Electronic Companion To "Overconfident Competing Newsvendors"

In this technical supplement, we verify the scope and applicability of our key results by exploring the effects of four modeling variations: partial spillover (§EC.1), inventory-dependent demand rationing (§EC.2), random demand splitting (§EC.3), and an expanded definition of overconfidence (§EC.4). The proofs for the technical results of this supplement are compiled in its appendix.

EC.1. Partial Spillover

In this extension, we consider the case in which only a fixed portion $\theta \in [0,1]$ of unsatisfied customers from one newsvendor is willing to visit the other newsvendor to assess the extent to which the qualitative results of §3-5 depend on the level of spillover demand between competing newsvendors. Note that when $\theta = 1$, this model reduces to the base model in §§3-5.

EC.1.1. Symmetric Overconfidence

Given demands (X_1, X_2) and order quantities (q_1, q_2), the expected profit for Newsvendor i is, for $i = 1, 2$,

$$\pi_i(q_1, q_2) = p\mathrm{E}[(X_i + \theta(X_{3-i} - q_{3-i})^+) \wedge q_i] - cq_i.$$

The (unbiased) equilibrium order quantities (q_1^c, q_2^c) satisfy

$$\mathrm{P}\left(X_i + \theta(X_{3-i} - q_{3-i}^c)^+ \leq q_i^c\right) = \beta, \tag{EC.1}$$

for $i = 1, 2$. Given overconfidence level α, Newsvendor i behaves as though random demands X_1 and X_2 are instead $D_1(\alpha)$ and $D_2(\alpha)$, respectively, where $D_i(\alpha) = \alpha\mu + (1-\alpha)X_i$ and $\mu = \mathrm{E}[X_1] = \mathrm{E}[X_2]$. Consequently, Newsvendor i behaves as though its game with its rival satisfies

$$\max_{q_i} \pi_i(\alpha) = p\mathrm{E}[(D_i(\alpha) + \theta(D_{3-i}(\alpha) - q_{3-i})^+) \wedge q_i] - cq_i, \tag{EC.2}$$

and

$$\max_{q_{3-i}} \pi_{3-i}(\alpha) = p\mathrm{E}[(D_{3-i}(\alpha) + \theta(D_i(\alpha) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}. \tag{EC.3}$$

As in §3, Lemma EC.1 characterizes the equilibrium solution to (EC.2)-(EC.3).

LEMMA EC.1. *The equilibrium ordering quantities* $(\hat{q}_1^c, \hat{q}_2^c)$ *for the overconfident newsvendor system (EC.2)-(EC.3) exist and are unique. Moreover, they are identical, and* $\hat{q}^c := \hat{q}_1^c = \hat{q}_2^c$ *satisfies* $P(D_1(\alpha)+\theta(D_2(\alpha)-\hat{q}^c)^+ \le \hat{q}^c) = P(D_2(\alpha)+\theta(D_1(\alpha)-\hat{q}^c)^+ \le \hat{q}^c) = \beta$.

As in §3, define $\hat{z}^c$ by $\hat{q}^c(\alpha) = \alpha\mu + (1-\alpha)\hat{z}^c$. Proposition EC.1 next characterizes $\hat{z}^c$, and shows that "pull-to-center" effect in our full spillover setting (Proposition 1) also holds in a partial spillover setting.

PROPOSITION EC.1. *The equilibrium* $\hat{z}^c$ *for the overconfident newsvendor system (EC.2)-(EC.3) is* $\hat{z}^c = g^{-1}(\beta) = q_n$. *Accordingly,* $d\hat{q}^c(\alpha)/d\alpha < 0$ *if and only if* $\beta > \hat{m}^c$ *for* $i = 1, 2$, *where* $\hat{m}^c := g(\mu) < \hat{m}$ *and* $g(y) := P[X_1 + \theta(X_2 - y)^+ \le y]$.

As in the full spillover case (Proposition 2), the impact of overconfidence on the profits of newsvendors depends on the critical value β in Proposition EC.2.

PROPOSITION EC.2. *Let* $\hat{\pi}^c(\alpha) = \hat{\pi}_1(\alpha) = \hat{\pi}_2(\alpha)$ *denote the equilibrium expected profits for the overconfident newsvendor system (EC.2)-(EC.3). Then,* $\hat{\pi}^c(\alpha)$ *is concave in* α. *Moreover: (a) if* $\beta \le \hat{m}^c$, *then* $\hat{\pi}^c(\alpha)$ *decreases in* α*; (b) if* $\hat{m}^c < \beta \le m$, *then* $\hat{\pi}^c(\alpha)$ *increases in* α, *where* $m := h(\mu)$ *and* $h(\cdot)$ *is defined as*

$$h(x) := P[X_1 + \theta(X_2 - x)^+ \le x] + \theta P[X_2 \ge x, X_1 + \theta X_2 \le (1+\theta)x]; \tag{EC.4}$$

(c) if $\beta > m$, *then,* $\hat{\pi}^c(\alpha)$ *increases in* α *for* $\alpha \in (0, \hat{\alpha}]$ *and then, decreases in* α *for* $\alpha \in [\hat{\alpha}, 1]$, *where* $\hat{\alpha} := \frac{g^{-1}(\beta) - h^{-1}(\beta)}{g^{-1}(\beta) - \mu}$.

Proposition EC.2 thus shows that overconfidence remains a potential positive force in this extension. Indeed, because $\hat{m}^c = \mathrm{P}(X_1 + \theta(X_2 - \mu)^+ \le \mu)$ decreases as θ increases, Proposition EC.2 implies that the greater is the spillover demand between competing newsvendors, the easier it is for overconfidence to yield a system-wide positive effect. Intuitively, this follows because the higher is the residual demand from its rival, the greater is the competitive force that derives a newsvendor to inflate its order quantity over the system optimal. And, the greater is that inflation, the more room is provided for the counterbalance benefit of overconfidence.

Lemma EC.2. *(a) An (unbiased) centralized planner orders less than unbiased competing newsvendors. That is,* $q_c^* < \hat{q}^c(\alpha = 0)$.

(b) Suppose $\alpha = 1$. *An (unbiased) centralized planner orders less than the overconfident competing newsvendors if* $\beta < m$ *but orders more than the competing overconfident newsvendors if* $\beta > m$. *That is,* $q_c^* < \hat{q}(\alpha = 1) \Longleftrightarrow \beta < m$.

Thus, similar to the full spillover case, overconfidence can coordinate the system.

Proposition EC.3. *If* $\beta > m$, *then competing newsvendors with overconfidence level* $\alpha = \hat{\alpha}$ *orders the same quantity that an (unbiased) central planner orders, i.e.,* $\hat{q}^c(\hat{\alpha}) = q_c^*$.

EC.1.2. Asymmetric Overconfidence

In this case with asymmetric overconfidence, Newsvendor i behaves as though its game with its rival were described by

$$\max_{q_i} \pi_i(\alpha_i) = p\mathrm{E}[(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \wedge q_i] - cq_i,$$

and

$$\max_{q_{3-i}} \pi_{3-i}(\alpha_i) = p\mathrm{E}[(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}.$$

Thus, Newsvendor i, $i = 1, 2$, solves $\mathrm{P}(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \le q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) - q_i)^+ \le q_{3-i}) = \beta$ to derive the equilibrium order quantities $(\hat{q}_1^c(\alpha_i), \hat{q}_2^c(\alpha_i))$. Accordingly, Newsvendor 1 orders $\hat{q}_1(\alpha_1)$ and anticipates Newsvendor 2 to order $\hat{q}_2(\alpha_1)$, whereas Newsvendor 2 orders $\hat{q}_2(\alpha_2)$ and anticipates Newsvendor 1 to order $\hat{q}_1(\alpha_2)$. The ensuing expected profits are

$$\hat{\pi}_i^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_i + \theta(X_{3-i} - \hat{q}_{3-i}^c(\alpha_{3-i}))^+ \wedge \hat{q}_i^c(\alpha_i)] - c\hat{q}_i^c(\alpha_i). \tag{EC.5}$$

Lemma EC.3. *Associated with the overconfidence levels* α_1 *and* α_2 *are the order quantities* $\hat{q}_i^c(\alpha_i) = \alpha_i\mu + (1 - \alpha_i)q_n$, *where* $q_n = g^{-1}(\beta)$ *as defined in Proposition EC.1.*

We next explore whether there exist asymmetric overconfidence combinations that would effectively coordinate the system. Toward that end, note that the unbiased central planner would order $2q_c^* = 2h^{-1}(\beta)$ for the system, whereas the total order quantity for the two competing newsvendors is $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\mu + (2 - \alpha_1 - \alpha_2)q_n$.

Proposition EC.4. *Suppose $\beta > m$, and that α_1 and α_2 satisfy $(\alpha_1+\alpha_2)/2 = \hat{\alpha}$, where $\hat{\alpha} = (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta)-\mu)$ as defined in Proposition EC.2. Then the sum of the newsvendors' orders equals that of a central planner: $\hat{q}_1^c(\alpha_1)+\hat{q}_2^c(\alpha_2) = 2q_c^*$.*

Proposition EC.4 not only extends Proposition EC.3 to the asymmetric bias case, but also verifies that $\hat{\alpha}$ represents the level of bias distortion that effectively counterbalances the impact of competition. Proposition EC.5 next reveals that a less biased newsvendor does not necessarily earn a higher expected profit than its more biased competitor as in Proposition 5.

Proposition EC.5. *With a partial spillover, if $\beta > \hat{m}^c$, then the less biased newsvendor earns a higher equilibrium expected profit than the more biased newsvendor, i.e., if $\beta > \hat{m}^c$, then $\alpha_1 < \alpha_2 \Longrightarrow \hat{\pi}_1^c(\alpha_1,\alpha_2) > \hat{\pi}_2^c(\alpha_1,\alpha_2)$. However, if $\beta < \hat{m}^c$, then the more biased newsvendor can earn a higher equilibrium expected profit than the less biased newsvendor.*

Recall that $\hat{m}^c$ decreases in the spillover ratio θ. Thus, Proposition EC.5 further reveals that although a smaller spillover demand makes it harder for overconfidence to have an overall positive effect (as per Proposition EC.2), it nevertheless makes it easier for the more biased newsvendor to outperform its rival.

Next, Proposition EC.6 demonstrates that this overconfidence advantage remains even if the biased newsvendor's competitor is a sophisticated newsvendor as in Proposition 6.

Proposition EC.6. *A sophisticated newsvendor can earn a lower expected profit in equilibrium than its biased competitor.*

EC.2. Inventory-Dependent Demand Rationing

In this extension, we consider the case in which the total market demand is a random variable X with a support $[0,\infty)$ and a distribution function $F(\cdot)$, and the realized demand is initially split between two firms based on their order quantities q_1 and q_2. In particular, the initial demand for Newsvendor i is $\frac{q_i}{q_1+q_2}X$, for $i = 1,2$.

We first consider the scenario in which both newsvendors are unbiased. This setting is an extension of the classic competitive newsvendor setting (Lippman and McCardle 1997). For order quantities (q_1, q_2), Newsvendor 1's effective demand is $\frac{q_1}{q_1+q_2}X + (\frac{q_2}{q_1+q_2}X - q_2)^+$, where the first term is Newsvendor 1's initial demand and the second term is the spillover demand from Newsvendor 2. It follows that Newsvendor 1's sales is

$$\left[\frac{q_1}{q_1+q_2}X + \left(\frac{q_2}{q_1+q_2}X - q_2\right)^+\right] \wedge q_1 = \begin{cases} \frac{q_1}{q_1+q_2}X & X \le q_1+q_2; \\ q_1 & X > q_1+q_2. \end{cases}$$

Similarly, we can obtain Newsvendor 2's sales for given order quantities (q_1, q_2). So, Newsvendor i's expected profit is

$$\pi_i(q_1,q_2) = -cq_i + p\int_{q_1+q_2} q_i dF(x) + p\int^{q_1+q_2} \frac{q_i}{q_1+q_2} x dF(x). \tag{EC.6}$$

It can be shown that $\pi_i(q_1,q_2)$ is strictly concave, and Nash equilibrium order quantities (q_1^c, q_2^c) exist and are unique with $q_1^c = q_2^c$. Let $q_n := q_1^c = q_2^c$ and $Q_n := 2q_n$. Then, Q_n satisfies

$$\frac{1}{2}F(Q_n) + \frac{1}{2Q_n}\int^{Q_n} F(x)dx = \beta. \tag{EC.7}$$

Next, we consider the case of overconfident newsvendors. In particular, Newsvendor i, with overconfidence level α_i, will act as if the total demand were $D(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X$. Then, $F_{D_i}(x) = F(\frac{x-\alpha_i\mu}{1-\alpha_i})$.

It follows from (EC.7) that the equilibrium order quantities $(\hat{q}_1^c, \hat{q}_2^c)$ satisfy

$$\frac{1}{2}F_{D_i}(\hat{Q}^c) + \frac{1}{2\hat{Q}^c}\int_{\alpha\mu}^{\hat{Q}^c} F_{D_i}(x)dx = \beta, \tag{EC.8}$$

where $\hat{Q}^c = 2\hat{q}_c = 2\hat{q}_1^c = 2\hat{q}_2^c$. That is, $\frac{1}{2}F(\frac{\hat{Q}^c-\alpha_i\mu}{1-\alpha_i}) + \frac{1-\alpha}{2\hat{Q}^c}\int_0^{\frac{\hat{Q}^c-\alpha_i\mu}{1-\alpha_i}} F(x)dx = \beta$. To emphasize the dependence of $\hat{Q}^c$ on α, we write $\hat{Q}^c$ as $\hat{Q}^c(\alpha)$ when necessary.

