

Dynamic Conditional Beta is Alive and Well in the Cross-Section
of Daily Stock Returns

Online Appendix

I. Maximum Likelihood Estimation of the Dynamic Conditional Beta

We estimate the conditional covariances of each stock with the market portfolio ($\sigma_{im,t+1}$) based on the mean-reverting dynamic conditional correlation (DCC) model of Engle (2002). Engle defines the conditional correlation between two random variables r_1 and r_2 that each has zero mean as

$$\rho_{12,t} = \frac{E_{t-1}(r_{1,t} \cdot r_{2,t})}{\sqrt{E_{t-1}(r_{1,t}^2) \cdot E_{t-1}(r_{2,t}^2)}}, \quad (\text{A.1})$$

where the returns are defined as the conditional standard deviation times the standardized disturbance:

$$\sigma_{i,t}^2 = E_{t-1}(r_{i,t}^2), r_{i,t} = \sigma_{i,t} \cdot u_{i,t}, (i = 1, 2) \quad (\text{A.2})$$

where $u_{i,t}$ is a standardized disturbance that has zero mean and variance one for each series. Equations (A.1) and (A.2) indicate that the conditional correlation is also the conditional covariance between the standardized disturbances:

$$\rho_{12,t} = \frac{E_{t-1}(u_{1,t} \cdot u_{2,t})}{\sqrt{E_{t-1}(u_{1,t}^2) \cdot E_{t-1}(u_{2,t}^2)}} = E_{t-1}(u_{1,t} \cdot u_{2,t}). \quad (\text{A.3})$$

The conditional covariance matrix of returns is defined as

$$H_t = D_t \cdot \rho_t \cdot D_t, \text{ where } D_t = \text{diag} \left\{ \sqrt{\sigma_{i,t}^2} \right\}, \quad (\text{A.4})$$

where ρ_t is the time-varying conditional correlation matrix:

$$E_{t-1}(u_t \cdot u_t') = D_t^{-1} \cdot H_t \cdot D_t^{-1} = \rho_t, \text{ where } u_t = D_t^{-1} \cdot r_t. \quad (\text{A.5})$$

Engle (2002) introduces a mean-reverting DCC model:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} \cdot q_{jj,t}}}, \quad (\text{A.6})$$

$$q_{ij,t} = \bar{\rho}_{ij} + a_1 \cdot (u_{i,t-1} \cdot u_{j,t-1} - \bar{\rho}_{ij}) + a_2 \cdot (q_{ij,t-1} - \bar{\rho}_{ij}), \quad (\text{A.7})$$

where $\bar{\rho}_{ij}$ is the unconditional correlation between $u_{i,t}$ and $u_{j,t}$. Equation (A.7) indicates that the conditional correlation is mean reverting towards $\bar{\rho}_{ij}$ as long as $a_1 + a_2 < 1$. Engle (2002) assumes that each asset follows a univariate GARCH process and writes the log likelihood function as:

$$\begin{aligned} L &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + \ln |H_t| + r_t' H_t^{-1} r_t) \\ &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + 2 \ln |H_t| + r_t' D_t^{-1} D_t^{-1} r_t - u_t' u_t + \ln |\rho_t| + u_t' \rho_t^{-1} u_t). \end{aligned} \quad (\text{A.8})$$

As shown in Engle (2002), letting the parameters in D_t be denoted by θ and the additional parameters in ρ_t be denoted by φ , equation (A.8) can be written as the sum of a volatility part and a correlation part:

$$L(\theta, \varphi) = L_V(\theta) + L_C(\theta, \varphi). \quad (\text{A.9})$$

The volatility term is

$$L_V(\theta) = -\frac{1}{2} \sum_{t=1}^T \left(n \ln(2\pi) + \ln |D_t|^2 + r_t' D_t^{-2} r_t \right), \quad (\text{A.10})$$

and the correlation component is

$$L_C(\theta, \varphi) = -\frac{1}{2} \sum_{t=1}^T \left(\ln |\rho_t| + u_t' \rho_t^{-1} u_t - u_t' u_t \right). \quad (\text{A.11})$$

The volatility part of the likelihood is the sum of individual GARCH likelihoods:

$$L_V(\theta) = -\frac{1}{2} \sum_{t=1}^T \sum_{i=1}^n \left(\ln(2\pi) + \ln(\sigma_{i,t}^2) + \frac{r_{i,t}^2}{\sigma_{i,t}^2} \right), \quad (\text{A.12})$$

which is jointly maximized by separately maximizing each term. The second part of the likelihood is used to estimate the correlation parameters. The two-step approach to maximizing the likelihood is to find

$$\hat{\theta} = \operatorname{argmax} \{L_V(\theta)\}, \quad (\text{A.13})$$

and then take this value as given in the second stage:

$$\max_{\varphi} \left\{ L_C(\hat{\theta}, \varphi) \right\}. \quad (\text{A.14})$$

We estimate the conditional covariances of each stock with the market portfolio using the maximum likelihood method described above. The conditional variance of the market portfolio is estimated by maximizing the univariate GARCH likelihood. As presented in equation (17), the dynamic conditional beta of stock i is defined as the conditional covariance between stock i and the market, divided by the conditional variance of the market.

II. Descriptions of the Control Variables

We use a large set of control variables in the cross-sectional asset pricing tests. Unless otherwise stated, all variables are measured on the portfolio formation date (i.e., day d). We require that a minimum of 200 (15) daily observations be available for all variables computed from daily data in the past 252 (21) trading days, respectively.

Size: The firm's size is computed as the product of the price per share and the number of shares outstanding (in million dollars).

Book-to-Market: Following Fama and French (1992, 1993, and 2000), the book-to-market equity ratio at the end of June of year t , denoted BM , is computed as the book value of stockholders' equity, plus deferred taxes and investment tax credit (if available), minus the book value of preferred stock for the fiscal year $t - 1$, scaled by the market value of equity at end of December of $t - 1$. Depending on availability, the redemption, liquidation, or par value (in that order) is used to estimate the book value of preferred stock.

