

Supplemental File:
“Supplier Competition and Cost Improvement”

1. Technical Analysis

1.1 Proof of Lemma 1:

The lemma applies to both the sole-sourcing case (where $S = \{i\}$, $i = 1$ and $j = \emptyset$) and the dual-sourcing case (where $S = \{1, 2\}$ and $j = S \setminus \{i\}$). The following proof applies to both cases. Consider a given effort profile $\mathbf{e} = (e_i)_{i \in S}$. Let $\hat{\pi}_i(\hat{\gamma}_i | \gamma_i)$ denote supplier i 's expected profit if he reports $\hat{\gamma}_i$ when his true cost equals γ_i . We have $\hat{\pi}_i(\hat{\gamma}_i | \gamma_i) = \bar{t}_i(\hat{\gamma}_i, \mathbf{e}) - \gamma_i \bar{q}_i(\hat{\gamma}_i, \mathbf{e})$, where $\bar{t}_i(\hat{\gamma}_i, \mathbf{e}) \equiv \mathbb{E}_{\gamma_j}[t_i(\hat{\gamma}_i, \gamma_j)]$ and $\bar{q}_i(\hat{\gamma}_i, \mathbf{e}) \equiv \mathbb{E}_{\gamma_j}[q_i(\hat{\gamma}_i, \gamma_j)]$. We adopt the standard mechanism design approach as in Myerson (1981), and provide a sketch of the proof as follows. By the Revelation Principle (Lemma 1 of Myerson 1981), we can focus on the truth-telling mechanisms without loss of generality. In a truth-telling mechanism each supplier i finds it optimal to report $\hat{\gamma}_i = \gamma_i$ regardless of the realized γ_i ; this is the so-called incentive compatibility constraint: $\hat{\pi}_i(\hat{\gamma}_i | \gamma_i) \leq \hat{\pi}_i(\gamma_i | \gamma_i), \forall \gamma_i, \hat{\gamma}_i$. Duplicating the proof of Lemma 2 in Myerson (1981), we can show that the incentive compatibility constraint is equivalent to the combination of the following two constraints: (a) $\bar{q}_i(\gamma_i, \mathbf{e})$ decreasing in γ_i ; (b) $\hat{\pi}_i(\gamma_i | \gamma_i) = \hat{\pi}_i(\bar{c} - e_i | \bar{c} - e_i) + \int_{\gamma_i}^{\bar{c} - e_i} \bar{q}_i(\rho, \mathbf{e}) d\rho$. Given (b), supplier i 's participation constraint (i.e., he will not reject the buyer's procurement mechanism) is equivalent to $\hat{\pi}_i(\bar{c} - e_i | \bar{c} - e_i) \geq 0$. Duplicating the proof of Lemma 3 in Myerson (1981), we can show that the constraint (b) implies that the buyer's expected profit in any truth-telling mechanism equals $\mathbb{E}[\sum_{i \in S} \{\hat{\pi}_i(\bar{c} - e_i | \bar{c} - e_i) + (r - J(\gamma_i, e_i))q_i(\gamma)\}]$. This implies that the optimal mechanism that maximizes the buyer's expected profit has $\hat{\pi}_i(\bar{c} - e_i | \bar{c} - e_i) = 0$, and $q_i(\gamma) = 1$ if and only if $r - J(\gamma_i, e_i) \geq \max\{0, \max_{i' \in S} \{r - J(\gamma_{i'}, e_{i'})\}\}$. Given the optimal $q_i(\gamma)$, the constraint (b) described above implies the descriptions of $t_i(\gamma)$ in the lemma, and equations (1)-(2).

1.2 Equilibrium analysis of the no-commitment mechanism

1.2.1 Unobservable effort

Sole sourcing: Lemma 1 implies that the sole-sourcing game equilibrium effort, denoted by e_S^u , satisfies the equation $ae_S^u = \bar{q}(e_S^u)$, where $\bar{q}(e)$ is the supplier's expected winning quantity with effort e . Note that $\bar{q}(e)$ equals 1 if $e > 2\Delta - r + \underline{c}$ and equals $\frac{r + e - \underline{c}}{2\Delta}$, otherwise (as the cut-off cost is $\xi(e) = \frac{r + e - \underline{c}}{2}$). Therefore, the supplier's equilibrium effort e_S^u and the buyer's expected profit Π_S^u are:

$$e_S^u = \begin{cases} \frac{1}{a}, & \text{if } a(2\Delta - r + \underline{c}) < 1; \\ \frac{r - \underline{c}}{2a\Delta - 1}, & \text{otherwise;} \end{cases} \quad \Pi_S^u = \begin{cases} r - \bar{c} - \frac{1}{a}, & \text{if } a(2\Delta - r + \underline{c}) < 1; \\ \frac{(r - \underline{c})^2 a^2 \Delta}{(2a\Delta - 1)^2}, & \text{otherwise.} \end{cases}$$

Dual sourcing. Lemma 1 implies that the dual-sourcing game equilibrium efforts of suppliers 1 and 2, denoted by $e_{D,1}^u$ and $e_{D,2}^u$, respectively, satisfy the equations $ae_{D,i}^u = \bar{q}_i(e_{D,1}^u, e_{D,2}^u), i \in \{1, 2\}$, where $\bar{q}_i(\mathbf{e})$ is supplier i 's expected winning quantity given the effort profile $\mathbf{e} = (e_1, e_2)$. We now

derive the expression of $\bar{q}_i(\mathbf{e})$. The expected winning quantity depends on the value range of e_1 and e_2 . For convenience, we call the supplier with the higher effort the cost leader, and denote his effort by $e_L \equiv \max(e_1, e_2)$; we call the supplier with the lower effort the cost follower, and denote his effort by $e_F \equiv \min(e_1, e_2)$. For given e_L and e_F , the winning probabilities of the leader and follower are, respectively: 1) $d_L^{(1)}(e_L, e_F)$ and $d_F^{(1)}(e_L, e_F)$ if $e_L - e_F < 2\Delta$ and $e_L < 2\Delta - r + \underline{c}$, 2) $d_L^{(2)}(e_L, e_F)$ and $d_F^{(2)}(e_L, e_F)$ if $e_L - e_F < 2\Delta$ and $e_L > 2\Delta - r + \underline{c}$, and 3) 1 and 0 if $e_L - e_F > 2\Delta$, where

$$\begin{aligned} d_L^{(1)}(e_L, e_F) &\equiv F\left(\underline{c} + \frac{e_L - e_F}{2}\right) + \int_{\underline{c} + \frac{e_L - e_F}{2}}^{\eta(e_L)} \left(1 - F\left(c - \frac{e_L - e_F}{2}\right)\right) f(c) dc, \\ d_F^{(1)}(e_L, e_F) &\equiv \int_{\underline{c}}^{\eta(e_F)} \left(1 - F\left(c + \frac{e_L - e_F}{2}\right)\right) f(c) dc, \\ d_L^{(2)}(e_L, e_F) &\equiv F\left(\underline{c} + \frac{e_L - e_F}{2}\right) + \int_{\underline{c} + \frac{e_L - e_F}{2}}^{\bar{c}} \left(1 - F\left(c - \frac{e_L - e_F}{2}\right)\right) f(c) dc, \\ d_F^{(2)}(e_L, e_F) &\equiv \int_{\underline{c}}^{\bar{c} - \frac{e_L - e_F}{2}} \left(1 - F\left(c + \frac{e_L - e_F}{2}\right)\right) f(c) dc = \frac{1}{2} \left(1 - \frac{e_L - e_F}{2\Delta}\right)^2. \end{aligned}$$

From supplier i 's perspective, $\bar{q}_i(\mathbf{e})$ can be characterized as follows: For $e_j < 2\Delta - r + \underline{c}$,

$$\bar{q}_i(\mathbf{e}) = \begin{cases} d_F^{(1)}(\mathbf{e}) & \text{if } e_i < e_j \\ d_L^{(1)}(\mathbf{e}) & \text{if } e_i \in [e_j, 2\Delta - r + \underline{c}] \\ d_L^{(2)}(\mathbf{e}) & \text{if } e_i \in [2\Delta - r + \underline{c}, e_j + 2\Delta] \\ 1 & \text{if } e_i > e_j + 2\Delta \end{cases},$$

and for $e_j > 2\Delta - r + \underline{c}$,

$$\bar{q}_i(\mathbf{e}) = \begin{cases} d_F^{(2)}(\mathbf{e}) & \text{if } e_i < e_j \\ d_L^{(2)}(\mathbf{e}) & \text{if } e_i \in [e_j, e_j + 2\Delta] \\ 1 & \text{if } e_i > e_j + 2\Delta \end{cases}.$$

Figure SF-1 illustrates supplier 1's expected winning quantity $\bar{q}_1(\mathbf{e})$.

In the following, we provide detailed derivations for the best-response functions $e_i^*(e_j)$ and the results in the paper's Table 2. To derive the best-response $e_i^*(e_j)$, we define several critical values of a : $\hat{a}_L^{(0)}(e) \equiv \frac{d_L^{(1)}(e, e)}{e}$, $\hat{a}_L^{(1)}(e) \equiv \frac{d_L^{(1)}(2\Delta - r + \underline{c}, e)}{2\Delta - r + \underline{c}}$, $\hat{a}_L^{(2)}(e) \equiv \frac{d_L^{(2)}(e + 2\Delta, e)}{e + 2\Delta}$, and $\tilde{a}_L^{(0)}(e) \equiv \frac{d_L^{(2)}(e, e)}{e}$, which are also illustrated in Figure SF-1.

The function $e_i^*(e_j)$ can be specified by:

- 1) If $e_j < 2\Delta - r + \underline{c}$ and $a \leq \hat{a}_L^{(2)}(e_j)$, e_i^* is defined by $ae_i^* = 1$, giving $e_i^* = \frac{1}{a}$.
- 2) If $e_j < 2\Delta - r + \underline{c}$ and $\hat{a}_L^{(2)}(e_j) < a \leq \min(\hat{a}_L^{(1)}(e_j), \tilde{a}_L^{(0)}(e_j))$, e_i^* is defined by $ae_i^* = d_L^{(2)}(e_i^*, e_j)$, giving

$$e_i^*(e_j) = e_L^{(2)}(e_j) \equiv e_j + 2\Delta \left(1 - 2\Delta a + \sqrt{(1 - 2\Delta a)^2 + 1 - 2ae_j}\right).$$

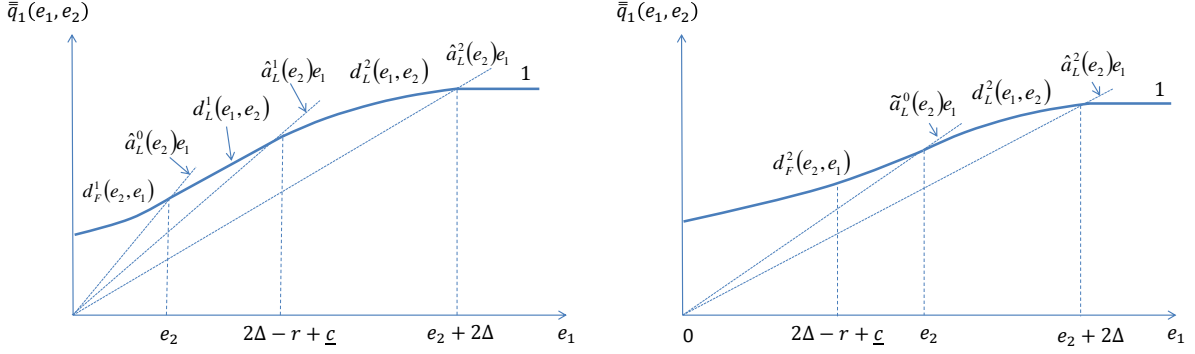


Figure SF-1: Expected winning quantity of supplier 1, $\bar{q}_1(e_1, e_2)$, as a function of e_1 for a given e_2 . The left plot is for $e_2 < 2\Delta - r + \underline{c}$ and right plot for $e_2 \geq 2\Delta - r + \underline{c}$.

3) If $e_j < 2\Delta - r + \underline{c}$ and $\hat{a}_L^{(1)}(e_j) < a \leq \hat{a}_L^{(0)}(e_j)$, e_i^* is defined by $ae_i^* = d_L^{(1)}(e_i^*, e_j)$, giving

$$e_i^*(e_j) = e_L^{(1)}(e_j) \equiv \frac{r - \underline{c} - \Delta \left(\frac{r - \underline{c} + e_j}{2\Delta} \right)^2}{2a\Delta - 1}.$$

4) If $e_j \leq 2\Delta - r + \underline{c}$ and $a > \hat{a}_L^{(0)}(e_j)$, e_i^* is defined by $ae_i^* = d_F^{(1)}(e_j, e_i^*)$, giving

$$e_i^*(e_j) = e_F^{(1)}(e_j) \equiv e_j + 2\Delta(2\Delta a - 1) - \sqrt{(e_j + 2\Delta(2\Delta a - 1))^2 - (r - \underline{c})(2(2\Delta - e_j) - r + \underline{c})}.$$

5) If $e_j > 2\Delta - r + \underline{c}$ and $a > \tilde{a}_L^{(0)}(e_j)$, e_i^* is defined by $ae_i^* = d_F^{(2)}(e_j, e_i^*)$, giving

$$e_i^*(e_j) = e_F^{(2)}(e_j) \equiv e_j - 2\Delta \left(1 - 2\Delta a + \sqrt{(1 - 2\Delta a)^2 - 1 + 2ae_j} \right).$$

It can be shown in all cases the best-response function $e_i^*(e_j)$ is decreasing, and is concave when $e_i^* > e_j$, convex when $e_i^* \leq e_j$. The symmetric fixed-point is given by $e_i^*(e_j) = e_j$, and may be reached in cases 4) and 5) as described above. It constitutes an equilibrium only if it is a contraction mapping, i.e., $\frac{de_i^*(e_j)}{de_j} > -1$ at the point. In case 4), the solution to $e_i^*(e_j) = e_j$ exists when $a \geq \hat{a} \equiv \frac{1}{2(2\Delta - r + \underline{c})}$. Under this condition, the symmetric solution is $e_j = 2\Delta \left(\sqrt{(2\Delta a - 1)^2 + 2a(r - \underline{c})} - (2\Delta a - 1) - \frac{r - \underline{c}}{2\Delta} \right)$, and $\frac{de_i^*(e_j)}{de_j} > -1$ is equivalent to $a > a_s^{(0)} \equiv \frac{1}{2\Delta} \left(1 + \frac{r - \underline{c}}{6\Delta} + \sqrt{\left(1 + \frac{r - \underline{c}}{6\Delta} \right)^2 - 1} \right)$. $a_s^{(0)} > \hat{a}$ if and only if $r - \underline{c} \leq \frac{3\Delta}{2}$. In case 5), the solution to $e_i^*(e_j) = e_j$ exists when $a \leq \hat{a}$. Under this condition, the symmetric solution is $e_j = \frac{1}{2a}$, and $\frac{de_i^*(e_j)}{de_j} > -1$ implies $a > \frac{1}{\Delta}$.

Define a_s such that $a_s = a_s^{(0)}$ if $r - \underline{c} \leq \frac{3\Delta}{2}$ and $a_s = \frac{1}{\Delta}$ otherwise. Then, for $a > a_s$, $\frac{de_i^*(e_j)}{de_j} > -1$ at $e_i^*(e_j) = e_j$ and hence for all e_j where $e_i^*(e_j) \leq e_j$. In this case, there is a unique effort equilibrium and it is symmetric. Otherwise, for $a \leq a_s$, we have $\frac{de_i^*(e_j)}{de_j} \leq -1$ at $e_i^*(e_j) = e_j$ and hence the symmetric fixed-point is not a contraction mapping. In addition, since $e_i^*\left(\frac{1}{a}\right) > 0$

and $e_i^*(0) \leq \frac{1}{a}$, there exists a unique solution to $e_i^*(e_j^*(e_i)) = e_i$ that has $e_j^*(e_i) > e_i$, resulting in a unique asymmetric equilibrium.