Next, we compare the two newsvendors' profits when their overconfidence levels are different $(\alpha_1 < \alpha_2)$. Thus, Newsvendor i's order quantity is $\hat{q}_i^c = \hat{Q}^c(\alpha_i)/2$, for $i = 1, 2$. So, by (EC.6), Newsvendor i's expected profit is $\hat{\pi}_i^c = -c\hat{q}_i^c + p\int_{\hat{q}_1^c+\hat{q}_2^c} \hat{q}_i^c dF(x) + p\int^{\hat{q}_1^c+\hat{q}_2^c} \frac{\hat{q}_i^c}{\hat{q}_1^c+\hat{q}_2^c} x dF(x)$. It follows that

$$\hat{\pi}_2^c - \hat{\pi}_1^c = c\hat{q}_1^c - p\int_{\hat{q}_1^c+\hat{q}_2^c} \hat{q}_1^c dF(x) - p\int^{\hat{q}_1^c+\hat{q}_2^c} \frac{\hat{q}_1^c}{\hat{q}_1^c+\hat{q}_2^c} x dF(x) - c\hat{q}_2^c + p\int_{\hat{q}_1^c+\hat{q}_2^c} \hat{q}_2^c dF(x)$$

$$
\begin{aligned}
&+p\int^{\hat{q}_1^c+\hat{q}_2^c}\frac{\hat{q}_2^c}{\hat{q}_1^c+\hat{q}_2^c}xdF(x)\\
&=-c(\hat{q}_2^c-\hat{q}_1^c)+p(\hat{q}_2^c-\hat{q}_1^c)\bar{F}(\hat{q}_1^c+\hat{q}_2^c)+\frac{p(\hat{q}_2^c-\hat{q}_1^c)}{\hat{q}_1^c+\hat{q}_2^c}\int^{\hat{q}_1^c+\hat{q}_2^c}xdF(x)\\
&=(\hat{q}_2^c-\hat{q}_1^c)\left[-c+p\bar{F}(\hat{q}_1^c+\hat{q}_2^c)+\frac{p}{\hat{q}_1^c+\hat{q}_2^c}\int^{\hat{q}_1^c+\hat{q}_2^c}xdF(x)\right]\\
&=(\hat{q}_2^c-\hat{q}_1^c)\left[-c+p\bar{F}(\hat{q}_1^c+\hat{q}_2^c)+\frac{p}{\hat{q}_1^c+\hat{q}_2^c}xF(x)|_0^{\hat{q}_1^c+\hat{q}_2^c}-\frac{p}{\hat{q}_1^c+\hat{q}_2^c}\int^{\hat{q}_1^c+\hat{q}_2^c}F(x)dx\right]\\
&=p\frac{\hat{q}_2^c-\hat{q}_1^c}{\hat{q}_1^c+\hat{q}_2^c}\left(\beta(\hat{q}_1^c+\hat{q}_2^c)-\int^{\hat{q}_1^c+\hat{q}_2^c}F(x)dx\right). \qquad \text{(EC.9)}
\end{aligned}
$$

In order to develop analytical insights, we assume that $X\sim U[0,1]$ in the remainder of §EC.2.

Hence, (EC.9) specializes to

$$
\hat{\pi}_2^c-\hat{\pi}_1^c=\begin{cases} p(\hat{q}_2^c-\hat{q}_1^c)(\beta-\frac{\hat{q}_1^c+\hat{q}_2^c}{2}) & \hat{q}_1^c+\hat{q}_2^c\le 1;\\ p\frac{\hat{q}_2^c-\hat{q}_1^c}{\hat{q}_1^c+\hat{q}_2^c}(\beta(\hat{q}_1^c+\hat{q}_2^c)-\frac{1}{2}) & \hat{q}_1^c+\hat{q}_2^c>1.\end{cases} \qquad \text{(EC.10)}
$$

Lemma EC.4 derives a closed-form solution for $\hat{q}_i^c(\alpha_i)$, the equilibrium order quantity of Newsvendor i with the overconfidence level α_i.

LEMMA EC.4. *Suppose $X\sim U[0,1]$. Then,*

$$
\hat{q}_i^c(\alpha_i)=\begin{cases}\frac{1}{6}\alpha_i+\frac{1}{3}\beta-\frac{1}{3}\alpha_i\beta+\frac{1}{6}h(\alpha_i,\beta) & \beta\le\frac{3-2\alpha_i}{2(2-\alpha_i)};\\ \frac{1}{8(1-\beta)} & \beta>\frac{3-2\alpha_i}{2(2-\alpha_i)},\end{cases}
$$

where

$$
h(\alpha_i,\beta)=\sqrt{4\alpha_i^2\beta^2-4\alpha_i^2\beta+\frac{1}{4}\alpha_i^2-8\alpha_i\beta^2+4\alpha_i\beta+4\beta^2}.
$$

EC.2.1. Symmetric Overconfidence

Assume $\alpha_1=a_2=\alpha$. Then, from Lemma EC.4, the equilibrium order quantity is

$$
\hat{q}^c(\alpha)=\hat{q}_i^c(\alpha)=\begin{cases}\frac{1}{6}\alpha+\frac{1}{3}\beta-\frac{1}{3}\alpha\beta+\frac{1}{6}h(\alpha,\beta) & \beta\le\frac{3-2\alpha}{2(2-\alpha)};\\ \frac{1}{8(1-\beta)} & \beta>\frac{3-2\alpha}{2(2-\alpha)}.\end{cases}
$$

And the equilibrium expected profit for each newsvendor is

$$
\hat{\pi}^c=\hat{\pi}_i^c=\begin{cases}-c\hat{q}^c+p\hat{q}^c(1-\hat{q}^c) & 2\hat{q}^c\le 1;\\ -c\hat{q}^c+\frac{1}{4}p & 2\hat{q}^c>1.\end{cases}
$$

Define β_c as the unique solution to the cubic equation $-16(1-\alpha)^2\beta^3+16(2\alpha-1)(\alpha-1)\beta^2+(16\alpha-17\alpha^2-4)\beta+\alpha^2=0$ in the interval $[\frac{1-2\alpha+\sqrt{3\alpha^2+1-3\alpha}}{4(1-\alpha)},\frac{1}{2}]$. Note that the existence and uniqueness of β_c are established in the proof of Proposition EC.7.

Proposition EC.7. *$\frac{\partial \hat{q}^c}{\partial \alpha} \geq 0$ if $\beta \leq \beta_c$, $\frac{\partial \hat{q}^c}{\partial \alpha} \leq 0$ if $\beta_c < \beta \leq \frac{3-2\alpha}{2(2-\alpha)}$, and $\frac{\partial \hat{q}^c}{\partial \alpha} = 0$ for $\beta > \frac{3-2\alpha}{2(2-\alpha)}$.*

Proposition EC.8 is the counterpart of Proposition 2 in §3. It characterizes how the overconfidence level affects a firm's expected profit at equilibrium, and demonstrates that overconfidence can benefit the newsvendor if $\beta > \beta_c$.

Proposition EC.8. *$\frac{\partial \hat{\pi}^c}{\partial \alpha} \leq 0$ if $\beta \leq \beta_c$; $\frac{\partial \hat{\pi}^c}{\partial \alpha} \geq 0$ if $\beta_c < \beta \leq \frac{3-2\alpha}{4-2\alpha}$; $\frac{\partial \hat{\pi}^c}{\partial \alpha} = 0$ if $\beta > \frac{3-2\alpha}{4-2\alpha}$.*

In this inventory-dependent demand rationing case, the demands for the two competing newsvendors are positively correlated. Yet, if a similar scenario of perfectly correlated demands were applied in §3, then overconfidence would never be beneficial because, in that case, $\hat{m}^c = m$ and $\hat{\alpha} = 0$ from Proposition 2. Thus, similar to the effects and intuition associated with spillover demand in the first extension, demand-rationing in this extension essentially enhances the beneficial effects of overconfidence from §3. Note, however, from the proof of Proposition EC.8, that the sum of the order quantities of the two overconfident newsvendors is strictly greater than the optimal total order quantity for the two products in the centralized system. Thus, the two overconfident newsvendors cannot achieve the first-best outcome when $X \sim U[0,1]$. We attribute this technicality to the linearity of the uniform distribution.

EC.2.2. Asymmetric Overconfidence

Proposition EC.9 is the counterpart of Proposition 5 in §4, and shows that the more overconfident newsvendor can earn a higher expected profit than the less overconfident one.

Proposition EC.9. *Suppose $X \sim U[0,1]$ and $\alpha_1 = 0$. Then, (i) for $\beta \in [0, 3/16]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ if and only if $\alpha_2 \leq \frac{8}{3}\beta\left(4 - 8\beta - \sqrt{2}\sqrt{32\beta^2 - 32\beta + 5}\right)$; (ii) for $\beta \in (3/16, 3/8]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ for all α_2; (iii) for $\beta \in (3/8, 1/2]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ if and only if $\alpha_2 \leq 4/3 - (8\beta - 2)^2/3$; (iv) for $\beta \in (1/2, 3/4]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ if and only if $\alpha_2 \geq \frac{1}{2\beta - 2}(4\beta - 3)$; (v) for $\beta > 3/4$. $\hat{\pi}_2^c - \hat{\pi}_1^c = 0$ for all α_2.*

The intuition behind Proposition EC.9 is similar to that behind Proposition 5: The more overconfident newsvendor can earn a higher expected profit than the less overconfident one because the former may uncannily place a more proper order than its rival. Geometrically, the shape of $\hat{q}_2^c$ as a function of α_2 depends on the value of β. When the critical ratio β is low ($\beta \leq 1/2$), $\hat{q}_2^c$ increases in α_2 when α_2 is low; consequently, $\hat{q}_2^c(\alpha_2) \geq \hat{q}_1^c(0)$ for low α_2. At the same time, both newsvendors tend to order less when β is small. These two factors imply that the more overconfident newsvendor (Newsvendor 2) earns a higher expected profit than the less overconfident one when the former's overconfidence level is low. When the critical ratio β is medium ($1/2 < \beta \leq 3/4$), $\hat{q}_2^c$ decreases in α_2. Consequently, the overconfident newsvendor earns a higher expected profit than its rival when its overconfident level is high. When the critical ratio β is high ($\beta > 3/4$), both newsvendors earn the same expected profit because they order the same quantity regardless of the more biased newsvendor's overconfidence level.

Next, we show that an overconfident newsvendor's profit even can be greater than that of a sophisticated rival. Toward that end, we study a case where Newsvendor 1 is unbiased ($\alpha_1 = 0$), and is fully cognizant of Newsvendor 2's overconfidence level α_2. By (EC.6), for $(q_1, \hat{q}_2^c)$, Newsvendor 1's expected profit is

$$\begin{aligned}\pi_1(q_1, \hat{q}_2^c) &= -cq_1 + p\int_{q_1+\hat{q}_2^c} q_1 dx + p\int^{q_1+\hat{q}_2^c} \frac{q_1}{q_1+\hat{q}_2^c} x dx \\ &= \begin{cases} (p-c)q_1 - p\frac{q_1}{2}(q_1+\hat{q}_2^c) & q_1+\hat{q}_2^c \leq 1; \\ \frac{1}{2}p\frac{q_1}{q_1+\hat{q}_2^c} - cq_1 & q_1+\hat{q}_2^c > 1. \end{cases}\end{aligned} \tag{EC.11}$$

Lemma EC.5. *Assume $X \sim U[0,1]$. Then, Newsvendor 1's equilibrium order quantity satisfies:*

$$\hat{q}_1^{fc} = \begin{cases} 0 & \beta \leq \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ \beta - \frac{1}{2}\hat{q}_2^c & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \leq \frac{3}{4}; \\ \frac{1}{2-2\beta}\left(\sqrt{2(1-\beta)\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right) & \beta > \frac{3}{4}. \end{cases} \tag{EC.12}$$

With (EC.11)-(EC.12), we have the two newsvendors' expected profits:

$$\hat{\pi}_1^{fc} = \begin{cases} 0 & \beta \leq \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ (p-c)\hat{q}_1^{fc} - p\frac{\hat{q}_1^{fc}}{2}(\hat{q}_1^{fc}+\hat{q}_2^c) & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \leq \frac{3}{4}; \\ -c\hat{q}_1^{fc} + \frac{1}{2}p\frac{\hat{q}_1^{fc}}{q_1+\hat{q}_2^c} & \beta > \frac{3}{4}. \end{cases}$$

$$\hat{\pi}_2^{fc} = \begin{cases} \hat{q}_2^c(p-c-p\hat{q}_2^c/2) & \beta \leq \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ (p-c)\hat{q}_2^c - p\frac{\hat{q}_2^c}{2}(\hat{q}_1^{fc}+\hat{q}_2^c) & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \leq \frac{3}{4}; \\ -c\hat{q}_2^c + \frac{1}{2}p\frac{\hat{q}_2^c}{\hat{q}_1^{fc}+\hat{q}_2^c} & \beta > \frac{3}{4}. \end{cases}$$

It follows that

$$\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \begin{cases} p\hat{q}_2^c(\beta - \hat{q}_2^c/2) & \beta \le \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ p(\hat{q}_2^c - \hat{q}_1^{fc})(\beta - \frac{1}{2}(\hat{q}_1^{fc} + \hat{q}_2^c)) & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \le \frac{3}{4}; \\ p(\hat{q}_2^c - \hat{q}_1^{fc})(1 - \beta + \frac{1}{2}\frac{1}{\hat{q}_1^{fc}+\hat{q}_2^c}) & \beta > \frac{3}{4}. \end{cases} \tag{EC.13}$$

As a result, analogous to Proposition 6 in §5, Proposition EC.10 shows that the even a sophisticated newsvendor can earn a lower expected profit in equilibrium than its biased rival.

PROPOSITION EC.10. *Suppose $X \sim U[0,1]$ and Newsvendor 1 is sophisticated. Then,*

$$\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \begin{cases} \le 0 & 0 < \beta \le \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ \ge 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \le \frac{3}{4}; \\ = 0 & \frac{3}{4} < \beta \le 1. \end{cases}$$

EC.3. Random Demand Splitting

Next, we consider a setting in which the market demand Y is split randomly between the two newsvendors. Specifically, Newsvendor i's demand is $X_i := r_i Y$, for $i = 1, 2$, where r_i is random with a support $[0,1]$ and $r_1 + r_2 = 1$ almost surely. Newsvendors are overconfident, and Newsvendor i behaves as though the demand is $D(\alpha_i) = \alpha_i \mu_i + (1-\alpha_i)X_i$, where $\mu_i = \mathrm{E}[r_i Y]$.

EC.3.1. Symmetric Overconfidence

As in §3, assume that newsvendors are identical in their overconfidence biases, i.e., $\alpha_1 = \alpha_2 = \alpha$. Then, Newsvendor i behaves as though its game with its rival were described by $\max_{q_i} \pi_i(\alpha) = p\mathrm{E}[(D_i(\alpha) + (D_{3-i}(\alpha) - q_{3-i})^+) \wedge q_i] - cq_i$, and $\max_{q_{3-i}} \pi_{3-i}(\alpha) = p\mathrm{E}[(D_{3-i}(\alpha) + (D_i(\alpha) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}$, where $D_i(\alpha) = \alpha\mu_i + (1-\alpha)X_i$. Thus, the equilibrium order quantities satisfy $\mathrm{P}(D_1(\alpha) + (D_2(\alpha) - \hat{q}_2^c)^+ \le \hat{q}_1^c) = \mathrm{P}(D_2(\alpha) + (D_1(\alpha) - \hat{q}_1^c)^+ \le \hat{q}_2^c) = \beta$. Accordingly, define $\hat{z}_i^c$ such that $\hat{q}_i^c(\alpha) = \alpha\mu_i + (1-\alpha)\hat{z}_i^c$. Then, $\mathrm{P}(X_1 + (X_2 - \hat{z}_2^c)^+ \le \hat{z}_1^c) = \mathrm{P}(X_2 + (X_1 - \hat{z}_1^c)^+ \le \hat{z}_2^c) = \beta$.