Reversal: Following Jegadeesh (1990), short-term reversal (REV) is defined as the stock return over the prior 21 trading days. In addition to the monthly return reversal, we also control for daily and weekly return reversals. Specifically, we control for 1-, 2-, 3-, 4-, and 5-day lagged returns in Fama-MacBeth regressions.

Momentum: Following Jegadeesh and Titman (1993), momentum (MOM) is the cumulative return of a stock in the past 252 trading days ending 5 trading days prior to the portfolio formation month (i.e., skipping the short-term reversal month).

Illiquidity: We use the Amihud's (2002) illiquidity measure and for each day we measure the illiquidity of a stock i over the past five days, denoted $ILLIQ$, as the average daily ratio of the absolute stock return to the dollar trading volume within a five-day period:

$$ILLIQ_i = \frac{1}{n} \sum_{n=1}^5 \left[\frac{|R_{i,d}|}{VOLD_{i,d}} \right], \quad (\text{A.15})$$

where $R_{i,d}$ and $VOLD_{i,d}$ are the daily return and dollar trading volume for stock i on day d , respectively.¹⁸

¹⁸At an earlier stage of the study, we use the original illiquidity measure of Amihud (2002) defined as the average daily ratio of the absolute stock return to the dollar trading volume within a 21-day period. The results from the 21-day $ILLIQ$ measure turn out to be similar to those reported in our tables.

Turnover: Share turnover, denoted $TURN$, is measured as the average daily ratio of the number of shares traded to the total shares outstanding within a five-day period. $TURN$ is an alternative measure of liquidity - the higher the share turnover, the easier for investors to buy or sell shares of the stock.

Co-skewness: Following Harvey and Siddique (2000), the firm's daily co-skewness ($COSKEW$) is defined as the estimate of γ_i in the regression using the daily return observations over the prior 21 trading days:

$$R_{i,d} - r_{f,d} = \alpha_i + \beta_i (R_{m,d} - r_{f,d}) + \gamma_i (R_{m,d} - r_{f,d})^2 + \epsilon_{i,d}, \quad (\text{A.16})$$

where $R_{i,d}$, $R_{f,d}$, and $R_{m,d}$ are the daily returns on stock i , the one-month Treasury bills, and the CRSP value-weighted index, respectively.

Idiosyncratic Volatility: Following Ang, Hodrick, Xing, and Zhang (2006), the daily idiosyncratic volatility of stock i ($IVOL$) is computed as the standard deviation of the residuals from the regression using daily returns over the prior 21 trading days:

$$R_{i,d} - r_{f,d} = \alpha_i + \beta_i (R_{m,d} - r_{f,d}) + \gamma_i SMB_d + \phi_i HML_d + \epsilon_{i,d}, \quad (\text{A.17})$$

where SMB_d and HML_d are, respectively, the daily size and book-to-market factors of Fama and French (1993).

Maximum Daily Return: Following Bali, Cakici, and Whitelaw (2011), the firm's extreme positive return (MAX) is defined as the average of five highest daily returns over the past 21 days.

Table A1
Testing the Difference between the Conditional and Unconditional Beta Premia

In Panel A, for each day, individual stocks are sorted into 10 univariate decile portfolios on the difference between the time-varying conditional beta and the unconditional beta estimated from either the CAPM ($DCC - CAPM$), the Fama-French model ($DCC - FF$), the Scholes-Williams model ($DCC - SW$), or the Dimson model ($DCC - Dimson$). Columns “RET” report the equal-weighted average excess returns for the decile portfolios formed on $DCC - CAPM$, $DCC - FF$, $DCC - SW$, and $DCC - Dimson$, respectively. Columns “ $DCC - CAPM$ ”, “ $DCC - FF$ ”, “ $DCC - SW$ ”, and “ $DCC - Dimson$ ” report the average difference between the conditional beta and the corresponding unconditional beta in each decile. The last two rows present the average return difference between Decile 10 and Decile 1 (High-Low) and alphas. Daily returns and alphas are converted into monthly percentage terms. Panel B reports the average slope coefficients from the Fama-MacBeth regressions of the one-day-ahead excess returns of individual stocks on the time-varying conditional beta estimates ($BETA_{DCC}$) after controlling for each unconditional beta measure. The Newey-West t -statistics are reported in parentheses. The sample covers the period from July 1963 to December 2013.

Panel A. Univariate sorts on the difference between the conditional beta and the unconditional beta												
Decile	RET	$DCC - CAPM$	RET	$DCC - FF$	RET	$DCC - SW$	RET	$DCC - Dimson$	RET	$DCC - SW$	RET	$DCC - Dimson$
1 (Low)	0.12	-0.61	-0.07	-0.86	-0.06	-0.88	-0.19	-1.57	0.23	-0.45	0.23	-0.86
2	0.14	-0.25	0.15	-0.45	0.14	-0.45	0.23	-0.86	0.42	-0.30	0.42	-0.59
3	0.17	-0.14	0.29	-0.31	0.28	-0.30	0.47	-0.41	0.58	-0.11	0.58	-0.25
4	0.20	-0.07	0.41	-0.21	0.38	-0.20	0.62	-0.11	0.60	0.06	0.60	0.04
5	0.28	-0.02	0.46	-0.13	0.45	-0.11	0.69	0.22	0.69	0.16	0.69	0.22
6	0.36	0.03	0.47	-0.05	0.50	-0.03	0.74	0.46	0.74	0.33	0.74	0.46
7	0.49	0.09	0.58	0.03	0.58	0.06	1.00	1.14	1.00	0.84	1.00	1.14
8	0.68	0.16	0.71	0.14	0.68	0.16	1.20		1.20		1.20	
9	1.03	0.29	0.84	0.30	0.84	0.33	(7.51)		(7.51)		(7.51)	
10 (High)	1.71	0.78	1.33	0.81	1.38	0.84	1.41		1.41		1.41	
High-Low	1.60		1.40		1.44		(6.89)		(7.97)		(7.51)	
Alpha	1.65		1.41		1.41		(6.64)		(6.94)		1.15	
			(7.09)		(6.94)						(6.73)	

