E-Companion

Extrapolative Beliefs in Perceptual and Economic Decisions: Evidence of a Common Mechanism

This E-companion contains supplementary material. Section 1 includes supplementary information on preprocessing the reaction time data and on the ordering of tasks in the experimental sessions. Section 2 provides robustness checks and extended statistical tests of the basic results from the PDT and EDT. Section 3 provides details on derivation of the prior probability from the DDM parameters. Section 4 contains the full set of instructions given to our experimental subjects.

1. Data preprocessing and order of experimental tasks

1.1 Preprocessing of reaction time data. The PDT consisted of 4 blocks of 300 trials each. RTs and error rates systematically increased over the course of each block, likely due to subjects' fatigue (See Fig. S1 and S2). To control for this, we removed a linear time trend, within each block, for each subject. All results and analyses in the text use this de-trended RT data and are robust to exclusion of the de-trending step.

1.2 On the ordering of the tasks in the experimental session. In our within-subjects design, the ordering of the tasks was not randomized and the PDT always took place first. We began the experiment with the PDT for all subjects because during pilot testing, we observed a sharp fatigue effect in subjects' response times in this task (see Fig. S1-S3). We were therefore concerned that this fatigue effect would vary between subjects if the PDT was administered later in the session for a subset of subjects. Furthermore, because subjects in the PDT were explicitly informed that the probability of seeing either shape was 0.5, we believe that any possible spillover effects between the two tasks should bias us against finding the extrapolation effect observed in the EDT (where subjects were not explicitly informed about the underlying random process).

2. Robustness checks and extended statistical tests

Table S1
MIXED MODEL LINEAR REGRESSION (SUB. RANDOM INTERCEPT & SLOPE), PDT

	Dependent variable:
	AdjustedRT (correct)
StreakLength	0.002***
	(0.001)
Continuation	0.020***
	(0.004)
StreakLength x Continuation	-0.015***
	(0.001)
Constant	0.394***
	(0.008)
Observations	42,476
Log Likelihood	36,343.610
Akaike Inf. Crit.	-72,663.220
Bayesian Inf. Crit.	-72,559.340
Note:	*p<0.1; **p<0.05; ***p<0.01

Table S2:
MIXED MODEL LOGISTIC REGRESSION (SUB. RANDOM INTERCEPT & SLOPE), PDT

	Dependent variable:
	Correct=1
StreakLength	-0.136***
	(0.028)
Continuation	-1.129***
	(0.103)
StreakLength x Continuation	0.642***
	(0.042)
Constant	3.240***
	(0.100)
Observations	44,992
Log Likelihood	-9,327.701
Akaike Inf. Crit.	18,677.400
Bayesian Inf. Crit.	18,773.260
Note:	*p<0.1; **p<0.05; ***p<0.01

The following matrices summarize the pairwise t-statistics that result from comparing mean RTs presented in Figures 2A and 2B (Each subject mean is used as single observation).

Table S3: Difference in mean continuation response times at different streak lengths

Continuation	**1**	**2**	**3**	**4**	**5**	**6**
1		t(37)=9.48 p<0.001	t(37)=10.53 p<0.001	t(37)=10.27 p<0.001	t(37)=6.99 p<0.001	t(37)=8.81 p<0.001
2			t(37)=7.99 p<0.001	t(37)=5.99 p<0.001	t(37)=4.15 p<0.001	t(37)=5.95 p<0.001
3				t(37)=2.09 p=0.04	t(37)=1.63 p=0.11	t(37)=3.05 p=0.004
4					t(37)=0.27 p=0.79	t(37)=1.34 p=0.19
5						t(37)=1.08 p=0.29
6						

Table S4: Difference in mean error rates at different streak lengths

Continuation	1	2	3	4	5	6
1		t(37)=7.20 p<0.001	t(37)=12.91 p<0.001	t(37)=9.99 p<0.001	t(37)=10.42 p<0.001	t(37)=-7.22 p<0.001
2			t(37)=-3.25 p<0.002	t(37)=-3.95 p<0.001	t(37)=-4.98 p<0.001	t(37)=-3.84 p<0.001
3				t(37)=-1.49 p=0.14	t(37)=-3.09 p=0.004	t(37)=-1.98 p=0.05
4					t(37)=-1.15 p=0.26	t(37)=-1.31 p=0.19
5						t(37)=-0.22 p=0.83
6						

Table S5: Difference in mean response times between continuation and violation trials

