Internet Appendix
for
"Correlations"

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This Internet Appendix provides additional results that are left out of the main text of the paper. The appendix is organized as follows: Section 1 presents proofs of the propositions in the main text of the paper. Section 2 performs an extensive principal component analysis and shows that ratio habits explain the first principal component in the time series of correlations and other asset pricing related time series. Section 3 shows the performance of the log price-dividend ratio as explanatory variable instead of ratio habits. Next, Section 4 presents regressions analysis that shows that model implied ratio habit or aggregate risk aversion predicts excess returns in-sample and out-of-sample. Finally, Section 5 presents regression analysis with portfolios sorted on size, book-to-market, and momentum instead of industry sorted portfolios.

1 Proofs of Propositions

Proof of Proposition 2

We solve for equilibrium using the martingale approach (see Cox and Huang (1989) and Karatzas et al. (1987)). Each investor solves the static optimization problem

$$\max_{C_j} E\left[\int_0^\infty e^{-\rho t}\frac{1}{1-\gamma_j}C_j(t)^{1-\gamma_j}X(t)^{\gamma_j-\eta}dt\right] \tag{1}$$

$$s.t.$$

$$E\left[\int_0^\infty \xi(t)C_j(t)dt\right] \le f_{Y,j}(0)E\left[\int_0^T \xi(t)C(t)dt\right], \tag{2}$$

where $f_{Y,j}(0) = \frac{Y_j(0)}{Y_L(0)+Y_H(0)}$ *is the initial wealth fraction of investor type* j*. Necessary and sufficient conditions for optimality are*

$$C_j(t) = \left(y_j e^{\rho t}X(t)^{\eta-\gamma_j}\xi(t)\right)^{-\frac{1}{\gamma_j}}, \tag{3}$$

where $y_j > 0$ *is such that*

$$E\left[\int_0^\infty \xi(t)\left(y_j e^{\rho t}X(t)^{\eta-\gamma_j}\xi(t)\right)^{-\frac{1}{\gamma_j}}dt\right] = f_{Y,j}(0)E\left[\int_0^\infty \xi(t)C(t)dt\right], \tag{4}$$

i.e., that the budget condition holds with equality. To solve for equilibrium, it is convenient to introduce an aggregate investor

$$u(C(t),X(t),t) = \max_{C_L(t),C_H(t)}\left\{\begin{array}{c} ae^{-\rho t}\frac{1}{1-\gamma_L}C_L(t)^{1-\gamma_L}X(t)^{\gamma_j-\eta} \\ +(1-a)\,e^{-\rho t}\frac{1}{1-\gamma_H}C_H(t)^{1-\gamma_H}X(t)^{\gamma_j-\eta}\end{array}\right\} \tag{5}$$

$$s.t.$$

$$C_L(t) + C_H(t) = C(t). \tag{6}$$

From the first-order conditions (FOC) of the aggregate investor's problem we have

$$ae^{-\rho t}\left(\frac{C_L(t)}{X(t)}\right)^{-\gamma_L} X(t)^{-\eta} = (1-a)\,e^{-\rho t}\left(\frac{C_H(t)}{X(t)}\right)^{-\gamma_H} X(t)^{-\eta}. \tag{7}$$

Defining the consumption share $f(t) = \frac{C_L(t)}{C(t)}$ *of L investors and imposing market clearing, Equation 6, we can rewrite Equation 7 as*

$$f(t) = \left(\frac{a}{1-a}\right)^{\frac{1}{\gamma_L}} e^{\left(\frac{\gamma_H}{\gamma_L}-1\right)\omega(t)} \left(1-f(t)\right)^{\frac{\gamma_H}{\gamma_L}}. \tag{8}$$

Proof of Proposition 4

First note that the utility function of the aggregate investor is defined through Equation 5. The coefficient of relative risk aversion is

$$\mathcal{R}(t) = -\frac{u_{CC}(C(t),X(t),t)}{u_C(C(t),X(t),t)}C(t), \tag{9}$$

where u_C *and* u_{CC} *denote the first and second partial derivative with respect to aggregate consumption, respectively. To calculate* $\mathcal{R}$*, we need to compute the partial derivatives of the aggregate investor's utility function. To this end, note that from the FOC of the aggregate investor problem we have that*

$$a\ u_{L,C}(C_L, X(t), t) = (1-a)\ u_{H,C}(C_H, X(t), t). \tag{10}$$

Consequently, we have that

$$\begin{aligned} u_C(C(t),X(t),t) &= a\ u_{L,C}(C_L,X(t),t)\frac{\partial C_L}{\partial C} + (1-a)\ u_{H,C}(C_H,X(t),t)\frac{\partial C_H}{\partial C} \\ &= a\ u_{L,C}(C_L,X(t),t)\left(\frac{\partial C_L}{\partial C} + \frac{\partial C_H}{\partial C}\right) \\ &= a\ u_{L,C}(C_L,X(t),t), \end{aligned} \tag{11}$$

where the second equality follows from Equation 10 and the third equality follows from differentiating both sides of the market clearing condition in Equation 6. Next we calculate the *second derivative of the aggregate investor's utility function*

$$u_{CC}(C(t), X(t), t) = a\ u_{L,CC}(C_L, X(t), t)\frac{\partial C_L}{\partial C}. \tag{12}$$