Note that $\hat{z}_1^c$ and $\hat{z}_2^c$ both are independent of α. Thus, $dq_i^c(\alpha)/d\alpha < 0$ if and only if $\hat{z}_i^c > \mu$. Moreover, note that $\hat{z}_i^c$ is Newsvendor i's order quantity when $\alpha = 0$. Thus, the pull-to-center effect (Proposition 1) holds, however, unlike in Proposition 1, we cannot provide a closed-form characterization of the understocking vs. overstocking condition resulting from overconfidence.

Next, note that the equilibrium system profit is $\hat{\pi}_1^c(\alpha) + \hat{\pi}_2^c(\alpha) = 2\mu(p-c) - (p-c)\mathrm{E}[X_1 + X_2 - \hat{q}_1^c(\alpha) - \hat{q}_2^c(\alpha)]^+ - c\mathrm{E}[\hat{q}_1^c(\alpha) + \hat{q}_2^c(\alpha) - X_1 - X_2]^+$. Thus,

$$\frac{d[\hat{\pi}_1^c(\alpha) + \hat{\pi}_2^c(\alpha)]}{d\alpha} = 2(\bar{z} - \mu)[(c-p)\bar{F}_{\bar{X}}((1-\alpha)\bar{z} + \alpha\mu) + cF_{\bar{X}}((1-\alpha)\bar{z} + \alpha\mu)]$$

$$= 2(\bar{z}-\mu)[(c-p)+pF_{\bar{X}}((1-\alpha)\bar{z}+\alpha\mu)]$$

$$= 2p(\bar{z}-\mu)[-\beta+F_{\bar{X}}((1-\alpha)\bar{z}+\alpha\mu)],$$

where $\bar{X} = Y/2$ and $\bar{z} = \frac{\hat{z}_1^c+\hat{z}_2^c}{2}$. As a result, $\frac{d^2[\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)]}{d\alpha^2} = -p(\bar{z}-\mu)^2 f_{\bar{X}}((1-\alpha)\bar{z}+\alpha\mu) < 0$, which indicates that the system equilibrium profit $\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)$ is concave in α. Moreover, $\frac{d[\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)]}{d\alpha}|_{\alpha=1} = p(\beta-m)(\mu-\bar{z}) > 0$ if $\bar{z} < \mu$. Therefore, if $\bar{z} < \mu$, then $\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)$ is an increasing function because of its concavity, which implies that the overall system can benefit from overconfidence (Proposition 2).

We next check whether overconfidence can lead to the first-best equilibrium (Proposition 3). Note that the central planner solves

$$\max_{q_1,q_2} p\mathrm{E}[(X_1+(X_2-q_2)^+)\wedge q_1]-cq_1+p\mathrm{E}[(X_2+(X_1-q_1)^+)\wedge q_2]-cq_2.$$

Consequently, if $F_{\bar{X}}(\gamma\bar{z}+(1-\gamma)\mu) = \beta$, then overconfidence can yield a first-best equilibrium (Proposition 3).

EC.3.2. Asymmetric Overconfidence

We next examine whether the more biased newsvendor can earn a higher expected profit than its less biased rival. For this purpose, we assume r_1 and r_2 are identically distributed, which guarantees that newsvendors have identical expected profits when they are symmetric on overconfidence levels. Furthermore, Newsvendor i behaves as though its game with its rival were described by $\max_{q_i}\pi_i(\alpha_i) = p\mathrm{E}[(D_i(\alpha_i)+(D_{3-i}(\alpha_i)-q_{3-i})^+)\wedge q_i]-cq_i$, and $\max_{q_{3-i}}\pi_{3-i}(\alpha_i) = p\mathrm{E}[(D_{3-i}(\alpha_i)+(D_i(\alpha_i)-q_i)^+)\wedge q_{3-i}]-cq_{3-i}$. As in §4, the stocking factors are identical, i.e., $\hat{z}^c := \hat{z}_1^c = \hat{z}_2^c$.

Proposition EC.11 shows that the less biased newsvendor can earn a lower expected profit than its rival, and this result remains true even if the less biased newsvendor is sophisticated.

Proposition EC.11. *Let $\hat{m}^c = P(X_1+(X_2-\bar{\mu})^+ \le \bar{\mu})$ with $\bar{\mu} = E[r_iY]$ for $i = 1,2$. If $\beta < \hat{m}^c$, then the more biased newsvendor can earn a higher expected profit than the less biased newsvendor with a random spliting. Furthermore, even a sophisticated newsvendor can earn a lower expected profit in equilibrium than its biased competitor.*

EC.4. Expanded Definition of Overconfidence

In this section, we extend our analysis to the case in which a newsvendor with demand X behaves as though the demand were $D(\alpha)=\alpha\rho+(1-\alpha)X$, where $\rho\geq 0$. Note that when $\rho=\mu$, this model reduces to the base model in §§3-5.

EC.4.1. Symmetric Overconfidence

Newsvendor i behaves as though its game with its rival satisfies

$$\max_{q_i}\pi_i(\alpha)=p\mathrm{E}[(D_i(\alpha)+(D_{3-i}(\alpha)-q_{3-i})^+)\wedge q_i]-cq_i, \tag{EC.14}$$

and

$$\max_{q_{3-i}}\pi_{3-i}(\alpha)=p\mathrm{E}[(D_{3-i}(\alpha)+(D_i(\alpha)-q_i)^+)\wedge q_{3-i}]-cq_{3-i}, \tag{EC.15}$$

where $D(\alpha_i)=\alpha_i\rho+(1-\alpha_i)X$ for $i=1,2$. The equilibrium ordering quantities $(\hat{q}_1^c,\hat{q}_2^c)$ for the overconfident newsvendor system (EC.14)-(EC.15) exist and are unique. Moreover, they are identical, and $\hat{q}^c:=\hat{q}_1^c=\hat{q}_2^c$ satisfies $\mathrm{P}(D_1(\alpha)+(D_2(\alpha)-\hat{q}^c)^+\leq\hat{q}^c)=\mathrm{P}(D_2(\alpha)+(D_1(\alpha)-\hat{q}^c)^+\leq\hat{q}^c)=\beta$. Note that the equilibrium $\hat{z}^c$ for the overconfident newsvendor system (EC.14)-(EC.15) is $\hat{z}^c=g^{-1}(\beta)=q_n$. Accordingly, $d\hat{q}^c(\alpha)/d\alpha<0$ if and only if $\beta>\hat{m}^c$ for $i=1,2$, where $\hat{m}^c:=g(\rho)$. Note that $\hat{m}^c$ is different from the one in §3, which is defined as $g(\mu)$.

Proposition EC.12. *Let $\hat{\pi}^c(\alpha)=\hat{\pi}_1(\alpha)=\hat{\pi}_2(\alpha)$ denote the equilibrium expected profits for the overconfident newsvendors. Then, $\hat{\pi}^c(\alpha)$ is concave in α. Moreover: (a) if $\beta\leq\hat{m}^c$, then $\hat{\pi}^c(\alpha)$ decreases in α; (b) if $\hat{m}^c<\beta<m$, then $\hat{\pi}^c(\alpha)$ increases in α, where $m:=F_{\bar{X}}(\rho)$ and $\bar{X}=\frac{X_1+X_2}{2}$; (c) if $m\leq\beta$, then, $\hat{\pi}^c(\alpha)$ increases in α for $\alpha\in(0,\hat{\alpha}]$ and then decreases in α for $\alpha\in[\hat{\alpha},1]$, where $\hat{\alpha}:=(g^{-1}(\beta)-F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta)-\rho)$.*

Proposition EC.12 thus expands Proposition 2 to the more general definition of overconfidence considered here. To further illustrate its applicability, suppose $\rho=q^*$. Then, $\hat{m}^c=g(q^*)<\beta=F_X(q^*)$, and $\beta<m\Longleftrightarrow F_X(q^*)<F_{\bar{X}}(q^*)$. Consequently, the condition for case (b) of Proposition EC.12 boils down to $F_X(q^*)<F_{\bar{X}}(q^*)$, which is equivalent to $q^*>0.5\Longleftrightarrow\beta>0.5$ when

$X \sim U[0,1]$. Thus, when $\beta > 0.5$ and $X \sim U[0,1]$, the newsvendors' expected profits are increasing with respect to α.

Proposition EC.13. *If $\beta > m$, then competing newsvendors with overconfidence level $\alpha = \hat{\alpha}$ order the same quantity that an (unbiased) central planner orders, i.e., $\hat{q}^c(\hat{\alpha}) = q_c^*$.*

Again, suppose $\rho = q^*$. In the coordinated system, the order quantity of the system is $2F_{\bar{X}}^{-1}(\beta)$. The system order quantity for overconfident and competitive newsvendors is $2[\alpha F_X^{-1}(\beta) + (1-\alpha)\hat{z}^c]$, where $g(\hat{z}^c) = \beta$. Note that when $X_i \sim U[0,1]$ for $i = 1,2$ and X_1 and X_2 are independent, $\alpha F_X^{-1}(\beta) + (1-\alpha)\hat{z}^c = \alpha\beta + (1-\alpha) = F_{\bar{X}}^{-1}(\beta)$ holds if $\alpha = \frac{2\sqrt{3}-3}{\sqrt{2}(\sqrt{6}-3\sqrt{\beta})}$ and $\beta \in [0, 1/2]$. Thus, overconfidence has the potential to coordinate the system when $\rho = q^*$.

EC.4.2. Asymmetric Overconfidence

We now study the case with asymmetric overconfidence levels. Analogous to (EC.14)-(EC.15), Newsvendor i behaves as though its game with its rival were described by $\max_{q_i} \pi_i(\alpha_i) = p\mathrm{E}[(D_i(\alpha_i) + (D_{3-i}(\alpha_i) - q_{3-i})^+) \wedge q_i] - cq_i$, and $\max_{q_{3-i}} \pi_{3-i}(\alpha_i) = p\mathrm{E}[(D_{3-i}(\alpha_i) + (D_i(\alpha_i) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}$.

Proposition EC.14. *If $\beta < \hat{m}^c$, then the more biased newsvendor can earn a higher expected profit than the less biased newsvendor. Furthermore, the more biased newsvendor even can earn a higher expected profit than a sophisticated rival.*

Proposition EC.14 reveals that a less biased newsvendor does not necessarily earn a higher expected profit than its more biased rival when $\rho > g^{-1}(\beta)$. This essentially means that the more biased newsvendor is more prone to earn a higher profit than the less biased one when the newsvendor behaves as though the actual demand is stochastically larger than it really is as is the case if $\rho > \mu$.

Appendix: Proofs for Electronic Companion To "Overconfident Competing Newsvendors"

Proof of Lemma EC.1 Following Lippman and McCardle (1997), the equilibrium ordering quantities $(\hat{q}_1^c, \hat{q}_2^c)$ exist and are unique. Moreover, from (3), they are identical because $\mathrm{P}(D_1(\alpha) +$

$\theta(D_2(\alpha)-\hat{q}^c)^+ < \hat{q}^c) = \mathrm{P}(D_2(\alpha)+\theta(D_1(\alpha)-\hat{q}^c)^+ < \hat{q}^c) = \beta$, where $\hat{q}^c = \hat{q}_1^c = \hat{q}_2^c$. Q.E.D.

Proof of Proposition EC.1 By the definition of $\hat{q}^c(\alpha)$, $\mathrm{P}[D_1(\alpha)+\theta(D_2(\alpha)-\hat{q}^c(\alpha))^+ \le \hat{q}^c(\alpha)] = \beta$. Note that $D_i(\alpha) = \alpha\mu + (1-\alpha)X_i$. So, $g(\hat{z}^c) = \mathrm{P}[X_1+\theta(X_2-\hat{z}^c)^+ \le \hat{z}^c] = \beta$. From the definition of q_n and the monotonicity of $g(\cdot)$, we know that $\hat{z}^c = g^{-1}(\beta) = q_n$. A key observation here is that q_n is independent of α. Thus, $d\hat{q}^c(\alpha)/d\alpha < 0$ if and only if $q_n > \mu$, which is identical to $\beta > \hat{m}^c = g(\mu)$ for $i = 1, 2$. Moreover, $\hat{m}^c < \hat{m}$ because $g(\mu) = \mathrm{P}[X_1+\theta(X_2-\mu)^+ \le \mu] < F_X(\mu) = \hat{m}$. Q.E.D.

Proof of Proposition EC.2 Because the equilibrium profits of the two newsvendors are identical, we have both newsvendors' expected equilibrium profits as $\hat{\pi}^c(\alpha) = -c\hat{q}^c(\alpha) + p\mathrm{E}[\hat{q}^c(\alpha)\wedge(X_1 + \theta(X_2-\hat{q}^c(\alpha))^+]$. We first show that $\hat{\pi}^c$ is concave in $\hat{q}^c$. Using the fact that

$$\frac{\partial \mathrm{E}[\hat{q}^c\wedge(X_1+\theta(X_2-\hat{q}^c)^+]}{\partial \hat{q}^c} = \mathrm{E}\left[\frac{\partial[\hat{q}^c\wedge(X_1+\theta(X_2-\hat{q}^c)^+]}{\partial \hat{q}^c}\right],$$

we obtain

$$\begin{aligned}
\frac{\partial \hat{\pi}^c}{\partial \hat{q}^c} &= p\int_{\hat{q}^c}\int_{\hat{q}^c-\theta(X_2-\hat{q}^c)} f(x_1,x_2)dx_1dx_2 + p\int_{\hat{q}^c}\int^{\hat{q}^c} f(x_1,x_2)dx_2dx_1 \\
&\quad - p\theta\int_{\hat{q}^c}\int^{\hat{q}^c-\theta(X_2-\hat{q}^c)} f(x_1,x_2)dx_1dx_2 - c \\
&= p - c - p\mathrm{P}[X_1+\theta(X_2-\hat{q}^c)^+ \le \hat{q}^c] - p\theta\mathrm{P}[X_2 \ge \hat{q}^c, X_1+\theta X_2 \le (1+\theta)\hat{q}^c] \\
&= p - c - ph(\hat{q}^c), \qquad \text{(EC.16)}
\end{aligned}$$

where the second equality is from $\int_{\hat{q}^c}\int_{\hat{q}^c-\theta(X_2-\hat{q}^c)} f(x_1,x_2)dx_1dx_2 + \int_{\hat{q}^c}\int^{\hat{q}^c} f(x_1,x_2)dx_2dx_1 = 1 - \mathrm{P}[X_1+\theta(X_2-\hat{q}^c)^+ \le \hat{q}^c]$. Note that $h(\cdot)$ increases in $\hat{q}^c$. Consequently, $\partial\hat{\pi}^c/\partial\hat{q}^c$ decreases in $\hat{q}^c$. Therefore, $\hat{\pi}^c$ is concave in $\hat{q}^c$. It follows that $\hat{\pi}^c(\alpha)$ is concave in α because $\hat{q}^c = \alpha\mu + (1-\alpha)\hat{z}^c$ is linear in α. Accordingly, there are three cases to consider.