Streak	1	2	3	4	5	6
	t(37)=0.45 p=0.65	t(37)=3.95 p<0.001	t(37)=5.86 p<0.001	t(37)=6.41 p<0.001	t(37)=6.56 p<0.001	t(37)=7.19 p<0.001

Table S6: Difference in mean error rates between continuation and violation trials

Streak	1	2	3	4	5	6
	t(37)=1.02 p=0.31	t(37)=3.65 p<0.001	t(37)=4.61 p<0.001	t(37)=4.94 p<0.001	t(37)=5.36 p<0.001	t(37)=4.95 p<0.001

Table S7:
MIXED MODEL LINEAR REGRESSION (SUB. RANDOM INTERCEPT), EDT

	Dependent variable:
	EDT beliefs
Streak	4.463***
	(0.586)
Constant	48.613***
	(0.669)
Observations	15,580
Log Likelihood	-71,203.040
Akaike Inf. Crit.	142,420.100
Bayesian Inf. Crit.	142,473.700
Note:	*p<0.1; **p<0.05; ***p<0.01

3. Derivation of Prior Decoding Technique

The equation that we use to derive the prior probability from the DDM parameters is based on the basic DDM, but it can be extended to the full DDM in a straightforward manner by integrating the original equation against the across trial variability parameters. In particular, we start from equation (A3) from Ratcliff and Smith (2004), which provides the probability of hitting the upper boundary in the basic DDM. In our setting, this corresponds to the probability of making the "correct" response:

(S1) $$q^{basic}(\mathrm{c,a,b,M,s}) = \frac{e^{\frac{-2Mc}{s^2}} - e^{\frac{-2Mb}{s^2}}}{e^{\frac{-2Ma}{s^2}} - e^{\frac{-2Mb}{s^2}}}.$$

In this expression, *c* represents the initial point which always lies between the lower boundary, b, and the upper boundary, a. The prior probability of choosing the correct response can be computed by assuming that the drift rate, M, tends to 0. In this case, the stimulus contains no decision-relevant information, and therefore the probability of responding correctly is a function of the prior probability and the noise alone. Calculating the limit as the drift rate approaches zero by applying L'Hopital's rule, we find that,

(S2) $$\lim_{M\to 0} q^{basic}(\mathrm{c,a,b,M,s}) = \frac{\mathrm{c}-b}{a-b}$$

Using this result, we can then compute the prior probability of hitting the upper boundary for the extended DDM by integrating $q^{basic}(\mathrm{c,a,b,M,s})$ against the across trial variability parameters, and then allowing M to get arbitrarily small. In particular, the probability of crossing the upper boundary under the extended DDM is:

(S3) $$q^{advanced}(\mathrm{c,a,b,M,s}) = \iiint q^{basic}(\mathrm{c,a,b,M,s}) f(c)g(M)h(T)\, dcdMdT,$$

where *f, g, and h* represent the probability density functions of the initial point, drift rate, and non-decision time, respectively. To compute the prior probability, we take the limit as the drift rate goes to 0.

(S4) $$\lim_{M\to 0} q^{advanced}(\mathrm{c,a,b,M,s}) =$$

$$\lim_{M\to 0} \iiint q^{basic}(\mathrm{c,a,b,M,s}) f(c)g(M)h(T)\, dcdMdT$$

$$= \iiint \lim_{M\to 0} q^{basic}(\mathrm{c,a,b,M,s}) f(c)g(M)h(T)\, dcdMdT$$

$$= \iiint \frac{\mathrm{c}-b}{a-b} f(c)g(M)h(T)\, dcdMdT$$

$$= \int \frac{\mathrm{c}-b}{a-b} f(c)dc$$

$$= \frac{\mathrm{c}-b}{a-b}$$

The second equality is justified by the Dominated Convergence Theorem, which allows us interchange the order of the limit and the integration. The fifth equality is based on the facts that *c* is uniformly distributed and the integrand $\frac{c-b}{a-b}$, is linear in c. Finally, for the case at hand, since we set the lower boundary b to zero, the prior probability of reaching the upper boundary under the extended DDM is given by $\frac{c}{a}$. Figure S5 provides results from simulations for three different levels of $\frac{c}{a}$, and shows that the probability of choosing the correct alternative does converge to $\frac{c}{a}$ as the drift rate approaches zero.

4. Instructions

Thank you for participating in this experiment.
For your participation you have already made $5. During the rest of the experiment you have the chance to make more money. Your final payoff for participating depends on your decisions in Parts I, II.

