Appendix. Online Companion to *Incentive Alignment and Coordination of Project Supply Chains* by S. Chen and H. Lee

A. Mathematical Proofs

Proof of Lemma 1. The result is derived from the first-order-condition of the manufacturer's newsvendor-type optimization problem (2). □

Proof of Proposition 1. First, we derive the supplier's best response $\widetilde{t}_2(w_2)$. It should satisfy

$$\alpha_2 C_2 - \alpha_2(W_2 - w_2) - \alpha_2 w_2 F_{V_1}(\widetilde{t}_2(w_2)) = 0, \text{ where } V_1 = R_1 - l_2.$$

Next, we consider the manufacturer's problem. The manufacturer will choose w_2 that satisfies

$$-\frac{\partial EP_2(\widetilde{t}_2(w_2), w_2, \alpha)}{\partial t_2}\frac{d\widetilde{t}_2(w_2)}{dw_2} - \frac{\partial EP_2(\widetilde{t}_2(w_2), w_2, \alpha)}{\partial w_2} - \alpha P_0 F_{V_1}(\widetilde{t}_2(w_2))\frac{d\widetilde{t}_2(w_2)}{dw_2} = 0. \quad (12)$$

But note that

$$\frac{\partial EP_2(\widetilde{t}_2(w_2), w_2, \alpha)}{\partial t_2} = -\alpha(W_2 - w_2) - \alpha w_2 F_{V_1}(\widetilde{t}_2(w_2)) = -\alpha C_2,$$

where the second equation comes from the supplier's best response function. Hence, equation (12) can be simplified as follows:

$$-\frac{\partial EP_2(\widetilde{t}_2(w_2), w_2, \alpha)}{\partial w_2} + \left[\alpha C_2 - \alpha P_0 F_{V_1}(\widetilde{t}_2(w_2))\right]\frac{d\widetilde{t}_2(w_2)}{dw_2} = 0. \quad (13)$$

Moreover, according to Lemma 1, the optimal start time of the delivery process for task 2 in the centralized supply chain, t_2^*, satisfies

$$\alpha C_2 - \alpha P_0 F_{V_1}(t_2^*) = 0.$$

Hence, at $\widetilde{t}_2(w_2) = t_2^*$, equation (13) becomes

$$-\frac{\partial EP_2(\widetilde{t}_2(w_2), w_2, \alpha)}{\partial w_2} = 0,$$

which, however, can never be satisfied because

$$-\frac{\partial EP_2(\widetilde{t}_2(w_2), w_2, \alpha)}{\partial w_2} = -\alpha E[(\widetilde{t}_2(w_2) + l_2) - \max(R_1, \widetilde{t}_2(w_2) + l_2) - R_2] > 0.$$

This inequality also implies that under the manufacturer's optimization problem, she will choose a $w_2 > w_2^*$ where $\widetilde{t}_2(w_2^*) = t_2^*$. Since $\widetilde{t}_2(w_2)$ is an increasing function of w_2 (which can be easily derived from the supplier's best response function); this means that the manufacturer will choose a w_2 such that $\widetilde{t}_2(w_2) > \widetilde{t}_2(w_2^*) = t_2^*$.

Finally, the no-delay fixed-price contract is equivalent to the case where $w_2 = 0$, whereas the delayed fix-price contract is equivalent to the case where $w_2 = W_2$. Hence, they are two extreme cases of the partially-delayed fixed-price contract and thus they cannot coordinate the channel. Mathematically, from the supplier's best response function, we can verify that when $w_2 = 0$, the supplier prefers to start as early as possible, which means that he is over-motivated; in contrast, when $w_2 = W_2$, the supplier's start time is equal to $F_{V_1}(C_2/W_2) < t_2^*$, which means that he is under-motivated. □

Proof of Proposition 2. The manufacturer's problem is given by (4). We derive the first-order condition of J_M with respect to T_2; that is,

$$-\frac{\partial EP_2(\widetilde{t}_2(T_2),T_2,\alpha)}{\partial T_2}-\frac{\partial EP_2(\widetilde{t}_2(T_2),T_2,\alpha)}{\partial t_2}\frac{d\widetilde{t}_2(T_2)}{dT_2}-\alpha P_0\frac{dE[CT(\widetilde{t}_2(T_2))]}{dt_2}\frac{d\widetilde{t}_2(T_2)}{dT_2}=0. \tag{14}$$