Table A1 – continued

Panel B. Fama-MacBeth cross-sectional regressions

	(1)	(2)	(3)	(4)
Intercept	0.0260 (2.78)	0.0273 (3.02)	0.0263 (2.92)	0.0229 (2.51)
$BETA_{DCC}$	0.0569 (6.50)	0.0417 (6.42)	0.0422 (6.16)	0.0273 (4.84)
$BETA_{CAPM}$	-0.0566 (-5.23)			
$BETA_{FF}$		-0.0368 (-4.31)		
$BETA_{SW}$			-0.0373 (-4.44)	
$BETA_{Dimson}$				-0.0169 (-4.21)

Table A2
Fama-MacBeth Cross-Sectional Regressions: Predicting one-day-ahead Returns with the Dynamic Conditional Beta for Different Stock Samples

The one-day-ahead excess returns of individual stocks are regressed every day on the time-varying conditional beta estimates ($BETA_{DCC}$) after controlling for a large set of cross-sectional return predictors. The control variables are the natural logarithm of firm's market capitalization (SIZE), book-to-market equity ratio (BM), momentum (MOM), Amihud's illiquidity measure (ILLIQ), share turnover (TURN), idiosyncratic volatility (IVOL), short-term reversal (REV), maximum daily return (MAX), and co-skewness (COSKEW). Nine stock samples are examined: (1) S&P500 stocks, (2) big stocks, (3) small stocks, (4) 500 largest stocks, (5) 1,000 largest stocks, (6) 500 most liquid stocks, (7) 1,000 most liquid stocks, (8) the pooled sample of 500 largest and 500 most liquid stocks, and (9) the pooled sample of 1,000 largest and 1,000 most liquid stocks. This table reports the average slope coefficients from the Fama-MacBeth regressions and the Newey-West t -statistics (in parentheses). The sample covers the period from July 1963 to December 2013.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0322 (1.28)	0.0470 (2.00)	-0.0218 (-1.00)	0.0534 (1.93)	0.0526 (2.27)	0.0704 (3.11)	0.0536 (2.43)	0.0389 (1.70)	0.0462 (2.15)
$BETA_{DCC}$	0.0221 (3.08)	0.0191 (2.83)	0.0238 (3.78)	0.0144 (1.99)	0.0192 (3.00)	0.0155 (2.16)	0.0211 (3.12)	0.0225 (3.10)	0.0221 (3.32)
SIZE	-0.0023 (-0.95)	-0.0027 (-1.03)	0.0066 (1.97)	-0.0032 (-1.15)	-0.0034 (-1.34)	-0.0055 (-2.22)	-0.0034 (-1.35)	-0.0023 (-0.96)	-0.0026 (-1.10)
BM	0.0048 (1.12)	0.0077 (1.97)	0.0322 (7.19)	0.0055 (1.40)	0.0095 (2.42)	0.0077 (2.01)	0.0100 (2.60)	0.0081 (2.11)	0.0102 (2.67)
MOM	0.0002 (1.37)	0.0003 (3.04)	0.0002 (3.19)	0.0003 (2.71)	0.0003 (3.49)	0.0003 (3.06)	0.0002 (2.72)	0.0002 (2.59)	0.0002 (2.79)
IVOL	-0.0153 (-3.05)	-0.0158 (-4.24)	0.0268 (8.48)	-0.0244 (-5.74)	-0.0150 (-4.08)	-0.0227 (-5.01)	-0.0129 (-2.93)	-0.0119 (-2.68)	-0.0112 (-2.52)
ILLIQ	-0.3346 (-0.76)	-0.4227 (-1.35)	0.0082 (6.73)	-2.5346 (-2.86)	-0.5785 (-2.10)	-1.5808 (-2.68)	-0.3114 (-1.53)	-0.5534 (-1.38)	-0.3658 (-1.68)
TURN	0.1174 (5.38)	0.0639 (5.50)	0.2036 (6.09)	0.0626 (4.93)	0.0708 (5.92)	0.0473 (5.48)	0.0607 (5.99)	0.0598 (5.65)	0.0683 (5.76)
REV	-0.0070 (-11.50)	-0.0061 (-11.92)	-0.0144 (-12.40)	-0.0065 (-13.03)	-0.0062 (-11.61)	-0.0057 (-12.40)	-0.0054 (-10.28)	-0.0061 (-11.84)	-0.0059 (-10.77)
MAX	0.0030 (0.78)	0.0038 (1.22)	-0.0228 (-10.18)	0.0113 (2.88)	0.0039 (1.29)	0.0129 (3.20)	0.0019 (0.55)	0.0036 (1.01)	0.0007 (0.21)
COSKEW	-0.0002 (-1.95)	-0.0002 (-2.08)	0.0000 (-0.63)	-0.0001 (-1.40)	-0.0001 (-2.07)	-0.0001 (-1.04)	-0.0001 (-1.64)	-0.0001 (-1.46)	-0.0001 (-1.55)

Table A3
Univariate Value-Weighted Portfolios of Stocks Sorted by the Dynamic Conditional Beta

For each day, stocks are sorted into 10 univariate decile portfolios based on the time-varying conditional beta ($BETA_{DCC}$), estimated using the dynamic conditional correlation (DCC) model of Engle (2002). The $BETA_{DCC}$ measures are estimated using daily returns over the prior 252 trading days with at least 200 observations available. This table reports the value-weighted average excess returns (RET) and the corresponding alphas. The last row presents the average return difference between the High-beta and Low-beta deciles (High-Low). Daily returns and alphas are converted into monthly percentage terms. The Newey-West t -statistics are reported in parentheses. The sample covers the period from July 1963 to December 2013.