Part I:

In this part of the task, you will see a sequence of shapes; each element of the sequence will be displayed one at a time. There are only two possible shapes: a white circle and a white square. Your task is to accurately classify which shape is currently being presented, as quickly as possible. If you see the **circle**, press the **right** arrow button, and if you see the **square**, press the **left** arrow button. The trials will be broken up into 4 separate blocks of 300 trials; after each block, you will have a 20 seconds break. Please use only one hand to enter both buttons.
For every shape you correctly classify, you will be paid 1 cent. If you classify all the shapes correctly, you will make 1200*0.01=$12.00. However, for every 0.05 second it takes you to respond, you will lose 0.1 cents. (you will have a maximum of 2 seconds/trial to respond). **Therefore, to make the most money possible, you should answer as quickly and as accurately as you can.**

In each trial, the chance that you will see a circle is ½, and the chance that you will see a square is ½. Shapes on previous trials have no influence on the shape in the current trials; in other words the shape you see on the current trial is completely independent of all other shapes you've already seen.
Before the real task starts, you will start with 5 practice trials.

Part II (instructions were given only after part I was completed)

We have studied large numbers of publicly traded companies, and constructed models of their performance patterns. Using these models, we created sequences to represent patterns of "surprises" (actual performance minus predicted performance). An upward movement indicates a "positive surprise," which results when the firm performs better than expected, and a downward movement indicates a "negative surprise" when the firm performs worse than expected.

In this task, you will see a sequence of 400 performance surprises from a typical company, and your job is to estimate whether the next performance surprise will be positive or negative. For each of the 400 periods, you will see the performance surprises of the last 14 periods on the screen.

In each period, you will be asked to give a price at which you would be willing to buy a share of stock in this company. If you buy the stock and see a positive surprise, the stock will pay you

$100. If you buy the stock and see a negative surprise, the stock will pay you $0. The important thing to understand is the following: the price you are willing to pay will, in general, not be the price you actually pay for the stock. Instead, the actual price of the stock will be drawn randomly between $0 and $100. If your willingness to pay is above this random price, you will pay the random price and receive a share of the company. If your willingness to pay is below the random price, you do not buy the share of the company. **In order to make the most money under this rule, the best thing for you to do is set the price equal to probability you think there will be a positive surprise.**

Examples

1. Suppose you believe that there will be a positive earnings surprise with 75% chance. You should then be willing to pay exactly $75 for this share; if the actual price is $50, then you will pay $50 for something that has a 75% chance of winning $100 which on average, will make you money. If instead the random price drawn was $90, the rule says that you will not buy this stock since $75<$90. This is good because you avoid paying $90 for something that has only a 75% chance of paying you $100.

2. Suppose you are certain (a 100% chance) that there will be positive performance surprise. Then you would be willing to pay any price between $0-$100 to buy this stock. The only way to guarantee that you buy this stock is to set your price exactly equal to $100. If you made a mistake and set the price of the stock to $90, then if the random price drawn is $92, you would not be able to buy the $92 stock, which has a 100% chance to pay $100.

3. Suppose you are certain that there will be negative performance surprise (0% chance of a positive surprise). Then you are not willing to pay any price to buy this stock. The only way to guarantee that you don't end up paying something for this stock, is to set your price exactly equal to $0. If instead, you made a mistake and entered $10, then if the actual price drawn was $8, you would end up paying $8 for a stock that has 0% chance of paying you.

In each period, you will be given $100 in experimental currency to buy a share of the stock. Since the maximum price you would ever pay for a share is $100, you will always have enough cash to buy a share of this stock, since you receive a new $100 endowment each period. Your payoff in each period will depend on the three things: your willingness to pay, the actual price, and whether there was a positive or negative surprise. To illustrate your payoffs consider the two scenarios.

If you believe there will be a positive surprise for sure, and your willingness to pay is $100, and the actual price drawn is $0, and there is actually a positive surprise, then you will end the period with $100 – $0 + $100= $200. That is, you will end the period with the $100 you started with, you don't pay any cost since the price was $0, and you earn $100 for buying the stock and having a positive earning surprise.

If you believe there will be a positive surprise with 60% chance, the actual price drawn is $30, and there is a positive surprise, then your total earnings this period will be $100-$30+$100=$170.

Your final earnings will be the sum of each of your individual period earnings, divided by 5,000. It is important to emphasize once more: **the only way to maximize your final earnings is to**

enter your willingness to pay equal to the probability you think there will be a positive surprise.