Rearranging the terms of equation (14), we obtain

$$\frac{\partial EP_2(\widetilde{t}_2(T_2),T_2,\alpha)}{\partial T_2}=-\Big(\frac{\partial EP_2(\widetilde{t}_2(T_2),T_2,\alpha)}{\partial t_2}+\alpha P_0\frac{dE[CT(\widetilde{t}_2(T_2))]}{dt_2}\Big)\frac{d\widetilde{t}_2(T_2)}{dT_2}.$$

Under the assumption that the manufacturer's problem has a unique optimizer, the optimal material delivery schedule from the manufacturer's persepctive, T_2^*, should satisfy the above equation, which is indeed equation (5). Therefore, to coordinate the channel, it is desired that $\widetilde{t}_2(T_2^*)=t_2^*$ where T_2^* satisfies equation (5). □

Proof of Corollary 1. From the supplier's optimization problem, we should have

$$\alpha_2 C_2+\frac{\partial EP_2(\widetilde{t}_2(T_2),T_2,\alpha_2)}{\partial t_2}=0.$$

Substituting it into equation (14) and assuming that $\alpha_2=\alpha$, equation (14) becomes

$$\frac{\partial EP_2(\widetilde{t}_2(T_2),T_2,\alpha)}{\partial T_2}=-\Big(-\alpha C_2+\alpha P_0\frac{dE[CT(\widetilde{t}_2(T_2))]}{dt_2}\Big)\frac{d\widetilde{t}_2(T_2)}{dT_2}. \tag{15}$$

Furthermore, the optimality condition of the centralized supply chain is

$$\alpha C_2-\alpha P_0\frac{dE[CT(t_2^*)]}{dt_2}=0.$$

Substituting this equation into equation (15), we obtain

$$\frac{\partial EP_2(t_2^*,T_2^*,\alpha)}{\partial T_2}=0,\ \text{where } \widetilde{t}_2(T_2^*)=t_2^*.$$

□

Proof of Theorem 1. According to the structure of the contract, from the supplier's perspective, the expected net payment from the manufacturer will be

$$EP_2(t_2,T_2,\alpha_2)=\underbrace{(W_2-C_2)}_{Down-payment}+\underbrace{C_2\Big(1-\alpha_2 E[\max(T_2,t_2+l_2)]\Big)}_{Balance}+\underbrace{b_2E[T_2-(t_2+l_2)]^+}_{Bonus}-\underbrace{p_2E[t_2+l_2-T_2]^+}_{Penalty}.$$

In this formulation, we assume that the down-payment is made at the beginning of the project without loss of generality. Moreover, the penalty is p_2 for each time unit of delay, where p_2 is the net present value of the penalty rate (discounted back to the beginning of the project). The explanation for the bonus is similar.

Substituting this equation into the supplier's optimization problem (3), it can be verified that $d\widetilde{t}_2(T_2)/dT_2=1$. Indeed, the first-order-condition of the supplier's problem is

$$\alpha_2C_2+\frac{\partial EP_2(t_2,T_2,\alpha_2)}{\partial t_2}=\alpha_2C_2+[-\alpha_2C_2-p_2+(p_2-b_2+\alpha_2C_2)G_2(T_2-t_2)]=0,$$

which implies that given T_2^* as stated in (6), the supplier's response will be

$$\widetilde{t}_2(T_2^*)=T_2^*-G_2^{-1}\Big(\frac{p_2}{p_2-b_2+\alpha_2C_2}\Big)=t_2^*.$$

Hence, condition (1) of Proposition 2 is satisfied.