Decile	RET	Alpha
1 (Low)	0.35 (2.01)	0.30 (1.82)
2	0.23 (1.40)	0.21 (1.35)
3	0.34 (1.92)	0.35 (2.07)
4	0.37 (2.08)	0.41 (2.43)
5	0.42 (2.42)	0.49 (2.97)
6	0.53 (3.05)	0.63 (3.65)
7	0.50 (2.64)	0.63 (3.36)
8	0.49 (2.19)	0.67 (3.12)
9	0.54 (1.93)	0.75 (3.06)
10 (High)	0.91 (2.74)	1.18 (4.45)
High–Low	0.56 (2.03)	0.89 (3.80)

Table A4

Fama-MacBeth Cross-Sectional Regressions: Predicting one-day-ahead Returns with the Dynamic Conditional Beta for the Full CRSP Universe

The one-day-ahead excess returns of all stocks in the CRSP universe are regressed every day on the time-varying conditional beta estimates ($BETA_{DCC}$) after controlling for a large set of cross-sectional return predictors. The control variables are the natural logarithm of firm's market capitalization (SIZE), book-to-market equity ratio (BM), momentum (MOM), Amihud's illiquidity measure (ILLIQ), share turnover (TURN), idiosyncratic volatility (IVOL), short-term reversal (REV), maximum daily return (MAX), and co-skewness (COSKEW). This table reports the average slope coefficients from the Fama-MacBeth regressions and the Newey-West t -statistics (in parentheses). The sample covers the period from July 1963 to December 2013.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Intercept	0.0225 (1.96)	0.0645 (3.44)	0.0975 (5.57)	0.0293 (1.61)	0.0649 (3.40)	0.0346 (1.54)	0.1936 (8.89)	0.0642 (3.41)	-0.0096 (-0.50)	-0.0334 (-1.68)
$BETA_{DCC}$	0.0188 (3.36)	0.0310 (5.31)	0.0344 (6.32)	0.0269 (4.87)	0.0241 (4.39)	0.0280 (4.07)	0.0612 (8.50)	0.0317 (5.43)	0.0251 (4.08)	0.0301 (3.84)
SIZE		-0.0108 (-3.97)	-0.0141 (-5.99)	-0.0040 (-1.60)	-0.0118 (-4.19)	-0.0035 (-1.20)	-0.0242 (-7.73)	-0.0108 (-3.97)	0.0046 (1.97)	0.0073 (2.99)
BM		0.0161 (5.22)	0.0145 (4.69)	0.0152 (4.99)	0.0157 (4.84)	0.0259 (6.59)	0.0112 (3.48)	0.0162 (5.20)	0.0270 (6.35)	0.0277 (6.46)
MOM		0.0004 (5.92)	0.0004 (5.63)	0.0004 (6.77)	0.0003 (5.51)	0.0003 (4.20)	0.0002 (3.66)	0.0004 (5.99)	0.0003 (3.49)	0.0004 (5.32)
IVOL			-0.0088 (-4.02)						0.0231 (7.73)	0.0213 (7.68)
ILLIQ				0.0482 (5.32)					0.0069 (7.29)	0.0108 (10.39)
TURN					0.0724 (5.01)				0.1599 (6.19)	0.1764 (6.33)
REV						-0.0119 (-12.79)			-0.0128 (-12.55)	-0.0034 (-7.27)
MAX							-0.0272 (-12.00)		-0.0215 (-10.34)	-0.0204 (-9.37)
COSKEW								-0.0001 (-1.19)	0.0000 (-0.60)	-0.0660 (-9.93)
REV1										-0.0139 (-6.91)
REV2										-0.0085 (-12.11)
REV3										-0.0057 (-9.76)
REV4										-0.0085 (-10.27)
REV5										0.0001 (1.40)

Table A5
Time-series and Cross-Sectional Relation between the DCC Beta and Equity Portfolio Returns

Entries report the common slope estimates (A) and their t -statistics (in parentheses) from the following system of equations, $R_{i,t+1} = C_i + A\sigma_{im,t+1} + \varepsilon_{i,t+1}, i = 1, 2, \dots, n$, where $R_{i,t+1}$ denotes the daily excess return on equity portfolio i at time $t + 1$, $R_{m,t+1}$ denotes the daily excess return on the market portfolio at time $t + 1$, and $\sigma_{im,t+1}$ is the time- t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$. C_i is the intercept for equity portfolio i and A is the common slope coefficient. Estimation is based on daily data on 11 different samples of equity portfolios over the sample period of July 1, 1963 to December 31, 2013; 10 size, 10 book-to-market (bm), 10 momentum (mom), 10 short-term reversal (str), 10 long-term reversal (ltr), 10 operating profitability (op), 10 investment (inv), 10 industry, 30 industry (ind), the pooled sample of 10size/10bm/10mom/10ind (total of 40) portfolios, the pooled sample of 10size/10bm/10mom/30ind (total of 60) portfolios, the pooled sample of 10size/10bm/10mom/10str/10ltr/10op/10inv/10ind (total of 80) portfolios, and the pooled sample of 10size/10bm/10mom/10str/10ltr/10op/10inv/30ind (total of 100) portfolios. This table reports the common slope coefficients and their t -statistics in parentheses. The t -statistics are corrected for heteroskedasticity and autocorrelation for each series and contemporaneous cross-correlations among the error terms.

Equity Portfolios	Common Slope
10 SIZE	0.6225 (2.02)
10 BM	1.3266 (4.36)
10 MOM	3.1063 (10.44)
10 IND	1.5338 (5.11)
10 STR	3.7253 (12.59)
10 LTR	1.0919 (3.32)
10 OP	1.3488 (4.03)
10 INV	1.2061 (3.54)
30 IND	2.1904 (8.78)
40 SIZE-BM-MOM-IND	1.8803 (9.69)
60 SIZE-BM-MOM-IND30	2.1297 (11.74)
80 SIZE-BM-MOM-IND-STR-LTR-OP-INV	2.3306 (15.27)
100 SIZE-BM-MOM-IND30-STR-LTR-OP-INV	2.4596 (16.78)

Table A6
Fama-MacBeth Cross-Sectional Regressions: Predicting Cumulative Returns over Two to Five days with the Conditional Beta for the S&P500 stocks and the pooled sample of 500 largest and most liquid stocks

The 2-, 3-, 4-, and 5-day cumulative excess returns of stocks in the S&P500 index (Panel A) and the pooled sample of the 500 largest and 500 most liquid stocks (Panel B) are regressed every day on the time-varying conditional beta estimates ($BETA_{DCC}$) after controlling for a large set of cross-sectional return predictors. This table reports the average slope coefficients from the Fama-MacBeth regressions and the Newey-West t -statistics (in parentheses). The sample covers the period from July 1963 to December 2013.