After every 50 trials, you will see your accumulated payoff, and will be allowed to take a short break. Before the real task starts, you will start with 5 practice trials.

Figure S1. Average reaction times across subjects and across four blocks of trials.

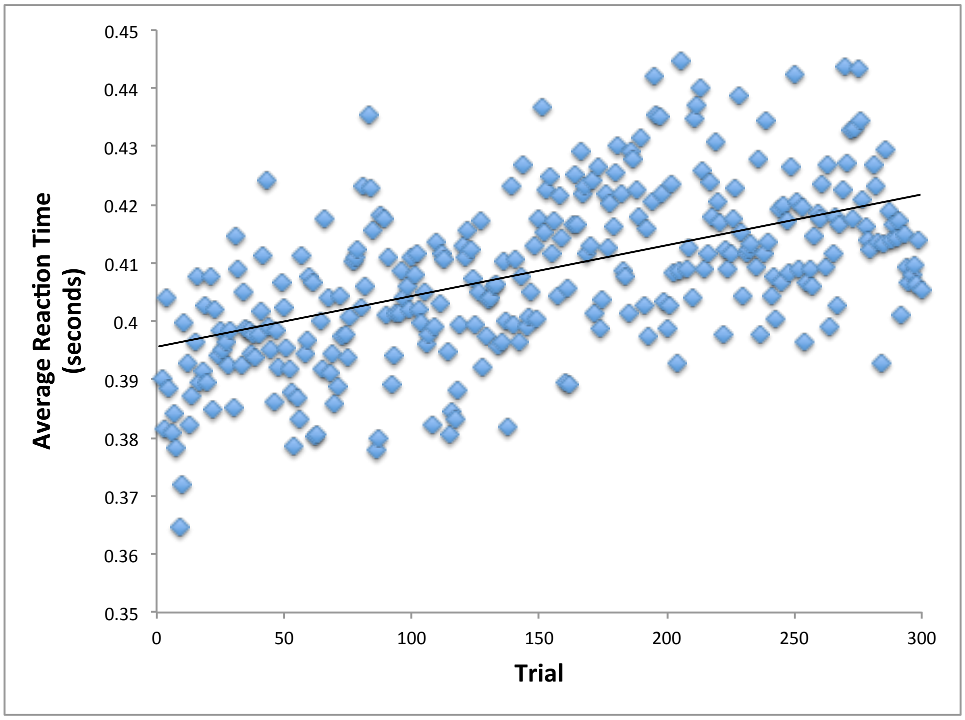


Figure S2. Average reaction times across subjects for each of the four blocks of trials (grouped by trials of 10).