Define the absolute risk aversion of investor type j as

$$\mathcal{A}_j(t) = -\frac{u_{j,CC}(C_j(t), X(t), t)}{u_{j,C}(C_j(t), X(t), t)}. \tag{13}$$

We have that

$$\begin{aligned}
\mathcal{A}(t) &= -\frac{u_{CC}(C(t), X(t), t)}{u_C(C(t), X(t), t)} \\
&= -\frac{a\ u_{L,CC}(C_L(t), X(t), t)}{a\ u_{L,C}(C_L(t), X(t), t)}\frac{\partial C_L}{\partial C} \\
&= \mathcal{A}_L(t)\frac{\partial C_L}{\partial C}.
\end{aligned} \tag{14}$$

Thus, we also have that $\frac{\partial C_L}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}$*. Similarly, we get that* $\frac{\partial C_H}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_H(t)}$*. Using the fact that* $\frac{\partial C_L}{\partial C} + \frac{\partial C_H}{\partial C} = 1$*, we obtain*

$$\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)} + \frac{\mathcal{A}(t)}{\mathcal{A}_H(t)} = 1, \tag{15}$$

or

$$\mathcal{A}(t) = \left(\frac{1}{\mathcal{A}_L(t)} + \frac{1}{\mathcal{A}_H(t)}\right)^{-1}. \tag{16}$$

Using $\mathcal{R}(t) = \mathcal{A}(t)C(t)$ *together with Equation 16, we find*

$$\begin{aligned}
\mathcal{R}(t) &= \mathcal{A}(t)C(t) \\
&= \left(\frac{1}{\mathcal{A}_L(t)} + \frac{1}{\mathcal{A}_H(t)}\right)^{-1} C(t) \\
&= \left(\frac{C_L}{C(t)\gamma_L} + \frac{C_H}{C(t)\gamma_H}\right)^{-1} \\
&= \left(\frac{1}{\gamma_L} f(t) + \frac{1}{\gamma_H}(1 - f(t))\right)^{-1}.
\end{aligned} \tag{17}$$

The absolute prudence of the representative investor, $\mathcal{P}^A(t)$*, is*

$$\mathcal{P}^A(t) = -\frac{u_{CCC}(C(t), X(t), t)}{u_{CC}(C(t), X(t), t)}. \tag{18}$$

Similarly, we define the absolute prudence of investor j as

$$\mathcal{P}_j^A(t) = -\frac{u_{j,CCC}(C_j(t), X(t), t)}{u_{j,CC}(C_j(t), X(t), t)}. \tag{19}$$

To evaluate Equation 18, we need to calculate $u_{CCC}(C(t), X(t), t)$

$$\begin{aligned} u_{CCC}(C(t), X(t), t) &= \frac{\partial^2 \left(a\ u_{L,C}(C_L(t), X(t), t)\right)}{\partial C^2} \\ &= \frac{\partial \left(a\ u_{L,CC}(C_L(t), X(t), t)\right) \frac{\partial C_L(t)}{\partial C}}{\partial C} \\ &= a\ u_{L,CCC}(C_L(t), X(t), t) \left(\frac{\partial C_L(t)}{\partial C}\right)^2 \\ &\quad + a\ u_{L,CC}(C_L(t), X(t), t) \frac{\partial^2 C_L(t)}{\partial C^2}. \end{aligned} \tag{20}$$

Similarly, we calculate

$$\begin{aligned} u_{CCC}(C(t), X(t), t) &= (1-a)\ u_{H,CCC}(C_H(t), X(t), t) \left(\frac{\partial C_H(t)}{\partial C}\right)^2 \\ &\quad + a\ u_{H,CC}(C_H(t), X(t), t) \frac{\partial^2 C_H(t)}{\partial C^2}. \end{aligned} \tag{21}$$

Using Equation 20 and Equation 21, allow to compute

$$\frac{\partial C_L(t)}{\partial C} \mathcal{P}^A(t) = -\frac{\partial^2 C_L(t)}{\partial C^2} + \mathcal{P}_L^A(t) \left(\frac{\partial C_L(t)}{\partial C}\right)^2 \tag{22}$$

and

$$\frac{\partial C_H(t)}{\partial C} \mathcal{P}^A(t) = -\frac{\partial^2 C_H(t)}{\partial C^2} + \mathcal{P}_H^A(t) \left(\frac{\partial C_H(t)}{\partial C}\right)^2. \tag{23}$$