Case (a): $\beta \le \hat{m}^c$. In this case, we evaluate $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=0}$. For $\alpha = 0$, $\hat{q}^c = \hat{z}^c$. So,

$$\begin{aligned}
\left.\frac{\partial \hat{\pi}^c}{\partial \hat{q}^c}\right|_{\alpha=0} &= p - c - p\mathrm{P}[X_1+\theta(X_2-\hat{z}^c)^+ \le \hat{z}^c] - p\theta\int_{\hat{z}^c}\int^{\hat{z}^c-\theta(X_2-\hat{z}^c)} f(x_1,x_2)dx_1dx_2 \\
&= p(\beta-\hat{m}^c) - p\theta\int_{\hat{z}^c}\int^{\hat{z}^c-\theta(X_2-\hat{z}^c)} f(x_1,x_2)dx_1dx_2 \\
&\le 0.
\end{aligned}$$

It follows that $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=0} = \left.\left[\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c} \times \frac{\partial\hat{q}^c}{\partial\alpha}\right]\right|_{\alpha=0} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=0} \times (\mu - \hat{z}^c) \leq 0$ because $\hat{z}^c \leq \mu \Longleftrightarrow \beta \leq \hat{m}^c$. Therefore, if $\beta \leq \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ is a decreasing function because of its concavity.

Case (b): $\hat{m}^c < \beta \leq m$. In this case, we evaluate $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=1}$. For $\alpha = 1$, $\hat{q}^c = \mu$. So,

$$\left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1} = p - c - p\mathrm{P}[X_1 + \theta(X_2-\mu)^+ \leq \mu] - p\theta\mathrm{P}[X_1 + \theta(X_2-\mu)^+ \leq \mu, X_1 \geq \mu]$$
$$= p\,(\beta - m) \leq 0.$$

Note that in this case, $\mu < \hat{z}^c$ because $\hat{m}^c < \beta$. Then, $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=1} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1}(\mu - \hat{z}^c) > 0$. Therefore, if $\hat{m}^c < \beta \leq m$, then $\hat{\pi}^c(\alpha)$ is an increasing function because of its concavity.

Case (c): $\beta > m$. In this case, $\left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=0} = p - c - ph(\hat{z}^c) = p[\beta - h(\hat{z}^c)] < 0$ because $g(\hat{z}^c) = \beta$ and $g(\cdot) \leq h(\cdot)$. As a result, $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=0}(\mu - \hat{z}^c) > 0$. Furthermore, $\left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1} = p(\beta - m) > 0$, which implies $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=1} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1}(\mu - \hat{z}^c) < 0$. Therefore, if $\beta > m$, then $\hat{\pi}^c(\alpha)$ is an increasing-decreasing function. To ensure $\frac{d\hat{\pi}^c(\alpha)}{d\alpha} = 0$, we must have $\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c} = 0$. Using (EC.16), we have $\hat{q}^c = h^{-1}(\beta)$, that is $\alpha\mu + (1-\alpha)\hat{z}^c = h^{-1}(\beta)$. Since $g(\hat{z}^c) = \beta$, we can conclude that $\frac{d\hat{\pi}^c(\alpha)}{d\alpha} = 0$ holds when $\alpha = \hat{\alpha} = (g^{-1}(\beta) - h^{-1}(\beta))/(g^{-1}(\beta) - \mu)$. Q.E.D.

Proof of Lemma EC.2 (a) In the competition case, the equilibrium order quantities $q_n = \hat{q}^c(\alpha = 0)$ satisfy (3). In this case, the order quantity q_n is higher than the order quantity q_c^* in the centralized system because q_c^* satisfies $h(q_c^*) = \beta$, q_n satisfies $g(q_n) = \beta$, and $h(y) \geq g(y)$ for all y.

(b) In the competition case, the infinitely overconfident newsvendors order μ, i.e., $\hat{q}(\alpha = 1) = \mu$. In the centralized system $h(q_c^*) = \beta$. Thus, $q_c^* < \hat{q}(\alpha = 1) = \mu$ if and only if $\beta < m$, where $m = h(\mu)$, as defined in (EC.4). Q.E.D.

Proof of Proposition EC.3 If $\beta > m$, then $h(\alpha\mu + (1-\alpha)\hat{z}^c) = \beta$ when $\alpha = \hat{\alpha}$ from Proposition EC.2. Thus, when $\alpha = \hat{\alpha}$, the competing newsvendors would order the same quantity that the central planner would order. Q.E.D.

Proof of Lemma EC.3 Because $D_1(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X_1$, $D_2(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X_2$ and the equilibrium solution satisfies $\mathrm{P}(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \leq q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) - q_i)^+ \leq q_{3-i}) = \beta$, Newsvendor i solves $\mathrm{P}(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \leq q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) -$

$q_i)^+ \leq q_{3-i}) = \beta$ to obtain the equilibrium order quantity $\hat{q}_i^c(\alpha_i) = \alpha_i\mu + (1-\alpha_i)\hat{z}_i$. Thus, $\mathrm{P}(X_i + \theta(X_{3-i} - \hat{z}_{3-i})^+) \leq \hat{z}_i) = \mathrm{P}(X_{3-i} + \theta(X_i - \hat{z}_i)^+ \leq \hat{z}_{3-i}) = \beta$. As a result, from the proof of Proposition EC.1, $\hat{z}_1 = \hat{z}_2 = q_n$ and $g(q_n) = \beta$. Q.E.D.

Proof of Proposition EC.4 For the overconfident newsvendors, the order quantities satisfy $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\mu + (2 - \alpha_1 - \alpha_2)q_n$. In contrast, the central planner's equilibrium order quantity is $2h^{-1}(\beta)$. Thus, if $\beta > m$, then the system can be coordinated as in Proposition EC.2. In particular, $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = 2q_c^*$ when $\alpha_1 + \alpha_2 = 2\hat{\alpha}$. Q.E.D.

Proof of Proposition EC.5 To prove Proposition EC.5, we first show the following two-part lemma. Lemma EC.5(a) Newsvendor i's equilibrium expected profit is decreasing in α_i (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_i} \leq 0$, for $i = 1, 2$), and Lemma EC.5(b) Newsvendor i's equilibrium expected profit is decreasing in α_{3-i} (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_{3-i}} < 0$ for $i = 1, 2$) if and only if $\beta < \hat{m}^c$. Toward that end, first note from (EC.5) and Lemma EC.3 that Newsvendor 1's profit is $\hat{\pi}_1^c(\alpha_1,\alpha_2) = p\mathrm{E}[(X_1 + \theta(X_2 - \hat{q}_2^c)^+) \wedge \hat{q}_1^c] - c\hat{q}_1^c$. Thus, $\frac{\partial \hat{\pi}_1^c}{\partial \hat{q}_1^c} = p\mathrm{P}[(X_1 + \theta(X_2 - \hat{q}_2^c)^+) \geq \hat{q}_1^c] - c$ and $\frac{\partial \hat{\pi}_1^c}{\partial \hat{q}_2^c} = -p\theta\mathrm{P}[X_1 + \theta(X_2 - \hat{q}_2^c) \leq \hat{q}_1^c, X_2 \geq \hat{q}_2^c]$. And, for $i = 1, 2$, $\frac{\partial \hat{q}_i^c}{\partial \alpha_i} = \mu - q_n$ and $\frac{\partial \hat{q}_i^c}{\partial \alpha_{3-i}} = 0$. Therefore,

$$\begin{aligned}\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} &= (\mu - q_n)p[\mathrm{P}(\alpha_1\mu + (1-\alpha_1)q_n < X_1 + \theta(X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+) - (1-\beta)] \\ &= (\mu - q_n)p[\beta - \mathrm{P}(\alpha_1\mu + (1-\alpha_1)q_n > X_1 + \theta(X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+)]. \qquad \text{(EC.17)}\end{aligned}$$

Accordingly:

Proof of Lemma EC.5(a): Without loss of generality, we let $i = 1$. First, if $q_n > \mu$, then $q_n > \hat{q}_i^c = \alpha_i\mu + (1-\alpha_i)q_n$, for $i = 1, 2$. So, $\mathrm{P}[X_1 + \theta(X_2 - \hat{q}_2^c)^+ \geq \hat{q}_1^c] < \mathrm{P}[X_1 + \theta(X_2 - q_n)^+ \geq q_n] = \beta$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} < 0$ if $q_n > \mu$. Second, if $q_n \leq \mu$, then $q_n \leq \hat{q}_i^c$, for $i = 1, 2$. So, $\mathrm{P}[X_1 + \theta(X_2 - \hat{q}_2^c)^+ \geq \hat{q}_1^c] \geq \mathrm{P}[X_1 + \theta(X_2 - q_n)^+ \geq q_n] = \beta$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} \leq 0$ if $q_n \leq \mu$. This completes the proof of Lemma EC.5(a).

Proof of Lemma EC.5(b) Without loss of generality, we let $i = 1$. Because $\frac{\partial \hat{\pi}_1^c}{\partial \hat{q}_2^c} < 0$, the sign of $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2}$ is determined by $q_n - \mu$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0 \Longleftrightarrow \mu > q_n$. Furthermore, from Proposition EC.1, $\mu > q_n \Longleftrightarrow \hat{m}^c > \beta$. This completes the proof of Lemma EC.5(b).

Note that if $\alpha_1 = \alpha_2$, then the newsvendors have identical expected profits. If $\alpha_1 < \alpha_2$ and $\beta > \hat{m}^c$, then Lemma EC.5(a) and Lemma EC.5(b) together imply that the less biased newsvendor has a higher expected profit. However, if $\beta < \hat{m}^c$, then we show that the more biased newsvendor can earn a higher expected profit, using an existence proof. To that end, suppose $\alpha_1 = 0$. Then,

$$\begin{aligned}\left.\frac{\partial \hat{\pi}_1^c(\alpha_1 = 0, \alpha_2)}{\partial \alpha_2}\right|_{\alpha_2=0} &= \left.\frac{\partial \hat{\pi}_1^c(\alpha_1 = 0, \alpha_2)}{\partial \hat{q}_2^c}\right|_{\alpha_2=0} (\mu - q_n) \\ &= -p\theta \mathrm{P}[(X_1 + \theta(X_2 - q_n)) \le q_n, X_2 \ge q_n](\mu - q_n) \\ &< 0.\end{aligned}$$

Similarly,

$$\begin{aligned}\left.\frac{\partial \hat{\pi}_2^c(\alpha_1 = 0, \alpha_2)}{\partial \alpha_2}\right|_{\alpha_2=0} &= \left.\frac{\partial \hat{\pi}_2^c(\alpha_1 = 0, \alpha_2)}{\partial \hat{q}_2^c}\right|_{\alpha_2=0} (\mu - q_n) \\ &= \{p\mathrm{P}[(X_1 + \theta(X_2 - q_n)) \ge q_n] - c\} (\mu - q_n) \\ &= 0.\end{aligned}$$

It follows that $\left[\frac{\partial \hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2} - \frac{\partial \hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2}\right]_{\alpha_2=0} < 0$. Note that $\hat{\pi}_1^c(\alpha_1 = 0, \alpha_2 = 0) = \hat{\pi}_2^c(\alpha_1 = 0, \alpha_2 = 0)$. So, there must exist some $\alpha_2 > 0$ such that $\hat{\pi}_2^c(\alpha_1 = 0, \alpha_2) > \hat{\pi}_1^c(\alpha_1 = 0, \alpha_2)$ if $\beta < \hat{m}^c$. The proof is complete. Q.E.D.

Proof of Proposition EC.6 Without loss of generality, define Newsvendor 2 as the biased newsvendor characterized by overconfidence level α_2 and define Newsvendor 1 as the sophisticated newsvendor. Accordingly, denote $\pi_1^{fc}(\alpha_2)$ and $\hat{\pi}_2^{fc}(\alpha_2)$ as the equilibrium expected profits of Newsvendor 1 and Newsvendor 2, respectively, for this case. Similarly, denote $q_1^{fc}(\alpha_2)$ and $\hat{q}_2^{fc}(\alpha_2)$ as the equilibrium order quantities of Newsvendor 1 and Newsvendor 2, respectively, for this case. Then, $\hat{q}_2^{fc}(\alpha_2) = \hat{q}_2^c(\alpha_2) = \alpha_2\mu + (1-\alpha_2)q_n$ from Lemma 4. So, $\frac{\partial \hat{q}_2^{fc}(\alpha_2)}{\partial \alpha_2} = \mu - q_n$. And $q_1^{fc}(\alpha_2)$ is determined by $\mathrm{P}(X_1 + \theta(X_2 - \hat{q}_2^{fc}(\alpha_2))^+ \le q_1^{fc}(\alpha_2)) = \beta$. Accordingly, Newsvendor 2's equilibrium expected profit can be written as $\hat{\pi}_2^{fc}(\alpha_2) = p\mathrm{E}[X_2 + \theta(X_1 - q_1^{fc})^+ \wedge \hat{q}_2^{fc}] - c\hat{q}_2^{fc}$, which implies that $\frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_1^{fc}} = -p\theta\mathrm{P}[X_2 + \theta(X_1 - q_1^{fc}) \le \hat{q}_2^{fc}, X_1 \ge q_1^{fc}]$ and $\frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_2^{fc}} = p\mathrm{P}[X_2 + \theta(X_1 - q_1^{fc})^+ \ge \hat{q}_2^{fc}] - c$. It follows that

$$\left.\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2=0} = \frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_1^{fc}} \times \left.\frac{\partial \hat{q}_1^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0} + \frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_2^{fc}} \times \left.\frac{\partial \hat{q}_2^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0}$$

$$= -p\theta \mathrm{P}[X_2 + \theta(X_1 - q_1^{fc}) \le \hat{q}_2^{fc}, X_1 \ge q_1^{fc}] \times \left.\frac{\partial \hat{q}_1^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0} +$$
$$\left\{p\mathrm{P}[X_2 + \theta(X_1 - q_1^{fc})^+ \ge \hat{q}_2^{fc}] - c\right\} \times (\mu - q_n)\big|_{\alpha_2=0}.$$

However, $p\mathrm{P}[X_2 + \theta(X_1 - q_1^{fc})^+ \ge \hat{q}_2^{fc}] - c\big|_{\alpha_2=0} = p\mathrm{P}[X_2 + \theta(X_1 - q_n)^+ \ge q_n] - c = 0$. So,

$$\left.\frac{d\hat{\pi}_2^{fc}}{d\alpha_2}\right|_{\alpha_2=0} = -p\theta\, \mathrm{P}[X_2 + \theta(X_1 - q_1^{fc}) \le \hat{q}_2^{fc}, X_1 \ge q_1^{fc}] \times \left.\frac{\partial \hat{q}_1^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0} > 0, \tag{EC.18}$$

where the inequality follows because $\beta < \hat{m}^c \Longrightarrow q_n < \mu$, $q_n < \mu \Longrightarrow \frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} = \frac{d\hat{q}_2^{c}(\alpha_2)}{d\alpha_2} = \mu - q_n < 0$, and $\frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} > 0 \Longrightarrow \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2} < 0$.