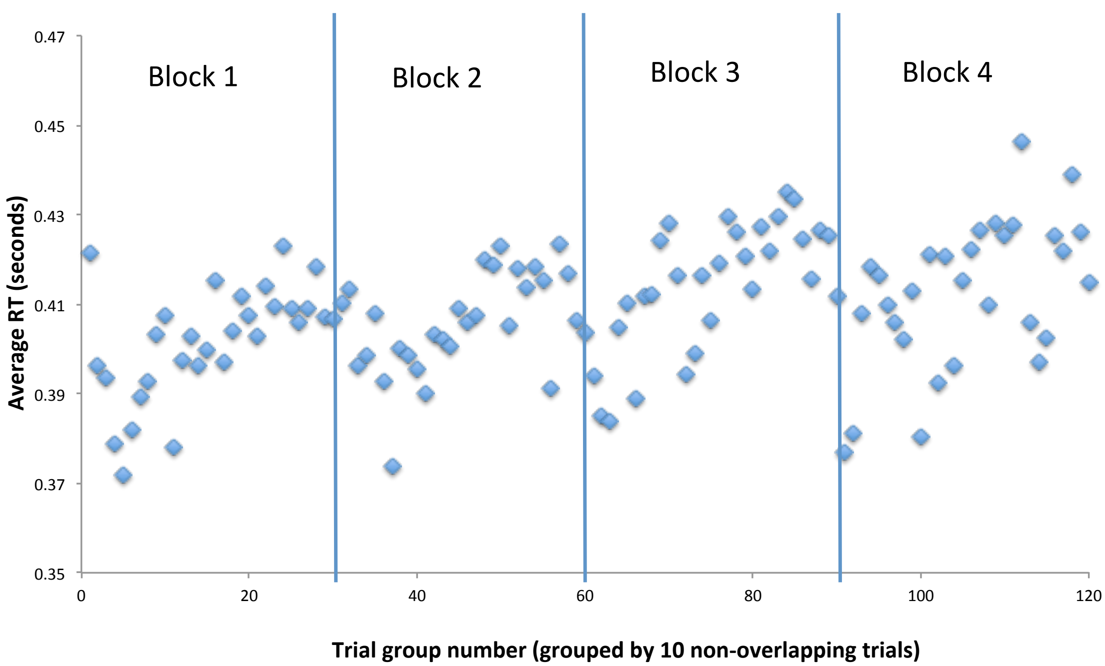


Figure S3. Average rate of correct responses across subjects for each of the four blocks of trials.

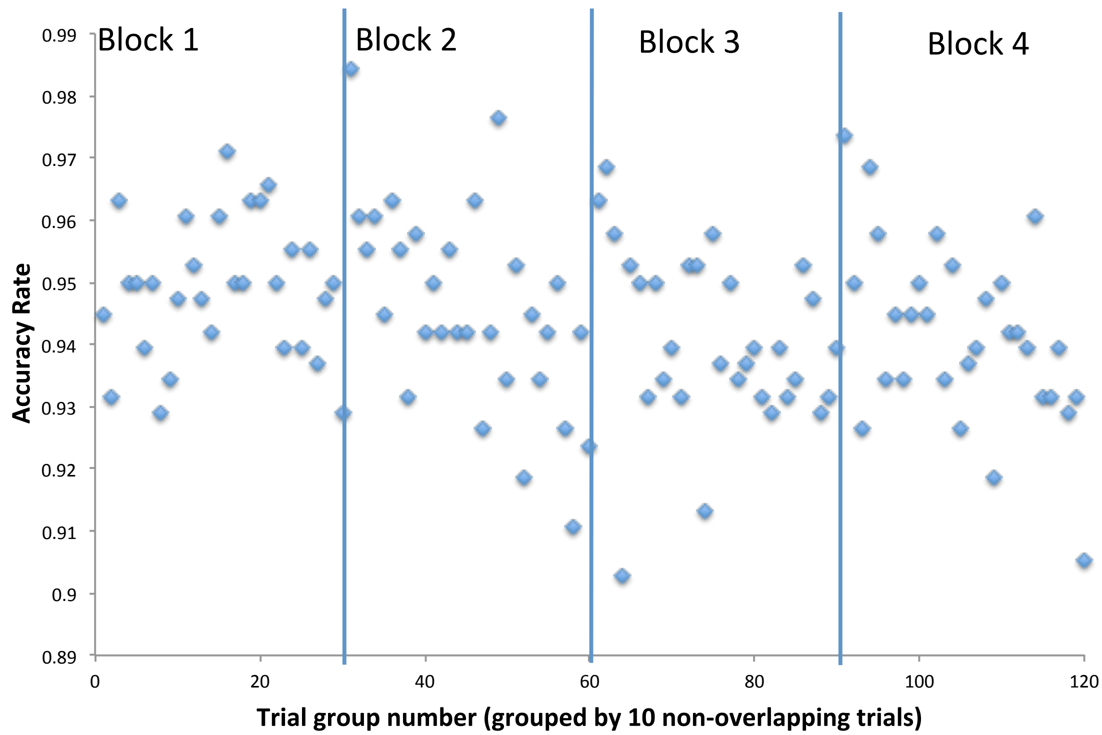


Figure S4. Average response times of correct and incorrect responses following "valid" cues (i.e., repetition following two or more repetitions or alternation following two or more alternations), 'invalid' cues (i.e., alternation following two or more repetitions or repetition following two or more alternations) and 'neutral' cues (all other trials).