Next, we characterize the asymmetric equilibrium, denoted by $(e_{D,1}^u, e_{D,2}^u)$. Given the expressions of $e_F^{(2)}(e_L)$ and $e_L^{(2)}(e_F)$, solving $e_F^{(2)}(e_{D,1}^u) = e_{D,2}^u$ and $e_L^{(2)}(e_{D,2}^u) = e_{D,1}^u$ leads to

$$e_{D,2}^u = \frac{(2\Delta a - 1)^2}{2a}, \quad e_{D,1}^u = \frac{2 - (2\Delta a - 1)^2}{2a}.$$

The constraint $e_{D,1}^u \geq 2\Delta - r + \underline{c}$ requires $a \leq \hat{a}_s \equiv \frac{(r-\underline{c}) + \sqrt{(r-\underline{c})^2 + 4\Delta^2}}{4\Delta^2}$, where $\hat{a}_s \leq a_s$ if and only if $r - \underline{c} \leq \frac{3}{2}\Delta$. Given the expressions of $e_F^{(1)}(e_L)$ and $e_L^{(1)}(e_F)$, solving $e_F^{(1)}(e_{D,1}^u) = e_{D,2}^u$ and $e_L^{(1)}(e_{D,2}^u) = e_{D,1}^u$ leads to

$$e_{D,2}^u = 2\Delta(2a\Delta - 1) - r + \underline{c}, \quad e_{D,1}^u = \frac{r - \underline{c} - \Delta(2a\Delta - 1)^2}{2a\Delta - 1}.$$

The constraint $e_{D,1}^u < 2\Delta - r + \underline{c}$ requires $a > \hat{a}_s$.

Therefore, the effort equilibrium can be summarized as follows:

Symmetric I (S1^u): $a_s < a < \hat{a}$. Both suppliers choose

$$e_{D,1}^u = e_{D,2}^u = \frac{1}{2a},$$

greater than $2\Delta - r + \underline{c}$. The buyer's profit is $\Pi_D^u = r - \bar{c} + e_D^u + \frac{\Delta}{3}$.

Symmetric II (S2^u): $a > \max(a_s, \hat{a})$. Both suppliers choose

$$e_{D,1}^u = e_{D,2}^u = 2\Delta \left(\sqrt{(2\Delta a - 1)^2 + 2a(r - \underline{c})} - (2\Delta a - 1) - \frac{r - \underline{c}}{2\Delta} \right),$$

less than $2\Delta - r + \underline{c}$. The buyer's profit is $\Pi_D^u = \frac{(r - \underline{c} + e_D^u)^2}{4\Delta} \left(2 - \frac{r - \underline{c} + e_D^u}{3\Delta} \right)$.

Asymmetric I (A1^u): $a < \min(\hat{a}_s, a_s)$. The supplier efforts are

$$e_{D,1}^u = \frac{(2\Delta a - 1)^2}{2a}, \quad e_{D,2}^u = \frac{2 - (2\Delta a - 1)^2}{2a},$$

where $e_{D,1}^u < e_{D,2}^u$ and $2\Delta - r + \underline{c} < e_{D,2}^u$. The buyer's profit is $\Pi_D^u = r - \bar{c} + e_{D,2}^u + \frac{\Delta}{3}(2\Delta a - 1)^3$.

Asymmetric II (A2^u): $\hat{a}_s < a < a_s$. The supplier efforts are

$$e_{D,1}^u = 2\Delta(2a\Delta - 1) - r + \underline{c}, \quad e_{D,2}^u = \frac{r - \underline{c} - \Delta(2a\Delta - 1)^2}{2a\Delta - 1},$$

where $e_{D,1}^u < e_{D,2}^u < 2\Delta - r + \underline{c}$. The buyer's profit is $\Pi_D^u = \frac{(r - \underline{c} + e_{D,2}^u)^2}{4\Delta} + \frac{1}{3\Delta^2} \left(\frac{r - \underline{c} + e_{D,1}^u}{2} \right)^3 + \left(1 - \frac{r - \underline{c} + e_{D,2}^u}{2\Delta} \right) \frac{(r - \underline{c} + e_{D,1}^u)^2}{4\Delta}$.

Full Proof of Proposition 2: Define $z \equiv \frac{r - \underline{c}}{2\Delta}$ and $x_i = F(\eta(e_i)) = \frac{r - \underline{c} + e_i}{2\Delta}$ for $i = L, F$. Define $y \equiv 2\Delta a > 1$. From the analysis of sole sourcing and dual sourcing in section 1.2.1, it is easy to

show that $\Pi_D^u - \Pi_S^u$ is increasing in a when $a > a_s$, i.e., when dual sourcing leads to a symmetric equilibrium: If $a > \max(a_s, \hat{a})$, we have

$$\left(\frac{d\Pi_D^u}{da} - \frac{d\Pi_S^u}{da} \right) / (2\Delta)^2 = \begin{cases} x_L(2-x_L) \left(\frac{y-1+z}{x_L+y-1} - 1 \right) + \frac{1}{y^2} & \text{if } y < \frac{1}{1-z} \\ x_L(2-x_L) \left(\frac{y-1+z}{x_L+y-1} - 1 \right) + \frac{yz^2}{(y-1)^3} & \text{else} \end{cases}.$$

Both pieces are decreasing in $y > 1$, and positive for $y \rightarrow \infty$, hence $\frac{d\Pi_D^u}{da} - \frac{d\Pi_S^u}{da} > 0$. If $a_s < a < \hat{a}$, we have

$$\left(\frac{d\Pi_D^u}{da} - \frac{d\Pi_S^u}{da} \right) / (2\Delta)^2 = \begin{cases} \frac{1}{2y^2} & \text{if } y < \frac{1}{1-z} \\ -\frac{1}{y^2} + \frac{yz^2}{(y-1)^3} & \text{else} \end{cases}.$$

With $y = 2\Delta a < 2\hat{a}\Delta = \frac{1}{2(1-z)} < \frac{1}{1-z}$, $\frac{d\Pi_D^u}{da} - \frac{d\Pi_S^u}{da} > 0$ follows.

Recall $a_s = a_s^{(0)}$ if $r - \underline{c} \leq \frac{3\Delta}{2}$ and $a_s = \frac{1}{\Delta}$ otherwise. We can see $a_s \leq \frac{1}{2\Delta - r + \underline{c}}$ for $\frac{r-\underline{c}}{\Delta} \in (1, 2)$. Therefore, when $a < a_s$, which implies an asymmetric equilibrium in dual sourcing, we have $\frac{d\Pi_S^u}{da} = -\frac{1}{a^2}$.

If further $a < \min(\hat{a}_s, a_s)$, from the analysis in section 1.2.1,

$$\frac{d\Pi_D^u}{da} - \frac{d\Pi_S^u}{da} = 8a\Delta^3(\Delta a - 1) + \frac{1}{2a^2} = \frac{16x^3(x-1) + 1}{2a^2}$$

for $x \equiv a\Delta$. Given $a < a_s \leq \frac{1}{\Delta}$ and $2a\Delta > 1$, we have $\frac{1}{2} < x < 1$. It is easy to show $16x^3(x-1) + 1$ is first negative and then positive as x increases on $[\frac{1}{2}, 1]$. Thus $\Pi_D^u - \Pi_S^u$ is quasi-convex for $a < \min(\hat{a}_s, a_s)$.

If $\hat{a}_s \leq a < a_s$, which only exists for $r - \underline{c} < \frac{3}{2}\Delta$, we have

$$\frac{d\Pi_D^u}{da} - \frac{d\Pi_S^u}{da} = \frac{2x^3}{a^2(2x-1)^3} \left(5(2x-1)^4 - v^2 - 2v(2x-1)^3 + \frac{1}{2x^3} \right)$$

for $x = a\Delta$ and $v \equiv \frac{r-\underline{c}}{\Delta} \in (1, \frac{3}{2})$. Recall $\hat{a}_s = \frac{v+\sqrt{v^2+4}}{4\Delta}$. Thus $x > \hat{a}_s\Delta = \frac{v+\sqrt{v^2+4}}{4}$. Since the expression in the big bracket above is increasing in $x \in [\frac{v+\sqrt{v^2+4}}{4}, 1]$, $\Pi_D^u - \Pi_S^u$ is quasi-convex.

It is easy to verify that $\Pi_D^u - \Pi_S^u$ is negative when $a = \hat{a}_s$ (for $r - \underline{c} < \frac{3}{2}\Delta$) or $a = a_s$, and $\Pi_D^u - \Pi_S^u = 0$ when $a = \frac{1}{2\Delta}$. Therefore, $\Pi_D^u - \Pi_S^u \leq 0$ for $a \in [\frac{1}{2\Delta}, a_s]$, i.e., when dual sourcing leads to an asymmetric effort equilibrium. Given that $\Pi_D^u - \Pi_S^u$ is increasing in $a > a_s$, $\Pi_D^u - \Pi_S^u > 0$ if and only if a is higher than a_s and sufficiently large. Since the a value at the indifference point decreases in Δ under symmetric equilibria, that also implies that higher Δ favors dual sourcing. ■

Proof of Proposition 4: When $a < \frac{1}{4(2\Delta - r + \underline{c})}$, $e_D^o > 2\Delta - r + \underline{c}$ and hence the buyer's profit in dual sourcing

$$\Pi_D^o = 2 \int_{\underline{c}}^{\bar{c}} (r - J(c, e_D^o)) (1 - F(c)) f(c) dc = r - \bar{c} + e_D^o + \frac{\Delta}{3} = r - \bar{c} + \frac{1}{4a} + \frac{\Delta}{3}.$$

When $a > \frac{1}{4(2\Delta - r + \underline{c})}$, $e_D^o < 2\Delta - r + \underline{c}$ and hence

$$\Pi_D^o = 2 \int_{\underline{c}}^{\eta(e_D^o)} (r - J(c, e_D^o)) (1 - F(c)) f(c) dc = \frac{(r - \underline{c} + e_D^o)^2}{4\Delta} \left(2 - \frac{r - \underline{c} + e_D^o}{3\Delta} \right).$$

The buyer's profit in sole sourcing

$$\Pi_S^o = (r + e - \eta(e)) F(\eta(e)) = \begin{cases} \Delta & \text{if } a < \frac{1}{2(2\Delta - r + \underline{c})} \\ 4\Delta \left(\frac{a(r - \underline{c})}{4a\Delta - 1} \right)^2 & \text{otherwise} \end{cases}$$

1) When $a < \frac{1}{4(2\Delta - r + \underline{c})}$, we have $\Pi_D^o - \Pi_S^o = r - \bar{c} + \frac{1}{4a} - \frac{2\Delta}{3} > 0$.

2) When $\frac{1}{4(2\Delta - r + \underline{c})} \leq a < \frac{1}{2(2\Delta - r + \underline{c})}$, we have $\Pi_S^o = \Delta$ and $\Pi_D^o = \frac{(r - \underline{c} + e_D^o)^2}{4\Delta} \left(2 - \frac{r - \underline{c} + e_D^o}{3\Delta} \right)$ with $\frac{d\Pi_D^o}{da} - \frac{d\Pi_S^o}{da} < 0$.

3) When $a \geq \frac{1}{2(2\Delta - r + \underline{c})}$, we have $\Pi_S^o = 4\Delta \left(\frac{a(r - \underline{c})}{4a\Delta - 1} \right)^2$ and $\Pi_D^o = \Delta x_D^2 \left(2 - \frac{2x_D}{3} \right)$ where

$$x_D \equiv \frac{r - \underline{c} + e_D^o}{2\Delta} = \sqrt{(4\Delta a - 1)^2 + 4a(r - \underline{c})} - (4\Delta a - 1).$$

Further, we can show that Π_D^o/Π_S^o strictly increases in a when $a \geq \frac{1}{2(2\Delta - r + \underline{c})}$, and $\Pi_D^o/\Pi_S^o > 1$ when $a \rightarrow \infty$. Therefore, if $\Pi_D^o > \Pi_S^o$ (or, equivalently $\Pi_D^o/\Pi_S^o > 1$) when $a = \frac{1}{2(2\Delta - r + \underline{c})}$, then $\Pi_D^o > \Pi_S^o$ for all a ; otherwise, there exist two thresholds a_1 and a_2 such that $a_1 < \frac{1}{2(2\Delta - r + \underline{c})} < a_2$ and $\Pi_D^o < \Pi_S^o$ if and only if $a \in (a_1, a_2)$. Finally, the condition $\Pi_D^o < \Pi_S^o$ at $a = \frac{1}{2(2\Delta - r + \underline{c})}$ holds if and only if $\frac{\Delta}{r - \underline{c}} > \frac{8z^2 - 15z - 2}{16z^2 - 30z - 18} \approx 0.743$, where $z > 0$ is the root of $z^3 - 3z^2 + \frac{15}{16} = 0$. ■

1.2.2 Observable effort

Sole sourcing. We derive the supplier's expected profit (with the effort disutility sunk) $\bar{\pi}_i(e) = \int_{\underline{c}}^{\eta(e)} F(c) dc$ and its derivative $\frac{\partial \bar{\pi}_i(e)}{\partial e}$ under all conditions of a and $r - \underline{c}$. We derive the equilibrium effort, denoted by e_S^o , by solving the first-order condition $\frac{\partial \bar{\pi}_i(e)}{\partial e}|_{e=e_S^o} = ae_S^o$. If $\eta(e) < \bar{c}$, i.e., $e < 2\Delta - r + \underline{c}$, then the supplier effort is such that $\pi(e) = \int_{\underline{c}}^{\eta(e)} F(c) dc$ with $\pi'(e) = \frac{r + e - \underline{c}}{4\Delta}$. Otherwise $\pi'(e) = 0$. Hence, for $a < \frac{1}{2(2\Delta - r + \underline{c})}$ (which implies $ae < \pi'(e)$ for effort e such that $\eta(e) \leq \bar{c}$), the effort is $2\Delta - r + \underline{c}$; otherwise, the effort is defined by $\frac{r + e - \underline{c}}{4\Delta} = ae$. To summarize, we have

$$e_S^o = \begin{cases} 2\Delta - r + \underline{c}, & \text{if } 2a(2\Delta - r + \underline{c}) < 1; \\ \frac{r - \underline{c}}{4a\Delta - 1}, & \text{otherwise.} \end{cases}$$

Dual sourcing. We derive each supplier i 's expected profit (with effort disutility sunk) $\bar{\pi}_i(\mathbf{e})$ given effort profile \mathbf{e} , and its derivative $\frac{\partial \bar{\pi}_i(\mathbf{e})}{\partial e_i}$ under all conditions of a and $r - \underline{c}$. Let e_F and e_L where $e_F \leq e_L$ be the supplier efforts of the cost leader and follower, and let π_F and π_L be the corresponding supplier profits. We first characterize $\frac{\partial \pi_L(e_L, e_F)}{\partial e_L}$ and $\frac{\partial \pi_F(e_L, e_F)}{\partial e_F}$ following equation (1) in the paper: 1) If $e_L - e_F > 2\Delta$, then $\frac{\partial \pi_L(e_L, e_F)}{\partial e_L} = \frac{\partial \pi_F(e_L, e_F)}{\partial e_F} = 0$; 2) If $e_L - e_F < 2\Delta$ and $e_L < 2\Delta - (r - \underline{c})$, then $\frac{\partial \pi_L(e_L, e_F)}{\partial e_L} = d_L^{(1)}(e_L, e_F)$ and $\frac{\partial \pi_F(e_L, e_F)}{\partial e_F} = d_F^{(1)}(e_L, e_F)$; 3) If $e_L - e_F < 2\Delta$ and $e_L > 2\Delta - (r - \underline{c})$, then $\frac{\partial \pi_L(e_L, e_F)}{\partial e_L} = d_L^{(2)}(e_L, e_F) > 0$ and $\frac{\partial \pi_F(e_L, e_F)}{\partial e_F} = d_F^{(2)}(e_L, e_F) > 0$, where we redefine

$$d_L^{(1)}(e_L, e_F) \equiv \frac{r + e_L - \underline{c}}{4\Delta} - \left(\frac{r + e_F - \underline{c}}{4\Delta} \right)^2,$$

$$d_F^{(1)}(e_L, e_F) \equiv \frac{r + e_F - \underline{c}}{4\Delta} \left(1 - \frac{r + e_L - \underline{c}}{4\Delta} - \frac{e_L - e_F}{4\Delta} \right),$$

$$d_L^{(2)}(e_L, e_F) \equiv \frac{1}{4} - \left(\frac{e_L - e_F}{4\Delta} \right)^2 > 0,$$

$$d_F^{(2)}(e_L, e_F) \equiv \left(\frac{1}{2} - \frac{e_L - e_F}{4\Delta} \right)^2 > 0.$$

From supplier i 's perspective, $\frac{\partial \pi_i(\mathbf{e})}{\partial e_i}$ can be characterized as follows: For $e_j < 2\Delta - r + \underline{c}$,

$$\frac{\partial \pi_i(\mathbf{e})}{\partial e_i} = \begin{cases} d_F^{(1)}(e_j, e_i) & \text{if } e_i < e_j \\ d_L^{(1)}(e_i, e_j) & \text{if } e_i \in [e_j, 2\Delta - r + \underline{c}] \\ d_L^{(2)}(e_i, e_j) & \text{if } e_i \in [2\Delta - r + \underline{c}, 2\Delta + e_j] \\ 0 & \text{if } e_i > 2\Delta + e_j \end{cases},$$

and for $e_j \geq 2\Delta - r + \underline{c}$,

$$\frac{\partial \pi_i(\mathbf{e})}{\partial e_i} = \begin{cases} 0 & \text{if } e_i < e_j - 2\Delta \\ d_F^{(2)}(e_j, e_i) & \text{if } e_i \in [e_j - 2\Delta, e_j] \\ d_L^{(2)}(e_i, e_j) & \text{if } e_i \in [e_j, 2\Delta + e_j] \\ 0 & \text{if } e_i > 2\Delta + e_j \end{cases}.$$

Let $e_i^*(e_j)$ be the best-response of e_i given e_j ; it is the unique solution to $\frac{\partial \pi_i(\mathbf{e})}{\partial e_i} = ae_i$. Note that the uniqueness is because ae_i can only cross $\frac{\partial \pi_i(\mathbf{e})}{\partial e_i}$ once from below given our assumption $a > \frac{1}{2\Delta}$. In the following, we derive $e_i^*(e_j)$ and solve the equilibrium effort, denoted by $(e_{D,1}^o, e_{D,2}^o)$, by solving the equation set $e_i^*(e_{D,j}^o) = e_{D,i}^o$ for $i = 1, 2$. We find the solution is always symmetric, i.e., $e_{D,1}^o = e_{D,2}^o \equiv e_D^o$, and it is always stable (i.e., $\frac{de_i^*(e_j)}{de_j} > -1, i = 1, 2$ at the point).