variable	A1. Cumulative returns over two days								A2. Cumulative returns over three days							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.1583 (4.18)	0.1907 (5.08)	0.1329 (2.97)	0.1333 (3.30)	0.1335 (3.16)	0.2967 (7.40)	0.1580 (4.11)	0.1013 (2.09)	0.2415 (4.24)	0.2879 (5.07)	0.2247 (3.38)	0.2100 (3.46)	0.2106 (3.37)	0.4273 (7.03)	0.2417 (4.18)	0.1812 (2.57)
$BETA_{DCC}$	0.0294 (2.65)	0.0360 (3.47)	0.0242 (2.09)	0.0198 (1.88)	0.0374 (2.97)	0.0833 (6.36)	0.0300 (2.74)	0.0205 (1.84)	0.0291 (1.88)	0.0379 (2.67)	0.0237 (1.48)	0.0165 (1.13)	0.0391 (2.23)	0.1008 (5.95)	0.0291 (1.89)	0.0129 (0.85)
SIZE	-0.0179 (-4.34)	-0.0207 (-5.27)	-0.0136 (-3.06)	-0.0154 (-3.63)	-0.0137 (-3.08)	-0.0295 (-6.50)	-0.0179 (-4.31)	-0.0077 (-1.67)	-0.0256 (-4.16)	-0.0297 (-5.06)	-0.0222 (-3.34)	-0.0225 (-3.53)	-0.0201 (-3.06)	-0.0411 (-6.17)	-0.0256 (-4.15)	-0.0140 (-2.09)
BM	0.0039 (0.57)	0.0029 (0.42)	0.0026 (0.37)	0.0049 (0.70)	0.0102 (1.25)	0.0022 (0.31)	0.0037 (0.53)	0.0088 (1.04)	0.0055 (0.53)	0.0043 (0.42)	0.0043 (0.42)	0.0068 (0.66)	0.0144 (1.19)	0.0032 (0.31)	0.0050 (0.49)	0.0133 (1.07)
MOM	0.0005 (2.12)	0.0005 (2.38)	0.0005 (2.26)	0.0005 (2.04)	0.0004 (1.46)	0.0004 (1.65)	0.0005 (2.22)	0.0003 (1.18)	0.0007 (2.06)	0.0008 (2.32)	0.0008 (2.15)	0.0007 (1.97)	0.0006 (1.48)	0.0006 (1.66)	0.0008 (2.15)	0.0004 (1.22)
IVOL	-0.0170 (-3.18)							-0.0399 (-4.67)	-0.0240 (-3.01)							-0.0628 (-5.11)
ILLIQ			-9.8348 (-1.38)					-0.7165 (-0.88)			-17.6987 (-1.68)					-1.3799 (-1.15)
TURN				0.1294 (3.92)				0.2059 (5.31)				0.1708 (3.68)				0.2779 (5.13)
REV					-0.0117 (-9.42)			-0.0136 (-10.88)					-0.0160 (-8.95)			-0.0192 (-10.06)
MAX						-0.0481 (-8.21)		0.0176 (2.61)						-0.0648 (-7.76)		0.0331 (3.39)
COSKEW							-0.0002 (-1.07)	-0.0002 (-1.24)							-0.0002 (-0.99)	-0.0002 (-0.94)

Table A6 – continued

variable	A3. Cumulative returns over four days								A4. Cumulative returns over five days							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.3229 (4.23)	0.3818 (5.02)	0.3157 (3.60)	0.2858 (3.54)	0.2875 (3.50)	0.5445 (6.74)	0.3242 (4.19)	0.2623 (2.87)	0.4046 (4.23)	0.4762 (5.01)	0.4045 (3.75)	0.3628 (3.61)	0.3666 (3.61)	0.6535 (6.51)	0.4064 (4.19)	0.3454 (3.13)
<i>BETA_{BC}</i>	0.0280 (1.38)	0.0384 (2.11)	0.0225 (1.09)	0.0132 (0.69)	0.0394 (1.75)	0.1131 (5.42)	0.0272 (1.35)	0.0070 (0.36)	0.0237 (0.95)	0.0357 (1.61)	0.0189 (0.74)	0.0075 (0.32)	0.0358 (1.30)	0.1193 (4.91)	0.0230 (0.92)	-0.0012 (-0.05)
SIZE	-0.0330 (-4.02)	-0.0381 (-4.87)	-0.0305 (-3.47)	-0.0293 (-3.47)	-0.0265 (-3.07)	-0.0514 (-5.88)	-0.0331 (-4.02)	-0.0205 (-2.34)	-0.0402 (-3.92)	-0.0462 (-4.75)	-0.0384 (-3.53)	-0.0360 (-3.43)	-0.0331 (-3.09)	-0.0609 (-5.65)	-0.0403 (-3.92)	-0.0272 (-2.54)
BM	0.0073 (0.53)	0.0058 (0.43)	0.0062 (0.45)	0.0090 (0.65)	0.0181 (1.15)	0.0045 (0.33)	0.0067 (0.49)	0.0175 (1.07)	0.0092 (0.54)	0.0076 (0.45)	0.0084 (0.49)	0.0112 (0.65)	0.0217 (1.12)	0.0061 (0.35)	0.0084 (0.49)	0.0212 (1.06)
MOM	0.0010 (2.04)	0.0010 (2.31)	0.0010 (2.10)	0.0009 (1.96)	0.0008 (1.53)	0.0008 (1.71)	0.0010 (2.13)	0.0006 (1.31)	0.0012 (2.00)	0.0013 (2.26)	0.0012 (2.05)	0.0011 (1.92)	0.0010 (1.54)	0.0010 (1.72)	0.0012 (2.09)	0.0008 (1.37)
IVOL		-0.0305 (-2.89)						-0.0807 (-5.05)		-0.0364 (-2.78)						-0.0962 (-5.02)
ILLIQ			-24.6911 (-1.80)					-1.7820 (-1.15)			-29.8553 (-1.82)					-2.0358 (-1.13)
TURN				0.2055 (3.54)				0.3390 (5.05)				0.2346 (3.48)				0.3913 (5.03)
REV					-0.0191 (-8.55)			-0.0233 (-9.40)					-0.0216 (-8.27)			-0.0266 (-9.03)
MAX						-0.0779 (-7.39)		0.0436 (3.41)						-0.0881 (-7.06)		0.0522 (3.37)
COSKEW							-0.0002 (-0.81)	-0.0001 (-0.59)							-0.0002 (-0.70)	-0.0001 (-0.33)