Adding up Equations 22 and 23 and noting that $\frac{\partial^2 C_L(t)}{\partial C^2} + \frac{\partial^2 C_H(t)}{\partial C^2} = 0$, we get

$$\mathcal{P}^A(t) = \mathcal{P}_L^A(t) \left(\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}\right)^2 + \mathcal{P}_H^A(t) \left(\frac{\mathcal{A}(t)}{\mathcal{A}_H(t)}\right)^2. \tag{24}$$

The relative prudence of the representative investor is

$$\begin{aligned} \mathcal{P}(t) &= \mathcal{P}^A(t) C(t) \\ &= (1+\gamma_L) \left(\frac{\mathcal{R}(t)}{\gamma_L}\right)^2 f(t) + (1+\gamma_H) \left(\frac{\mathcal{R}(t)}{\gamma_H}\right)^2 (1-f(t)). \end{aligned} \tag{25}$$

Proposition IA-1

Consumers' consumption dynamics evolve according to

$$\begin{aligned} dC_j(t) &= C_j(t)\left(\mu_{C_j}(t)dt + \sigma_{C_j}(t)dZ_C(t)\right), \\ \text{where} \quad \mu_{C_j}(t) &= \left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\mu_C(t) + \left(1 - \frac{\mathcal{R}(t)}{\gamma_j}\right)\lambda\omega(t) \\ &\quad + \frac{1}{2}\left[(1+\gamma_j)\left(\frac{\mathcal{R}(t)}{\gamma_j}\right) - \mathcal{P}(t)\right]\left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\sigma_C^2, \\ \sigma_{C_j}(t) &= \left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\sigma_C. \end{aligned} \tag{26}$$

Proof of Proposition IA-1

First, note that the individual consumption is only a function of aggregate consumption C and the habit level X. By Ito's lemma we have

$$dC_j(t) = \frac{\partial C_j(t)}{\partial C}dC(t) + \frac{\partial C_j(t)}{\partial X}dX(t) + \frac{1}{2}\frac{\partial^2 C_j(t)}{\partial C^2}\left(dC(t)\right)^2. \tag{27}$$

To evaluate Equation 27, we need the partial derivatives $\frac{\partial C_j(t)}{\partial C}$, $\frac{\partial C_j(t)}{\partial X}$ and $\frac{\partial^2 C_j(t)}{\partial C^2}$. From the proof of Proposition 2 we have that

$$\frac{\partial C_j(t)}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}, \tag{28}$$

and

$$\frac{\partial^2 C_j(t)}{\partial C^2} = \mathcal{P}_j^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}\right)^2 - \mathcal{P}^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}\right). \tag{29}$$

Next, we compute

$$\begin{aligned} \frac{\partial C_L(t)}{\partial X} &= \frac{\partial f(t)C(t)}{\partial X} \\ &= C(t)\frac{\partial f(t)}{\partial X} + f(t)\frac{\partial C(t)}{\partial X} \\ &= C(t)\frac{\partial f(t)}{\partial \omega}\frac{\partial \omega}{\partial X} \\ &= -C_L(t)\left(\frac{\mathcal{R}(t)}{\gamma_L} - 1\right)\frac{1}{X(t)}, \end{aligned} \tag{30}$$

where in the above we have used the fact that $\frac{\partial \omega}{\partial X} = -\frac{1}{X(t)}$ and $\frac{\partial f(t)}{\partial \omega} = f(t)\left(\frac{\mathcal{R}(t)}{\gamma_L} - 1\right)$. Similarly, we get that

$$\frac{\partial C_H(t)}{\partial X} = -C_H(t)\left(\frac{\mathcal{R}(t)}{\gamma_H} - 1\right)\frac{1}{X(t)}. \tag{31}$$

Inserting the partial derivatives together with the dynamics of C and X into Equation (27) yields the proposition.

Proof of Proposition 6

The expression for the state price density follows from the standard result that the state price density is proportional to the marginal utility of the representative investor

$$\xi(t) = \frac{u_C(C(t), X(t), t)}{u_C(C(0), X(0), 0)}. \tag{32}$$

The dynamics of the state price density follow, Duffie (2001),

$$\frac{d\xi(t)}{\xi(t)} - (r(t)dt + \theta(t)dZ_C(t)). \tag{33}$$

Next, applying Ito's lemma to $u_C(C(t), X(t), t)$ we obtain

$$\begin{aligned} du_C(C(t), X(t), t) &= u_{Ct}(C(t), X(t), t)dt + u_{CC}(C(t), X(t), t)dC(t) + u_{CX}(C(t), X(t), t)dX(t) \\ &\quad + \frac{1}{2}u_{CCC}(C(t), X(t), t)\left(dC(t)\right)^2 \\ &= \left(u_{CC}(C(t), X(t), t)C(t)\mu_C(t) + u_{CX}(C(t), X(t), t)X(t)\lambda\omega(t)\right)dt \\ &\quad + \left(\frac{1}{2}u_{CCC}(C(t), X(t), t)C(t)^2\sigma_C^2 + u_{Ct}(C(t), X(t), t)\right)dt \\ &\quad + u_{CC}(C(t), X(t), t)C(t)\sigma_C dZ_C(t). \end{aligned} \tag{34}$$