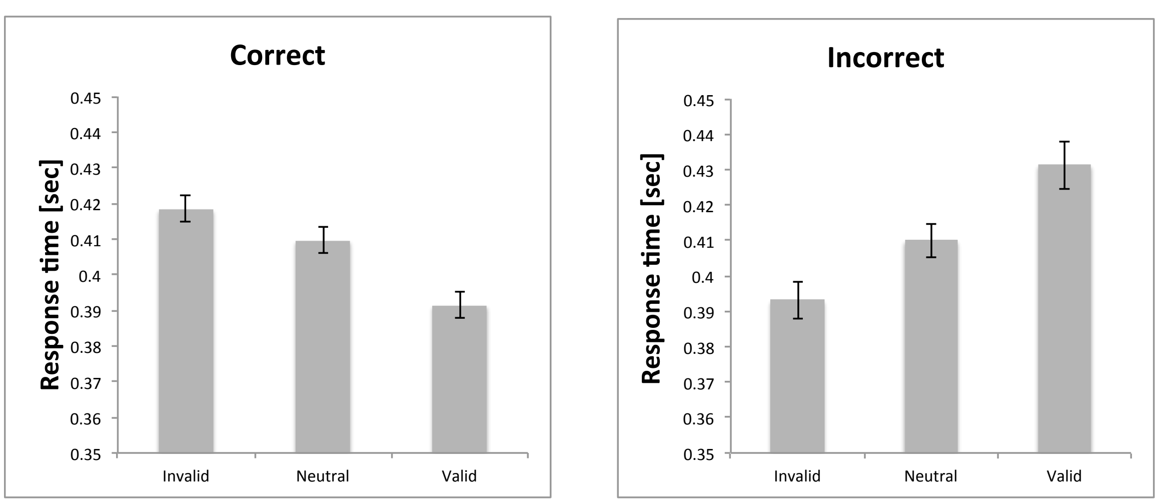
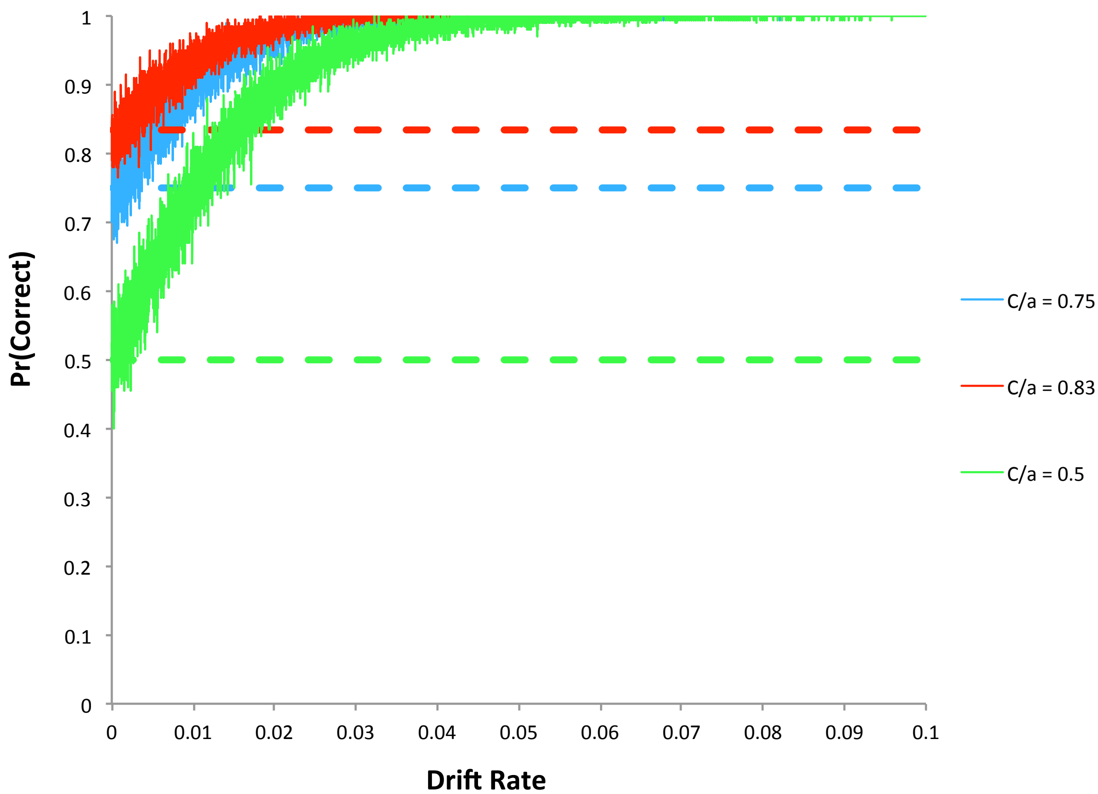


Figure S5. Simulations of choosing the "correct" alternative as a function of the drift rate. For a given level of within trial noise, as the drift rate tends to zero, the probability of choosing the correct alternative converges to C/a. Three different levels of C/a are shown: 0.5, 0.75, and 0.83.



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