Furthermore, since

$$\frac{\partial EP_2(t_2,T_2,\alpha)}{\partial t_2} = -\alpha C_2 - p_2 + (p_2 - b_2 + \alpha C_2)G_2(T_2 - t_2),$$

and

$$\frac{\partial EP_2(t_2,T_2,\alpha)}{\partial T_2} = p_2 - (p_2 - b_2 + \alpha C_2)G_2(T_2 - t_2).$$

Then at T_2^* and $t_2^* = \widetilde{t}_2(T_2^*)$,

$$\frac{\partial EP_2(t_2^*,T_2^*,\alpha)}{\partial T_2} + \Big(\frac{\partial EP_2(t_2^*,T_2^*,\alpha)}{\partial t_2} + \alpha P_0 \frac{dE[CT(t_2^*)]}{dt_2}\Big)\frac{d\widetilde{t}_2(T_2^*)}{dT_2} = -\alpha C_2 + \alpha P_0 \frac{dE[CT(t_2^*)]}{dt_2} = 0,$$

where the second equation comes from the optimality condition of the centralized supply chain.

Therefore, condition (2) of Proposition 2 is also satisfied and channel coordination is achieved. □

Proof of Proposition 3. Here we provide only the proof of the two-task-in-series model, since the proof of the two-task-in-parallel model is quite similar. First, consider the expected channel profit.

$$J_{SC}^* = -C_2(1 - \alpha t_2^*) + P_0 - \alpha P_0 E[\max(R_1, t_2^* + l_2) + R_2].$$

Note that

$$\begin{aligned} E[\max(R_1, t_2 + l_2)] &= E[\max(V_1, t_2)] + E[l_2] \\ &= t_2 F_{V_1}(t_2) + \mu_{V_1}\overline{\Phi}\Big(\frac{t_2 - \mu_{V_1}}{\sigma_{V_1}}\Big) + \sigma_{V_1}\phi\Big(\frac{t_2 - \mu_{V_1}}{\sigma_{V_1}}\Big) + E[l_2], \end{aligned}$$

where $V_1 = R_1 - l_2$ and thus $\mu_{V_1} = \mu_1 - \tau_2$ and $\sigma_{V_1} = \sqrt{\sigma_1^2 + s_2^2} = m_2$. Φ and ϕ are the c.d.f. and p.d.f. of the standard normal distribution.

Since t_2^* is the optimal solution of the centralized system satisfying $F_{V_1}(t_2^*) = C_2/P_0$,

$$\begin{aligned} E[\max(R_1, t_2^* + l_2)] &= t_2^* F_{V_1}(t_2^*) + \mu_{V_1}\overline{\Phi}\Big(\frac{t_2^* - \mu_{V_1}}{\sigma_{V_1}}\Big) + \sigma_{V_1}\phi\Big(\frac{t_2^* - \mu_{V_1}}{\sigma_{V_1}}\Big) + E[l_2] \\ &= \mu_{V_1} + \sigma_{V_1} Z\Phi(Z) + \sigma_{V_1}\phi(Z) + \tau_2, \end{aligned}$$

where $Z = \Phi^{-1}(C_2/P_0)$. Substituting the above equation into the expression of J_{SC}^*, we obtain

$$J_{SC}^* = P_0 - C_2 - \alpha P_0(\mu_1 + \mu_2) + \alpha C_2(\mu_1 - \tau_2) - \alpha P_0 m_2 \phi(Z).$$

Next, consider the supplier's expected profit (under the assumption that $\alpha_2 = \alpha$)

$$J_S^* = -C_2(1 - \alpha t_2^*) + EP_2(t_2^*, T_2^*, \alpha),$$

where

$$\begin{aligned} EP_2(t_2^*,T_2^*,\alpha) &= (W_2 - C_2) + C_2\Big(1 - \alpha E[\max(T_2^*, t_2^* + l_2)]\Big) + b_2 E[T_2^* - (t_2^* + l_2)]^+ - p_2 E[t_2^* + l_2 - T_2^*]^+ \\ &= W_2 - (p_2 + \alpha C_2 - b_2)E[t_2^* + l_2 - T_2^*]^+ - \alpha C_2 T_2^* + b_2(T_2^* - t_2^* - \tau_2). \end{aligned}$$