To evaluate Equation 34, we need in addition to $u_{CC}(C(t), X(t), t)$ also expressions for $u_{Ct}(C(t), X(t), t)$ and $u_{CX}(C(t), X(t), t)$. First note that

$$u_{Ct}(C(t), X(t), t) = -\rho u_C(C(t), X(t), t). \tag{35}$$

Next, we calculate $u_{CX}(C(t), X(t), t)$ as follows

$$\begin{aligned} u_{CX}(C(t), X(t), t) &= \frac{\partial a\ u_{L,C}(C_L(t), X(t), t)}{\partial X} \\ &= (\gamma_L - \eta)\, a\ u_{L,C}(C_L(t), X(t), t)X(t)^{-1} \\ &\quad + \gamma_L \frac{C(t)}{C_L(t)} f'(t) a\ u_{L,C}(C_L(t), X(t), t)X(t)^{-1} \\ &= \left(\gamma_L - \eta + \gamma_L \frac{C(t)}{C_L(t)}\left[\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)} - f(t)\right]\right) a\ u_{L,C}(C_L(t), X(t), t)X(t)^{-1} \\ &= (\mathcal{R}(t) - \eta)\, u_C(C(t), X(t), t)X(t)^{-1}. \end{aligned} \tag{36}$$

Since $f(t) = f(\omega(t))$, its derivative is given by $f'(t) = \frac{df(\omega(t))}{d\omega}$. Next, we use the fact that $\frac{\partial C_L(t)}{\partial C(t)} = \frac{\partial f(t)C(t)}{\partial C(t)} = f(t) + C(t)f'(t)\frac{\partial \omega(t)}{\partial C(t)} = f(t) + f'(t)$ together with $\frac{\partial C_L(t)}{\partial C(t)} = \frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}$. Now, note that we have that

$$u_{CCC}(C(t), X(t), t) = u_C(C(t), X(t), t)\mathcal{R}(t)\mathcal{P}(t)\frac{1}{C(t)^2}. \tag{37}$$

Inserting Equations 12, 35, 36 and 37 together with the corresponding dynamics of $C(t)$ and $X(t)$ into Equation 34 we get

$$\begin{aligned}\frac{du_C(C(t), X(t), t)}{u_C(C(t), X(t), t)} &= -\left(\rho + \eta\lambda\omega(t) + \mathcal{R}(t)(\mu_C - \lambda\omega(t)) - \frac{1}{2}\mathcal{R}(t)\mathcal{P}(t)\sigma_C^2\right)dt \\ &\quad -\mathcal{R}(t)\sigma_C dZ_C(t). \end{aligned} \tag{38}$$

Finally, matching the drift and diffusion coefficients in Equation 33 with Equation 38 we obtain

$$r(t) = \rho + \eta\lambda\omega(t) + \mathcal{R}(t)(\mu_C - \lambda\omega(t)) - \frac{1}{2}\mathcal{R}(t)\mathcal{P}(t)\sigma_C^2 \tag{39}$$

$$\theta(t) = \mathcal{R}(t)\sigma_C. \tag{40}$$

2 Empirics – Principal Component Analysis

We calculate the first principal component of the forty-five correlation series (PCA CORR), the ten series of 3-year ahead excess returns (PCA EXR), the ten series of standard deviations (PCA STD) and the ten series of quadratic variations of turnover (PCA QV) separately. To calculate the first principal component of all the series, we compute the average of the four sets of series to reduce the impact of the forty-five correlation series and obtain from the averages the first principal component (PCA TOTAL). First, we regress the first principal component of these four series onto model implied external relative habit. Second, we regress the first principal component from all the series, PCA TOTAL, onto model implied external relative habit.[1]

Table 1 shows the results from these principal component regressions. All regression coefficients show negative sign consistent with a heterogeneous consumer version of the model.

[1]The first principal component explains 54%, 86%, 87% and 71% of the variation in the 45 correlation coefficients, 10 standard deviations, 10 quadratic variations of turnover and 10 three years ahead excess returns. The first principal component explains 51% of the variation of the averages of the four series.

Further, all coefficient estimates for external relative habit show highly significant Newey-West corrected t-statistics. The adjusted R-squared range from 14.06% to 40.95%. Our results regarding excess returns are essentially unchanged if we correct the nominal short rate with expected inflation instead of realized inflation.[2] Overall, we conclude that signs of the coefficients as well as the explanatory power of the regressions support our theory.

3 Empirics – Log Price-Dividend Ratio Regressions

In our model the log price-dividend ratio is increasing in ω, i.e., the relation represents a one-to-one mapping. Indeed, the log price-dividend ratio leads to comparable results for the principal component analysis, Table 2, as well as for regressions that explain the averages of the series we study, Table 3.