Table A6 – continued

variable	B1. Cumulative returns over two days								B2. Cumulative returns over three days							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.1318 (3.64)	0.1667 (4.63)	0.1257 (3.02)	0.1293 (3.47)	0.1193 (3.03)	0.2490 (6.42)	0.1333 (3.67)	0.1082 (2.49)	0.2105 (3.84)	0.2610 (4.79)	0.2222 (3.58)	0.2134 (3.79)	0.1953 (3.31)	0.3702 (6.34)	0.2132 (3.88)	0.1993 (3.13)
<i>BETA_{DCC}</i>	0.0299 (2.59)	0.0376 (3.53)	0.0263 (2.23)	0.0259 (2.33)	0.0361 (2.75)	0.0798 (6.39)	0.0304 (2.65)	0.0235 (2.07)	0.0323 (1.96)	0.0438 (2.96)	0.0295 (1.77)	0.0280 (1.77)	0.0398 (2.16)	0.1006 (6.03)	0.0326 (1.99)	0.0219 (1.41)
SIZE	-0.0145 (-3.36)	-0.0176 (-4.25)	-0.0124 (-2.73)	-0.0141 (-3.26)	-0.0121 (-2.61)	-0.0243 (-5.16)	-0.0147 (-3.43)	-0.0078 (-1.70)	-0.0222 (-3.43)	-0.0266 (-4.29)	-0.0220 (-3.24)	-0.0223 (-3.44)	-0.0189 (-2.76)	-0.0354 (-5.11)	-0.0226 (-3.51)	-0.0157 (-2.30)
BM	0.0122 (1.83)	0.0107 (1.62)	0.0107 (1.62)	0.0119 (1.83)	0.0160 (2.09)	0.0086 (1.28)	0.0119 (1.79)	0.0144 (1.92)	0.0163 (1.67)	0.0146 (1.51)	0.0152 (1.57)	0.0157 (1.65)	0.0218 (1.96)	0.0117 (1.19)	0.0159 (1.64)	0.0208 (1.91)
MOM	0.0005 (2.76)	0.0006 (3.15)	0.0006 (2.99)	0.0005 (2.64)	0.0005 (2.60)	0.0005 (2.78)	0.0005 (2.93)	0.0004 (2.33)	0.0008 (2.68)	0.0009 (3.07)	0.0008 (2.86)	0.0007 (2.59)	0.0008 (2.57)	0.0008 (2.71)	0.0008 (2.82)	0.0007 (2.33)
IVOL		-0.0172 (-3.75)					-0.0337 (-4.52)			-0.0257 (-3.75)					-0.0531 (-4.99)	
ILLIQ			-17.7736 (-2.82)				-1.0801 (-1.57)				-28.1426 (-3.00)					-1.8875 (-1.88)
TURN				0.0445 (3.41)			0.1071 (5.61)					0.0561 (2.98)				0.1453 (5.38)
REV					-0.0101 (-9.59)		-0.0121 (-11.10)						-0.0138 (-9.04)			-0.0169 (-10.16)
MAX						-0.0418 (-8.37)	0.0192 (3.05)							-0.0575 (-8.00)		0.0331 (3.65)
COSKEW							-0.0001 (-0.69)								-0.0002 (-0.70)	0.0000 (-0.15)

Table A6 – continued

variable	B3. Cumulative returns over four days								B4. Cumulative returns over five days							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.2862 (3.88)	0.3518 (4.83)	0.3219 (3.88)	0.2968 (3.94)	0.2684 (3.42)	0.4785 (6.20)	0.2898 (3.93)	0.2939 (3.50)	0.3643 (3.94)	0.4425 (4.86)	0.4226 (4.13)	0.3825 (4.08)	0.3452 (3.55)	0.5822 (6.09)	0.3666 (3.96)	0.3908 (3.82)
<i>BETA_{BCC}</i>	0.0346 (1.60)	0.0499 (2.60)	0.0335 (1.54)	0.0307 (1.49)	0.0430 (1.80)	0.1172 (5.60)	0.0349 (1.62)	0.0231 (1.16)	0.0321 (1.21)	0.0507 (2.16)	0.0328 (1.23)	0.0294 (1.17)	0.0406 (1.40)	0.1260 (5.13)	0.0327 (1.23)	0.0205 (0.87)
SIZE	-0.0295 (-3.42)	-0.0350 (-4.25)	-0.0319 (-3.52)	-0.0304 (-3.53)	-0.0256 (-2.83)	-0.0453 (-5.00)	-0.0300 (-3.50)	-0.0241 (-2.67)	-0.0367 (-3.39)	-0.0430 (-4.18)	-0.0416 (-3.69)	-0.0384 (-3.57)	-0.0324 (-2.88)	-0.0545 (-4.88)	-0.0369 (-3.44)	-0.0326 (-2.91)
BM	0.0204 (1.58)	0.0183 (1.44)	0.0195 (1.53)	0.0192 (1.53)	0.0272 (1.86)	0.0148 (1.15)	0.0198 (1.54)	0.0263 (1.85)	0.0244 (1.53)	0.0222 (1.41)	0.0235 (1.49)	0.0226 (1.46)	0.0323 (1.80)	0.0179 (1.13)	0.0239 (1.50)	0.0313 (1.79)
MOM	0.0010 (2.70)	0.0011 (3.09)	0.0011 (2.85)	0.0010 (2.64)	0.0011 (2.63)	0.0011 (2.76)	0.0011 (2.84)	0.0009 (2.41)	0.0013 (2.67)	0.0014 (3.07)	0.0013 (2.81)	0.0012 (2.62)	0.0013 (2.61)	0.0013 (2.77)	0.0013 (2.82)	0.0012 (2.46)
IVOL		-0.0340 (-3.79)						-0.0680 (-4.89)		-0.0411 (-3.70)						-0.0825 (-4.94)
ILLIQ			-39.4777 (-3.13)					-2.6182 (-2.04)			-49.0036 (-3.10)					-3.2586 (-2.15)
TURN				0.0648 (2.64)				0.1769 (5.13)				0.0717 (2.43)				0.2018 (4.97)
REV					-0.0165 (-8.58)			-0.0202 (-9.33)						-0.0184 (-8.21)		-0.0229 (-8.84)
MAX						-0.0698 (-7.79)		0.0421 (3.48)						-0.0796 (-7.47)		0.0508 (3.41)
COSKEW							-0.0002 (-0.56)	0.0000 (0.16)							-0.0001 (-0.45)	0.0001 (0.29)