Since $T_2^* - t_2^* = G_2^{-1}\big(p_2/(p_2 + \alpha C_2 - b_2)\big)$, $T_2^* = t_2^* + \tau_2 + s_2 z$ where $z = \Phi^{-1}\big(p_2/(p_2 + \alpha C_2 - b_2)\big)$. Then,

$$\begin{aligned} EP_2(t_2^*,T_2^*,\alpha) &= W_2 - (p_2 + \alpha C_2 - b_2)E[t_2^* + l_2 - T_2^*]^+ - \alpha C_2 T_2^* + b_2(T_2^* - t_2^* - \tau_2) \\ &= W_2 - (p_2 + \alpha C_2 - b_2)\Big(s_2\phi(z) - s_2 z\overline{\Phi}(z)\Big) - \alpha C_2(\mu_1 + m_2 Z + s_2 z) + b_2 s_2 z \\ &= W_2 - (p_2 + \alpha C_2 - b_2)s_2\phi(z) - \alpha C_2(\mu_1 + m_2 Z). \end{aligned}$$

Substituting it into the supplier's expected profit formulation, we obtain

$$J_S^* = -C_2(1-\alpha t_2^*) + EP_2(t_2^*, T_2^*, \alpha) = W_2 - C_2 - \alpha C_2 \tau_2 - (p_2 + \alpha C_2 - b_2) s_2 \phi(z).$$

Finally, the manufacturer's expected profit can be obtained by brutal force calculation; alternatively, one can also verify that it is equal to $J_{SC}^* - J_S^*$ (under the assumption that $\alpha = \alpha_2$). □

Proof of Corollary 2. The result about W_2 is trivial. We will prove the monotonicity property of J_S^* (and thus J_M^*) with respect to p_2 (the proof for b_2 is similar). Define $M(x) = (x+a)\phi(\Phi^{-1}(\frac{x}{x+a}))$, where $a > 0$.

$$M'(x) \triangleq m(x) = \phi(\Phi^{-1}(\frac{x}{x+a})) - \frac{a}{x+a}\Phi^{-1}(\frac{x}{x+a}).$$

It can be verified that $m'(x) < 0$ because

$$m'(x) = -\frac{a^2}{(x+a)^3}\frac{1}{\phi(\Phi^{-1}(\frac{x}{x+a}))}.$$

Moreover, we prove that $\text{limit}_{x\to+\infty} m(x) = 0$ (using L'Hôpital's Rule). Hence, $m(x) > 0$, meaning that $M(x)$ is increasing in x for $x \in [0, +\infty]$. Therefore, J_M^* is increasing in p_2 and J_S^* is decreasing in p_2. □

Proof of Corollary 3. The first part of this corollary is easy to see based on the expressions of the expected channel profit stated in Proposition 3. To understand the second part of this corollary, note that J_{SC}^* is a decreasing function of m_2 for the sequential topology and of m_2' for the parallel topology. Hence, the impact of s_2, σ_1, and σ_2 on J_{SC}^* is explained by the impact of them on m_2 (or m_2'). In particular, m_2 (or m_2') increases as s_2 increases. Also, m_2 (or m_2') increases as σ_1 and σ_2 increases if $\rho = 0$; if $\rho \neq 0$, however, m_2' is not monotone in σ_1 and σ_2. □

Proof of Corollary 4. The result is straightforward based on the supplier's profit in Proposition 3. □

Proof of Corollary 5. The proof of this corollary is similar to that of Corollary 3. □

Proof of Theorem 2. First, we provide the problem formulation as follows:

In the centralized supply chain, the central planner's problem is

$$J_{SC}^* = \max_{(t_2,\cdots,t_N)\in\mathbb{R}^{N-1}} \sum_{n=2}^{n=N} -C_n(1-\alpha t_n) + P_0 - \alpha P_0 E[CT(t_2,\cdots,t_N)] - p_0 E[CT(t_2,\cdots,t_N) - T_0]^+, \quad (16)$$

where the decision is about the start times of the material delivery processes for all tasks, and $CT(t_2,\cdots,t_N)$ denotes the project completion time given those start times. The expectation is with respect to the distributions of all on-site tasks and delivery lead-time. Note that in addition to the fixed revenue P_0, the manufacturer may face a project deadline T_0 as well as a delay penalty at a rate of p_0 for each time unit of delay (p_0 is the net present value of the penalty rate discounted back to the beginning of the project). If $p_0 = 0$, the problem is equivalent to the situation where there is only a fixed revenue P_0 upon completion of the project.