4 Predicting Excess Returns In-Sample and Out-of-Sample

Model implied relative consumption forecasts excess returns in-sample and out-of-sample. Because relative consumption does not include the level of market prices, it is unlikely to produce spurious results. In the sense that relative consumption forecasts excess returns in the model, it is a natural predictor of stock market returns.[3]

The relation between the excess return on the market portfolio and relative consumption is

$$Corr_t(E_t(dR_M(t) - r(t)), \omega(t)) < 0, \tag{41}$$

which is negative for most of the distribution of ω. Hence, on average, the model implies a negative relation between expected excess returns and relative consumption. A discrete time formulation implies the following slope coefficient in a predictive regression

$$\beta_t = \frac{Cov_t(E_t(R_{M,t+1} - r_t), \omega_t)}{Var_t(\omega_t)}, \tag{42}$$

which is negative whenever Equation 41 is negative. Therefore, the model predicts on average negative relations between relative consumption and expected excess return of the market portfolio as well as other portfolios.

The first three rows of Table 4 show the coefficient estimate, Newey-West corrected t-statistics and the adjusted R-squared for in-sample regressions using relative consumption

[2]Model implied external relative habit and the real short rate show statistically significant negative relation.

[3]Cooper and Priestley (2009) argue that it is important to link predictability to economic fundamentals.

from the heterogeneous investor economy as the predictive variable. The table contains 3 sets of regressions: 1 year excesses returns, 3 year excess returns and 5 year excess returns with each set containing regressions for 10 industries using Kenneth French's industry classification and the market excess return. Regressions include a constant with data ranging from 1927 to 2009. The predictive impact of relative consumption is statistically significant at least at the 10 percent level except for the following sectors: Non-Durables (five year horizon), Energy (one and five year horizons), and Health and Utils for all horizons. Importantly, all coefficients appear with negative sign favoring the model with heterogeneity in risk aversion. The adjusted R-squared statistics indicate that regressions with significant coefficients explain at least 2.8 percent of the variation in excess industry and market returns. Overall, adjusted R-squared statistics first increase when the prediction horizon increases, 3 year versus 1 year, but decrease when the prediction horizon is 5 years.

Next, we ask whether predictive regressions perform also out-of-sample by making nested forecast comparisons. The comparisons are between a model which includes only the constant and a model which includes a constant and relative consumption as a predictor. Theil's U, in the fourth row for each prediction horizon in Table 4, is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number larger than one for Theil's U indicates that the restricted model (with only a constant) has a lower root-mean-squared error than the model with relative consumption as an explanatory variable. The root-mean-squared error of the regressions which include relative consumption are always lower than the regressions with a constant, except for Non-Durables (three and five year horizons) and Health and Utilities for all horizons. Another standard out-of-sample test is the MSE-F statistic which tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The fifth row for each prediction horizon in Table 4 shows the test statistics from our data. The ENC-NEW statistic, in the 6th row for each prediction horizon, tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. At the 10% level, we obtain a picture very similar to the previous results.

5 Empirics – Alternative Portfolio Sorts

In the main body of the paper we calibrate the model to ten industry portfolios. According to our model, stock market correlations, standard deviations and expected returns as well as quadratic variation of portfolio policies have negative relation with ω. This negative relation, however, is independent of portfolio sorts. Therefore, we repeat the regression from the main text of the paper and the predictive regression in Section 4, but use alternative portfolio sorts. In particular, we consider ten portfolios sorted on size, book-to-market and momentum, respectively. The Tables 5 - 10 confirm the prediction of our model and, thus, show that our empirical results in the main text of the paper are not an artifact of industry sorted portfolios.

References

Cooper, I., and R. Priestley, 2009, "Time-Varying Risk Premiums and the Output Gap," *Review of Financial Studies*, 22, 2601–2633.

Cox, J. C., and C. F. Huang, 1989, "Optimal Consumption and Portfolio Policies when Asset Prices Follow a Diffusion Process," *Journal of Economic Theory*, 49, 33–83.

Duffie, D., 2001, *Dynamic Asset Pricing Theory*, Princeton University Press.

Karatzas, I., J. P. Lehoczky, and S. E. Shreve, 1987, "Optimal Consumption and Portfolio Decisions for a 'Small Investor' on a Finite Horizon," *SIAM Journal of Control and Optimization*, 25, 1557–1586.