Next, note that Newsvendor 1's equilibrium expected profit can be written as $\pi_1^{fc}(\alpha_2) = p\mathrm{E}[(X_1 + \theta(X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+) \wedge q_1^{fc}] - cq_1^{fc}$. Thus, by the envelope theorem,

$$\frac{d\pi_1^{fc}(\alpha_2)}{d\alpha_2} = p\theta \mathrm{P}\left[X_1 + \theta\left(X_2 - \alpha_2\mu - (1-\alpha_2)q_n\right) < q_1^{fc}, X_2 > \alpha_2\mu + (1-\alpha_2)q_n\right](q_n - \mu) < 0, \tag{EC.19}$$

where the inequality holds because, again, $\beta < \hat{m}^c \Longrightarrow q_n < \mu$ from Proposition EC.1. It follows from (EC.18) and (EC.19) that $[\frac{d\hat{\pi}_2^{fc}}{d\alpha_2} - \frac{d\pi_1^{fc}}{d\alpha_2}]_{\alpha_2=0} > 0$. Hence, there must exist some $\alpha_2 > 0$ such that $\hat{\pi}_2^{fc}(\alpha_2) > \pi_1^{fc}(\alpha_2)$ because $\hat{\pi}_1^{fc}(\alpha_2 = 0) = \hat{\pi}_2^{fc}(\alpha_2 = 0)$. Q.E.D.

Proof of Lemma EC.4. For $X \sim U[0,1]$, $D(\alpha_i) = \alpha_i/2 + (1-\alpha_i)X$ because $\mathrm{E}[X] = 1/2$. Hence,

$$F_D(x) = F\left(\frac{x - \alpha_i\mu}{1-\alpha_i}\right) = \begin{cases} 0 & x \le \frac{\alpha_i}{2}; \\ \frac{x - \alpha_i/2}{1-\alpha_i} & \frac{\alpha_i}{2} < x \le 1 - \frac{\alpha_i}{2}; \\ 1 & x > 1 - \frac{\alpha_i}{2}. \end{cases}$$

Substituting $1 - \alpha_i/2$ for $\hat{Q}^c$ in the left-hand side of (EC.8), we obtain

$$\frac{F_D(1-\alpha_i/2)}{2} + \frac{1}{2(1-\alpha_i/2)}\int_{\frac{\alpha_i}{2}}^{1-\frac{\alpha_i}{2}} F_D(x)dx = \frac{3-2\alpha_i}{2(2-\alpha_i)}.$$

So, if $\beta > \frac{3-2\alpha_i}{2(2-\alpha_i)}$, $\hat{Q}^c > 1 - \alpha_i/2$; and if $\beta \le \frac{3-2\alpha_i}{2(2-\alpha_i)}$, $\hat{Q}^c \le 1 - \alpha_i/2$. Suppose $\hat{Q}^c \in [\alpha_i/2, 1-\alpha_i/2]$. Then, (EC.8) becomes $\frac{1}{2}\frac{Q - \alpha_i/2}{1-\alpha_i} + \frac{1}{2Q}\int_{\alpha_i/2}^{Q} \frac{x - \alpha_i/2}{1-\alpha_i}dx = \beta$, which is a quadratic equation in Q, and the left-hand side of the equation increases in Q. Thus, $\hat{Q}^c = \frac{1}{3}\alpha_i + \frac{2}{3}\beta - \frac{2}{3}\alpha_i\beta + \frac{1}{3}h(\alpha_i, \beta)$. Suppose $\hat{Q}^c > 1 - \alpha_i/2$. Then, (EC.8) becomes

$$\frac{1}{2} + \frac{1}{2Q}\left(\int_{\alpha_i/2}^{1-\alpha_i/2} \frac{x - \alpha_i/2}{1-\alpha_i}dx + \int_{1-\alpha_i/2}^{Q} dx\right) = \beta,$$

with the solution $\hat{Q}^c = \frac{1}{4(1-\beta)}$. The desired result then follows because $\hat{q}^c = \hat{Q}^c/2$. Q.E.D.

Proof of Proposition EC.7. Clearly, $\partial \hat{q}_i^c(\alpha)/\partial\alpha = 0$ for $\beta > \frac{3-2\alpha}{2(2-\alpha)}$. For $\beta \le \frac{3-2\alpha}{2(2-\alpha)}$, $\hat{q}_i^c(\alpha) = \frac{1}{6}\alpha + \frac{1}{3}\beta - \frac{1}{3}\alpha\beta + \frac{1}{6}h(\alpha,\beta)$, and

$$\frac{\partial \hat{q}_i^c(\alpha)}{\partial\alpha} = \frac{1}{12\sqrt{16\alpha^2\beta^2 - 16\alpha^2\beta + \alpha^2 - 32\alpha\beta^2 + 16\alpha\beta + 16\beta^2}} \times \Big[\underbrace{(2-4\beta)\sqrt{16\alpha^2\beta^2 - 16\alpha^2\beta + \alpha^2 - 32\alpha\beta^2 + 16\alpha\beta + 16\beta^2}}_{A} + \underbrace{\alpha + 8\beta - 16\beta^2 - 16\alpha\beta + 16\alpha\beta^2}_{B}\Big].$$

Note that $B \ge 0 \Longleftrightarrow \beta \le \frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right)$. So we consider two following cases.

Case 1: $\beta \le \frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right)$. Note that $\beta \le 1/2$. So, $A > 0$ and $B > 0$, which implies that $\partial\hat{q}_i^c/\partial\alpha > 0$.

Case 2: $\beta > \frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right)$. Note that $\frac{3-2\alpha}{2(2-\alpha)} \ge 1/2$, we need to consider two sub-cases. *Case 2A*: $\frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right) < \beta \le 0.5$. Then, $A > 0$ and $B < 0$, and we compare their squares:

$$\begin{aligned}&(1-2\beta)^2(16\alpha^2\beta^2 - 16\alpha^2\beta + \alpha^2 - 32\alpha\beta^2 + 16\alpha\beta + 16\beta^2) - (\alpha + 8\beta - 16\beta^2 - 16\alpha\beta + 16\alpha\beta^2)^2 \\ &= 12\beta\Big[\underbrace{\left(1 - 16\beta^3 + 32\beta^2 - 17\beta\right)\alpha^2 - \left(48\beta^2 - 32\beta^3 - 16\beta\right)\alpha - \left(16\beta^3 - 16\beta^2 + 4\beta\right)}_{C}\Big].\end{aligned}$$

Note that C is a cubic function of β, $\frac{\partial C}{\partial\beta}$ is a quadratic function of β, and $\frac{\partial C}{\partial\beta} < 0$ for $\frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right) < \beta \le 0.5$. Since $C|_{\beta=\frac{1}{4(1-\alpha)}\left(1-2\alpha+\sqrt{1-3\alpha+3\alpha^2}\right)} > 0$ and $C|_{\beta=0.5} < 0$, there must exist a unique solution $\beta_c \in \left[\frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right), 1/2\right]$ such that $C \ge 0 \Longleftrightarrow \beta \le \beta_c$. Consequently, $\frac{\partial\hat{q}_i^c}{\partial c} \ge 0 \Longleftrightarrow \beta \le \beta_c$. *Case 2B*: $1/2 < \beta \le \frac{3-2\alpha}{4-2\alpha}$. In this case, $\frac{\partial\hat{q}_i^c}{\partial\alpha} < 0$ because $A < 0$ and $B < 0$. Q.E.D.

Proof of Proposition EC.8. Because $\hat{\pi}_i^c$ is independent of α when $\beta > \frac{3-2\alpha}{2(2-\alpha)}$, it suffices to consider the case $\beta \le \frac{3-2\alpha}{2(2-\alpha)}$, in which $\hat{q}_i^c(\alpha) = \frac{1}{6}\alpha + \frac{1}{3}\beta - \frac{1}{3}\alpha\beta + \frac{1}{6}\sqrt{4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2}$ by Lemma EC.4. Furthermore, for $\beta \le \frac{3-2\alpha}{2(2-\alpha)}$, $\hat{q}_i^c(\alpha) - \frac{1}{2} = \frac{1}{6}\left[\sqrt{4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2} - (3 - \alpha - 2\beta + 2\alpha\beta)\right] < 0$ because $3 - \alpha - 2\beta + 2\alpha\beta \ge 0$, and $\left(4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2\right) - (3 - \alpha - 2\beta + 2\alpha\beta)^2 = 6\alpha + 12(1-\alpha)\beta - \frac{3}{4}\alpha^2 - 9 < 0$. In addition, because $\hat{\pi}_i^c = -c\hat{q}_i{}^c + p\hat{q}_i^c(1-\hat{q}_i^c)$, $\frac{\partial\hat{\pi}_i^c}{\partial\alpha} = \frac{\partial\hat{\pi}_i^c}{\partial\hat{q}_i^c} \times \frac{\partial\hat{q}_i^c}{\partial\alpha} = p[\beta - 2\hat{q}_i^c] \times \frac{\partial\hat{q}_i^c}{\partial\alpha}$, where

$\beta - 2\hat{q}_i^c = \frac{1}{3}(\beta - \alpha + 2\alpha\beta - \sqrt{4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2})$. If $\beta \leq \frac{\alpha}{1+2\alpha} \Longleftrightarrow \beta - \alpha + 2\alpha\beta \geq 0$, then $\beta - 2\hat{q}_i^c < 0$. If $\beta > \frac{\alpha}{1+2\alpha}$, define $f(\beta) := (\beta - \alpha + 2\alpha\beta)^2 - (4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2) = (12\alpha - 3)\beta^2 - 6\alpha\beta + \frac{3}{4}\alpha^2$. Note that $f\left(\frac{\alpha}{1+2\alpha}\right) = \frac{3}{4}\frac{\alpha^2}{(2\alpha+1)^2}(4\alpha^2 + 4\alpha - 11) < 0, f'\left(\frac{\alpha}{1+2\alpha}\right) = 12\alpha\frac{\alpha-1}{2\alpha+1} < 0$, and $f\left(\frac{3-2\alpha}{2(2-\alpha)}\right) = \frac{3}{4}\frac{(\alpha-1)^2}{(\alpha-2)^2}(\alpha^2 + 6\alpha - 9) < 0$. Consequently, $f(\beta) < 0$ for $\beta \in [\frac{\alpha}{1+2\alpha}, \frac{3-2\alpha}{2(2-\alpha)}]$. In summary, $\beta - 2\hat{q}_i^c \leq 0$ when $\beta \leq \frac{3-2\alpha}{2(2-\alpha)}$. Appealing to Proposition EC.7, we complete the proof. Q.E.D.

Proof of Proposition EC.9 When $\alpha_1 = 0$, from Lemma EC.4,

$$\hat{q}_1^c = \begin{cases} \frac{2}{3}\beta & \beta \leq \frac{3}{4}; \\ \frac{1}{8(1-\beta)} & \beta > \frac{3}{4}. \end{cases} \tag{EC.20}$$

We partition the interval $[0, 1]$ into four disjoint sub-intervals to facilitate the comparison of $\hat{\pi}_1^c$ and $\hat{\pi}_2^c$: $[0, \frac{3-\alpha_2}{8}]$, $(\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$, $(\frac{3-2\alpha_2}{2(2-\alpha_2)}, \frac{3}{4}]$, and $(\frac{3}{4}, 1]$. For each sub-interval, we sign three expressions to compare $\hat{\pi}_1^c$ and $\hat{\pi}_2^c$. First, we sign $\hat{q}_1^c + \hat{q}_2^c - 1$; second, we sign $\hat{q}_1^c - \hat{q}_2^c$; and third, depending on the sign of $\hat{q}_1^c + \hat{q}_2^c - 1$, we sign either $\beta - \frac{\hat{q}_1^c + \hat{q}_2^c}{2}$ or $\beta(\hat{q}_1^c + \hat{q}_2^c) - \frac{1}{2}$. From (EC.10), we see that the signs of $\hat{q}_1^c + \hat{q}_2^c - 1$, $\hat{q}_1^c - \hat{q}_2^c$, and $\beta - \frac{\hat{q}_1^c + \hat{q}_2^c}{2}$ or $\beta(\hat{q}_1^c + \hat{q}_2^c) - \frac{1}{2}$, taken together, completely determine the sign of $\hat{\pi}_2^c - \hat{\pi}_1^c$.