In the decentralized supply chain, the manufacturer determines the material delivery schedules $(T_2,\cdots,T_N)$, and supplier n solves the following problem:

$$J_{S,n}(T_n) = \max_{t_n\in\mathbb{R}} -C_n(1-\alpha_n t_n) + EP_n(t_n, T_n, \alpha_n), \quad (17)$$

where $EP_n(t_n, T_n, \alpha_n)$ is the expected amount of the net discounted payment to the supplier (from the supplier's view, the cash flows are discounted at a rate of α_n).

Given all suppliers' best response functions $(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_n(T_n))$, the manufacturer's problem is

$$J_M = \max_{(T_2,\cdots,T_N)\in\mathbb{R}^{N-1}} \; -\sum_{n=2}^{n=N} EP_n(\widetilde{t}_n(T_n),T_n,\alpha) + P_0 - \alpha P_0 E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_n))] \\ -p_0 E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_N)) - T_0]^+. \tag{18}$$

First, we examine the structural property of the centralized problem.

Lemma 2. *$CT(t_2,\cdots,t_N)$ is a convex function of $(t_2,\cdots,t_N)$ for any directed acyclic topology.*

Proof of Lemma 2. Let ST_n denote the (earliest) start time of task n and CT_n denote the (earliest) completion time of task n. We have $CT_n = ST_n + R_n$, and $ST_n = \max_{i\in\mathcal{P}_n}\{CT_i, l_n + t_n\}$, where $\mathcal{P}_n$ denotes the set of all the preceding project tasks for task n. We now show that $CT_n : \mathbb{R}^{N-1} \to \mathbb{R}, \forall n \in \{1,\cdots,N\}$, is a convex function of $(l_2,\cdots,l_N)$ by induction.

First, define $\mathcal{A} = \emptyset$ and $\mathcal{B} = \{1,\cdots,N\}$. We start with the project tasks whose preceding sets are empty; i.e., for task n with $\mathcal{P}_n = \emptyset$. Since $CT_n = l_n + t_n + R_n$ for those tasks, they are obviously convex functions of $(l_2,\cdots,l_N)$. Then, we move the indices of those tasks from $\mathcal{B}$ to $\mathcal{A}$. Next, we consider the tasks whose preceding tasks are all in set $\mathcal{A}$. For example, if task m is such a task, then $CT_m = \max_{i\in\mathcal{P}_m}\{CT_i, l_m + t_m\}$, where $\mathcal{P}_m$ is a subset of $\mathcal{A}$. If we have proved that CT_i's are all convex functions of $(l_2,\cdots,l_n)$ for $i \in \mathcal{A}$, then CT_m is also a convex function, since the point-wise maximum of convex functions is also convex (see also, Boyd and Vandenberghe, 2004, page 80). Then, we move the indices of those tasks from $\mathcal{B}$ to $\mathcal{A}$.

Repeat the proof procedure until $\mathcal{A} = \{1,\cdots,N\}$ and $\mathcal{B} = \emptyset$, then we can show that CT_n is a convex function for all $n \in \{1,\cdots,N\}$. Note that $CT(l_2,\cdots,l_N)$ is the completion time of the entire project, which is equal to $max\{CT_1,\cdots,CT_N\}$. Therefore, $CT(l_2,\cdots,l_N)$ is a convex function. □

As a result of Lemma 2, $E[CT(t_2,\cdots,t_N)]$ and $E[CT(t_2,\cdots,t_N) - T_0]^+$ are both convex functions of $(t_2,\cdots,t_N)$. Therefore, the optimization problem of the centralized supply chain is a convex optimization problem. There must exist (at least) one optimal solution to this optimization problem. Let $(t_2^*,\cdots,t_N^*)$ denote such an optimal solution. Then, mimicking the proof of Proposition 2 and Theorem 1, channel coordination by using the delivery-schedule-based contract can be proved. More specifically, the proof of channel coordination is shown below.