Table 1: **Empirics — Principal Component Analysis.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied external relative habit as explanatory variable for the first principal component of industry market correlations (PCA CORR), 3-year ahead expected excess returns (PCA EXR), standard deviations (PCA STDV), quadratic variations of industry turnover (PCA QV), and the first principal component of the four means of the respective series (PCA TOTAL). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover. Model implied external relative habit is linearly interpolated from the heterogeneous consumer model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	PCA CORR	PCA EXR	PCA STDV	PCA QV	PCA TOTAL
Intercept	0.5387	0.3617	0.3232	0.4423	0.1687
	(2.9547)	(3.6869)	(3.0444)	(6.7586)	(4.7019)
Model implied external habit, ω	-3.3680	-2.2556	-2.0205	-2.7656	-1.0524
	(-3.0384)	(-4.0211)	(-3.3327)	(-7.9762)	(-4.8871)
Adjusted R-squared	0.1406	0.1501	0.4095	0.2392	0.2719

Table 2: **Empirics with the Log Price-Dividend Ratio — Principal Component Analysis.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied log Price-Dividend ratio (pd) as explanatory variable of the first principal component of industry market correlations (PCA CORR), 3-year ahead expected excess returns (PCA EXR), standard deviations (PCA STDV), quadratic variations of industry turnover (PCA QV), and the first principal component of the four means of the respective series (PCA TOTAL). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	PCA CORR	PCA EXR	PCA STDV	PCA QV	PCA TOTAL
Intercept	2.3299	0.8976	0.3122	1.5327	0.5088
	(8.1551)	(5.3126)	(2.3408)	(13.0112)	(8.7718)
pd-ratio	-0.70161	-0.27175	-0.094015	-0.46155	-0.15404
	(-8.1361)	(-5.3423)	(-2.5066)	(-14.1999)	(-8.7895)
Adjusted R-squared	0.41733	0.14183	0.059489	0.45431	0.37974

Table 3: **Empirics with the Log Price-Dividend Ratio — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied log Price-Dividend ratio (pd) as explanatory variable of the average of industry market correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), standard deviations (Av. STDV), and quadratic variations of industry turnover (Av. QV). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover is estimated by a GARCH(1,1) model based on log changes in turnover. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV	Av. QV
Intercept	1.007	0.3739	0.29082	0.59651
	(25.3129)	(6.6404)	(7.1048)	(18.2762)
pd-ratio	-0.094924	-0.089417	-0.028203	-0.11399
	(-7.9479)	(-5.1959)	(-2.4477)	(-12.6464)
Adjusted R-squared	0.38805	0.14389	0.057661	0.47446

Table 4: **Empirics — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 industry returns, and the market return on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 industry returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other	Mkt
1 year	Coefficient	-0.7645	-2.2214	-1.3620	-0.5428	-1.0977	-0.8883	-1.1093	-0.3020	-0.1509	-0.8508	-0.8969
	t-statistics	-2.7532	-3.7688	-3.2671	-1.4148	-2.2542	-2.0395	-3.7377	-0.9408	-0.2421	-2.3849	-2.8392
	Adj. R-squared	0.0375	0.1101	0.0828	0.0065	0.0348	0.0527	0.0526	-0.0061	-0.0108	0.0280	0.0457
	Theil's U	0.9731	0.8976	0.9225	0.9970	0.9603	0.9711	0.9550	0.9966	1.0073	0.9648	0.9536
	MSE-F	2.2993	9.8919	7.1780	0.2493	3.4613	2.4753	3.9562	0.2815	-0.5932	3.0437	4.0887
	ENC-NEW	1.6596	6.5249	5.2799	0.3866	1.9220	1.5971	2.5484	0.1757	-0.2791	1.8737	2.5655
3 year	Coefficient	-1.2595	-4.3025	-3.0497	-1.3942	-2.9738	-2.1803	-1.9299	-0.8658	-0.6338	-1.8276	-2.0394
	t-statistics	-2.0500	-3.3514	-3.6276	-1.8990	-4.1732	-4.6367	-3.3049	-1.4796	-0.8203	-3.2314	-3.8722
	Adj. R-squared	0.0386	0.2071	0.1817	0.0331	0.1000	0.1154	0.0710	0.0048	-0.0024	0.0561	0.1008
	Theil's U	0.9729	0.8257	0.8085	0.9896	0.9025	0.9491	0.9464	0.9859	0.9845	0.9344	0.8994
	MSE-F	2.2630	18.6636	21.1884	0.8427	9.1083	4.4055	4.6619	1.1552	1.2670	5.8098	9.4460
	ENC-NEW	1.9320	13.6465	17.5287	1.3477	5.3968	3.2575	3.4095	0.6535	0.7912	3.7521	6.4276
5 year	Coefficient	-1.2952	-4.9389	-3.4826	-1.0383	-3.5356	-2.5932	-2.2884	-1.3108	-0.8938	-1.9686	-2.2946
	t-statistics	-1.5359	-3.5891	-3.1063	-0.9750	-3.3304	-4.7007	-3.1951	-1.3886	-1.3221	-2.0412	-2.8981
	Adj. R-squared	0.0258	0.2259	0.1845	0.0053	0.1043	0.1087	0.0845	0.0118	0.0014	0.0443	0.0939
	Theil's U	0.9726	0.7830	0.7734	0.9784	0.9079	0.9453	0.9294	0.9863	0.9772	0.9420	0.8856
	MSE-F	2.2283	24.6203	26.2042	1.7443	8.3154	4.6483	6.1528	1.0913	1.8392	4.9469	10.7276
	ENC-NEW	1.5420	16.1885	18.2481	1.2092	4.8803	3.4253	4.2000	0.64116	1.2122	3.0579	6.7181