In particular, we redefine several critical values of a : $\hat{a}_L^{(0)}(e) \equiv \frac{d_L^{(1)}(e, e)}{e}$, $\hat{a}_L^{(1)}(e) \equiv \frac{d_L^{(1)}(2\Delta - r + \underline{c}, e)}{(2\Delta - r + \underline{c})}$, $\hat{a}_L^{(2)}(e) \equiv \frac{d_L^{(2)}(2\Delta - r + \underline{c}, e)}{(2\Delta - r + \underline{c})}$, and $\tilde{a}_L^{(0)}(e) \equiv \frac{d_L^{(2)}(e, e)}{e}$. It can be shown for $e < 2\Delta - r + \underline{c}$, $\hat{a}_L^{(2)}(e) \leq \hat{a}_L^{(1)}(e) \leq \hat{a}_L^{(0)}(e)$ (where equality is achieved with $e = 2\Delta - r + \underline{c}$), $\hat{a}_L^{(0)}(e)$ and $\hat{a}_L^{(1)}(e)$ decreasing in e , and $\hat{a}_L^{(2)}(e)$ increasing in e . Let $e_i^*(e_j)$ be the best response of e_i given e_j . Then:

1) If $a \leq \hat{a}_L^{(2)}(e_j)$ and $e_j < 2\Delta - r + \underline{c}$, or $a \leq \tilde{a}_L^{(0)}(e_j)$ and $e_j > 2\Delta - r + \underline{c}$, e_i^* is defined by $ae_i^* = d_L^{(2)}(e_i^*, e_j)$, giving

$$e_i^*(e_j) = e_L^{(2)}(e_F) \equiv e_F + 2\Delta \left(\sqrt{(4\Delta a)^2 - 4ae_F + 1 - 4\Delta a} \right),$$

which is increasing in e_j . Note $e_L^{(2)}(0) = 2\Delta \left(\sqrt{(4\Delta a)^2 + 1 - 4\Delta a} \right)$ is less than 2Δ because $e_L^{(2)}(0) > 2\Delta$ would mean $\sqrt{(4\Delta a)^2 + 1 - 4\Delta a} > 1$, which is not possible.

2) If $\hat{a}_L^{(2)}(e_j) < a \leq \hat{a}_L^{(1)}(e_j)$ and $e_j < 2\Delta - r + \underline{c}$, e_i^* is equal to $2\Delta - r + \underline{c}$.

3) If $\hat{a}_L^{(1)}(e_j) < a \leq \hat{a}_L^{(0)}(e_j)$ and $e_j < 2\Delta - r + \underline{c}$, e_i^* is defined by $ae_i^* = d_L^{(1)}(e_i^*, e_j)$, giving

$$e_i^*(e_j) = e_L^{(1)}(e_j) \equiv \frac{r - \underline{c} - 4\Delta \left(\frac{r + e_j - \underline{c}}{4\Delta} \right)^2}{4\Delta a - 1},$$

which is decreasing in e_j with $e_L'(e_j) > -1$ (since $\frac{r+e_j-\underline{c}}{2\Delta(4\Delta a-1)} < \frac{1}{4\Delta a-1} < 1$).

4) If $a > \widehat{a}_L^{(0)}(e_j)$ and $e_j < 2\Delta - r + \underline{c}$, e_i^* is defined by $ae_i^* = d_F^{(1)}(e_j, e_i^*)$, giving

$$e_i^*(e_j) = e_F^{(1)}(e_j) \equiv e_j + 2\Delta \left(4\Delta a - 1 - \sqrt{\left(4\Delta a - 1 + \frac{r+e_j-\underline{c}}{2\Delta} \right)^2 - 4a(r-\underline{c})} \right),$$

which is decreasing in e_j with $e_F'(e_j) > -1$.

5) If $a > \widetilde{a}_L^{(0)}(e_j)$ and $e_j > 2\Delta - r + \underline{c}$, e_i^* is defined by $ae_i^* = d_F^{(2)}(e_j, e_i^*)$, giving

$$e_i^*(e_j) = e_F^{(2)}(e_j) \equiv e_j + 2\Delta \left(4\Delta a - 1 - \sqrt{(4\Delta a - 1)^2 + 4ae_j - 1} \right),$$

which is decreasing in e_j . Then, $e_i^*(e_j) = e_j$ gives $e_j = \frac{1}{4a}$ where $e_F^{(2)}(e_j) > -1$.

The symmetric fixed point of the best-response functions, $e_i^*(e_j) = e_j$, may be reached in the above cases 4) or 5). In case 4), we have $e_F'(e_j) > -1$ for the entire range. In case 5), the solution is $e_j = \frac{1}{4a}$, at which $e_F^{(2)}(e_j) > -1$. Thus the symmetric fixed point is contraction mapping, hence an equilibrium. It can be characterized as follows:

$$e_D^o = \begin{cases} \frac{1}{4a} & \text{if } 4a(2\Delta - r + \underline{c}) < 1 \\ 2\Delta \sqrt{(4\Delta a - 1)^2 + 4a(r - \underline{c})} - 2\Delta(4\Delta a - 1) - r + \underline{c} & \text{else} \end{cases}.$$

1.3 Equilibrium analysis of the full-commitment mechanism

Full proof of Lemma 3

We first prove part (ii). For any mechanism that induces supplier efforts $\mathbf{e} \equiv (e_i)_{i \in S}$, we can find a truth-telling mechanism that induces the same efforts based on the revelation principle; hence we restrict to truth-telling mechanisms. Given the effort e_j of supplier $j = S \setminus i$, the truth-telling constraint for supplier i implies that supplier i 's profit from the mechanism, before deducting the effort disutility, equals $u_i(\gamma_i, e_j) = \int_{\gamma_i}^{\bar{\gamma}} \bar{q}_i(\rho, e_j) d\rho + \bar{u}_i(\bar{\gamma}, e_j)$, where $\bar{q}_i(\gamma_i, e_j) \equiv \int_{\gamma_j} \bar{q}_i(\boldsymbol{\gamma}) g(\gamma_j, e_j) d\gamma_j$ is supplier i 's expected quantity for realized cost γ_i , and $\bar{u}_i(\bar{\gamma}, e_j)$ is the supplier's profit for the highest possible cost $\bar{\gamma}$ (for any effort). Then supplier i 's expected profit with effort e_i equals $U_i(e_i, e_j) = \int_{\gamma_i=\underline{c}-e_i}^{\bar{c}-e_i} u_i(\gamma_i) g(\gamma_i, e_i) d\gamma_i = \int_{\gamma_i=\underline{c}-e_i}^{\bar{c}-e_i} \int_{\gamma_i}^{\bar{\gamma}} \bar{q}_i(\rho, e_j) d\rho g(\gamma_i, e_i) d\gamma_i + \bar{u}_i(\bar{\gamma}, e_j)$, which, by changing the order of integration, can be transformed to

$$U_i(e_i, e_j) = \int_{\gamma_i=\underline{c}-e_i}^{\bar{c}-e_i} \bar{q}_i(\gamma_i, e_j) G(\gamma_i, e_i) d\gamma_i + \int_{\bar{c}-e_i}^{\bar{\gamma}} \bar{q}_i(\gamma_i, e_j) d\gamma_i + \bar{u}_i(\bar{\gamma}, e_j). \quad (\text{SF-1})$$

Then supplier i 's total profit is $\pi_i(e_i, e_j) = U_i(e_i, e_j) - \varphi(e_i)$. The best-response e_i solves $\frac{\partial}{\partial e_i} U_i(e_i, e_j) = ae_i$. For $\bar{\bar{q}}_i(\mathbf{e}) \equiv \int_{\gamma_i} \bar{q}_i(\gamma_i, e_j) g(\gamma_i, e_i) d\gamma_i$, it can be shown $\frac{\partial}{\partial e_i} U_i(e_i, e_j) = \bar{\bar{q}}_i(\mathbf{e})$ based on $G(\gamma_i, e_i) = F(\gamma_i + e_i)$. Hence, the supplier efforts \mathbf{e} induced by the procurement mechanism is defined by: $ae_i = \bar{\bar{q}}_i(\mathbf{e})$ for $i \in S$.

Next, we prove part (i) by analyzing the buyer's optimal mechanism given the response of efforts. The buyer's expected profit Π equals the expected supply chain profit $\sum_{i \in S} \left[\int_{\gamma_i} (r - \gamma_i) \bar{q}_i(\gamma_i, e_j) g(\gamma_i, e_i) d\gamma_i \right]$

minus the suppliers' expected profit $\sum_{i \in S} U_i(\mathbf{e})$. Replacing $U_i(\mathbf{e})$ with (SF-1), the buyer's profit can be written as $\Pi = \int_{\boldsymbol{\gamma}} \sum_{i \in S} [(r - J(\gamma_i, e_i)) q_i(\boldsymbol{\gamma})] g(\boldsymbol{\gamma}, \mathbf{e}) d\boldsymbol{\gamma} - \sum_{i \in S} \left[\int_{\bar{c} - e_i}^{\bar{\gamma}} \bar{q}_i(\gamma_i, e_j) d\gamma_i + \bar{u}_i(\bar{\gamma}, e_j) \right]$, where $g(\boldsymbol{\gamma}, \mathbf{e})$ is the joint probability density of $\boldsymbol{\gamma} \equiv (\gamma_i)_{i \in S}$ given \mathbf{e} . In maximizing her expected profit, the buyer always sets $q_i(\boldsymbol{\gamma}) = 0$ for any $\gamma_i > \bar{c} - e_i$ along with $\bar{u}_i(\bar{\gamma}, e_j) = 0$. The buyer's profit then becomes $\Pi = \int_{\boldsymbol{\gamma}} \sum_{i \in S} [(r - J(\gamma_i, e_i)) q_i(\boldsymbol{\gamma})] g(\boldsymbol{\gamma}, \mathbf{e}) d\boldsymbol{\gamma}$. For $\mathbf{c} \equiv (c_i)_{i \in S}$, let $f(\mathbf{c})$ be the joint probability density of \mathbf{c} and further define $\tilde{J}(\mathbf{c}) = c + \frac{F(\mathbf{c})}{f(\mathbf{c})}$. Given $J(c_i - e_i, e_i) = \tilde{J}(c_i) - e_i$ and $e_i = \frac{1}{a} \bar{q}_i(\mathbf{e}) = \frac{1}{a} \int_{\mathbf{c}} q_i(\mathbf{c} - \mathbf{e}) f(\mathbf{c}) d\mathbf{c}$ for $i \in S$, Π can be further transformed to

$$\Pi = \int_{\mathbf{c}} \sum_{i \in S} [(r - \tilde{J}(c_i)) q_i(\mathbf{c} - \mathbf{e})] f(\mathbf{c}) d\mathbf{c} + \sum_{i \in S} \frac{1}{a} \left(\int_{\mathbf{c}} q_i(\mathbf{c} - \mathbf{e}) f(\mathbf{c}) d\mathbf{c} \right)^2. \quad (\text{SF-2})$$

Consider a new feasible mechanism with quantity rule $(q'_i(\boldsymbol{\gamma}))_{i \in S}$ that results in supplier efforts $\mathbf{e}' \equiv (e'_i)_{i \in S}$. Define $\varepsilon_i(\mathbf{c}) \equiv q'_i(\mathbf{c} - \mathbf{e}') - q_i(\mathbf{c} - \mathbf{e})$, $i \in S$. Then, following (SF-2),

$$\begin{aligned} \Pi' &= \int_{\mathbf{c}} \sum_{i \in S} [(r - \tilde{J}(c_i)) (q_i(\mathbf{c} - \mathbf{e}) + \varepsilon_i(\mathbf{c}))] f(\mathbf{c}) d\mathbf{c} + \sum_{i \in S} \frac{1}{a} \left(\int_{\mathbf{c}} (q_i(\mathbf{c} - \mathbf{e}) + \varepsilon_i(\mathbf{c})) f(\mathbf{c}) d\mathbf{c} \right)^2, \\ \Pi' - \Pi &= \int_{\mathbf{c}} \sum_{i \in S} [(r - \tilde{J}(c_i) + 2e_i) \varepsilon_i(\mathbf{c})] f(\mathbf{c}) d\mathbf{c} + \frac{1}{a} \sum_{i \in S} \left(\int_{\mathbf{c}} \varepsilon_i(\mathbf{c}) f(\mathbf{c}) d\mathbf{c} \right)^2, \\ \lim_{\varepsilon_i(\cdot) \rightarrow 0} (\Pi' - \Pi) &= \Phi \equiv \int_{\mathbf{c}} \sum_{i \in S} [(r - \tilde{J}(c_i) + 2e_i) \varepsilon_i(\mathbf{c})] f(\mathbf{c}) d\mathbf{c}. \end{aligned} \quad (\text{SF-3})$$

Therefore, the sign of $\lim_{\varepsilon_i \rightarrow 0} (\Pi' - \Pi)$ is determined by the sign of Φ .