Case 1: $\beta \in [0, \frac{3-\alpha_2}{8}]$. For this case, we present a detailed proof to illustrate the steps of analysis taken to derive the desired results. The analysis for other cases are similar, and are therefore only outlined for space. When $\beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$, by (EC.20) and Lemma EC.4, the equilibrium order quantities are

$$\hat{q}_1^c = \frac{2}{3}\beta, \tag{EC.21}$$

$$\hat{q}_2^c = \frac{1}{6}\alpha_2 + \frac{1}{3}\beta - \frac{1}{3}\alpha_2\beta + \frac{1}{6}h(\alpha_2, \beta). \tag{EC.22}$$

We first sign $\hat{q}_1^c + \hat{q}_2^c - 1$. By (EC.21) and (EC.22), $\hat{q}_1^c + \hat{q}_2^c - 1 = \frac{1}{6}[h(\alpha_2, \beta) - (6 - 6\beta - \alpha_2 + 2\alpha_2\beta)]$. Note that $6 - 6\beta - \alpha_2 + 2\alpha_2\beta > 0$ for $\beta \in [0, \frac{3-\alpha_2}{8}]$. So, $\hat{q}_1^c + \hat{q}_2^c - 1$ has the same sign as $h^2(\alpha_2, \beta) - (6 - 6\beta - \alpha_2 + 2\alpha_2\beta)^2$. By the definition of $h(\alpha_2, \beta)$ and collecting terms, we obtain $J(\beta) := h^2(\alpha_2, \beta) - (6 - 6\beta - \alpha_2 + 2\alpha_2\beta)^2 = (16\alpha_2 - 32)\beta^2 + (72 - 32\alpha_2)\beta + \left(12\alpha_2 - \frac{3}{4}\alpha_2^2 - 36\right)$. To sign $J(\beta)$ for $\beta \in [0, \frac{3-\alpha_2}{8}]$, we check its values and its first-order derivatives at the boundaries:

$$J\left(\frac{3-\alpha_2}{8}\right) = \frac{1}{4}\alpha_2^3 + \frac{5}{4}\alpha_2^2 - \frac{15}{4}\alpha_2 - \frac{27}{2} \le 0, \tag{EC.23}$$

$$J'\left(\frac{3-\alpha_2}{8}\right) = -4\alpha_2^2 - 12\alpha_2 + 48 \ge 0. \tag{EC.24}$$

Note that the graph of $J(\beta)$ is a downward-opening parabola. It then follows from (EC.23) and (EC.24) that $J(\beta) \le 0$ for $\beta \in [0, \frac{3-\alpha_2}{8}]$, which implies that $\hat{q}_1^c + \hat{q}_2^c \le 1$. Consequently, by (EC.10),

$$\hat{\pi}_2^c - \hat{\pi}_1^c = p(\hat{q}_2^c - \hat{q}_1^c)\left(\beta - \frac{\hat{q}_1^c + \hat{q}_2^c}{2}\right). \tag{EC.25}$$

We next sign $\hat{q}_1^c - \hat{q}_2^c$. By (EC.21) and (EC.22), $\hat{q}_2^c - \hat{q}_1^c = \frac{1}{6}h(\alpha_2,\beta) - (-\frac{1}{6}\alpha_2 + \frac{1}{3}\beta + \frac{1}{3}\alpha_2\beta)$. For $\beta \in [0, \frac{3-\alpha_2}{8}]$, $-\frac{1}{6}\alpha_2 + \frac{1}{3}\beta + \frac{1}{3}\alpha_2\beta \ge 0 \Longleftrightarrow \beta \ge \frac{\alpha_2}{2\alpha_2+2}$. Consequently, $\hat{q}_2^c - \hat{q}_1^c \ge 0$ for $\beta \le \frac{\alpha_2}{2\alpha_2+2}$, and $\hat{q}_2^c - \hat{q}_1^c$ has the same sign as $h^2(\alpha_2,\beta) - (-\alpha_2 + 2\beta + 2\alpha_2\beta)^2$ for $\beta \ge \frac{\alpha_2}{2\alpha_2+2}$. However, $h^2(\alpha_2,\beta) - (-\alpha_2 + 2\beta + 2\alpha_2\beta)^2 = -\frac{1}{4}\alpha_2\left(64\beta^2 - 32\beta + 3\alpha_2\right)$. Using properties of quadratic functions, we can verify that for $\beta \in [\frac{\alpha_2}{2\alpha_2+2}, \frac{3-\alpha_2}{8}]$, $64\beta^2 - 32\beta + 3\alpha_2 < 0$, which implies that $\hat{q}_2^c > \hat{q}_1^c$. In summary, for $\beta \in [0, \frac{3-\alpha_2}{8}]$,

$$\hat{q}_2^c > \hat{q}_1^c. \tag{EC.26}$$

We last sign $\beta - \frac{\hat{q}_1^c + \hat{q}_2^c}{2}$. First, for $\beta \le \frac{\alpha_2}{6+2\alpha_2}$, $\hat{q}_1^c + \hat{q}_2^c - 2\beta = \frac{1}{6}[h(\alpha_2,\beta) - (-\alpha_2 + 6\beta + 2\alpha_2\beta)] \ge 0$ because $-\alpha_2 + 6\beta + 2\alpha_2\beta \ge 0 \Longleftrightarrow \beta \ge \frac{\alpha_2}{6+2\alpha_2}$. For $\beta > \frac{\alpha_2}{6+2\alpha_2}$, $h^2(\alpha_2,\beta) - (-\alpha_2 + 6\beta + 2\alpha_2\beta)^2 = \left(-32\alpha_2 - 32\right)\beta^2 + 16\alpha_2\beta - \frac{3}{4}\alpha_2^2$. Again, using properties of quadratic function, we can verify that

$$\hat{q}_1^c + \hat{q}_2^c - 2\beta = \begin{cases} > 0 & \frac{\alpha_2}{6+2\alpha_2} < \beta \le \frac{8\alpha_2 + 2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}; \\ \le 0 & \beta > \frac{8\alpha_2 + 2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}. \end{cases}$$

In summary, for $\beta \in [0, \frac{3-\alpha_2}{8}]$,

$$\hat{q}_1^c + \hat{q}_2^c - 2\beta = \begin{cases} > 0 & \beta \le \frac{8\alpha_2 + 2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}; \\ \le 0 & \beta > \frac{8\alpha_2 + 2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}. \end{cases} \tag{EC.27}$$

Using (EC.25)-(EC.27), we obtain

$$\hat{\pi}_2^c - \hat{\pi}_1^c = \begin{cases} \le 0 & \beta \le \frac{8\alpha_2 + 2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}; \\ \ge 0 & \beta > \frac{8\alpha_2 + 2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}. \end{cases}$$

Case 2: $\beta \in (\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$. In this case, $\hat{q}_1^c = \frac{2}{3}\beta$ and $\hat{q}_2^c = \frac{1}{6}\alpha_2 + \frac{1}{3}\beta - \frac{1}{3}\alpha_2\beta + \frac{1}{6}h(\alpha_2,\beta)$.

First, $\hat{q}_1^c+\hat{q}_2^c-1=\frac{1}{6}\left[h(\alpha_2,\beta)-(6-\alpha_2-6\beta+2\alpha_2\beta)\right]$. Note that $6-\alpha_2-6\beta+2\alpha_2\beta\geq 0$ for $\beta\in(\frac{3-\alpha_2}{8},\frac{3-2\alpha_2}{2(2-\alpha_2)}]$. So, we show $h^2(\alpha_2,\beta)-(6-\alpha_2-6\beta+2\alpha_2\beta)^2=(16\alpha_2-32)\,\beta^2+(72-32\alpha_2)\,\beta+\left(12\alpha_2-\frac{3}{4}\alpha_2^2-36\right)<0$ for $\beta\in(\frac{3-\alpha_2}{8},\frac{3-2\alpha_2}{2(2-\alpha_2)})$ using the properties of quadratic functions. Therefore, $\hat{q}_1^c+\hat{q}_2^c<1$. It follows from (EC.10) that for $\beta\in(\frac{3-\alpha_2}{8},\frac{3-2\alpha_2}{2(2-\alpha_2)}]$,

$$\hat{\pi}_2^c-\hat{\pi}_1^c=p(\hat{q}_2^c-\hat{q}_1^c)\big(\beta-\frac{\hat{q}_1^c+\hat{q}_2^c}{2}\big). \tag{EC.28}$$

Second, $2\beta-(\hat{q}_1^c+\hat{q}_2^c)=6\beta-\alpha_2+2\alpha_2\beta-h(\alpha_2,\beta)$ where $6\beta-\alpha_2+2\alpha_2\beta>0$ for $\beta\in(\frac{3-\alpha_2}{8},\frac{3-2\alpha_2}{2(2-\alpha_2)}]$. From properties of quadratic functions, $(6\beta-\alpha_2+2\alpha_2\beta)^2-h^2(\alpha_2,\beta)=32\beta^2\alpha_2+32\beta^2-16\beta\alpha_2+\frac{3}{4}\alpha_2^2>0$ for $\beta\in(\frac{3-\alpha_2}{8},\frac{3-2\alpha_2}{2(2-\alpha_2)}]$. Thus,

$$2\beta>(\hat{q}_1^c+\hat{q}_2^c). \tag{EC.29}$$

Third, $\hat{q}_2^c-\hat{q}_1^c=h(\alpha_2,\beta)-(2\beta-\alpha_2+2\alpha_2\beta)$. Note that for $\beta\in(\frac{3-\alpha_2}{8},\frac{3-2\alpha_2}{2(2-\alpha_2)}]$, $2\beta-\alpha_2+2\alpha_2\beta\geq 0$ and $h^2(\alpha_2,\beta)-(2\beta-\alpha_2+2\alpha_2\beta)^2=\frac{1}{4}\alpha_2\left(-64\beta^2+32\beta-3\alpha_2\right)\geq 0\Longleftrightarrow\beta\leq\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}$. Consequently,

$$\hat{q}_2^c-\hat{q}_1^c=\begin{cases}\geq 0 & \frac{3-\alpha_2}{8}<\beta\leq\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4};\\ <0 & \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}<\beta\leq\frac{3-2\alpha_2}{2(2-\alpha_2)}.\end{cases} \tag{EC.30}$$

From (EC.28)-(EC.30), we see that

$$\hat{\pi}_2^c-\hat{\pi}_1^c=\begin{cases}\geq 0 & \frac{3-\alpha_2}{8}<\beta\leq\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4};\\ \leq 0 & \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}<\beta\leq\frac{3-2\alpha_2}{2(2-\alpha_2)}.\end{cases}$$

Case 3: $\beta\in(\frac{3-2\alpha_2}{2(2-\alpha_2)},3/4]$. In this case, by (EC.20) and Lemma EC.4, $\hat{q}_1^c=\frac{2}{3}\beta$ and $\hat{q}_2^c=\frac{1}{8(1-\beta)}$. First, $\frac{2}{3}\beta+\frac{1}{4(1-\beta)}\geq 1\Longleftrightarrow\beta\geq\frac{5}{4}-\frac{1}{4}\sqrt{7}$. Consequently, we consider two sub-cases. For $\beta\in(\frac{3-2\alpha_2}{2(2-\alpha_2)},\frac{5}{4}-\frac{1}{4}\sqrt{7}]$, $\hat{\pi}_2^c\geq\hat{\pi}_1^c$ because $2\beta\geq\hat{q}_1^c+\hat{q}_2^c$ and $\hat{q}_1^c\leq\hat{q}_2^c$. For $\beta\in(\frac{5}{4}-\frac{1}{4}\sqrt{7},\frac{3}{4}]$, $\hat{\pi}_2^c\geq\hat{\pi}_1^c$ because $\beta(\hat{q}_1^c+\hat{q}_2^c)-\frac{1}{2}\geq 0$ and $\hat{q}_1^c\leq\hat{q}_2^c$.

Case 4: $\beta>\frac{3}{4}$. In this case, $\hat{q}_1^c=\hat{q}_2^c=\frac{1}{8(1-\beta)}$. Consequently, $\hat{\pi}_1^c=\hat{\pi}_2^c$.

Combining all four cases, we obtain

$$\hat{\pi}_2^c-\hat{\pi}_1^c=\begin{cases}\leq 0 & 0<\beta\leq\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16\alpha_2+16};\\ \geq 0 & \frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16\alpha_2+16}<\beta\leq\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4};\\ \leq 0 & \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}<\beta\leq\frac{3-2\alpha_2}{2(2-\alpha_2)};\\ \geq 0 & \frac{3-2\alpha_2}{2(2-\alpha_2)}<\beta\leq\frac{3}{4};\\ =0 & \frac{3}{4}<\beta\leq 1.\end{cases}$$

Notice 1) for $\alpha_2 \in [0,1]$, $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} \leq \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4} \leq \frac{3-2\alpha_2}{2(2-\alpha_2)} \leq \frac{3}{4}$, and 2) as a_2 increases from 0 to 1, $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2}$ increases from 0 to 3/16, $\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}$ decreases from 1/2 to 3/8, and $\frac{3-2\alpha_2}{2(2-\alpha_2)}$ decreases from 3/4 to 1/2.

For $\beta \in [0,3/16]$, if $\alpha_2 \leq \frac{8}{3}\beta\left(4-8\beta-\sqrt{2}\sqrt{32\beta^2-32\beta+5}\right)$, then $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} \leq \beta \leq \frac{3-\alpha_2}{8}$, which implies $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$, and if $\alpha_2 \geq \frac{8}{3}\beta\left(4-8\beta-\sqrt{2}\sqrt{32\beta^2-32\beta+5}\right)$, then $\beta \leq \frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \leq 0$.

For $\beta \in (3/16,3/8]$, $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} < \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}$ holds for all $\alpha_2 \in [0,1]$. Therefore, for $\beta \in (3/16,3/8]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ for all α_2.

For $\beta \in [3/8,1/2]$, if $\alpha_2 \leq 4/3-(8\beta-2)^2/3$ then $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} \leq \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$; and if $\alpha_2 \geq 4/3-(8\beta-2)^2/3$, then $\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4} \leq \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \leq 0$.

For $\beta \in [1/2,3/4]$, if $\alpha_2 \leq \frac{1}{2\beta-2}(4\beta-3)$, $\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4} < \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \leq 0$. If $\alpha_2 \geq \frac{1}{2\beta-2}(4\beta-3)$, $\frac{3-2\alpha_2}{2(2-\alpha_2)} < \beta \leq \frac{3}{4}$ which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$.

For $\beta \geq 3/4$. $\hat{\pi}_2^c - \hat{\pi}_1^c = 0$ for all α_2.

The desired result then follows. Q.E.D.