Table A7
Dynamic Conditional Beta, Investor Attention, and Buying Intensity

Panel A reports the equal-weighted average excess returns (RET) and the alphas of decile portfolios formed based on the daily fraction of buyer-initiated trades (BUY). Panel B reports the equal-weighted average excess returns (RET) and the alphas of decile portfolios sorted by DCC-beta. Panel C reports the equal-weighted average excess returns and the alphas of decile portfolios sorted by DCC-beta after controlling for the fraction of buyer-initiated trades (BUY). In Panels A through C, daily returns and alphas are converted into monthly percentage terms. The Newey-West t -statistics are reported in parentheses. The sample covers the period from January 1993 to December 2013.

Panel A. Univariate Portfolios of BUY			Panel B. Univariate Portfolios of β^{DCC}			Panel C. Bivariate Portfolio of β^{DCC} controlling for BUY		
Decile	RET	Alpha	Decile	RET	Alpha	Decile	RET	Alpha
Low BUY	-0.07 (-0.25)	0.09 (0.30)	Low β^{DCC}	0.19 (1.10)	0.18 (1.09)	Low β^{DCC}	0.53 (1.96)	0.64 (2.69)
2	0.39 (1.43)	0.62 (1.99)	2	0.36 (1.63)	0.35 (1.58)	2	0.45 (1.73)	0.53 (2.03)
3	0.68 (2.44)	0.86 (2.59)	3	0.35 (1.64)	0.34 (1.64)	3	0.38 (1.43)	0.47 (1.68)
4	0.78 (2.76)	0.95 (2.78)	4	0.33 (1.47)	0.34 (1.53)	4	0.37 (1.32)	0.45 (1.44)
5	0.97 (3.64)	1.05 (3.18)	5	0.38 (1.81)	0.42 (1.98)	5	0.44 (1.51)	0.51 (1.52)
6	0.99 (3.55)	1.05 (3.03)	6	0.39 (1.23)	0.45 (2.03)	6	0.48 (1.68)	0.55 (1.66)
7	1.10 (3.83)	1.14 (3.22)	7	0.44 (1.44)	0.49 (2.15)	7	0.54 (1.88)	0.60 (1.66)
8	1.11 (3.79)	1.08 (3.21)	8	0.41 (1.80)	0.46 (1.77)	8	0.56 (1.91)	0.62 (1.65)
9	1.01 (3.33)	0.99 (2.90)	9	0.45 (2.07)	0.54 (1.63)	9	0.81 (2.47)	0.85 (2.02)
High BUY	0.95 (3.02)	0.99 (2.79)	High β^{DCC}	0.82 (1.86)	0.92 (2.25)	Low β^{DCC}	1.14 (2.78)	1.15 (2.28)
High-Low	1.02 (7.21)	0.90 (7.06)	High-Low	0.63 (1.70)	0.73 (1.85)	High-Low	0.60 (1.73)	0.51 (1.36)