We show that, if the mechanism $(q_i(\boldsymbol{\gamma}))_{i \in S}$ violates the conditions specified in part (i), it can always be improved by constructing a new feasible mechanism $(q'_i(\boldsymbol{\gamma}))_{i \in S}$ that produces $\Phi > 0$ and hence $\Pi' > \Pi$. This is achieved by first constructing $\varepsilon_i(\cdot) \rightarrow 0$ that leads to $\Phi > 0$. Then let $\mathbf{e}' = \mathbf{e} + \int_{\mathbf{c}} \varepsilon_i(\mathbf{c}) f(\mathbf{c}) d\mathbf{c}$, and construct the new mechanism $q'_i(\boldsymbol{\gamma}) = q_i(\boldsymbol{\gamma} + \mathbf{e}' - \mathbf{e}) + \varepsilon_i(\boldsymbol{\gamma} + \mathbf{e}')$. Given $\varepsilon_i(\cdot)$ specified as follows, the new mechanism is feasible and implements $\varepsilon_i(\cdot)$, inducing efforts \mathbf{e}' .

i) If there exist innate costs $\tilde{\mathbf{c}} \equiv (\tilde{c}_i)_{i \in S}$ with positive probability measure such that $r - \tilde{J}(\tilde{c}_i) + 2e_i < 0$ and $q_i(\tilde{\mathbf{c}} - \mathbf{e}) > 0$, let $\varepsilon_i(\tilde{\mathbf{c}}) < 0$, $\varepsilon_i(\mathbf{c}) \leq 0$ for all $c_i > \tilde{c}_i$ and $\varepsilon_i(\mathbf{c}) = 0$ for all $c_i < \tilde{c}_i$, with $\int_{c_j} \varepsilon_i(\mathbf{c}) f(c_j) dc_j + \bar{q}_i(c_i - e_i, e_j)$ remaining decreasing in c_i .

ii) If there exist innate costs $\tilde{\mathbf{c}}$ with positive probability measure such that $r - \tilde{J}(\tilde{c}_i) + 2e_i > 0$ and $\sum_{i \in S} q_i(\tilde{\mathbf{c}} - \mathbf{e}) < 1$, let $\varepsilon_i(\tilde{\mathbf{c}}) > 0$, $\varepsilon_i(\mathbf{c}) \geq 0$ for all $c_i < \tilde{c}_i$ and $\varepsilon_i(\mathbf{c}) = 0$ for all $c_i > \tilde{c}_i$, with $\int_{c_j} \varepsilon_i(\mathbf{c}) f(c_j) dc_j + \bar{q}_i(c_i - e_i, e_j)$ remaining decreasing in c_i .

iii) For dual sourcing, if there exist innate costs $\tilde{\mathbf{c}}$ with positive probability measure such that $r - \tilde{J}(\tilde{c}_i) + 2e_i > r - \tilde{J}(\tilde{c}_j) + 2e_j > 0$ and $q_i(\tilde{\mathbf{c}} - \mathbf{e}) < q_j(\tilde{\mathbf{c}} - \mathbf{e})$, let $\varepsilon_i(\tilde{\mathbf{c}}) = -\varepsilon_j(\tilde{\mathbf{c}}) > 0$, $\varepsilon_i(\mathbf{c}) = -\varepsilon_j(\mathbf{c}) \geq 0$ for all \mathbf{c} with $c_i < \tilde{c}_i$ and $c_j > \tilde{c}_j$, and $\varepsilon_i(\mathbf{c}) = \varepsilon_j(\mathbf{c}) = 0$ otherwise, with $\int_{c_j} \varepsilon_i(\mathbf{c}) f(c_j) dc_j + \bar{q}_i(c_i - e_i, e_j)$ remaining decreasing in c_i , $i \in S$. ■

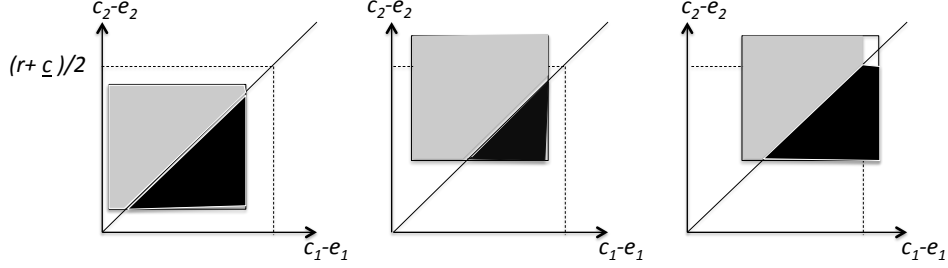


Figure SF-2: In the full-comment mechanism three scenarios that have different expressions of $\bar{q}_i^*(e_1^f, e_2^f)$

Derivations for the results in Table 4 of the paper

In the proof of Lemma 3(ii) we show that the first-order derivative of supplier i 's expected profit with respect to his effort choice e_i , i.e., $\frac{\partial U_i(e_i, e_{-i})}{\partial e_i}$, equals $\bar{q}_i(\mathbf{e})$ given \mathbf{e} . This implies $\frac{\partial^2}{\partial e_i^2} U_i(e_i, e_{-i}) = \frac{1}{\Delta} [\bar{q}_i(\underline{c} - e_i; \mathbf{e}) - \bar{q}_i(\bar{c} - e_i; \mathbf{e})] \leq \frac{1}{\Delta}$.

When $a\Delta \geq 1$:

$\frac{\partial^2}{\partial e_i^2} U_i(e_i, e_{-i}) \leq a$ always holds; therefore, supplier i 's expected profit is always concave with respect to e_i . It suffices to examine the first-order conditions to find supplier i 's best-response effort e_i^f .

Assume $e_1^f \geq e_2^f$ without loss of generality. We have three possible scenarios, depending on the values of $\bar{c} - e_i^f$ and $\frac{r+c}{2}$, as illustrated in Figure SF-2, where the equilibrium allocation quantities $\bar{q}_i^*(e_1^f, e_2^f)$ have different expressions.

In the first two scenarios, i.e., when $\bar{c} - e_1^f \leq \frac{r+c}{2}$, we have $\bar{q}_1^*(e_1^f, e_2^f) + \bar{q}_2^*(e_1^f, e_2^f) = 1$,

$$ae_2^f = \bar{q}_2^*(e_1^f, e_2^f) = [1 - \frac{e_1^f - e_2^f}{\Delta}]^2 / 2, \text{ and}$$

$$ae_1^f = \bar{q}_1^*(e_1^f, e_2^f) = 1 - \bar{q}_2^*(e_1^f, e_2^f).$$

Equivalently, we have

$$a(e_1^f + e_2^f) = 1 \text{ and } a(e_1^f - e_2^f) = 2(e_1^f - e_2^f)/\Delta - (e_1^f - e_2^f)^2/\Delta^2,$$

which admit two set of solutions:

$$e_1^f = e_2^f = \frac{1}{2a};$$

$$e_2^f = \frac{(a\Delta - 1)^2}{2a}, e_1^f = \frac{1}{a} - \frac{(a\Delta - 1)^2}{2a}, \text{ only when } a\Delta < 2.$$

For stableness, we need

$$\frac{de_2^*(e_1)}{de_1} \Big|_{e_1=e_1^f, e_2=e_2^f} = \frac{e_1^f - e_2^f - \Delta}{a\Delta^2 - \Delta + e_1^f - e_2^f} > -1 \text{ and}$$

$$\left. \frac{de_1^*(e_2)}{de_2} \right|_{e_1=e_1^f, e_2=e_2^f} = \frac{e_1^f - e_2^f - \Delta}{a\Delta^2 - \Delta + e_1^f - e_2^f} > -1,$$

which imply that the symmetric solution is unstable when $a\Delta < 2$, is stable when $a\Delta \geq 2$. When $a\Delta = 2$, the asymmetric solution and the symmetric solution coincide. Note: the best-response function $e_i^*(e_j)$ is given by $ae_i^*(e_j) = \bar{q}_i^*(e_i^*(e_j), e_j)$.

In the last scenario, i.e., when $\bar{c} - e_1^f > \frac{r+c}{2}$, we have $\bar{q}_1^*(e_1^f, e_2^f) + \bar{q}_2^*(e_1^f, e_2^f) < 1$,

$$ae_2^f = \bar{q}_2^*(e_1^f, e_2^f) = [1 - \frac{e_1^f - e_2^f}{\Delta}]^2/2 - [\frac{(\bar{c} - e_1^f) - \frac{r+c}{2}}{\Delta}]^2/2, \text{ and}$$

$$ae_1^f = \bar{q}_1^*(e_1^f, e_2^f) = \frac{\frac{r+c}{2} - (c - e_1^f)}{\Delta} - [\frac{\frac{r+c}{2} - (c - e_2^f)}{\Delta}]^2/2,$$

or, equivalently

$$a(e_1^f + e_2^f) = 1 - [\bar{c} - \frac{r+c}{2} - e_1^f][\bar{c} - \frac{r+c}{2} - e_2^f]/\Delta^2$$

$$\text{and } a(e_1^f - e_2^f) = (e_1^f - e_2^f)(\Delta + e_2^f + \frac{r-c}{2})/\Delta^2.$$

It admits a symmetric solution

$$e_1^f = e_2^f = \kappa - a\Delta^2 \pm \Delta\sqrt{a^2\Delta^2 - 2a\kappa + 1}, \text{ where } \kappa = \bar{c} - \frac{r+c}{2}.$$

Given that $e_1^f < \kappa$, the solution above must take the ‘-’ sign if $\kappa < \frac{1}{2a}$ and it takes the ‘+’ sign, otherwise. It also admits an asymmetric solution

$$e_2^f = a\Delta^2 - \Delta - \frac{r-c}{2} \text{ and } e_1^f = \frac{r-c}{2(a\Delta-1)} - \frac{(a\Delta-1)\Delta}{2}.$$

For stability, we need

$$\left. \frac{de_1^*(e_2)}{de_2} \right|_{e_1=e_1^f, e_2=e_2^f} = -\frac{e_2^f + \frac{r-c}{2}}{a\Delta^2 - \Delta} > -1 \text{ and}$$

$$\left. \frac{de_2^*(e_1)}{de_1} \right|_{e_1=e_1^f, e_2=e_2^f} = -\frac{e_2^f + \frac{r-c}{2}}{a\Delta^2 - \Delta + e_1^f - e_2^f} > -1.$$

At the symmetric solution, we have

$$\left. \frac{de_1^*(e_2)}{de_2} \right|_{e_1=e_1^f, e_2=e_2^f} = \left. \frac{de_2^*(e_1)}{de_1} \right|_{e_1=e_1^f, e_2=e_2^f} = -\frac{(1-a\Delta) \pm \sqrt{a^2\Delta^2 - 2a\kappa + 1}}{a\Delta - 1} > -1,$$

which implies $a(r-c) < 3(a\Delta-1)^2$.

When $a\Delta < 1$:

If a dual-sourcing outcome (i.e., interior solution of e_1^f and e_2^f) is in equilibrium, e_1^f and e_2^f must satisfy the first-order conditions that we have examined. Note that by their boundary conditions equilibrium types $S1^f$, $S2^f$ and $A2^f$ all require $a\Delta \geq 1$; equilibrium type $A1^f$ does not hold when $a\Delta < 1$ because it implies $e_1^f - e_2^f \geq \Delta$, which in turn implies $q_2^f = 0$ and hence $e_2^f = 0$ (a

contradiction to the expression of e_2 in A1^f). Therefore, the results from the analysis with first-order conditions suggest that dual-sourcing outcome can not be in equilibrium when $a\Delta < 1$; in other words, $e_2^f = 0$ must hold in equilibrium when $a\Delta < 1$.

Given that $e_2^f = 0$, the first-order derivative of supplier 1's expected profit equals

$$1 - (\max\{0, 1 - \frac{e_1}{\Delta}\})^2/2 - ae_1,$$

which can be shown always positive for $e_1 < \frac{1}{a}$ and negative for $e_1 > \frac{1}{a}$. Thus, the best response is $e_1 = \frac{1}{a}$ when $e_2 = 0$. When $e_1^f = \frac{1}{a}$ and $e_2^f = 0$, we have $q_2^f(\gamma) = 0$ and $q_1^f(\gamma) = 1$ for all γ , i.e. the sole-sourcing outcome in equilibrium.

Proof of Proposition 6

Part (i) can be verified by the results in Table 4 in the paper and is illustrated in Figure 5(b), where the threshold $a_s^f = 1 + \sqrt{\frac{a(r-c)}{3}}$ when $r - c \leq \frac{3\Delta}{2}$ and $a_s^f = \frac{2}{\Delta}$ when $r - c > \frac{3\Delta}{2}$. To prove part (ii), consider the sole-sourcing case: Per Lemma 3, the equilibrium effort e_S^f must satisfy the equation set $ae_S^f = \bar{q}^*(e_S^f) = Prob(c_i - e_S^f < \frac{r+c}{2})$. The solution is $e_S^f \equiv \min\{\frac{1}{a}, \frac{r-c}{2(a\Delta-1)}\}$, which decreases in a and is always higher than e_1^f in Table 4 of the paper. Part (iii) holds because sole-sourcing is equivalent to the special case of dual-sourcing in which the buyer sets $q_2(\gamma) = 0, \forall \gamma$. As Table 4 shows, when the buyer optimizes the mechanism, the optimal mechanism has: when $a\Delta < 1$, the optimal dual-sourcing reduces to sole-sourcing; when $a\Delta \geq 1$ (S1^f, A1^f, S2^f, A2^f), however, the mechanism has both e_1^f and e_2^f positive (and each supplier i has positive winning probability given that the probability equals e_i^f/a), which means dual-sourcing yields strictly higher profit than sole-sourcing. ■

1.4 Analysis of the first-best scenario

Suppose the buyer owns one supplier (i.e., counterpart of sole-sourcing). The buyer has the cost γ from $[\underline{c} - e, \bar{c} - e]$ and her expected total profit equals $E[\max\{0, r - \gamma\}] - ae^2/2$, which is maximized by the first-best effort level, denoted by $e_S^{FB} = \min\{\frac{r-c}{a\Delta-1}, \frac{1}{a}\}$.

Suppose the buyer owns two suppliers (i.e., counterpart of dual-sourcing). She has two random draws, γ_1 from cost distribution $[\underline{c} - e_1, \bar{c} - e_1]$ and γ_2 from cost distribution $[\underline{c} - e_2, \bar{c} - e_2]$. Assume $e_1 \geq e_2$ without loss of generality. Her expected total profit is $E[\max\{0, r - \gamma_1, r - \gamma_2\}] - ae_1^2/2 - ae_2^2/2$. We solve the optimal effort e_1^{FB} and e_2^{FB} , present them in Table 3 in the paper and illustrate them in Figure 5(a).

1.5 Equilibrium analysis of the partial-commitment mechanism

1.5.1 Sole sourcing

Buyer's best-response reserve price. Given any supplier's effort level e , if the buyer sets the reserve price ξ , her expected profit equals $(r - \xi)[\xi - (\underline{c} - e)]/\Delta$. The first-order condition of ξ gives the best-response reserve price $\xi(e) = \min\{\bar{c} - e, \frac{r+c-e}{2}\}$.

Unobservable effort case. Denote the supplier's best response when the buyer chooses reserve price ξ by $e(\xi)$. If the supplier chooses e , his expected profit equals $\frac{1}{\Delta} \int_{\underline{c}-e}^{\xi} (\xi - \gamma) d\gamma - ae^2/2$, which is maximized by $e(\xi) = \frac{\xi - \underline{c}}{a\Delta - 1}$. The equilibrium effort, denoted by $e_S^{u,p}$, has $e(\xi(e_S^{u,p})) = e_S^{u,p}$; this implies $e_S^{u,p} = \min\{\frac{r - \underline{c}}{2a\Delta - 1}, \frac{1}{a}\}$.

Observable effort case. The supplier chooses $e \geq 0$ to maximize $\frac{1}{\Delta} \int_{\underline{c}-e}^{\xi(e)} (\xi(e) - \gamma) d\gamma - ae^2/2$. The first-order condition gives the equilibrium effort, denoted by $e_S^{o,p} = \min\{[2\Delta - (r - \underline{c})]^+, \frac{r - \underline{c}}{4a\Delta - 1}\}$.

Sole sourcing equilibrium effort comparison. It is easy to verify that $e_S^{o,p} < e_S^{u,p}$.

1.5.2 Dual sourcing

Buyer's best-response reserve prices. Given any supplier effort profile (e_1, e_2) , the buyer's expected profit with reserve prices $\xi_i, i = 1, 2$, equals

$$\frac{1}{\Delta^2} \left[\int_{\underline{c}-e_1}^{\xi_1} \int_{\max\{\underline{c}-e_2, \gamma_1\}}^{\bar{c}-e_2} (r - \min\{\gamma_2, \xi_1\}) d\gamma_2 d\gamma_1 + \int_{\underline{c}-e_2}^{\xi_2} \int_{\gamma_2}^{\bar{c}-e_1} (r - \min\{\gamma_1, \xi_2\}) d\gamma_1 d\gamma_2 \right].$$

The first-order conditions imply the optimal $\xi_i(e_i) = \min\{\frac{r + \underline{c} - e_i}{2}, \bar{c} - e_1\}$.