Let $J_M(T_2,\cdots,T_N)$ denote the objective function of the manufacturer's optimization problem. The first-order-partial-derivatives of $J_M(T_2,\cdots,T_N)$ are as follows: for any $n \in \{2,\cdots,N\}$

$$\frac{\partial J_M(T_2,\cdots,T_N)}{\partial T_n} = -\frac{\partial EP_n(\widetilde{t}_n(T_n),T_n,\alpha)}{\partial T_n} - \frac{\partial EP_n(\widetilde{t}_n(T_n),T_n,\alpha)}{\partial t_n}\frac{d\widetilde{t}_n(T_n)}{dT_n} \\ -\alpha P_0 \frac{\partial E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_N))]}{\partial t_n}\frac{d\widetilde{t}_n(T_n)}{dT_n} - p_0 \frac{\partial E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_N)) - T_0]^+}{\partial t_n}\frac{d\widetilde{t}_n(T_n)}{dT_n}$$

It can be verified that $d\widetilde{t}_n(T_n)/dT_n = 1$ and $\partial EP_n(\widetilde{t}_n(T_n),T_n,\alpha)/\partial T_n + \partial EP_n(\widetilde{t}_n(T_n),T_n,\alpha)/\partial t_n = -\alpha C_n$. Thus, the first-order-partial derivatives of $J_M(T_2,\cdots,T_N)$ are: $\forall n \in \{2,\cdots,N\}$

$$\frac{\partial J_M(T_2,\cdots,T_N)}{\partial T_n} = \alpha C_n - \alpha P_0 \frac{\partial E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_N))]}{\partial t_n} - p_0 \frac{\partial E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_N)) - T_0]^+}{\partial t_n}. \tag{19}$$

Then, the second-order-derivatives of $J_M(T_2,\cdots,T_N)$: $\forall m,n \in \{2,\cdots,N\}$

$$\frac{\partial^2 J_M(T_2,\cdots,T_N)}{\partial T_n \partial T_m} = -\alpha P_0 \frac{\partial^2 E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_N))]}{\partial t_n \partial t_m} - p_0 \frac{\partial^2 E[CT(\widetilde{t}_2(T_2),\cdots,\widetilde{t}_N(T_N)) - T_0]^+}{\partial t_n \partial t_m}.$$

Therefore, the concavity of $J_M(T_2,\cdots,T_N)$ is guaranteed by the convexity of $E[CT(t_2,\cdots,t_N)]$ and $E[CT(t_2,\cdots,t_N)-T_0]^+$. Furthermore, the first-order-conditions of the manufacturer's problem are exactly the same as the first-order-conditions of the first-best solutions. Therefore, in the decentralized supply chain, the manufacturer's profit is maximized by setting each T_n^* such that $\widetilde{t}_n(T_n^*)=t_n^*$. That is, $T_n^*=t_n^*+G_n^{-1}\big(p_n/(p_n+\alpha_nC_n-b_n)\big)$. □

Proof of Corollary 6. Under the coordinating contract, the maximized expected profit of supplier n is:

$$\begin{aligned}
J_{S,n}^* &= -C_n(1-\alpha_nt_n^*)+EP_n(t_n^*,T_n^*,\alpha_n)\\
&= -C_n(1-\alpha_nt_n^*)+W_n-\alpha_nC_nE[\max(T_n^*,t_n^*+l_n)]+b_nE[T_n^*-t_n^*-l_n]^+-p_nE[t_n^*+l_n-T_n^*]^+\\
&= W_n-C_n-\alpha_nC_nE[\max(T_n^*-t_n^*,l_n)]+b_nE[T_n^*-t_n^*-l_n]^+-p_nE[t_n^*+l_n-T_n^*]^+\\
&= W_n-C_n-\alpha_nC_n\tau_n-(\alpha_nC_n-b_n)E[(T_n^*-t_n^*)-l_n]^+-p_nE[l_n-(T_n^*-t_n^*)]^+.
\end{aligned}$$