Table 5: **Size Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten size sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Size sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.92608	0.17465	0.3775
	(134.7149)	(7.4878)	(9.2808)
Relative consumption, ω	-0.083886	-0.6404	-0.92581
	(-2.057)	(-4.7258)	(-3.957)
Adjusted R-squared	0.038919	0.10923	0.44339

Table 6: **BM Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten book-to-market sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Book-to-market sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.89778	0.28147	0.37096
	(82.2972)	(4.2825)	(9.1625)
Relative consumption, ω	-0.19937	-0.91342	-0.96231
	(-3.1788)	(-2.5318)	(-4.1551)
Adjusted R-squared	0.10979	0.096109	0.49111

Table 7: **Momentum Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten momentum sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Momentum sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.8573	0.21641	0.3113
	(60.3286)	(4.8501)	(8.0865)
Relative consumption, ω	-0.18107	-0.40609	-0.63045
	(-2.135)	(-1.6217)	(-2.9175)
Adjusted R-squared	0.052038	0.029092	0.28267

Table 8: **Size Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 size sorted portfolios on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 size sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-2.5331	-2.3642	-2.0631	-1.766	-1.5606	-1.522	-1.3315	-1.2363	-1.0641	-0.76187
	t-statistics	-3.7912	-3.9303	-3.4419	-3.678	-3.542	-3.5886	-3.4128	-3.3881	-3.0504	-2.4094
	Adj. R-squared	0.10335	0.12343	0.11171	0.093707	0.084875	0.086283	0.069929	0.072247	0.057857	0.034455
	Theil's U	0.90734	0.90559	0.90648	0.91603	0.93167	0.91258	0.94562	0.9418	0.94773	0.96313
	MSE-F	8.587	8.7749	8.6791	7.6691	6.0827	8.0307	4.7331	5.0969	4.5336	3.1212
	ENC-NEW	6.6204	7.1416	7.0036	5.7212	4.5479	5.6783	3.5434	3.7448	3.1748	1.8997
3 year	Coefficient	-5.4825	-5.0902	-4.4132	-3.915	-3.2542	-3.4706	-2.627	-2.4402	-2.3836	-1.9029
	t-statistics	-2.9939	-3.211	-3.0312	-3.7668	-3.4398	-4.2756	-3.0354	-2.9019	-3.4775	-4.1285
	Adj. R-squared	0.13887	0.17569	0.1748	0.17274	0.14305	0.17962	0.1134	0.12062	0.11901	0.089665
	Theil's U	0.88625	0.85769	0.86816	0.8739	0.89822	0.86268	0.91162	0.88741	0.88705	0.91651
	MSE-F	10.6536	14.0158	12.7447	12.0672	9.3394	13.4044	7.9286	10.5242	10.5642	7.4286
	ENC-NEW	9.312	12.8505	12.1593	10.5713	8.3344	11.5743	6.8144	8.5001	8.1435	4.7055
5 year	Coefficient	-6.2674	-5.5616	-4.7686	-4.5781	-3.6692	-4.0739	-2.7641	-2.5305	-2.668	-2.2446
	t-statistics	-2.6992	-2.7431	-2.7775	-3.4134	-3.1552	-3.8067	-2.4722	-2.4207	-2.7209	-2.9326
	Adj. R-squared	0.12387	0.15503	0.157	0.17529	0.14107	0.19233	0.10297	0.10742	0.11654	0.08405
	Theil's U	0.9001	0.86455	0.87416	0.87178	0.89524	0.85526	0.88538	0.86682	0.85659	0.90839
	MSE-F	8.9036	12.8402	11.7284	12.0001	9.4139	13.9507	10.4762	12.5739	13.7892	8.0513
	ENC-NEW	7.0617	10.0728	9.5515	9.7094	7.7083	11.1572	7.4234	8.497	9.217	4.7559