Unobservable effort case. When the effort is unobservable, the buyer chooses the reserve price and the suppliers choose their effort simultaneously. Consider the equilibrium denoted by $(e_1^{u,p}, e_2^{u,p}; \xi_1, \xi_2)$. We solve the equilibrium by combining all three players' best-response functions. Assume $e_1^{u,p} \geq e_2^{u,p}$ without loss of generality. We must have the buyer's best-response functions: $\xi_i(\mathbf{e}) = \min\{\frac{r + \underline{c} - e_i}{2}, \bar{c} - e_1\}, i = 1, 2$, where bold symbol \mathbf{e} denotes (e_1, e_2) . Given any ξ_1, ξ_2 and e_j , supplier i solves e_i that maximizes his expected profit:

$$\begin{aligned} \max_{e_1} \frac{1}{\Delta^2} \int_{\underline{c}-e_1}^{\xi_1} \int_{\max\{\underline{c}-e_2, \gamma_1\}}^{\bar{c}-e_2} (\min\{\gamma_2, \xi_1\} - \gamma_1) d\gamma_2 d\gamma_1 - ae_1^2/2; \\ \max_{e_2} \frac{1}{\Delta^2} \int_{\underline{c}-e_2}^{\xi_2} \int_{\gamma_2}^{\bar{c}-e_1} (\min\{\gamma_1, \xi_2\} - \gamma_2) d\gamma_1 d\gamma_2 - ae_2^2/2. \end{aligned}$$

The first-order derivatives with respect to e_i are $\bar{q}_i(\mathbf{e}, \boldsymbol{\xi}) - ae_i$, where bold symbol $\boldsymbol{\xi}$ denotes (ξ_1, ξ_2) . It can be shown that when $a\Delta \geq 1$, the second-order derivatives are always non-positive, and hence we can focus on the first-order conditions; namely, the equilibrium effort $e_i^{u,p}$ must satisfy:

$$ae_1^{u,p} = \bar{q}_1^{u,p}(\mathbf{e}^{u,p}, \boldsymbol{\xi}) \equiv \frac{\xi_1 - \underline{c} + e_1^{u,p}}{\Delta} - \frac{(\xi_1 - \underline{c} + e_2^{u,p})^2}{2\Delta^2}; \quad (\text{SF-4})$$

$$ae_2^{u,p} = \bar{q}_2^{u,p}(\mathbf{e}^{u,p}, \boldsymbol{\xi}) \equiv \frac{(\bar{c} - e_1^{u,p} - \underline{c} + e_2^{u,p})^2}{2\Delta^2} - \frac{(\bar{c} - e_1^{u,p} - \xi_2)^2}{2\Delta^2}, \quad (\text{SF-5})$$

$$\text{where } \xi_i = \min\left\{\frac{r + \underline{c} - e_i^{u,p}}{2}, \bar{c} - e_1^{u,p}\right\}, i = 1, 2.$$

In the following, we derive the results in Table 5 of the paper. To solve the equilibrium $(e_1^{u,p}, e_2^{u,p}; \xi_1, \xi_2)$ defined by the equations above we consider three possible cases that differ in the expressions of the best-response reserve price $\xi_i(\mathbf{e})$.

Case 1: $\xi_i = \bar{c} - e_1^{u,p}$ for both $i = 1, 2$. In this case, we must have $e_1^{u,p} > 2\Delta - (r - \underline{c})$. Solve the equations that define the suppliers' best-response, we obtain two potential solutions.

A potential symmetric solution has

$$e_1^{u,p} = e_2^{u,p} = \frac{1}{2a} \quad (\text{Label S1}^{u,p}).$$

Its conditions include

$$e_1^{u,p} > 2\Delta - (r - \underline{c}) \longrightarrow (r - \underline{c}) > 2\Delta - \frac{1}{2a}.$$

A potential asymmetric solution has

$$e_1^{u,p} = \frac{2 - (a\Delta - 1)^2}{2a}; e_2^{u,p} = \frac{(a\Delta - 1)^2}{2a} \quad (\text{Label A1}^{u,p}).$$

Its conditions include:

$$e_1^{u,p} > e_2^{u,p} \longrightarrow a\Delta < 2;$$

$$e_1^{u,p} > 2\Delta - (r - \underline{c}) \longrightarrow (r - \underline{c}) > \frac{a^2\Delta^2 + 2a\Delta - 1}{2a}.$$

We derive the slope of the best-response functions:

$$\frac{de_1^*}{de_2} = \frac{de_2^*}{de_1} \Big|_{e_1=e_1^{u,p}, e_2=e_2^{u,p}} = \frac{-(1 - e_1 + e_2)}{a - 1 + e_1 - e_2} < 0.$$

The stability conditions (i.e., $\frac{de_1^*}{de_2} > -1$ and $\frac{de_2^*}{de_1} > -1$) for $S1^{u,p}$ require $a\Delta > 2$.

Case 2: $\xi_i = \frac{r + \underline{c} - e_i^{u,p}}{2}$ for both $i = 1, 2$. In this case, we must have $2e_1^{u,p} - e_2^{u,p} < 2\Delta - (r - \underline{c})$. Solve the equations that define the suppliers' best-response, we obtain two potential solutions.

A potential symmetric solution has

$$e_1^{u,p} = e_2^{u,p} = 2\Delta - (r - \underline{c}) - 4a\Delta^2 + 2\Delta\sqrt{2a(r - \underline{c}) + 4a^2\Delta^2 - 4a\Delta + 1} \quad (\text{Label S2}^{u,p}).$$

Its conditions include

$$e_2^{u,p} \geq 0 \longrightarrow 0 \leq (r - \underline{c}) \leq 4\Delta;$$

$$2e_1^{u,p} - e_2^{u,p} \leq 2\Delta - (r - \underline{c}) \longrightarrow r - \underline{c} \leq 2\Delta - \frac{1}{2a};$$

and the stability condition requires

$$\frac{de_1^*}{de_2} \Big|_{e_1=e_1^{u,p}, e_2=e_2^{u,p}} = \frac{-2(r - \underline{c} - e_1^{u,p} + 2e_2^{u,p})}{4a\Delta^2 - 2\Delta - (r - \underline{c}) + e_1^{u,p} - 2e_2^{u,p}} > -1 \longrightarrow (r - \underline{c}) < \frac{7}{18a}(2a\Delta - 1)^2$$

$$\frac{de_2^*}{de_1} \Big|_{e_1=e_1^{u,p}, e_2=e_2^{u,p}} = \frac{-2(r - \underline{c} + e_2^{u,p})}{4a\Delta^2 - 2\Delta + 2e_1^{u,p} - 3e_2^{u,p} - (r - \underline{c})} > -1 \longrightarrow (r - \underline{c}) < \frac{7}{18a}(2a\Delta - 1)^2.$$

A potential asymmetric solution has

$$e_1^{u,p} = \frac{14\sigma}{25} - (r - \underline{c}) - \frac{116a\Delta^2}{25} + \frac{58\Delta}{25}$$

$$e_2^{u,p} = \frac{2\sigma}{25} - (r - \underline{c}) + \frac{12a\Delta^2}{25} - \frac{6\Delta}{25}, \quad (\text{Label A2}^{u,p})$$

where

$$\sigma \equiv \Delta \sqrt{50a(r - \underline{c}) + 9(2a\Delta - 1)^2}.$$

The condition $e_1^{u,p} \geq e_2^{u,p}$ implies $a(r - \underline{c}) \geq \frac{7}{18}(2a\Delta - 1)^2$. The condition

$$2e_1^{u,p} - e_2^{u,p} < 2\Delta - (r - \underline{c}) \longrightarrow r - \underline{c} < \frac{352a^2\Delta^2 - 108a\Delta - 9}{338a}.$$

It can be verify that the stableness conditions always hold given the previous conditions.

Case 3: $\xi_1 = \frac{r + \underline{c} - e_1^{u,p}}{2}$ and $\xi_2 = \bar{c} - e_1^{u,p}$. In this case, we must have $e_1^{u,p} \leq 2\Delta - (r - \underline{c})$ and $2e_1^{u,p} - e_2^{u,p} > 2\Delta - (r - \underline{c})$. By the conditions, there is no symmetric equilibrium. There exists an asymmetric equilibrium, which does not have closed-form expressions of $e_1^{u,p}$ and $e_2^{u,p}$. We label this equilibrium by $A3^{u,p}$. The existence of the equilibrium is based on the continuity of the best-response curves as $r - \underline{c}$ increases between the $A2^{u,p}$ right boundary and $A1^{u,p}$ left boundary.

Observable effort case. When the effort is observable, the suppliers choose their effort in anticipation that the buyer chooses the reserve prices contingent on her observation of the effort profile. Consider the equilibrium denoted by $(e_1^{o,p}, e_2^{o,p}; \xi_1, \xi_2)$. Assume $e_1^{o,p} \geq e_2^{o,p}$ without loss of generality. We must have $\xi_i = \xi(\mathbf{e}^{o,p})$, where

$$\xi(\mathbf{e}) \equiv \min\left\{\frac{r + \underline{c} - e_i}{2}, \bar{c} - e_1\right\}, i = 1, 2; \forall \mathbf{e}.$$

Let $\bar{\pi}_i^{o,p}(\mathbf{e})$ denote supplier i 's expected profit excluding the effort cost. We have

$$\begin{aligned} \bar{\pi}_1^{o,p}(\mathbf{e}) &= \frac{1}{\Delta^2} \int_{\underline{c} - e_1}^{\xi_1(\mathbf{e})} \int_{\max\{\underline{c} - e_2, \gamma_1\}}^{\bar{c} - e_2} (\min\{\gamma_2, \xi_1(\mathbf{e})\} - \gamma_1) d\gamma_2 d\gamma_1; \\ \bar{\pi}_2^{o,p}(\mathbf{e}) &= \frac{1}{\Delta^2} \int_{\underline{c} - e_2}^{\xi_2(\mathbf{e})} \int_{\gamma_2}^{\bar{c} - e_1} (\min\{\gamma_1, \xi_2(\mathbf{e})\} - \gamma_2) d\gamma_1 d\gamma_2. \end{aligned}$$

We can derive the first-order derivatives:

$$\begin{aligned} \frac{\partial \bar{\pi}_1^{o,p}(\mathbf{e})}{\partial e_1} &\equiv \frac{(\xi_1 - \underline{c} + e_1)^2 - (e_1 - e_2)^2}{2\Delta^2} + \left(1 + \frac{d\xi_1}{de_1}\right) \frac{(\bar{c} - e_2 - \xi_1)(\xi_1 - \underline{c} + e_1)}{\Delta^2}; \\ \frac{\partial \bar{\pi}_2^{o,p}(\mathbf{e})}{\partial e_2} &\equiv \frac{(\xi_2 - \underline{c} + e_2)^2}{2\Delta^2} + \left(1 + \frac{d\xi_2}{de_2}\right) \frac{(\bar{c} - e_1 - \xi_2)(\xi_2 - \underline{c} + e_2)}{\Delta^2}. \end{aligned}$$

Note that

$$\frac{\partial \bar{\pi}_2^{o,p}(\mathbf{e})}{\partial e_2} \leq \frac{(\xi_2 - \underline{c} + e_2)^2}{2\Delta^2} + \frac{(\bar{c} - e_1 - \xi_2)(\xi_2 - \underline{c} + e_2)}{\Delta^2} = \bar{q}_2(\mathbf{e}), \quad (\text{SF-6})$$

$$\frac{\partial \bar{\pi}_1^{o,p}(\mathbf{e})}{\partial e_1} \leq \frac{(\xi_1 - \underline{c} + e_1)^2 - (e_1 - e_2)^2}{2\Delta^2} + \frac{1}{2} \frac{(\bar{c} - e_2 - \xi_1)(\xi_1 - \underline{c} + e_1)}{\Delta^2} < \bar{q}_1(\mathbf{e}). \quad (\text{SF-7})$$

Namely, $\frac{\partial \bar{\pi}_i^{o,p}(\mathbf{e})}{\partial e_i} \leq \bar{q}_i(\mathbf{e})$: given any \mathbf{e} , supplier i 's marginal effort return is always less than his winning probability.

Note: $\frac{\partial \bar{\pi}_i^{o,p}(\mathbf{e})}{\partial e_i}$ can be discontinuous because so is $\frac{d\xi_i}{de_i}$ when and only when ξ_i shifts from $\frac{r + \underline{c} - e_i}{2}$ to $\bar{c} - e_1$. Therefore, supplier i 's best-response $e_i^*(e_j)$ is the largest e_i such that $\frac{\partial \bar{\pi}_i^{o,p}(\mathbf{e})}{\partial e_i} \geq a e_i$.

In the following, we provide detailed derivations for the results in Table 5. To solve the equilibrium $(e_1^{o,p}, e_2^{o,p}; \xi_1, \xi_2)$ via the first-order conditions defined above. We consider three possible cases that differ in the expressions of the best-response reserve price ξ_i .

Case 1: $\xi_i = \bar{c} - e_1^{o,p}$ for both $i = 1, 2$. In this case, we must have $e_1^{o,p} > 2\Delta - (r - \underline{c})$. The first-order conditions yield:

$$ae_1^{o,p} = \frac{\Delta^2 - (e_1^{o,p} - e_2^{o,p})^2}{2\Delta^2};$$

$$ae_2^{o,p} = \frac{(\Delta - e_1^{o,p} + e_2^{o,p})^2}{2\Delta^2}.$$

They imply:

$$\frac{de_1^*}{de_2} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} = \frac{e_1^{o,p} - e_2^{o,p}}{a\Delta^2 + e_1^{o,p} - e_2^{o,p}};$$

$$\frac{de_2^*}{de_1} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} = \frac{e_1^{o,p} - e_2^{o,p} - \Delta}{a\Delta^2 - \Delta + e_1^{o,p} - e_2^{o,p}}.$$

A potential symmetric equilibrium

$$e_1^{o,p} = e_2^{o,p} = \frac{1}{2a} \quad (\text{Label S1}^{o,p}).$$

Its conditions include

$$e_1^{o,p} \geq 2\Delta - (r - \underline{c}) \longrightarrow r - \underline{c} \geq 2\Delta - \frac{1}{2a}$$

It is easy to verify $\frac{de_1^*}{de_2} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} = 0$ and $\frac{de_2^*}{de_1} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} = \frac{1}{1-a\Delta}$, and hence the equilibrium is always stable.

A potential asymmetric equilibrium

$$e_1^{o,p} = \Delta - \frac{a\Delta^2}{2}; e_2^{o,p} = \frac{a\Delta^2}{2}.$$

Its conditions include

$$e_1^{o,p} > e_2^{o,p} \longrightarrow a\Delta < 1.$$

However, $\frac{de_1^*}{de_2} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} > 0$ and $\frac{de_2^*}{de_1} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}}$ approaches negative infinity; this always imply unstableness. We discard this equilibrium solution.

Case 2: $\xi_i = \frac{r+\underline{c}-e_i^{o,p}}{2}$ for both $i = 1, 2$. In this case, we must have $2e_1^{o,p} - e_2^{o,p} < 2\Delta - (r - \underline{c})$. The first-order conditions yield

$$ae_1^{o,p} = \frac{(r - \underline{c} + e_1^{o,p})^2 - 4(e_1^{o,p} - e_2^{o,p})^2}{8\Delta^2} + \frac{(r - \underline{c} + e_1^{o,p})(2\Delta - 2e_2^{o,p} + e_1^{o,p} - (r - \underline{c}))}{8\Delta^2}; \quad (\text{SF-8})$$

$$ae_2^{o,p} = \frac{(r - \underline{c} + e_2^{o,p})^2}{8\Delta^2} + \frac{(r - \underline{c} + e_2^{o,p})(2\Delta - 2e_1^{o,p} + e_2^{o,p} - (r - \underline{c}))}{8\Delta^2}. \quad (\text{SF-9})$$

They imply the following

$$\frac{de_1^*}{de_2} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} = \frac{3e_1^{o,p} - 4e_2^{o,p} - (r - \underline{c})}{4a\Delta^2 + 2e_1^{o,p} - 3e_2^{o,p} - (r - \underline{c}) - \Delta};$$

$$\frac{de_2^*}{de_1} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} = \frac{-e_2^{o,p} - (r - \underline{c})}{4a\Delta^2 + e_1^{o,p} - 2e_2^{o,p} - (r - \underline{c}) - \Delta}.$$

A potential symmetric solution is

$$e_1^{o,p} = e_2^{o,p} = \frac{r - \underline{c}}{4a\Delta - 1} \quad (\text{Label } S2^{o,p})$$

The conditions include:

$$2e_1^{o,p} - e_2^{o,p} \leq 2\Delta - (r - \underline{c}) \longrightarrow r - \underline{c} \leq 2\Delta - \frac{1}{2a};$$

$$\frac{de_1^*}{de_2} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} > -1, \frac{de_2^*}{de_1} \Big|_{e_1=e_1^{o,p}, e_2=e_2^{o,p}} > -1 \longrightarrow r - \underline{c} < \frac{(4a\Delta - 1)^2}{8a}.$$

A potential asymmetric solution is:

$$\begin{aligned} e_1^{o,p} &= \Delta - 4a\Delta^2 - (r - \underline{c}) + 3\Delta\sqrt{2a(r - \underline{c})} ; \\ e_2^{o,p} &= -(r - \underline{c}) + \Delta\sqrt{2a(r - \underline{c})} \quad (\text{Label } A2^{o,p}) \end{aligned}$$

Its conditions include:

$$e_1^{o,p} > e_2^{o,p} \longrightarrow r - \underline{c} > \frac{(4a\Delta - 1)^2}{8a\Delta^2};$$

$$2e_1^{o,p} - e_2^{o,p} \leq 2\Delta - (r - \underline{c}) \longrightarrow r - \underline{c} \leq \frac{32a\Delta}{25}.$$

Overall, the binding ones are

$$\frac{(4a\Delta - 1)^2}{8a\Delta^2} < r - \underline{c} \leq \frac{32a\Delta}{25}.$$

Case 3: $\hat{\gamma}_1 = \frac{r + \underline{c} - e_1^{o,p}}{2}$ and $\hat{\gamma}_2 = \bar{c} - e_1^{o,p}$. In this case, we must have $e_1^{o,p} \leq 2\Delta - (r - \underline{c})$ and $2e_1^{o,p} - e_2^{o,p} > 2\Delta - (r - \underline{c})$. By the conditions, there is no symmetric equilibrium. There exists an asymmetric equilibrium, which does not have closed-form expressions of $e_1^{o,p}$ and $e_2^{o,p}$. We label this equilibrium by $A3^{o,p}$. The existence of the equilibrium is based on the continuity of the best-response curves as $r - \underline{c}$ increases between the $A2^{o,p}$ right boundary and $A1^{o,p}$ left boundary.