Proof of Lemma EC.5 When choosing its order quantity, Newsvendor 1 knows that Newsvendor 2 orders $\hat{q}_2^c$. Therefore, Newsvendor 1 chooses q_1 to maximize $\pi_1(q_1,\hat{q}_2^c)$ defined in (EC.11). Note that $\pi_1(q_1,\hat{q}_2^c)$ is a piece-wise function with two segments joined at $q_1 = 1-\hat{q}_2^c$. For $q_1 \geq 1-\hat{q}_2^c$, $\left.\frac{\partial \pi_1}{\partial q_1}\right|_{q_1=1-\hat{q}_2^c} = \frac{1}{2}p\hat{q}_2^c - c = p(\beta+\frac{1}{2}\hat{q}_2^c-1)$ because $\frac{\partial \pi_1}{\partial q_1} = \frac{p\hat{q}_2^c}{2(q_1+\hat{q}_2^c)^2} - c$. For $q_1 \leq 1-\hat{q}_2^c$, $\frac{\partial \pi_1}{\partial q_1}|_{q_1=1-\hat{q}_2^c} = p(\beta-1+\frac{1}{2}\hat{q}_2^c)$ because $\frac{\partial \pi_1}{\partial q_1} = p(\beta-q_1-\frac{1}{2}\hat{q}_2^c)$. So, the two segments of $\pi_1(q_1,\hat{q}_2^c)$ have equal first-order derivatives at the point they join. Consequently, Newsvendor 1's equilibrium order quantity $\hat{q}_1^{fc}$ satisfies

$$\hat{q}_1^{fc} = \begin{cases} \leq 1-\hat{q}_2^c & \beta-1+\frac{1}{2}\hat{q}_2^c \leq 0; \\ > 1-\hat{q}_2^c & \beta-1+\frac{1}{2}\hat{q}_2^c > 0. \end{cases} \tag{EC.31}$$

Suppose $\beta-1+\frac{1}{2}\hat{q}_2^c > 0$. Solving the corresponding first-order condition $\frac{\partial \pi_1}{\partial q_1} = -c+\frac{1}{2}p\frac{\hat{q}_2^c}{(q_1+\hat{q}_2^c)^2} = 0$ yields

$$q_1 = \frac{1}{2-2\beta}\left(\sqrt{2\,(1-\beta)\,\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right). \tag{EC.32}$$

Suppose $\beta - 1 + \frac{1}{2}\hat{q}_2^c \leq 0$. Then solving the corresponding first-order condition yields:

$$q_1 = \beta - \frac{1}{2}\hat{q}_2^c. \tag{EC.33}$$

Hence, by (EC.31)-(EC.33), Newsvendor 1's order quantity $\hat{q}_1^{fc}$ satisfies

$$\hat{q}_1^{fc} = \begin{cases} \max(\beta - \frac{1}{2}\hat{q}_2^c, 0) & \hat{q}_2^c \leq 2(1-\beta); \\ \frac{1}{2-2\beta}\left(\sqrt{2\left(1-\beta\right)\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right) & \hat{q}_2^c > 2(1-\beta). \end{cases} \tag{EC.34}$$

We next compare $\hat{q}_2^c$ and $2(1-\beta)$. Recall from Lemma EC.4,

$$\hat{q}_2^c = \begin{cases} \frac{1}{6}\alpha_2 + \frac{1}{3}\beta - \frac{1}{3}\alpha_2\beta + \frac{1}{6}h(\alpha_2,\beta) & \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}; \\ \frac{1}{8(1-\beta)} & \beta > \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases}$$

Suppose $\beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$. Then, $\hat{q}_2^c - 2(1-\beta) = \frac{1}{6}\left[h(\alpha_2,\beta) - ((-14+2\alpha_2)\beta + 12 - \alpha_2)\right]$. It can be verified that $(-14+2\alpha_2)\beta + 12 - \alpha_2 \geq 0$ for $\beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$. So, $\hat{q}_2^c - 2(1-\beta)$ and $h^2(\alpha_2,\beta) - ((-14+2\alpha_2)\beta + 12 - \alpha_2)^2$ have identical signs. Using some algebraic operations and properties of quadratic functions, we find that $h^2(\alpha_2,\beta) - ((-14+2\alpha_2)\beta+12-\alpha_2)^2 < 0$, which implies that $\hat{q}_2^c \leq 2(1-\beta)$. Suppose $\beta > \frac{3-2\alpha_2}{2(2-\alpha_2)}$. Then, $\hat{q}_2^c - 2(1-\beta) = \frac{1}{8(1-\beta)} - 2(1-\beta) \geq 0 \Longleftrightarrow \beta \geq 3/4$. In summary

$$\hat{q}_2^c = \begin{cases} \leq 2(1-\beta) & \beta \leq \frac{3}{4}; \\ \geq 2(1-\beta) & \beta > \frac{3}{4}. \end{cases} \tag{EC.35}$$

Combining (EC.34) and (EC.35), we obtain

$$\hat{q}_1^{fc} = \begin{cases} \max(\beta - \frac{1}{2}\hat{q}_2^c, 0) & \beta \leq \frac{3}{4}; \\ \frac{1}{2-2\beta}\left(\sqrt{2\left(1-\beta\right)\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right) & \beta > \frac{3}{4}. \end{cases} \tag{EC.36}$$

To completely characterize $\hat{q}_1^{fc}$, we need to compare $\beta - \frac{1}{2}\hat{q}_2^c$ and 0. Suppose $\beta \in (\frac{3-2\alpha_2}{2(2-\alpha_2)}, \frac{3}{4}]$. Then, $\hat{q}_2^c = \frac{1}{8(1-\beta)}$ by Lemma EC.4, which implies that $\beta - \frac{1}{2}\hat{q}_2^c \geq 0$. Suppose $\beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$. Then, by Lemma EC.4, $\beta - \frac{1}{2}\hat{q}_2^c$ and $(10\beta - \alpha_2 + 2\alpha_2\beta) - h(\alpha_2,\beta)$ have the same sign. If $\beta \leq \frac{\alpha_2}{10+2\alpha_2}$, then $10\beta - \alpha_2 + 2\alpha_2\beta \leq 0$, which implies that $\beta - \frac{1}{2}\hat{q}_2^c \leq 0$. If $\beta > \frac{\alpha_2}{10+2\alpha_2} \Longleftrightarrow 10\beta - \alpha_2 + 2\alpha_2\beta \geq 0$, we can use properties of quadratic functions to show that $(10\beta - \alpha_2 + 2\alpha_2\beta)^2 - h^2(\alpha_2,\beta) \geq 0 \Longleftrightarrow \beta \geq \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$. So, $\beta - \frac{1}{2}\hat{q}_2^c \geq 0 \Longleftrightarrow \beta \geq \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$. In summary,

$$\beta - \frac{1}{2}\hat{q}_2^c = \begin{cases} \leq 0 & \beta \leq \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}; \\ \geq 0 & \beta > \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}. \end{cases} \tag{EC.37}$$

Applying (EC.37) to (EC.36), we complete the proof. Q.E.D.

Proof of Proposition EC.10 We have three cases. Case 1: $\beta \le \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$. In this case, $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = p\hat{q}_2^c(\beta - \hat{q}_2^c/2)$ by (EC.13). So, $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} \le 0 \Longleftrightarrow \beta \le \hat{q}_2^c/2$, which holds because $\beta \le \hat{q}_2^c/2$ when $\beta \le \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$ (EC.37).

Case 2: $\alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{3}{4}$. By (EC.13), $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \frac{p}{2}\left(\frac{3}{2}\hat{q}_2^c - \beta\right)\left(\beta - \frac{\hat{q}_2^c}{2}\right) > 0 \Longleftrightarrow \frac{1}{2}\hat{q}_2^c < \beta < \frac{3}{2}\hat{q}_2^c$. Note that $\frac{1}{2}\hat{q}_2^c < \beta$ holds for all $\beta > \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$; see (EC.37). We only need to rank β and $\frac{3}{2}\hat{q}_2^c$. Recall that by Lemma EC.4,

$$\hat{q}_2^c = \begin{cases} \frac{1}{6}\alpha_2 + \frac{1}{3}\beta - \frac{1}{3}\alpha_2\beta + \frac{1}{6}h(\alpha_2,\beta) & \beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}; \\ \frac{1}{8(1-\beta)} & \beta > \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases}$$

So, we consider two sub-cases. Case 2.1: $\alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}$. In this subcase, $\frac{3\hat{q}_2^c}{2} - \beta = \frac{1}{4}h(\alpha_2,\beta) - (-\frac{1}{4}\alpha_2 + \frac{1}{2}\beta + \frac{1}{2}\alpha_2\beta) \ge 0$ because $-\frac{1}{4}\alpha_2 + \frac{1}{2}\beta + \frac{1}{2}\alpha_2\beta \le 0$ for $\beta \in (\alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}, \frac{\alpha_2}{2(1+\alpha_2)}]$. For $\beta \in (\frac{\alpha_2}{2(1+\alpha_2)}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$, $\frac{3\hat{q}_2^c}{2} - \beta \ge 0 \Longleftrightarrow \beta \le \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}$ because of the properties of quadratic functions. Hence,

$$\frac{3\hat{q}_2^c}{2} - \beta = \begin{cases} \ge 0 & \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ \le 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases}$$

Case 2.2: $\frac{3-2\alpha_2}{2(2-\alpha_2)} < \beta \le \frac{3}{4}$. In this subcase $\frac{3\hat{q}_2^c}{2} - \beta = \frac{3}{16(1-\beta)} - \beta \le 0$. In summary, for Case 2,

$$\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \begin{cases} \le 0 & \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ \ge 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \le \frac{3}{4}. \end{cases}$$

Case 3: $\beta > 3/4$. In this case, $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = p(\hat{q}_2^c - \hat{q}_1^{fc})\left(1 - \beta + \frac{1}{2(\hat{q}_1^{fc} + \hat{q}_2^c)}\right) = 0$ because $\hat{q}_2^c - \hat{q}_1^{fc} = 0$ from Lemma EC.4 and (EC.12). Q.E.D.

Proof of Proposition EC.11 To prove Proposition EC.11, we first show the following three-part lemma. Lemma EC.11(a) $\mu > \hat{z}^c \Longleftrightarrow \hat{m}^c > \beta$, Lemma EC.11(b) Newsvendor i's equilibrium expected profit is decreasing in α_i (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_i} \le 0$, for $i = 1, 2$), and Lemma EC.11(c) Newsvendor i's equilibrium expected profit is decreasing in α_{3-i} (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_{3-i}} < 0$ for $i = 1, 2$) if and only if $\beta < \hat{m}^c$.

Proof of Lemma EC.11(a): In the equilibrium, $\mathrm{P}(X_1 + (X_2 - \hat{z}_2^c)^+ \le \hat{z}_1^c) = \mathrm{P}(X_2 + (X_1 - \hat{z}_1^c)^+ \le \hat{z}_2^c) = \beta$, where $X_1 = r_1 Y$ and $X_2 = r_2 Y$. When r_1 and r_2 are identically distributed, X_1 and X_2 are identically distributed. As a result, $\hat{z}^c = \hat{z}_1^c = \hat{z}_2^c$, and $\bar{\mu} = \mu_1 = \mu_2$. Thus, $dq_i^c(\alpha)/d\alpha < 0$ if and only if $\hat{z}^c > \bar{\mu}$, which implies that $\hat{z}^c > \bar{\mu}$ if and only if $\beta > \hat{m}^c$.

Proof of Lemma EC.11(b): Without loss of generality, we let $i = 1$. First note that Newsvendor 1's profit is $\hat{\pi}_1^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+ \wedge (\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c)] - c[\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c]$. Thus,

$$\begin{aligned}\frac{\partial \hat{\pi}_1^c(\alpha_1, \alpha_2)}{\partial \alpha_1} &= (\bar{\mu} - \hat{z}^c)p[\mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c < X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) - (1-\beta)] \\ &= (\bar{\mu} - \hat{z}^c)p[\beta - \mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+)]. \qquad \text{(EC.38)}\end{aligned}$$

First, if $\hat{z}^c > \bar{\mu}$, then $\mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c < \hat{z}^c$, the second inequality is from $\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c < \hat{z}^c$, and the equality is from Lemma 4. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} < 0$ if $\hat{z}^c > \bar{\mu}$. Second, if $\hat{z}^c \leq \bar{\mu}$, then $\mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c \geq X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) \geq \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) \geq \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c \geq \hat{z}^c$, the second inequality is from $\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c \geq \hat{z}^c$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} \leq 0$ if $\hat{z}^c \leq \bar{\mu}$. In conclusion, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} \leq 0$.

Proof of Lemma EC.11(c) Without loss of generality, we let $i = 1$. Note that

$$\frac{\partial \hat{\pi}_1^c(\alpha_1, \alpha_2)}{\partial \alpha_2} = p(\hat{z}^c - \bar{\mu})\mathrm{P}[\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c, X_2 \geq \alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c].$$

Because $\mathrm{P}[\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - (1-\alpha_2)\hat{z}^c - \alpha_2\bar{\mu}, X_2 \geq \alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c] \geq 0$, the sign of $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2}$ is determined by $\hat{z}^c - \bar{\mu}$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0 \Longleftrightarrow \bar{\mu} > \hat{z}^c$. Furthermore, from Lemma EC.11(a), $\bar{\mu} > \hat{z}^c \Longleftrightarrow \hat{m}^c > \beta$.

Note that if $\alpha_1 = \alpha_2$, then the newsvendors have identical expected profits. Next, we show that the less biased newsvendor can have a lower expected profit. To that end, suppose $\alpha_1 = 0$. Then, from (EC.38) and Lemma EC.11(c), $\frac{\partial \hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2} - \frac{\partial \hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2}\Big|_{\alpha_2=0} = (\bar{\mu} - \hat{z}^c)p[\beta - \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+)] \quad + p(\bar{\mu} - \hat{z}^c)\mathrm{P}(\hat{z}^c > X_1 + X_2 - \hat{z}^c, X_2 \geq \hat{z}^c) > 0$ because $\mathrm{P}(X_1 + (X_2 - \hat{z}^c)^+ < \hat{z}^c) = \beta$ and $\beta < \hat{m}^c \Longrightarrow \bar{\mu} > \hat{z}^c$ from Lemma (a). As a result, the more biased newsvendor (Newsvendor 2) can have a higher expected profit than the less biased newsvendor (Newsvendor 1).

Next, we show that a sophisticated newsvendor can earn a lower expected profit than its overconfident competitor. Define Newsvendor 2 as the biased newsvendor characterized by overconfidence level α_2 and define Newsvendor 1 as the unbiased newsvendor cognizant of its competitor's α_2. Accordingly, denote $\pi_1^{fc}(\alpha_2)$ and $\hat{\pi}_2^{fc}(\alpha_2)$ as the equilibrium expected profits of Newsvendor 1 and Newsvendor 2, respectively, for this case. Similarly, denote $q_1^{fc}(\alpha_2)$ and $\hat{q}_2^{fc}(\alpha_2)$ as the equilibrium order quantities of Newsvendor 1 and Newsvendor 2, respectively, for this case. Then, $\hat{q}_2^{fc}(\alpha_2) = \hat{q}_2^c(\alpha_2) = \alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c$, and $q_1^{fc}(\alpha_2)$ is defined implicitly by $\mathrm{P}[X_1 + (X_2 - \hat{q}_2^{fc}(\alpha_2))^+ \le q_1^{fc}(\alpha_2)] = \beta$.