Since $T_n^*-t_n^*=G_n^{-1}\big(p_n/(p_n+\alpha_nC_n-b_n)\big)$ by Theorem 2, it is obvious that $J_{S,n}^*$ depends only on the distribution of l_n whose c.d.f. is $G_n(\cdot)$. Furthermore, if l_n is normally distributed with mean τ_n and standard deviation s_n, the expected profit of supplier n is

$$J_{S,n}^*=W_n-C_n-\alpha_nC_n\tau_n-(p_n+\alpha_nC_n-b_n)s_n\phi\Big(\Phi^{-1}\big(\frac{p_n}{p_n+\alpha_nC_n-b_n}\big)\Big).$$

Therefore, $J_{S,n}^*$ will increase if τ_n or s_n were reduced. □

Proof of Proposition 4. To prove that a biased estimate of the mean of the delivery lead-time does not affect channel coordination but a biased estimate of the variance of the delivery lead-time does, we can verify that if the delivery lead-time is normally distributed, equation (8) implies

$$\begin{aligned}
\widetilde{t}_2(T_{2'}^*) &= t_{2'}^*+G_{2'}^{-1}\Big(\frac{p_2}{p_2+\alpha_2C_2-b_2}\Big)-G_2^{-1}\Big(\frac{p_2}{p_2+\alpha_2C_2-b_2}\Big)\\
&= \Big[(\mu_1-\tau_2')+\sqrt{\sigma_1^2+s_{2'}^2}Z\Big]+(\tau_2'+s_{2'}z)-(\tau_2+s_2z)\\
&= \mu_1-\tau_2+\sqrt{\sigma_1^2+s_{2'}^2}Z+(s_{2'}-s_2)z,
\end{aligned}$$

whereas

$$t_2^*=\mu_1-\tau_2+\sqrt{\sigma_1^2+s_2^2}Z,$$

where $Z=\Phi^{-1}(C_2/P_0)$ and $z=\Phi^{-1}(p_2/(p_2+\alpha_2C_2-b_2))$. And, τ_2' (and s_2') is the mean (and standard deviation) of the manufacturer's estimate of the delivery lead-time, which is different from the supplier's estimate. Hence, $\widetilde{t}_2(T_{2'}^*)=t_2^*$ if and only if $s_2'=s_2$ regardless of τ_2'.

Next, we show that under the delivery-schedule-based contract, there is no incentive for the supplier to conceal his estimate of the delivery lead-time from the manufacturer. Suppose that the supplier's estimate of the delivery lead-time has a distribution with c.d.f. G_2, but he informs the manufacturer of G_2' instead. Consequently, the manufacturer will determine a targeted material delivery date, $T_{2'}^*$, provided that she trusts the supplier's information on the distribution of the delivery lead-time G_2'. Then, the supplier's optimal expected profit is given by

$$J_S(T_{2'}^*)=-C_2(1-\alpha_2\widetilde{t}_2(T_{2'}^*))+EP_2(\widetilde{t}_2(T_{2'}^*),T_{2'}^*,\alpha_2).$$

To examine the impact of telling the manufacturer a biased distribution of the delivery lead-time with a biased standard deviation s'_2, we derive

$$dJ_S(T^*_{2'})/ds'_2 = \alpha_2 C_2 \frac{d\widetilde{t}_2(T^*_{2'})}{ds'_2} + \frac{\partial EP_2(\widetilde{t}_2(T^*_{2'}), T^*_{2'}, \alpha_2)}{\partial t_2} \frac{d\widetilde{t}_2(T^*_{2'})}{ds'_2} + \frac{\partial EP_2(\widetilde{t}_2(T^*_{2'}), T^*_{2'}, \alpha_2)}{\partial T_2} \frac{dT^*_{2'}}{ds'_2}$$

But we also have $\alpha_2 C_2 + \partial EP_2(\widetilde{t}_2(T^*_{2'}), T^*_{2'}, \alpha_2)/\partial t_2 = 0$ from the first-order-condition of the supplier's optimization problem. Furthermore, we find that $\partial EP_2(\widetilde{t}_2(T^*_{2'}), T^*_{2'}, \alpha_2)/\partial T_2 = 0$, because

$$\frac{\partial EP_2(\widetilde{t}_2(T^*_{2'}), T^*_{2'}, \alpha_2)}{\partial T_2} = p_2 - (p_2 + \alpha_2 C_2 - b_2) G_2^{-1}\big(T^*_{2'} - \widetilde{t}_2(T^*_{2'}|l_2)\big) = 0.$$