1.6 Proof of Proposition 7

Part (i): It can be verified by the results in Table 5 and is illustrated in Figure 5(c-d).

Part (ii) for unobservable effort: The sole-sourcing equilibrium effort $e_S^{u,p}$ solves $ae = \text{Prob}(c - e \leq \xi(e))$ for the variable e , and the dual-sourcing equilibrium effort $e_{D,1}^{u,p}$ solves $ae = \text{Prob}(c_1 - e \leq \min\{\xi(e), c_2 - e_{D,2}^{u,p}\})$ for the variable e . Part (ii) holds because $\text{Prob}(c_1 - e \leq \min\{\xi(e), c_2 - e_{D,2}^{u,p}\}) \leq \text{Prob}(c - e \leq \xi(e))$ for all e and any $e_{D,2}^{u,p}$.

Part (ii) for unobservable effort: We analyze each region in Figure 5(d).

Region $S1^{o,p}$. It is easy to show that $e_{D,1}^{o,p} = \frac{1}{2a} \geq e_S^{o,p} = [2\Delta - (r - \underline{c})]^+$, where the equality holds only on the the boundary of $S1$.

Region $S2^{o,p}$. It occurs that $e_{D,1}^{o,p} = \frac{r - \underline{c}}{4a\Delta - 1} = e_S^{o,p}$.

Regions $A2^{o,p}$ and $A3^{o,p}$. In both regions, $e_S^{o,p} = \frac{r - \underline{c}}{4a\Delta - 1}$, and the equilibrium effort $(e_{D,1}^{o,p}, e_{D,2}^{o,p})$ satisfy the supplier 1's best response curve: $ae_1 = \frac{(r - \underline{c} + e_1)^2 - 4(e_1 - e_2)^2}{8\Delta^2} + \frac{(r - \underline{c} + e_1)(2\Delta - 2e_2 + e_1 - (r - \underline{c}))}{8\Delta^2}$.

This implies $(a - \frac{1}{4\Delta})(e_{D,1}^{o,p} - e_S^{o,p}) = \frac{e_{D,1}^{o,p} - e_{D,2}^{o,p}}{4\Delta}(r - \underline{c} - e_{D,1}^{o,p} + 2e_{D,2}^{o,p}) \geq 0$, which in turn implies $e_{D,1}^{o,p} \geq e_S^{o,p}$ if $r - \underline{c} - e_{D,1}^{o,p} + 2e_{D,2}^{o,p} \geq 0$ holds. Otherwise, if $r - \underline{c} - e_{D,1}^{o,p} + 2e_{D,2}^{o,p} < 0$, we have $\xi_1 = \frac{r - \underline{c} - e_{D,1}^{o,p}}{2} < \underline{c} - e_{D,2}^{o,p}$, which implies $\xi_1 < \gamma_2$ for sure and thus supplier 1's expected profit given $e_{D,1}^{u,p}$ equals $\int_{\underline{c} - e_{D,1}^{u,p}}^{\xi_1(e_{D,1}^{u,p})} (\xi_1(e_{D,1}^{u,p}) - \gamma_1)g(\gamma_1|e_{D,1}^{u,p})d\gamma_1 - a(e_{D,1}^{u,p})^2$, where $\xi_1(e_{D,1}^{u,p}) = \frac{r - \underline{c} - e_{D,1}^{u,p}}{2}$. Applying the first-order condition, we have $e_{D,1}^{u,p} = \frac{r - \underline{c}}{4a\Delta - 1}$, equal to $e_S^{o,p}$.

Part (iii): We compare corresponding points (with equal a and $r - \underline{c}$) between Figure 5(c) and (d).

The following are clear observations: 1) $S1^{u,p}$ is a subset of $S1^{o,p}$, and the equilibrium effort level in the overlapped region equals; 2) $S2^{u,p}$ is a subset of $S2^{o,p}$, and the equilibrium effort level in the overlapped region is higher in $S2^{u,p}$. Therefore, part (iv) holds in the regions of $S1^{u,p}$ and $S2^{u,p}$. We examine the other regions as follows

Region $S1^{o,p}$ overlapped with $A1^{u,p}$. Let $\Pi^{u,p}(e_{D,1}^{u,p}, e_{D,2}^{u,p})$ and $\Pi^{o,p}(e_{D,1}^{o,p}, e_{D,2}^{o,p})$ denote the buyer's equilibrium expected profit in $A1^{u,p}$ and $S1^{o,p}$, respectively. We can show that $\Pi^{u,p}(e_{D,1}^{u,p}, e_{D,2}^{u,p}) = r - \underline{c} + \frac{(e_{D,1}^{u,p} - e_{D,2}^{u,p})^3}{6\Delta^2} + \frac{1}{2a} - \frac{2}{3}\Delta$ and $\Pi^{o,p}(e_{D,1}^{o,p}, e_{D,2}^{o,p}) = r - \underline{c} + \frac{(e_{D,1}^{o,p} - e_{D,2}^{o,p})^3}{6\Delta^2} + \frac{1}{2a} - \frac{2}{3}\Delta$. The former is higher because $e_{D,1}^{u,p} \geq e_{D,2}^{u,p}$ in $A1^{u,p}$ and $e_{D,1}^{o,p} = e_{D,2}^{o,p}$ in $S1^{o,p}$.

Region $S2^{o,p}$ overlapped with $A1^{u,p}$. The equilibrium effort in Region $S2^{o,p}$ equals $\frac{r - \underline{c}}{4a\Delta - 1}$, which is less than that in Region $S1^{o,p}$ (i.e., $\frac{1}{2a}$); therefore, the buyer's expected profit is lower in $S2^{o,p}$ than $S1^{o,p}$. This implies higher buyer profit in $A1^{u,p}$ than the overlapped $S2^{o,p}$ because we just proved that the buyer's expected payoff is higher under $A1^{u,p}$ than $S1^{o,p}$.

Regions $S2^{o,p}$ overlapped with $A2^{u,p}$ and $A3^{u,p}$. In the proof of part (iv) we will show that the buyer's expected profit when the suppliers choose the equilibrium effort levels in $A2^{u,p}$ and $A3^{u,p}$ is higher than the expected profit if the suppliers' effort choices share the same expression as in Region $S2^{u,p}$. The latter is higher than the buyer's expected profit in $S2^{o,p}$ because the suppliers' equilibrium effort is higher in $S2^{u,p}$ than in $S2^{o,p}$. Therefore, the buyer's expected equilibrium profit is higher in $A2^{u,p}$ and $A3^{u,p}$ than the overlapped region of $S2^{o,p}$.

Regions $A2^{o,p}$ and $A3^{o,p}$ overlapped with $A1^{u,p}$. In the proof of part (iv) we will show that the buyer's expected profit in $A1^{u,p}$ equals to the buyer's expected profit under the optimal full-commitment mechanism. It exceeds the buyer's expected profit in $A2^{o,p}$ and $A3^{o,p}$, because we can show that the buyer can always design a suboptimal full-commitment mechanism that induces the same allocation outcome (contingent on the realized \mathbf{x}) as in $A2^{o,p}$ and $A3^{o,p}$ but higher supplier effort levels. In particular, in the equilibrium of $A2^{o,p}$ and $A3^{o,p}$, the buyer allocates the contract to i if and only if $c_i - e_i^{o,p} < c_j - e_j^{o,p}$ and $c_i - e_i^{o,p} < \frac{r - \underline{c} - e_i^{o,p}}{2}$; let $(\tilde{q}_1^{o,p}(c_1, c_2), \tilde{q}_2^{o,p}(c_1, c_2))$ denote the implied allocation rule in the space of (c_1, c_2) . Per the proof of Lemma 3, the buyer's expected profit in $A2^{o,p}$ and $A3^{o,p}$ equals: $\Pi^{o,p} = \int \left\{ \sum_{i=1,2} [r - \tilde{J}(c_i) + e_i^{o,p}] \tilde{q}_i^{o,p}(\mathbf{c}) \right\} f(\mathbf{c}) d\mathbf{c}$. Now, consider that the buyer commits to using $(\tilde{q}_1^{o,p}, \tilde{q}_2^{o,p})$ in the unobservable effort case. Her expected profit equals $\Pi^{u,p} = \int \left\{ \sum_{i=1,2} [r - \tilde{J}(c_i) + \tilde{e}_i^{u,p}] \tilde{q}_i^{o,p}(\mathbf{c}) \right\} f(\mathbf{c}) d\mathbf{c}$, where $\tilde{e}_i^{u,p}$ is supplier i 's equilibrium effort choice in the mechanism $(\tilde{q}_1^{o,p}, \tilde{q}_2^{o,p})$. To prove $\Pi^{u,p} > \Pi^{o,p}$, it suffices to show $\tilde{e}_i^{u,p} \geq \tilde{e}_i^{o,p}$: It holds because 1) $a\tilde{e}_i^{u,p}$ equals $\bar{q}_i(\tilde{e}_1^{u,p}, \tilde{e}_2^{u,p})$, i.e., supplier i 's winning probability under the mechanism $(\tilde{q}_1^{o,p}, \tilde{q}_2^{o,p})$, where as $a\tilde{e}_i^{o,p}$ equals $\frac{\partial \bar{\pi}_i^{o,p}(\mathbf{e})}{\partial e_i} |_{(\tilde{e}_1^{o,p}, \tilde{e}_2^{o,p})}$ and 2) $\bar{q}_i(\mathbf{e}) \geq \frac{\partial \bar{\pi}_i^{o,p}(\mathbf{e})}{\partial e_i}$ for all \mathbf{e} (i.e., inequalities (SF-6)-

(SF-7)).

Part (iv) for observable effort: It directly follows from part (ii).

Part (iv) for unobservable effort: To prove part (iv) for the unobservable effort case, we analyze each region in Figure 5(c).

Figure 5(c): Regions $A1^{u,p}$ and $S1^{u,p}$. In both regions, the equilibrium outcome is the same as in the optimal full-commitment mechanism case (i.e., $e_i^{u,p} = e_i^f$ and $\xi_i^{u,p} = \xi_i^f = \bar{c} - e_{D,1}$ for $i = 1, 2$); for verification, one compares Tables 4 and 5. Therefore, the buyer's expected profit in dual-sourcing equals that in the full-commitment case. By Proposition 6(iii), the buyer has higher expected profit in the $A1^{u,p}$ and $S1^{u,p}$ dual-sourcing equilibrium than sole-sourcing.

Figure 5(c): Region $S2^{u,p}$. We can derive the buyer's expected profit under dual-sourcing $(\frac{1}{2} - \frac{e_{D,1}^{u,p} + r - \underline{c}}{12\Delta})(e_{D,1}^{u,p} + r - \underline{c})^2 / \Delta^2$. In region $S2^{u,p}$, the buyer's profit under sole sourcing equals $r - \underline{c} + \frac{1}{a} - \Delta \leq [\frac{a(r-\underline{c})}{2a\Delta-1}]^2$ if $e_S^{u,p} = \frac{1}{a}$, and equals $[\frac{a(r-\underline{c})}{2a\Delta-1}]^2$, otherwise (i.e., where $e_S^{u,p} = \frac{r-\underline{c}}{2a\Delta-1}$). The buyer's profit is always higher under dual sourcing because $(\frac{1}{2} - \frac{e_{D,1}^{u,p} + r - \underline{c}}{12\Delta})(e_{D,1}^{u,p} + r - \underline{c})^2 / \Delta^2 = y^2(\frac{1}{2} - \frac{y}{12}) \geq [\frac{a(r-\underline{c})}{2a\Delta-1}]^2 = y^2[\frac{y}{8(2a\Delta-1)} - \frac{1}{2}]^2$, where $y \equiv \frac{e_{D,1}^{u,p} + r - \underline{c}}{\Delta} \in [0, 2]$ in region $S2^{u,p}$.

Figure 5(c): Regions $A2^{u,p}$ and $A3^{u,p}$. Let $\Pi^{u,p}(e_{D,1}, e_{D,2})$ denote the buyer's expected profit when supplier i chooses effort e_i , $i = 1, 2$. In regions $A2^{u,p}$ and $A3^{u,p}$, we can show $\Pi^{u,p}(e_{D,1}^{u,p}, e_{D,2}^{u,p}) > \Pi^{u,p}(e_{S2}, e_{S2})$, where $e_{S2} \equiv 2\Delta - (r - \underline{c}) - 4a\Delta^2 + 2\Delta\sqrt{4a^2\Delta^2 - 4a\Delta + 2a(r - \underline{c}) + 1}$; namely, the buyer's expected profit when the suppliers choose the equilibrium effort levels is higher than the expected profit if the suppliers' effort choices share the same expression as in Region $S2^{u,p}$. Our proof above for region $S2^{u,p}$ extends to show that the buyer's expected profit given $e_1 = e_2 = e_{S2}$ is higher than the sole-sourcing equilibrium profit. Therefore, her dual-sourcing equilibrium profit in regions $A2^{u,p}$ and $A3^{u,p}$ is higher than sole-sourcing.

We now prove that $\Pi^{u,p}(e_1^{u,p}, e_2^{u,p}) > \Pi^{u,p}(e_{S2}, e_{S2})$, where $\Pi^{u,p}(e_1, e_2)$ denotes the buyer's expected profit when supplier i chooses effort e_i , $i = 1, 2$ and $e_{S2} \equiv 2\Delta - (r - \underline{c}) - 4a\Delta^2 + 2\Delta\sqrt{4a^2\Delta^2 - 4a\Delta + 2a(r - \underline{c}) + 1}$. Note that (e_{S2}, e_{S2}) is the unstable fixed-point when $(a, r - \underline{c})$ falls in regions A2 and A3, and $(e_1^{u,p}, e_2^{u,p})$ is the stable fixed-point. We can derive the partial derivatives:

$$\begin{aligned} \frac{\partial \Pi^{u,p}(e_1, e_2)}{\partial e_1} &= \bar{q}_1(e_1, e_2) + (r - e_1 + 2e_2) + \frac{(r - e_1 + 2e_2)^2}{4} - \frac{(r + e_2)^2}{4}; \\ \frac{\partial \Pi^{u,p}(e_1, e_2)}{\partial e_2} &= \bar{q}_2(e_1, e_2) + \frac{(r + e_2)^2}{4}. \end{aligned}$$

We can compute $\Pi^{u,p}(e_1^{u,p}, e_2^{u,p}) - \Pi^{u,p}(e_{S2}, e_{S2})$ by a path integral through any path \mathcal{L} from (e_{S2}, e_{S2}) to $(e_1^{u,p}, e_2^{u,p})$:

$$\Pi^{u,p}(e_1^{u,p}, e_2^{u,p}) - \Pi^{u,p}(e_{S2}, e_{S2}) = \int_{\mathcal{L}} \left[\frac{\partial \Pi^{u,p}(e_1, e_2)}{\partial e_1} + \frac{\partial \Pi^{u,p}(e_1, e_2)}{\partial e_2} \right] d\mathcal{L}.$$

Consider the linear path $e_1(t) = e_{S2} + (e_1^{u,p} - e_{S2})t$ and $e_2(t) = e_{S2} + (e_2^{u,p} - e_{S2})t$ with t from 0 to 1, and we have

$$\Pi^{u,p}(e_1^{u,p}, e_2^{u,p}) - \Pi^{u,p}(e_{S2}, e_{S2}) = \int_{t=0}^1 \frac{d\Pi^{u,p}(e_1(t), e_2(t))}{dt} dt,$$

where

$$\frac{d\Pi^{u,p}(e_1(t), e_2(t))}{dt} = \left[\frac{\partial \Pi^{u,p}(e_1, e_2)}{\partial e_1} (e_1^{u,p} - e_{S2}) + \frac{\partial \Pi^{u,p}(e_1, e_2)}{\partial e_2} (e_2^{u,p} - e_{S2}) \right].$$

To prove that $\Pi^{u,p}(e_1^{u,p}, e_2^{u,p}) - \Pi^{u,p}(e_{S2}, e_{S2}) > 0$, it suffices to show $\frac{d\Pi^{u,p}(e_1(t), e_2(t))}{dt} > 0, \forall t \in [0, 1]$, which is true because $\frac{d^2 \Pi^{u,p}(e_1(t), e_2(t))}{dt^2} = -(e_{S2} - e_2^{u,p})^2 (e_1^{u,p} - e_2^{u,p} + \frac{e_1^{u,p} - e_{S2}}{2}) < 0$, and we can verify that $\frac{d\Pi^{u,p}(e_1(t), e_2(t))}{dt}|_{t=1} > 0$.