Table 9: **BM Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 book-to-market sorted portfolios on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 book-to-market sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-0.82169	-0.8233	-0.76847	-1.1144	-0.90286	-1.2255	-0.97298	-1.1637	-1.206	-1.5153
	t-statistics	-2.3926	-2.8631	-2.898	-2.8092	-2.32	-2.3933	-2.4825	-2.7033	-2.4605	-2.5837
	Adj. R-squared	0.026426	0.041171	0.034161	0.064196	0.036934	0.07056	0.036174	0.046697	0.044599	0.050486
	Theil's U	0.97336	0.95999	0.97043	0.96016	0.9571	0.93103	0.96008	0.93666	0.92978	0.9342
	MSE-F	2.2191	3.4037	2.4746	3.3882	3.6665	6.1463	3.3957	5.5929	6.2696	5.8331
	ENC-NEW	1.3681	2.1445	1.7087	2.7441	2.4577	4.3464	2.3702	3.7847	4.2616	4.0595
3 year	Coefficient	-1.9894	-1.8852	-1.4569	-2.436	-1.8727	-3.5065	-2.1905	-2.4929	-2.5124	-2.3348
	t-statistics	-3.1101	-3.8753	-2.9266	-3.003	-2.6954	-4.4321	-3.7471	-3.3561	-3.0991	-1.8417
	Adj. R-squared	0.07155	0.10998	0.059748	0.13227	0.066723	0.19868	0.09042	0.098653	0.084035	0.043883
	Theil's U	0.9377	0.90651	0.95494	0.93532	0.90063	0.84483	0.91744	0.85973	0.88695	0.90971
	MSE-F	5.3543	8.4587	3.7672	5.5802	9.081	15.6425	7.3346	13.7645	10.5757	8.126
	ENC-NEW	3.4442	5.7063	2.9289	5.9387	5.9431	14.7487	5.9752	10.2236	8.0908	5.7096
5 year	Coefficient	-2.4954	-2.3527	-1.5835	-2.4631	-1.7428	-4.1236	-2.4591	-2.4624	-2.6212	-1.3322
	t-statistics	-2.911	-3.1794	-2.3926	-2.3457	-1.6343	-3.9262	-2.6903	-2.2095	-2.3383	-0.75529
	Adj. R-squared	0.083094	0.12797	0.054045	0.10096	0.036314	0.19746	0.088374	0.067935	0.073144	0.0020927
	Theil's U	0.92249	0.88364	0.93753	0.91574	0.93415	0.80051	0.89156	0.8689	0.88681	0.96878
	MSE-F	6.6544	10.6666	5.2324	7.3151	5.5462	21.2986	9.8059	12.3314	10.3195	2.4883
	ENC-NEW	3.9994	6.7739	3.3904	5.7852	3.2813	17.3771	6.8504	7.7807	7.1821	1.5034

Table 10: **Momentum Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 momentum sorted portfolios on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 momentum sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-1.4134	-1.3645	-0.64953	-1.1966	-0.81324	-0.98901	-0.91562	-0.90842	-1.0725	-0.74576
	t-statistics	-3.2985	-3.1765	-1.8873	-3.3075	-2.4874	-3.2436	-2.5844	-2.4358	-2.7167	-1.5307
	Adj. R-squared	0.042124	0.063928	0.010663	0.068581	0.037633	0.053421	0.047874	0.04295	0.053237	0.0091011
	Theil's U	0.95863	0.95785	0.98744	0.94953	0.95497	0.94866	0.93964	0.95353	0.95291	0.98042
	MSE-F	3.527	3.5974	1.0238	4.3657	3.8612	4.4468	5.3037	3.9942	4.0511	1.6139
	ENC-NEW	2.2202	2.6941	0.67696	3.3608	2.3963	2.9122	3.3162	2.6541	2.8495	0.97364
3 year	Coefficient	-2.4182	-3.2404	-1.2261	-2.4368	-1.7952	-2.072	-2.3744	-1.9882	-2.3738	-1.9526
	t-statistics	-3.1389	-4.0428	-2.8487	-3.2629	-4.1023	-3.7515	-5.4936	-2.7736	-2.912	-2.4118
	Adj. R-squared	0.060771	0.16318	0.019003	0.11417	0.090235	0.098443	0.1265	0.081617	0.1037	0.043638
	Theil's U	0.94223	0.91922	0.98599	0.90756	0.89059	0.89092	0.85984	0.91074	0.91304	0.95019
	MSE-F	4.9292	7.1561	1.1161	8.3493	10.1706	10.1344	13.7511	8.0195	7.7828	4.1957
	ENC-NEW	3.3809	7.9184	1.0365	7.205	6.6044	6.8477	9.4183	5.2956	5.9867	2.7626
5 year	Coefficient	-3.0006	-4.2482	-1.6043	-2.776	-2.1352	-2.1092	-3.0671	-2.0398	-2.1098	-1.8455
	t-statistics	-2.5646	-4.3167	-2.664	-2.8204	-2.9244	-2.6365	-4.5601	-1.9102	-1.8279	-1.8869
	Adj. R-squared	0.071098	0.20035	0.026245	0.11867	0.08461	0.071213	0.1501	0.058496	0.058741	0.028411
	Theil's U	0.92636	0.85332	0.96323	0.85252	0.87129	0.89095	0.81768	0.93075	0.93319	0.95987
	MSE-F	6.2819	14.1862	2.9563	14.2846	12.0557	9.8713	18.8358	5.8653	5.636	3.2442
	ENC-NEW	4.1534	13.0081	2.1192	10.0006	7.2018	5.9492	12.3747	3.5552	3.6923	1.9515