Thus, $dJ_S(T^*_{2'})/ds'_2$. It would not benefit the supplier at all if he did not inform the manufacturer of his true estimate of the delivery lead-time. □

Proposition 5. *For the two-task-in-series base model,*

$$\begin{cases} J^*_{SC} = (P_0 - C_2) + \alpha C_2 \mu_1 - \alpha_2 C_2 \tau_2 - \alpha P_0 \left[\mu_1 + \mu_2 + m_2 \phi(Z)\right] + (\alpha - \alpha_2) s_2 (\phi(z) + p_2 z) C_2, \\ J^*_S = (W_2 - C_2) - \alpha_2 C_2 \tau_2 - (p_2 + \alpha_2 C_2 - b_2) s_2 \phi(z), \\ J^*_M = (P_0 - W_2) + \alpha C_2 \mu_1 - \alpha P_0 \left[\mu_1 + \mu_2 + m_2 \phi(Z)\right] + (p_2 + \alpha C_2 - b_2) s_2 \phi(z) + s_2 z (\alpha - \alpha_2) p_2 C_2. \end{cases}$$

While for the two-task-in-parallel base model,

$$\begin{cases} J^*_{SC} = (P_0 - C_2) + \alpha C_2 (\mu_1 - \mu_2) - \alpha_2 C_2 \tau_2 - \alpha P_0 \left[\mu_1 + m'_2 \phi(Z)\right] + (\alpha - \alpha_2) s_2 (\phi(z) + p_2 z) C_2, \\ J^*_S = (W_2 - C_2) - \alpha C_2 \tau_2 - (p_2 + \alpha C_2 - b_2) s_2 \phi(z), \\ J^*_M = (P_0 - W_2) + \alpha C_2 (\mu_1 - \mu_2) - \alpha P_0 \left[\mu_1 + m'_2 \phi(Z)\right] + (p_2 + \alpha C_2 - b_2) s_2 \phi(z) + s_2 z (\alpha - \alpha_2) p_2 C_2, \end{cases}$$

where $Z = \Phi^{-1}(\frac{C_2}{P_0})$, $z = \Phi^{-1}\left(\frac{p_2}{p_2 + \alpha_2 C_2 - b_2}\right)$, $m_2 = \sqrt{\sigma_1^2 + s_2^2}$, *and* $m'_2 = \sqrt{\sigma_1^2 + \sigma_2^2 + s_2^2 - 2\rho\sigma_1\sigma_2}$.

Based on the results of Proposition 5, we find that most key insights of Corollary 3-5 remain the same even when the costs of capital are not equal.

From the supplier's perspective, his profit depends only on the distribution of his own delivery lead-time, and, in particular, the supplier's profit is a decreasing function of the mean and variance of the lead-time – the same result has been obtained in Corollary 4.

From the manufacturer's perspective, her profit is a decreasing function of the mean of the on-site tasks, but it is independent of the mean of the delivery lead-time, provided that all other parameters are fixed; the same interesting result has been stated in Corollary 5. Moreover, it is the same conclusion as in Corollary 5 that in general, the manufacturer's expected profit is not monotone in s_2, σ_1, and σ_2, but if the distributions of the on-site tasks are independent, then the manufacturer's profit is a decreasing function of σ_1 and σ_2.

For the channel profit, the result is similar to that of Corollary 3. Specifically, the channel profit is a decreasing function of the mean duration of the on-site tasks and the mean delivery lead-time. In general, the channel profit is not monotone in the variance of the on-site tasks, but if those tasks are independent, then the channel profit is a decreasing function of the variance of the on-site tasks, which is also the same as in Corollary 3. However, under the assumption of unequal costs of capital, the channel profit is not monotone in the variance of the delivery lead-time, which is the only difference of the results under the assumption of unequal costs of capital from those results under the assumption of equal costs of capital.