2. Generality of results

2.1 General cost distribution F

In this subsection we show that the following main results of our paper hold when the innate cost distribution $F(c)$ is a general distributions with increasing $F(c)/(c)$. First, if the disutility parameter a is sufficiently small, then the symmetric pure strategy fixed point is not stable and an asymmetric pure strategy equilibrium exists. Second, the asymmetric equilibrium region expands as the buyer's commitment level increases, i.e., changes the mechanism from no-commitment, to partial-commitment, and finally to full-commitment. Third, dual sourcing becomes more favorable relative to sole sourcing as the buyer's commitment level increases.

We provide an outline of our analysis. In step 1, we derive the symmetric fixed point and show it is stable (and hence is an equilibrium) if and only if $a \geq a_s^{mech}$, where a_s^{mech} is a positive threshold that depends on the mechanism $mech$. Let $mech = u$ denote the no-commitment mechanism unobservable effort case, $mech = p$ denote the partial-commitment mechanism unobservable effort case, and $mech = f$ denote the full-commitment mechanism. In particular, we show the unstableness of the symmetric fixed point when $a < a_s^{mech}$ by showing the derivative of each supplier's best response at the symmetric fixed point is small than -1 . In step 2, we show that there exists a stable asymmetric equilibrium when $a < a_s^{mech}$. In step 3, we show the impact of the buyer's commitment on the asymmetric equilibrium region by showing the thresholds under the three cases (no-commitment mechanism with unobservable effort, partial-commitment mechanism with unobservable effort, and full-commitment mechanism) are ranked, namely, $a_s^u \leq a_s^p \leq a_s^f$. In step 4, we show the impact of the buyer's commitment on her preference to dual sourcing.

Define the following notations. Given any distribution $F(c)$, denote the virtual costs $J_1 \equiv c_1 + \frac{F(c_1)}{f(c_1)}$ and $J_2 \equiv c_2 + \frac{F(c_2)}{f(c_2)}$. Thus, $J(\gamma_i, e_i) = J_i - e_i$ for $i = 1, 2$. Let \underline{J} and \bar{J} denote $\underline{c} + \frac{F(\underline{c})}{f(\underline{c})}$ and $\bar{c} + \frac{F(\bar{c})}{f(\bar{c})}$, respectively. Let L and l denote the cdf and pdf of random variable J_i .

2.1.1 Step 1: symmetric fixed point and its derivative

Denote the symmetric fixed point by $e_1 = e_2 = \hat{e}$. It is the unique intersection of $e_1 = e_1^*(e_2)$ of $e_1 = e_2$. Note that Lemma 2 in the paper extends to the case with general F because its proof (in the paper Appendix) does not use the assumption F being uniform. Lemma 2 implies given e_2 the

best-response of supplier 1, denoted by e_1^* , satisfies $ae_1^* = \bar{q}_1(e_1^*, e_2)$, which in turn implies

$$\bar{q}_1(\hat{e}, \hat{e}) = a\hat{e}, \text{ and} \quad (\text{SF-10})$$

$$\frac{de_1^*}{de_2} = \frac{1}{a} \frac{\partial \bar{q}_1(e_1^*, e_2)}{\partial e_2}. \quad (\text{SF-11})$$

In the following, we derive $\bar{q}_1(e_1, e_2)$ and $\frac{de_1^*}{de_2}$ under each of the three mechanisms, and show that the existence of threshold a_s^{mech} .

No-commitment mechanism with unobservable effort

When $e_1 \geq e_2$, we have:

$$\bar{q}_1(e_1, e_2) = \int_{\underline{J}}^{J_1 - e_1 \leq \underline{J} - e_2} dL(J_1) + \int_{J_1 - e_1 > \underline{J} - e_2}^{J_1 - e_1 \leq \min\{r, \bar{J} - e_1\}} \int_{J_2 - e_2 > J_1 - e_1}^{\bar{J}} dL(J_2) dL(J_1); \quad (\text{SF-12})$$

$$\begin{aligned} \frac{de_1^*}{de_2} &= \frac{1}{a} \frac{\partial \bar{q}_1(e_1^*, e_2)}{\partial e_2} \\ &= -\frac{1}{a} \int_{\underline{J} - e_2 + e_1}^{\min\{r + e_1, \bar{J}\}} l(J_1 - e_1 + e_2) dL(J_1). \end{aligned} \quad (\text{SF-13})$$

When $e_1 < e_2$, we have:

$$\begin{aligned} \bar{q}_1(e_1, e_2) &= \int_{\underline{J}}^{J_1 - e_1 \leq \min\{r, \bar{J} - e_2\}} \int_{J_2 - e_2 > J_1 - e_1}^{\bar{J}} dL(J_2) dL(J_1), \\ &= \int_{\underline{J}}^{\min\{e_1 + r, \bar{J} - e_2 + e_1\}} \int_{J_1 - e_1 + e_2}^{\bar{J}} dL(J_2) dL(J_1); \\ \frac{de_1^*}{de_2} &= \frac{1}{a} \frac{\partial \bar{q}_1(e_1^*, e_2)}{\partial e_2} \\ &= -\frac{1}{a} \int_{\underline{J}}^{\min\{r, \bar{J} - e_2\}} l(J - e_1 + e_2) dL(J). \end{aligned}$$

By equations (SF-10) and (SF-12), the symmetric fixed point, denoted by \hat{e}^u , is the unique intersection of ae and $\bar{q}_1(e, e)$:

$$\begin{aligned} a\hat{e}^u &= \bar{q}_1(\hat{e}^u, \hat{e}^u) = \int_{\underline{J}}^{\min\{\hat{e}^u + r, \bar{J}\}} \int_{J_1}^{\bar{J}} dL(J_2) dL(J_1) \\ &= \begin{cases} L(r + \hat{e}^u) - \frac{1}{2}L^2(r + \hat{e}^u) & \text{if } \bar{J} - \hat{e}^u \geq r \\ \frac{1}{2}, & \text{if } \bar{J} - \hat{e}^u < r. \end{cases} \end{aligned} \quad (\text{SF-14})$$

Equation (SF-14) implies that both \hat{e}^u and $a\hat{e}^u$ continuously decrease as a increases: When a is sufficiently low, $\hat{e}^u = \frac{1}{2a}$ and \hat{e}^u goes to zero as a approaches infinity.

By equation (SF-13) the derivative of the best-response curve at the symmetric fixed point equals:

$$\frac{de_1^*}{de_2} \Big|_{e_1=e_2=\hat{e}^u} = \frac{\partial \bar{q}_1(e_1, e_2)}{\partial e_2} \Big|_{e_1=e_2=\hat{e}^u} = -\frac{1}{a} \int_{\underline{J}}^{\min\{\hat{e}^u + r, \bar{J}\}} l(J) dL(J),$$

which increases in a because \hat{e}^u decreases in a . Therefore, there exists a threshold

$$a_s^u = \int_{\underline{J}}^{\min\{\hat{e}^u+r, \bar{J}\}} l(J) dL(J), \quad (\text{SF-15})$$

such that $\frac{de_1^*}{de_2}|_{e_1=e_2=\hat{e}^u} > -1$ if and only if $a > a_s^u$.

Full-commitment mechanism

Similarly, we study the full-commitment mechanism case. When $e_1 \geq e_2$, we have

$$\bar{q}_1(e_1, e_2) = \int_{\underline{J}}^{J_1-2e_1 \leq J-2e_2} dL(J_1) + \int_{J_1-2e_1 > \underline{J}-2e_2}^{J_1-2e_1 \leq \min\{r, \bar{J}-2e_1\}} \int_{J_2-2e_2 > J_1-2e_1}^{\bar{J}} dL(J_2) dL(J_1), \quad (\text{SF-16})$$

$$\begin{aligned} \frac{de_1^*}{de_2} &= \frac{1}{a} \frac{\partial \bar{q}_1(e_1^*, e_2)}{\partial e_2} \\ &= -\frac{2}{a} \int_{\underline{J}-2e_2+2e_1}^{\min\{r+2e_1, \bar{J}\}} l(J_1 - 2e_1 + 2e_2) dL(J_1) \end{aligned} \quad (\text{SF-17})$$

When $e_1 < e_2$, we have

$$\begin{aligned} \bar{q}_1(e_1, e_2) &= \int_{\underline{J}}^{J_1-2e_1 \leq \min\{r, \bar{J}-2e_2\}} \int_{J_2-2e_2 > J_1-2e_1}^{\bar{J}} dL(J_2) dL(J_1), \\ &= \int_{\underline{J}}^{\min\{2e_1+r, \bar{J}-2e_2+2e_1\}} \int_{J_1-2e_1+2e_2}^{\bar{J}} dL(J_2) dL(J_1) \\ \frac{de_1^*}{de_2} &= \frac{1}{a} \frac{\partial \bar{q}_1(e_1^*, e_2)}{\partial e_2} \\ &= -\frac{2}{a} \int_{\underline{J}}^{\min\{r+2e_1, \bar{J}-2e_2+2e_1\}} l(J - 2e_1 + 2e_2) dL(J) \end{aligned}$$

By equations (SF-10) and (SF-16), the symmetric fixed point, denoted by \hat{e}^f , is the unique intersection of ae and $\bar{q}_1(e, e)$:

$$\begin{aligned} a\hat{e}^f &= \bar{q}_1(\hat{e}^f, \hat{e}^f) = \int_{\underline{J}}^{\min\{2\hat{e}^f+r, \bar{J}\}} \int_{J_1}^{\bar{J}} dL(J_2) dL(J_1) \\ &= \begin{cases} L(r + 2\hat{e}^f) - \frac{1}{2}L^2(r + 2\hat{e}^f) & \text{if } \bar{J} - 2\hat{e}^f \geq r \\ \frac{1}{2}, & \text{if } \bar{J} - 2\hat{e}^f < r. \end{cases} \end{aligned} \quad (\text{SF-18})$$

It is easy to show that both \hat{e}^f and $a\hat{e}^f$ continuously decrease as a increases: When a is sufficiently low, $\hat{e}^f = \frac{1}{2a}$ and \hat{e}^f goes to zero as a approaches infinity.

By equation (SF-17) the derivative of the best-response curve at the symmetric fixed point equals:

$$\frac{de_1^*}{de_2}|_{e_1=e_2=\hat{e}^f} = \frac{\partial \bar{q}_1(e_1, e_2)}{\partial e_2}|_{e_1=e_2=\hat{e}^f} = -\frac{2}{a} \int_{\underline{J}}^{\min\{2\hat{e}^f+r, \bar{J}\}} l(J) dL(J),$$

which increases in a because \hat{e}^f decreases in a . Therefore, $\frac{de_1^*}{de_2}|_{e_1=e_2=\hat{e}^f} > -1$ if and only if

$$a > a_s^f = 2 \int_{\underline{J}}^{\min\{2\hat{e}^f+r, \bar{J}\}} l(J) dL(J). \quad (\text{SF-19})$$

Partial-commitment mechanism with unobservable effort

Similarly, we study the partial-commitment mechanism case. When $e_1 \geq e_2$, we have

$$\bar{q}_1(e_1, e_2) = \int_{\underline{J}}^{J_1 - 2e_1 \leq \underline{J} - 2e_2} dL(J_1) + \int_{J_1 - 2e_1 > \underline{J} - 2e_2}^{J_1 - e_1 \leq \min\{r, \bar{J} - e_1\}} \int_{J_2 - 2e_2 > J_1 - 2e_1}^{\bar{J}} dL(J_2) dL(J_1), \quad (\text{SF-20})$$

$$\begin{aligned} \frac{de_1^*}{de_2} &= \frac{1}{a} \frac{\partial \bar{q}_1(e_1^*, e_2)}{\partial e_2} \\ &= -\frac{2}{a} \int_{\underline{J} - 2e_2 + 2e_1}^{\min\{r + e_1, \bar{J}\}} l(J_1 - 2e_1 + 2e_2) dL(J_1) \end{aligned} \quad (\text{SF-21})$$

When $e_1 < e_2$, we have

$$\begin{aligned} \bar{q}_1(e_1, e_2) &= \int_{\underline{J}}^{J_1 - e_1 \leq \min\{r, \bar{J} - 2e_2 + e_1\}} \int_{J_2 - 2e_2 > J_1 - 2e_1}^{\bar{J}} dL(J_2) dL(J_1), \\ &= \int_{\underline{J}}^{\min\{e_1 + r, \bar{J} - 2e_2 + 2e_1\}} \int_{J_1 - 2e_1 + 2e_2}^{\bar{J}} dL(J_2) dL(J_1) \\ \frac{de_1^*}{de_2} &= \frac{1}{a} \frac{\partial \bar{q}_1(e_1^*, e_2)}{\partial e_2} \\ &= -\frac{2}{a} \int_{\underline{J}}^{\min\{r + e_1, \bar{J} - 2e_2 + 2e_1\}} l(J - 2e_1 + 2e_2) dL(J) \end{aligned}$$

By equations (SF-10) and (SF-20), the symmetric fixed point, denoted by $\hat{e}^{u,p}$, is the unique intersection of ae and $\bar{q}_1(e, e)$:

$$\begin{aligned} a\hat{e}^{u,p} &= \bar{q}_1(\hat{e}^{u,p}, \hat{e}^{u,p}) = \int_{\underline{J}}^{\min\{\hat{e}^{u,p} + r, \bar{J}\}} \int_{J_1}^{\bar{J}} dL(J_2) dL(J_1) \\ &= \begin{cases} L(r + \hat{e}^{u,p}) - \frac{1}{2}L^2(r + \hat{e}^{u,p}) & \text{if } \bar{J} - \hat{e}^{u,p} \geq r \\ \frac{1}{2}, & \text{if } \bar{J} - \hat{e}^{u,p} < r. \end{cases} \end{aligned} \quad (\text{SF-22})$$

It is easy to show that both $\hat{e}^{u,p}$ and $a\hat{e}^{u,p}$ continuously decrease as a increases: When a is sufficiently low, $\hat{e}^{u,p} = \frac{1}{2a}$ and $\hat{e}^{u,p}$ goes to zero as a approaches infinity.

By equation (SF-21) the derivative of the best-response curve at the symmetric fixed point equals

$$\frac{de_1^*}{de_2} \Big|_{e_1=e_2=\hat{e}^{u,p}} = \frac{\partial \bar{q}_1(e_1, e_2)}{\partial e_2} \Big|_{e_1=e_2=\hat{e}^{u,p}} = -\frac{2}{a} \int_{\underline{J}}^{\min\{\hat{e}^{u,p} + r, \bar{J}\}} l(J) dL(J),$$

which increases in a because $\hat{e}^{u,p}$ decreases in a . Therefore, $\frac{de_1^*}{de_2} \Big|_{e_1=e_2=\hat{e}^{u,p}} > -1$ if and only if

$$a > a_s^{u,p} = 2 \int_{\underline{J}}^{\min\{\hat{e}^{u,p} + r, \bar{J}\}} l(J) dL(J). \quad (\text{SF-23})$$

2.1.2 Step 2: Asymmetric equilibrium

Consider the case $a < a_s^{mech}$. Our analysis in step 1 implies that $\frac{de_1^*}{de_2^*}|_{e_1=e_2=\hat{e}^{u,p}} < -1$. By symmetry, we have $\frac{de_2^*}{de_1^*}|_{e_1=e_2=\hat{e}^{u,p}} < -1$. This, together with the fact that $e_2^*(e_1)$ decreases and goes to zero as e_1 increases, imply that $e_2^*(e_1)$ must cross $e_1^*(e_2)$ at least once from below as e_1 increases in $[\hat{e}, \infty)$. Such a crossing point constitutes a stable asymmetric equilibrium. The stableness is given by the property that $e_2^*(e_1)$ must cross $e_1^*(e_2)$ from below.

2.1.3 Step 3: Impact of commitment on a_s^{mech}

Comparing equations (SF-14), (SF-18) and (SF-22), one can easily see $\bar{q}_1(\hat{e}^u, \hat{e}^u) = \bar{q}_1(\hat{e}^{u,p}, \hat{e}^{u,p}) < \bar{q}_1(\hat{e}^f, \hat{e}^f)$, and hence $\hat{e}^u = \hat{e}^{u,p} < \hat{e}^f$. This, together with the comparisons across equations (SF-15), (SF-23) and (SF-19), imply that $a_s^f > a_s^p > a_s$.

