left(\bar{\theta}\alpha_H\,(\alpha_H+N) + \left(1-\bar{\theta}\right)\alpha_L\,(\alpha_L+N)\right), \tag{34}$$

which is higher than in the main model by a factor $2-p$. As in Lemma 1, the website's expected profit cannot be higher than that in the symmetric information benchmark, and can be equal to it only in a separating equilibrium that replicates the symmetric information benchmark outcome. As in Lemma 2, a separating equilibrium does not exist in the subgame with deal sales not displayed, because the type L merchant will always mimic the type H merchant in such an equilibrium to sell to all new consumers and matched consumers at a higher price.

Consider a separating equilibrium in the subgame with deal sales displayed in which $d_t = (2-p)\,\alpha_t$ for $t \in \{H, L\}$. At the deal price $(2-p)\,\alpha_H$, frequent- and late-new consumers buy the deal iff period 1 sales is not below $\tau \in (\tau_L, \tau_H]$, where τ_t is as in Lemma 4. They always buy at the deal price $(2-p)\,\alpha_L$. All matched consumers buy the deal at either equilibrium deal price from either merchant type. Early-new consumers buy the deal at either equilibrium deal price. The type L merchant's expected profit is

$$\Pi_L = \lambda\alpha_L\,(\alpha_L + N)\,(2-p) + pN\alpha_L. \tag{35}$$

If it mimics the type H merchant, then it cannot sell to frequent- and late-new consumers. Its expected profit is

$$\Pi_L^{'} = \lambda\alpha_H\left(\alpha_L + \tfrac{1}{2}\beta N\right)(2-p) + \tfrac{1}{2}p\beta N\alpha_L. \tag{36}$$

For mimicking to be unattractive, we require $\Pi_L \geq \Pi_L^{'}$, which yields

$$\alpha_L^2 + (N-\alpha_H)\,\alpha_L - \tfrac{1}{2}\alpha_H\beta N + \frac{p}{\lambda\,(2-p)}N\alpha_L\left(1-\tfrac{1}{2}\beta\right) \;\;\geq\;\; 0. \tag{37}$$

Comparing equation (37) with the non-mimicking constraint in equation (9), we note that the former is less stringent as there is an additional positive term $\frac{p}{\lambda(2-p)}N\alpha_L\left(1-\frac{1}{2}\beta\right)$ on the LHS. Thus, with repeat purchases, displaying deal sales supports a distortionless separating equilibrium for a larger range of parameters.

Lower Bound for Regular Price

We first show that if $p \leq \alpha_H$ then website profit from displaying deal sales can be lower than in our main analysis. Then, we obtain the lower bound $\bar{p}$ for the regular price below which the website will not display deal sales in equilibrium. We consider $p \in (\bar{\alpha}, \alpha_H]$. In this case, the equilibrium in the subgame with deal sales not displayed is the same as in the main analysis. In the subgame with deal sales displayed, the regular price can constrain the type H merchant's deal price in a separating equilibrium since $d_H < p \leq \alpha_H$. In particular, distortionless separation cannot occur. Consequently, if $\alpha_L \geq \alpha_2$ (such that the equilibrium in the main analysis is distortionless separation), then the equilibrium deal prices are $d_H = p - \epsilon$ and $d_L = \alpha_L$ for $\epsilon > 0$ arbitrarily small.[1] Similarly, the equilibrium deal prices are $d_H = p - \epsilon$ and $d_L = \alpha_L$ if the equilibrium in the

[1] By definition, a deal is a discount to the regular price. Allowing for deal price to be higher than the regular price will not affect our results since experienced consumers will then not buy the deal, and, therefore, neither will new consumers.

main model is a separating equilibrium with price distortion in which $d_H = d_H^* \geq p$. Consequently, website expected profit in a separating equilibrium can be lower than in the main model. The lower the regular price, the less attractive it is for the website to display deal sales.

To obtain the lower bound $\bar{p}$ for the regular price below which the website will not display deal sales, we consider the following limiting case: $d_H = \bar{p} - \epsilon$ and $d_L = \alpha_L$ in the separating equilibrium in the subgame with deal sales displayed, and the website is indifferent between displaying and not displaying deal sales. For the separating equilibrium to occur, we require that mimicking is unattractive for the type L merchant

$$\alpha_L (\alpha_L + N) \geq \bar{p} \left(\alpha_L + \tfrac{1}{2} N \beta\right). \tag{38}$$

We remark that condition (38) will hold if $\beta \to 0$ since $N > 1 > \bar{p}$. We further note that it is profitable for the type H merchant to offer a deal since d_H is arbitrarily close to p.

We next examine the conditions for the website to be indifferent about displaying and not displaying deal sales. Consider the subgame with deal sales not displayed. If only the type L merchant offers a deal, then the website strictly prefers to display deal sales since doing so increases type H merchant deal revenue without affecting type L merchant deal revenue. Hence, the website cannot be indifferent between displaying and not displaying sales. Therefore, the equilibrium must be a pooling equilibrium in which $d_H = d_L = \bar{\alpha}$. We require that it is profitable for the type H merchant to offer a deal

$$\bar{\alpha} (\alpha_H + N) \quad \geq \quad \bar{p} \alpha_H, \tag{39}$$

and that the website is indifferent between displaying and not displaying deal sales

$$\bar{\alpha} (N + \bar{\alpha}) \quad = \quad \bar{\theta} \bar{p} (N + \alpha_H) + \left(1 - \bar{\theta}\right) \alpha_L (N + \alpha_L). \tag{40}$$

Since conditions (38) and (39) only impose an upper bound, we obtain the lower bound from condition (40) as

$$\bar{p} \quad = \quad \frac{\bar{\alpha}(N+\bar{\alpha}) - \left(1-\bar{\theta}\right)\alpha_L (N+\alpha_L)}{\bar{\theta}(N+\alpha_H)}. \tag{41}$$

We note that $\bar{p} > \bar{\alpha}$ since the RHS in condition (40) is strictly lower than the LHS if $\bar{p} = \bar{\alpha}$.

We remark that if $p \in (\alpha_L, \bar{\alpha}]$, then the equilibrium deal price with deal sales not displayed will be $d_H = d_L = p - \epsilon$, where $\epsilon > 0$ arbitrarily small. The website will not display deal sales since this can only reduce deal revenue from the type L merchant – either the type L merchant charges a deal price $d_L = \alpha_L < \bar{\alpha}$ or $d_L = p - \epsilon$, and frequent- and late-new consumers do not buy the deal in period 2.