Accordingly, Newsvendor 2's equilibrium expected profit can be written as $\hat{\pi}_2^{fc}(\alpha_2) = p\mathrm{E}[(X_2 + (X_1 - q_1^{fc}(\alpha_2))^+) \wedge (\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c)] - c[\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c]$, which implies that $\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2} = (\mu - \hat{z}^c)p\{\mathrm{P}[\alpha_2\mu + (1-\alpha_2)\hat{z}^c < X_2 + (X_1 - q_1^{fc}(\alpha_2))^+] - (1-\beta)\} - \mathrm{E}\left[\mathbb{1}_{X_2+(X_1-q_1^{fc}(\alpha_2))^+<\alpha_2\bar{\mu}+(1-\alpha_2)\hat{z}^c, X_1>q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right] = (\bar{\mu} - \hat{z}^c)p\{\beta - \mathrm{P}[\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c > X_2 + (X_1 - q_1^{fc}(\alpha_2))^+]\} - \mathrm{E}\left[\mathbb{1}_{X_2+X_1-q_1^{fc}(\alpha_2)<\alpha_2\bar{\mu}+(1-\alpha_2)\hat{z}^c, X_1>q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right]$. This, in turns, implies that $\left.\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2=0} = (\bar{\mu} - \hat{z}^c)p\{\beta - \mathrm{P}[\hat{z}^c > X_2 + (X_1 - \hat{z}^c)^+]\} - \mathrm{E}[\mathbb{1}_{X_2+X_1-\hat{z}^c<\hat{z}^c, X_1>\hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] = -\mathrm{E}[\mathbb{1}_{X_2+X_1-\hat{z}^c<\hat{z}^c, X_1>\hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] > 0$, where the second equality follows because $\mathrm{P}[X_2 + (X_1 - \hat{z}^c)^+ < \hat{z}^c] = \beta$, and the inequality follows because $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \bar{\mu}$ from Lemma EC.11(a), $\hat{z}^c < \bar{\mu} \Longrightarrow \frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} = \frac{d\hat{q}_2^c(\alpha_2)}{d\alpha_2} = \bar{\mu} - \hat{z}^c < 0$, and $\frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} > 0 \Longrightarrow \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2} < 0$.

Next, note that Newsvendor 1's equilibrium expected profit can be written as $\pi_1^{fc}(\alpha_2) = p\mathrm{E}[(X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) \wedge q_1^{fc}(\alpha_2)] - cq_1^{fc}(\alpha_2)$. Thus, from the envelope theorem, $\frac{d\pi_1^{fc}(\alpha_2)}{d\alpha_2} = p\mathrm{E}\left[\mathbb{1}_{X_1+X_2-\alpha_2\bar{\mu}-(1-\alpha_2)\hat{z}^c<q_1^{fc}(\alpha_2), X_2>\alpha_2\bar{\mu}+(1-\alpha_2)\hat{z}^c}(\hat{z}^c - \bar{\mu})\right] < 0$, where the inequality is because, again, $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \bar{\mu}$ from Lemma EC.11(a).

Taken together, Newsvendor 2's equilibrium expected profit is strictly increasing in α_2 around $\alpha_2 = 0$, whereas Newsvendor 1's equilibrium expected profit is strictly decreasing in α_2 around $\alpha_2 = 0$. However, at $\alpha_2 = 0$, $\hat{\pi}_2^{fc}(\alpha_2) = \pi_1^{fc}(\alpha_2)$. Thus, all told, this means that $\hat{\pi}_2^{fc}(\alpha_2) > \pi_1^{fc}(\alpha_2)$ around $\alpha_2 = 0$, which suffices to complete the proof. Q.E.D.

Proof of Proposition EC.12 Because the equilibrium profits of the two newsvendors are identical, we have both newsvendors' expected equilibrium profits as $\hat{\pi}^c(\alpha) = \mu(p-c) - \frac{p-c}{2}\mathrm{E}[X_1 + X_2 -$

$2\hat{q}^c(\alpha)]^+ - \frac{c}{2}\mathrm{E}[2\hat{q}^c(\alpha) - X_1 - X_2]^+ = \mu(p-c) - (p-c)\mathrm{E}[\bar{X} - \alpha\rho - (1-\alpha)\hat{z}^c]^+ - c\mathrm{E}[\alpha\rho + (1-\alpha)\hat{z}^c - \bar{X}]^+$. Thus,

$$\begin{aligned}\frac{d\hat{\pi}^c(\alpha)}{d\alpha} &= (p-c)\bar{F}_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)(\rho - \hat{z}^c) + cF_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c)(\hat{z}^c - \rho)\\ &= (\rho - \hat{z}^c)[(p-c)\bar{F}_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c) - cF_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)]\\ &= (\rho - \hat{z}^c)[(p-c) - pF_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)]\\ &= p[\beta - F_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)](\rho - \hat{z}^c),\end{aligned}$$

and $\frac{d^2\hat{\pi}^c(\alpha)}{d\alpha^2} = -p(\hat{z}^c - \rho)^2 f_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c) < 0$. As a result, $\hat{\pi}^c(\alpha)$ is concave in α. Accordingly, there are three cases to consider.

Case (a): If $\beta \le \hat{m}^c$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = p(\beta - F_{\bar{X}}(\hat{z}^c))(\rho - \hat{z}^c) \le 0$ because $\hat{z}^c \le \rho$ and $\beta = g(\hat{z}^c) \le F_{\bar{X}}(\hat{z}^c)$. Therefore, if $\beta \le \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ is a decreasing function because of its concavity.

Case (b): If $\hat{m}^c < \beta \le m$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=1} = p(\beta - m)(\rho - \hat{z}^c) \ge 0$ because $\hat{z}^c > \rho \Longleftrightarrow \beta > \hat{m}^c$. Therefore, if $\hat{m}^c < \beta \le m$, then $\hat{\pi}^c(\alpha)$ is an increasing function because of its concavity.

Case (c): If $\beta > m$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = p(\beta - F_{\bar{X}}(\hat{z}^c))(\rho - \hat{z}^c) > 0$ because $g(\hat{z}^c) = \beta$ and $g(\cdot) \le F_{\bar{X}}(\cdot)$. Furthermore, $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=1} = p(\beta - m)(\rho - \hat{z}^c) < 0$. Therefore, if $\beta > m$, then $\hat{\pi}^c(\alpha)$ is an increasing-decreasing function. In order to let $\frac{d\hat{\pi}^c(\alpha)}{d\alpha} = 0$, we have $\beta - F_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c) = 0$. Since $g(\hat{z}^c) = \beta$, we can conclude that $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \rho)$. Q.E.D.

Proof of Proposition EC.13 For the overconfident newsvendors, the order quantities satisfy $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\rho + (2 - \alpha_1 - \alpha_2)q_n$. In contrast, the central planner's equilibrium order quantity is $2F_{\bar{X}}^{-1}(\beta)$. Thus, if $\beta > m$, then the system can be coordinated. In particular, $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = 2q_c^*$ when $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \mu)$. Q.E.D.

Proof of Proposition EC.14 To prove Proposition EC.14, we first show the following three-part lemma. Lemma EC.14(a) $\rho > \hat{z}^c \Longleftrightarrow \hat{m}^c > \beta$, Lemma EC.14(b) Newsvendor i's equilibrium expected profit is decreasing in α_i (i.e., $\frac{\partial\hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial\alpha_i} \le 0$, for $i = 1, 2$), and Lemma EC.14(c) Newsvendor i's equilibrium expected profit is decreasing in α_{3-i} (i.e., $\frac{\partial\hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial\alpha_{3-i}} < 0$ for $i = 1, 2$) if and only if $\beta < \hat{m}^c$.

Proof of Lemma EC.14(a): In the equilibrium, $g(z_1^c) = g(z_2^c) = \beta$. As a result, $\hat{z}^c = \hat{z}_1^c = \hat{z}_2^c$. Thus, $dq_i^c(\alpha)/d\alpha < 0 \iff \hat{z}^c > \rho \iff \beta > \hat{m}^c$.

Proof of Lemma EC.14(b): Without loss of generality, we let $i = 1$. First note that Newsvendor 1's profit is $\hat{\pi}_1^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+ \wedge (\alpha_1\rho + (1-\alpha_1)\hat{z}^c)] - c[\alpha_1\rho + (1-\alpha_1)\hat{z}^c]$. Thus,

$$\begin{aligned}\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_1} &= (\rho - \hat{z}^c)p[\mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c < X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) - (1-\beta)] \\ &= (\rho - \hat{z}^c)p[\beta - \mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+)]. \end{aligned} \quad \text{(EC.39)}$$

First, if $\hat{z}^c > \rho$, then $\mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\rho + (1-\alpha_1)\hat{z}^c < \hat{z}^c$, the second inequality is from $\alpha_2\rho + (1-\alpha_2)\hat{z}^c < \hat{z}^c$, and the equality is from Lemma 4. As a result, $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_1} < 0$ if $\hat{z}^c > \rho$. Second, if $\hat{z}^c \le \rho$, then $\mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c \ge X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) \ge \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) \ge \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\rho + (1-\alpha_1)\hat{z}^c \ge \hat{z}^c$, the second inequality is from $\alpha_2\rho + (1-\alpha_2)\hat{z}^c \ge \hat{z}^c$. As a result, $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_1} \le 0$ if $\hat{z}^c \le \rho$.

Proof of Lemma EC.14(c) Without loss of generality, we let $i = 1$. Note that $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_2} = p(\hat{z}^c - \rho)\mathrm{P}[\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c, X_2 \ge \alpha_2\rho + (1-\alpha_2)\hat{z}^c]$. Because $\mathrm{P}[\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - (1-\alpha_2)\hat{z}^c - \alpha_2\rho, X_2 \ge \alpha_2\rho + (1-\alpha_2)\hat{z}^c] \ge 0$, the sign of $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_2}$ is determined by $\hat{z}^c - \rho$. As a result, $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_2} < 0 \iff \rho > \hat{z}^c$. Furthermore, from Lemma EC.14(a), $\rho > \hat{z}^c \iff \hat{m}^c > \beta$.

If $\alpha_1 = \alpha_2$, then the newsvendors have identical expected profits. We next show that if $\beta < \hat{m}^c$, then the less biased newsvendor can earn a lower expected profit. To that end, suppose $\alpha_1 = 0$. Then, from (EC.39) and Lemma EC.14(c), $\frac{\partial\hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial\alpha_2} - \frac{\partial\hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial\alpha_2}\Big|_{\alpha_2=0} = (\rho - \hat{z}^c)p[\beta - \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+)] \quad + p(\rho - \hat{z}^c)\mathrm{P}(\hat{z}^c > X_1 + X_2 - \hat{z}^c, X_2 \ge \hat{z}^c) > 0$ because $\mathrm{P}(X_1 + (X_2 - \hat{z}^c)^+ < \hat{z}^c) = \beta$ and $\beta < \hat{m}^c \implies \rho > \hat{z}^c$ from Lemma EC.14(a). As a result, the more biased newsvendor (Newsvendor 2) can earn a higher expected profit than the less biased newsvendor (Newsvendor 1).

Now, define Newsvendor 2 as the biased newsvendor with overconfidence level α_2 and define Newsvendor 1 as a sophisticated newsvendor. Accordingly, denote $\pi_1^{fc}(\alpha_2)$ and $\hat{\pi}_2^{fc}(\alpha_2)$ as the equilibrium expected profits of Newsvendor 1 and Newsvendor 2, respectively, for this case. Similarly, denote $q_1^{fc}(\alpha_2)$ and $\hat{q}_2^{fc}(\alpha_2)$ as the equilibrium order quantities of Newsvendor 1 and Newsvendor 2, respectively, for this case. Then, $\hat{q}_2^{fc}(\alpha_2) = \hat{q}_2^{c}(\alpha_2) = \alpha_2\rho + (1-\alpha_2)\hat{z}^c$, and $q_1^{fc}(\alpha_2)$ is defined implicitly by

$$\mathrm{P}(X_1 + (X_2 - \hat{q}_2^{fc}(\alpha_2))^+ \leq q_1^{fc}(\alpha_2)) = \beta. \tag{EC.40}$$

Accordingly, Newsvendor 2's equilibrium expected profit $\hat{\pi}_2^{fc}(\alpha_2) = p\mathrm{E}\left[\left(X_2 + (X_1 - q_1^{fc}(\alpha_2))^+\right) \wedge (\alpha_2\rho + (1-\alpha_2)\hat{z}^c)\right] - c[\alpha_2\rho + (1-\alpha_2)\hat{z}^c]$, which implies that $\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2} = (\rho - \hat{z}^c)[\left\{\mathrm{P}\left[\alpha_2\rho + (1-\alpha_2)\hat{z}^c < X_2 + (X_1 - q_1^{fc}(\alpha_2))^+\right] - (1-\beta)\right\} - \mathrm{E}\left[\mathbb{1}_{X_2+(X_1-q_1^{fc}(\alpha_2))^+<\alpha_2\rho+(1-\alpha_2)\hat{z}^c, X_1>q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right] = (\rho - \hat{z}^c)p\{\beta - \mathrm{P}[\alpha_2\rho + (1-\alpha_2)\hat{z}^c > X_2 + (X_1 - q_1^{fc}(\alpha_2))^+]\} - \mathrm{E}\left[\mathbb{1}_{X_2+X_1-q_1^{fc}(\alpha_2)<\alpha_2\rho+(1-\alpha_2)\hat{z}^c, X_1>q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right]$. This, in turns, implies that $\left.\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2=0} = (\rho - \hat{z}^c)p\{\beta - \mathrm{P}[\hat{z}^c > X_2 + (X_1 - \hat{z}^c)^+]\} - \mathrm{E}[\mathbb{1}_{X_2+X_1-\hat{z}^c<\hat{z}^c, X_1>\hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] = -\mathrm{E}[\mathbb{1}_{X_2+X_1-\hat{z}^c<\hat{z}^c, X_1>\hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] > 0$, where the second equality follows because $\mathrm{P}\left[X_2 + (X_1 - \hat{z}^c)^+ < \hat{z}^c\right] = \beta$, and the inequality follows because $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \rho$, $\hat{z}^c < \rho \Longrightarrow \frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} = \frac{d\hat{q}_2^{c}(\alpha_2)}{d\alpha_2} = \rho - \hat{z}^c < 0$, and $\frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} > 0 \Longrightarrow \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2} < 0$ from (EC.40).

Next, note that Newsvendor 1's equilibrium expected profit can be written as $\pi_1^{fc}(\alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+ \wedge q_1^{fc}(\alpha_2)] - cq_1^{fc}(\alpha_2)$. Thus, from the envelope theorem, $\frac{d\pi_1^{fc}(\alpha_2)}{d\alpha_2} = p\mathrm{E}\left[\mathbb{1}_{X_1+X_2-\alpha_2\rho-(1-\alpha_2)\hat{z}^c<q_1^{fc}(\alpha_2), X_2>\alpha_2\rho+(1-\alpha_2)\hat{z}^c}(\hat{z}^c - \rho)\right] < 0$, where the inequality is because, again, $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \rho$ from Lemma EC.14(a).

Taken together, Newsvendor 2's equilibrium expected profit is strictly increasing in α_2 around $\alpha_2 = 0$, whereas Newsvendor 1's equilibrium expected profit is strictly decreasing in α_2 around $\alpha_2 = 0$. However, at $\alpha_2 = 0$, $\hat{\pi}_2^{fc}(\alpha_2) = \pi_1^{fc}(\alpha_2)$. Thus, all told, this means that $\hat{\pi}_2^{fc}(\alpha_2) > \pi_1^{fc}(\alpha_2)$ around $\alpha_2 = 0$, which suffices to establish Proposition EC.14. Q.E.D.