2.1.4 Step 4: Impact of commitment on dual- vs. sole- sourcing

We now show that dual sourcing becomes more preferable to sole sourcing as the buyer's commitment level increases, i.e., changes from the no-commitment mechanism, to the partial-commitment mechanism, and finally to the full-commitment mechanism.

Note that the buyer always prefers dual sourcing to sole sourcing under the full-commitment mechanism, as stated in Lemma 3, which holds under general F because its proof (in §1.3 of this supplemental file) holds under general F . Therefore, it suffices to show that dual sourcing is more preferable under the partial-commitment mechanism than under the no-commitment mechanism.

We compare the partial-commitment mechanism with the no-commitment mechanism. They share the same reserve price(s) and differ only in terms of rules of comparing two suppliers (if dual sourcing). In other words, they are effectively the same in the sole sourcing scenario. In contrast, in the dual sourcing scenario the buyer receives higher ex ante expected profit under the partial-commitment mechanism than under the no-commitment mechanism, because the former uses the same measure (i.e., $J(\gamma_i, e_i) - e_i$) as in the full-commitment mechanism to compare two suppliers' bids and this gives higher ex ante expected profit. We can prove it by a similar proof of Lemma 3. In particular, first, we duplicate the proof of Lemma 3 up to equation (SF-3); next, we let $\{q_i, i = 1, 2\}$ denote the allocation rules of the no-commitment mechanism; then, we construct $\{q'_i, i = 1, 2\}$ per procedure iii) as in the Lemma 3 proof, by which we obtain $\{q'_i, i = 1, 2\}$ equal to allocation rule of the partial-commitment mechanism. Note that in this process, the cut-off rule remains unchanged, meaning that $\{q'_i, i = 1, 2\}$ shares the same reserve prices as in $\{q_i, i = 1, 2\}$.

To summarize, if the buyer chooses dual sourcing, her ex ante expected profit is higher under the partial-commitment mechanism than under the no-commitment mechanism; if the buyer chooses sole sourcing, her ex ante expected profit equal under the two mechanisms. Therefore, dual sourcing is more preferable under the partial-commitment mechanism than under the no-commitment mechanism.

2.2 Symmetric mixed strategy equilibrium

Our paper shows that the symmetric pure strategy fixed point is not stable and an asymmetric pure strategy equilibrium exists if the disutility parameter a is sufficiently small. This result applies to the following scenarios: 1) no-commitment mechanism with unobservable effort; 2) partial-commitment mechanism; 3) full-commitment mechanism. In this section, we show that any symmetric mixed strategy fixed point reduces to the symmetric pure strategy fixed point and therefore, it is without loss of generality to not consider symmetric mixed strategy equilibrium. We focus on scenario 1), and by similar analysis we can show the result applies to other scenarios.

Suppose there exists a symmetric mixed strategy equilibrium, in which both suppliers randomize their effort choices in an interval $[\underline{e}, \bar{e}]$ with $\underline{e} \leq \bar{e}$. By definition, in the mixed strategy equilibrium, each supplier is indifferent in choosing any point $e \in [\underline{e}, \bar{e}]$.

Consider supplier 2's problem. Let $\bar{\pi}_2(e_1, e_2)$ denote the supplier's ex ante expected profit from the procurement mechanism (excluding the effort cost) given (e_1, e_2) ; thus, if supplier 2 chooses effort e , his ex ante expected profit equals

$$E_{e_1} [\bar{\pi}_2(e_1, e)] - ae^2/2.$$

Supplier 2 is indifferent with choosing $e \in [\underline{e}, \bar{e}]$ if and only if

$$\frac{\partial E_{e_1} [\bar{\pi}_2(e_1, e)]}{\partial e} - ae = 0, \forall e \in [\underline{e}, \bar{e}]. \quad (\text{SF-24})$$

We can show that $\frac{\partial E_{e_1} [\bar{\pi}_2(e_1, e)]}{\partial e} = E_{e_1} \left[\frac{\partial \bar{\pi}_2(e_1, e)}{\partial e} \right] = E_{e_1} [\bar{q}_2(e_1, e)]$, where $\bar{q}_2(e_1, e)$ denotes supplier 2's expected winning probability given (e_1, e) . Therefore, equation (SF-24) holds if and only if

$$E_{e_1} [\bar{q}_2(e_1, e)] - ae = 0, \forall e \in [\underline{e}, \bar{e}].$$

which holds if and only if

$$\frac{\partial E_{e_1} [\bar{q}_2(e_1, e)]}{\partial e} = a, \forall e \in [\underline{e}, \bar{e}]. \quad (\text{SF-25})$$

In the following, we show that (SF-25) can hold if and only if $\bar{e} = \underline{e}$, which means that there does not exist a symmetric mixed strategy in the strict sense (i.e., with $\underline{e} < \bar{e}$).

Given any distribution $F(c)$, denote the virtual costs $J_1 \equiv c_1 + \frac{F(c_1)}{f(c_1)}$ and $J_2 \equiv c_2 + \frac{F(c_2)}{f(c_2)}$. Thus, $J(\gamma_i, e_i) = J_i - e_i$ for $i = 1, 2$. Let \underline{J} and \bar{J} denote $\underline{c} + \frac{F(\underline{c})}{f(\underline{c})}$ and $\bar{c} + \frac{F(\bar{c})}{f(\bar{c})}$, respectively. Let L and l denote the cdf and pdf of random variable J_i .

In the no-commitment mechanism case with unobservable effort, we have

$$\begin{aligned} E_{e_1} [\bar{q}_2(e_1, e)] &= E_{e_1} \left[\int_{J_2 - e \leq \min\{r, \bar{J} - e\}} \int_{J_1 - e_1 > J_2 - e} dL(J_1) dL(J_2) \right] \\ &= E_{e_1} \left[\int_{\min\{r+e, \bar{J}\}} \int_{J_2 - e + e_1} dL(J_1) dL(J_2) \right]. \end{aligned}$$

When F is the uniform distribution $U[\underline{c}, \bar{c}]$, the distribution L is the uniform distribution $U[2\underline{c}, 2\bar{c}]$. It is easy to show that equation (SF-25) can hold only if $\underline{e} = \bar{e}$. For example, if $r > \bar{J}$ then

$$\frac{\partial E_{e_1}[\bar{q}_2(e_1, e)]}{\partial e} = \frac{1 - 2H(e)}{2\Delta},$$

which cannot be a constant as e varies in $[\underline{e}, \bar{e}]$ regardless of H (i.e., the distribution of e_1), except for the trivial case with $\underline{e} = \bar{e}$.

2.3 Minimum quantity commitment

In this subsection, we study the scenario in which the buyer commits to awarding a minimum quantity k_i to supplier i , $i = 1, 2$, with $k_1 + k_2 = k$. The buyer determines the allocation of the remaining quantity $1 - k$ between the suppliers along with the payments to the suppliers in the procurement mechanism announced after suppliers exert efforts. The procurement mechanism will use the same rule $q_i(\gamma)$ based on the virtual costs to allocate the remaining quantity; supplier i 's total quantity then becomes $q_i^K(\gamma) \equiv k_i + (1 - k)q_i(\gamma)$. The payment rule is designed to ensure incentive compatibility and interim participation: $t_i^K(\gamma) = \gamma_i q_i^K(\gamma) + \int_{\gamma_i}^{\bar{c} - e_i} \bar{q}_i^K(\rho, \mathbf{e}) d\rho$, where $\bar{q}_i^K(\rho, \mathbf{e}) \equiv \mathbb{E}_{\gamma_j | \mathbf{e}} [q_i^K(\gamma)]$. This gives the supplier expected profits

$$\pi_i^K(\gamma_i, \mathbf{e}) = k_i(\bar{c} - e_i - \gamma_i) + (1 - k)\pi_i(\gamma_i, \mathbf{e}),$$

and the buyer expected profits

$$\Pi^K(\mathbf{e}) = \sum_{i \in S} k_i(r - \bar{c} + e_i) + (1 - k)\Pi(\mathbf{e}),$$

where $\pi_i(\gamma_i, \mathbf{e})$ and $\Pi(\mathbf{e})$ are profits in the no-commitment model as specified in Equations 1 and 2 in the paper. For the committed quantity, the buyer loses the screening power and hence has to pay according to the upper bound cost, but can potentially gain the benefit of higher supplier effort.

2.3.1 Unobservable effort

It remains true that the equilibrium of efforts $\mathbf{e} = (e_i)_{i \in S}$ is based on the expected quantity purchased from each supplier: $\bar{q}_i^K(\mathbf{e}) \equiv (1 - k)\bar{q}_i(\mathbf{e}) + k_i = ae_i$ for $i \in S$, where $\bar{q}_i(\mathbf{e})$ is the original quantity expected by supplier i without quantity commitment. By committing a higher quantity, the buyer increases the supplier's incentive to improve as it reduces the quantity risk faced by the supplier, thereby guaranteeing the supplier a larger return of investment.

For given quantity commitment k_i , supplier i expects a higher expected quantity in sole sourcing than does in dual sourcing with the same effort. Therefore, with the quantity commitment fixed, sole sourcing leads to higher effort than dual sourcing as is the case without commitment.

In the following we analyze the endogenous quantity commitment decision.

Sole sourcing

Let the minimum quantity commitment be k . Following the procurement mechanism analyzed in the paper, the expected quantity of the supplier with effort e , $\bar{q}^K(e)$, equals 1 if $e > 2\Delta - r + \underline{c}$ (no cut-off), and $k + (1 - k) \frac{r+e-\underline{c}}{2\Delta}$ if $e < 2\Delta - r + \underline{c}$ (positive probability of cut-off). This further leads to the supplier effort $e_S^{u,K}$ equal to $\frac{1}{a}$ if $a < \frac{1}{2\Delta - r + \underline{c}}$, and equal to $\frac{(1-k)(r-\underline{c})+2\Delta k}{2a\Delta - 1 + k}$ otherwise. The buyer's profit is $\Pi_S^{u,K} = (1 - k) \left(r - \eta \left(e_S^{u,K} \right) + e_S^{u,K} \right) F \left(\eta \left(e_S^{u,K} \right) \right) + k \left(r - \bar{c} + e_S^{u,K} \right)$, which equals $r - \bar{c} + \frac{1}{a}$ if $a < \frac{1}{2\Delta - r + \underline{c}}$, and otherwise equals $(1 - k) \frac{(r - \underline{c} + e_S^{u,K})^2}{4\Delta} + k \left(r - \bar{c} + e_S^{u,K} \right)$.

We observe that $\Pi_S^{u,K}$ is convex in k . Therefore, the buyer either commits all quantity or does not commit at all. We find that the former policy (committing all quantity) is optimal when a is small or r is large, and the latter (zero commitment) is optimal in the opposite cases.

Dual sourcing

Assume $k_1 \geq k_2$ without loss of generality. Again, following the analysis of dual sourcing in the no-commitment model, the best-response of e_i given e_j , $e_i^*(e_j)$, is the unique solution to $(1 - k_1 - k_2) \bar{q}_i(e) + k_i = ae_i$ on e_i . Define several critical values of a : $\hat{a}_L^{(0)}(e) \equiv \frac{(1-k)d_L^{(1)}(e,e)+k_i}{e}$, $\hat{a}_L^{(1)}(e) \equiv \frac{(1-k)d_L^{(1)}(2\Delta-r+\underline{c},e)+k_i}{2\Delta-r+\underline{c}}$, $\hat{a}_L^{(2)}(e) \equiv \frac{(1-k)d_L^{(2)}(e+2\Delta,e)+k_i}{e+2\Delta}$, and $\tilde{a}_L^{(0)}(e) \equiv \frac{(1-k)d_L^{(2)}(e,e)+k_i}{e}$, that are similar to those illustrated in Figure SF-1 (after shifting the horizontal axis upward by k_i). Then $e_i^*(e_j)$ can be specified by:

- 1) If $a \leq \hat{a}_L^{(2)}(e_j)$, e_i^* is defined by $ae_i^* - k_i = 1 - k$, giving $e_i^* = \frac{1-k_j}{a}$.
- 2) If $\hat{a}_L^{(2)}(e_j) < a \leq \min \left(\hat{a}_L^{(1)}(e_j), \tilde{a}_L^{(0)}(e_j) \right)$, e_i^* is defined by $ae_i^* - k_i = (1 - k) d_L^{(2)}(e_i^*, e_j)$.
- 3) If $e_j < 2\Delta - r + \underline{c}$ and $\hat{a}_L^{(1)}(e_j) < a \leq \hat{a}_L^{(0)}(e_j)$, e_i^* is defined by $ae_i^* - k_i = (1 - k) d_L^{(1)}(e_i^*, e_j)$.
- 4) If $e_j \leq 2\Delta - r + \underline{c}$ and $a > \hat{a}_L^{(0)}(e_j)$, e_i^* is defined by $ae_i^* - k_i = (1 - k) d_F^{(1)}(e_j, e_i^*)$.
- 5) If $e_j > 2\Delta - r + \underline{c}$ and $a > \tilde{a}_L^{(0)}(e_j)$, e_i^* is defined by $ae_i^* - k_i = (1 - k) d_F^{(2)}(e_j, e_i^*)$.

We observe that again the buyer's profit is convex in the quantity commitment. The optimal is achieved with no quantity commitment with both suppliers, or committing all quantity to one supplier. The former reduces to our no-commitment model in the paper, and the later reduces to sole-sourcing with quantity commitment equal to one.

Comparing sole sourcing and dual sourcing

As the effort disutility a increases, we find that sole sourcing shifts from committing all quantity to zero quantity, and dual sourcing shifts from committing all quantity with a single supplier to zero commitment with both suppliers. The shift occurs earlier (at smaller a) in dual sourcing than in sole sourcing, meaning quantity commitment is more favorable when there is no alternative choice of suppliers. This also suggests that quantity commitment enhances the preference of dual sourcing over sole sourcing, allowing the former to (weakly) dominate the latter in the buyer profits. Figure SF-3 illustrates the buyer's profits from sole sourcing and dual sourcing with or without quantity commitment.

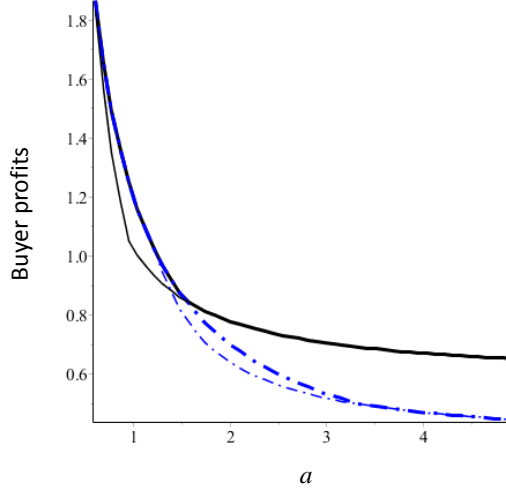


Figure SF-3: Buyer’s expected profits in dual sourcing and sole sourcing with minimum quantity commitment based on the value of effort disutility a . The solid lines are profits from dual sourcing and dotdashed lines are profits from sole sourcing. The thick lines are for the situation when quantity commitment is possible, and thin lines for the original case when there is no quantity commitment (the no-commitment model in the paper).

2.3.2 Observable effort

Define $\bar{\pi}_i^K(\mathbf{e}) \equiv E_{c_i} [\pi_i^K(c_i - e_i, \mathbf{e})] = (1 - k)\bar{\pi}_i(\mathbf{e}) + k_i(\bar{c} - \mu)$ where $\mu = E[c_i]$. It remains true that the equilibrium efforts are established by the equations $\frac{\partial \bar{\pi}_i^K(\mathbf{e})}{\partial e_i} = ae_i$, $i \in S$. Note $\frac{\partial \bar{\pi}_i^K(\mathbf{e})}{\partial e_i} = (1 - k)\frac{\partial \bar{\pi}_i(\mathbf{e})}{\partial e_i}$ decreases in k . That means quantity commitment will only *reduce* the incentive of supplier effort, as the supplier would not need to work as hard to “earn” that quantity. That also means, the supplier efforts will only depend on the non-committed quantity; the committed quantity lifts the supplier’s profit without increasing effort. Therefore, the buyer does not benefit from quantity commitment, and the optimal quantity commitment for the buyer is zero. In other words, with observable supplier effort, the buyer will choose *not* to commit a minimum quantity with a supplier. Therefore, quantity commitment benefits the buyer only when the effort is unobservable; hence the detrimental effect of effort information remains.