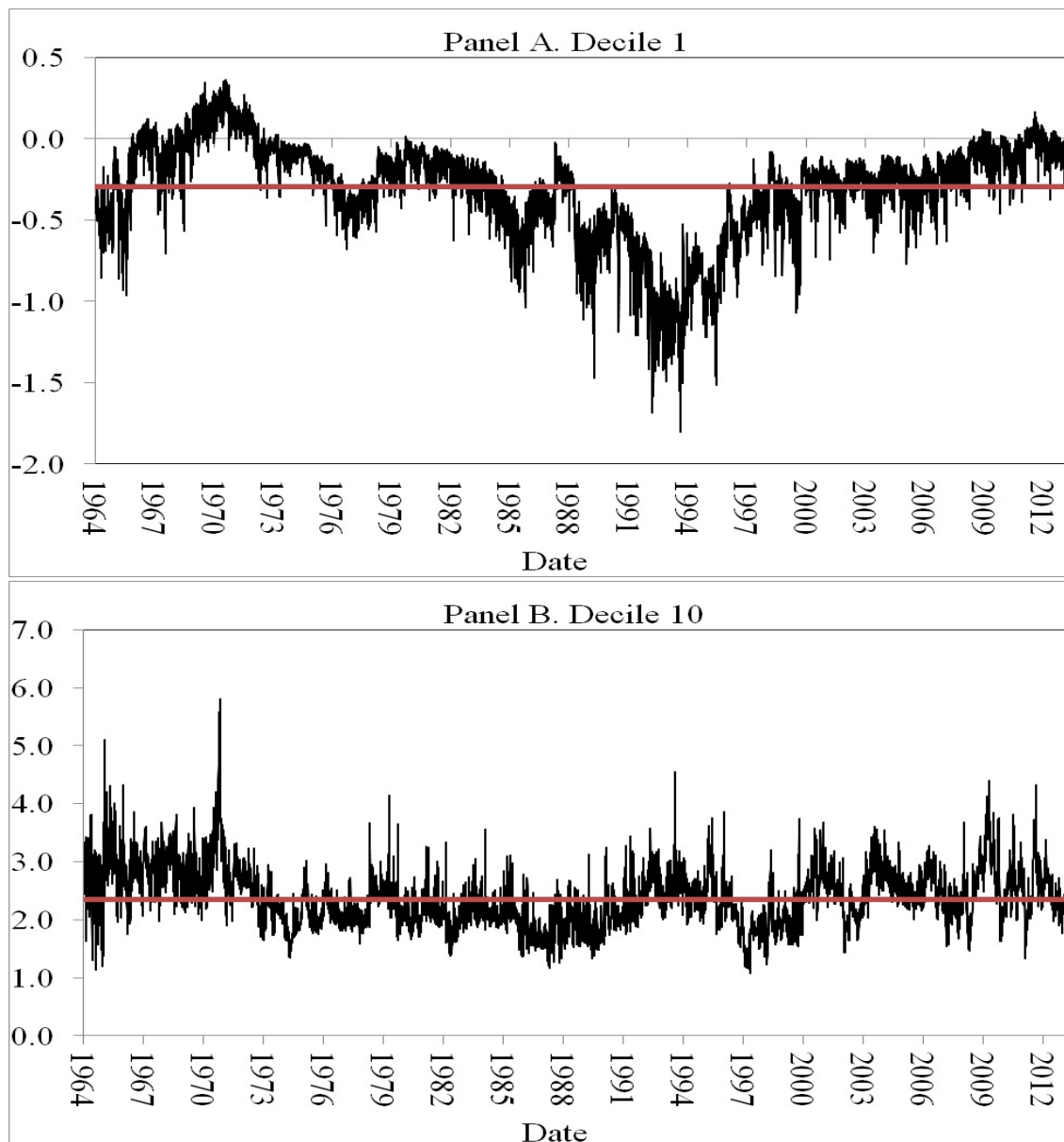
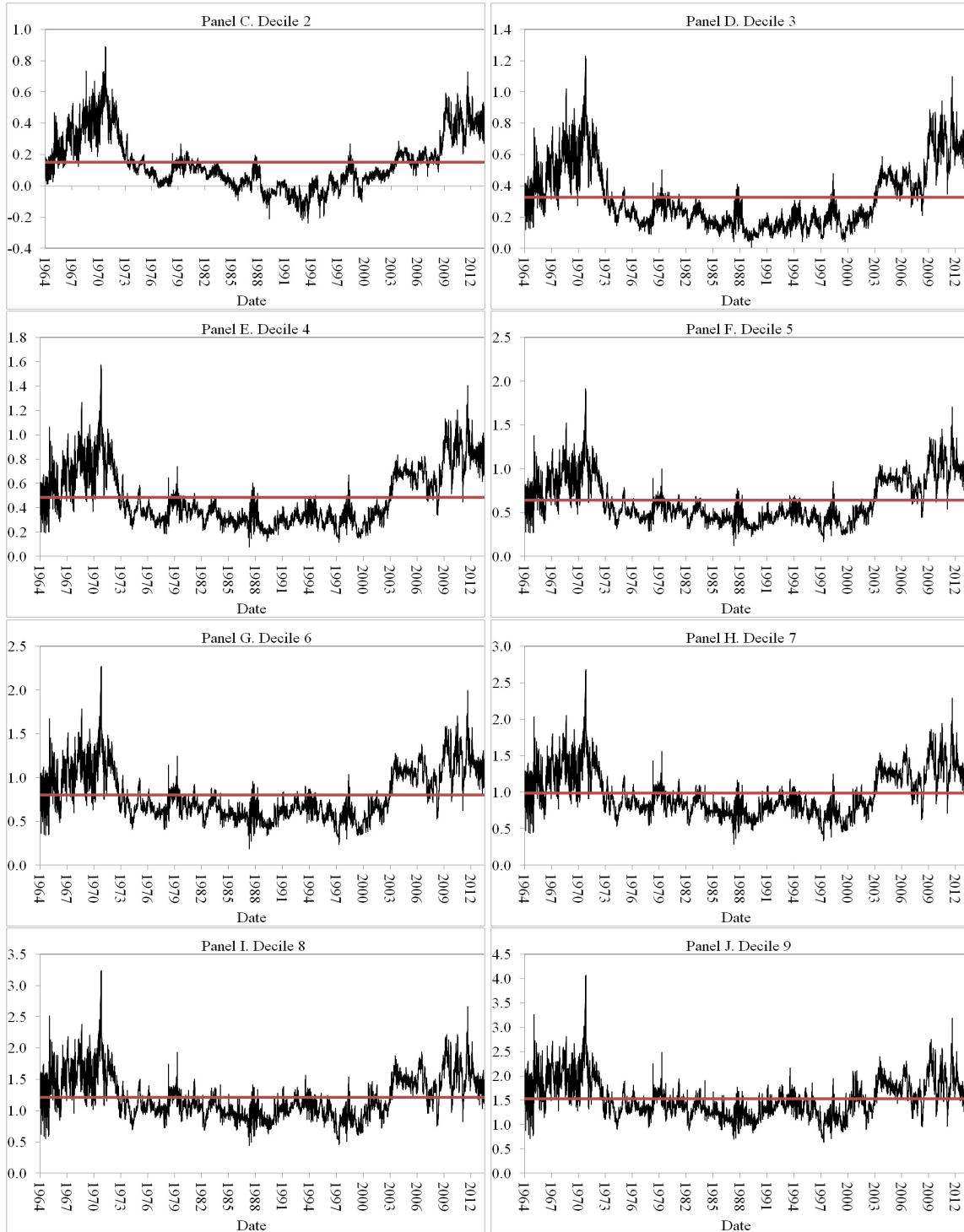


Figure A1. Daily conditional beta. For each day, individual stocks are sorted into deciles on the daily conditional market beta, estimated based on the dynamic conditional correlation (DCC) model of Engle (2002) using daily returns over the prior 252 trading days with at least 200 observations available, and updated on a daily basis. Panels A and B, respectively, depict the daily DCC beta (the curved line) for Decile 1 and Decile 10, computed by averaging the daily DCC beta across all stocks within each decile, and the corresponding time-series average of each portfolio's daily DCC beta (the straight line). Panels C to J depict the daily DCC beta and the time-series average for deciles 2 to 9. The sample period is from July 1963 to December 2013.

Figure A1 – continued



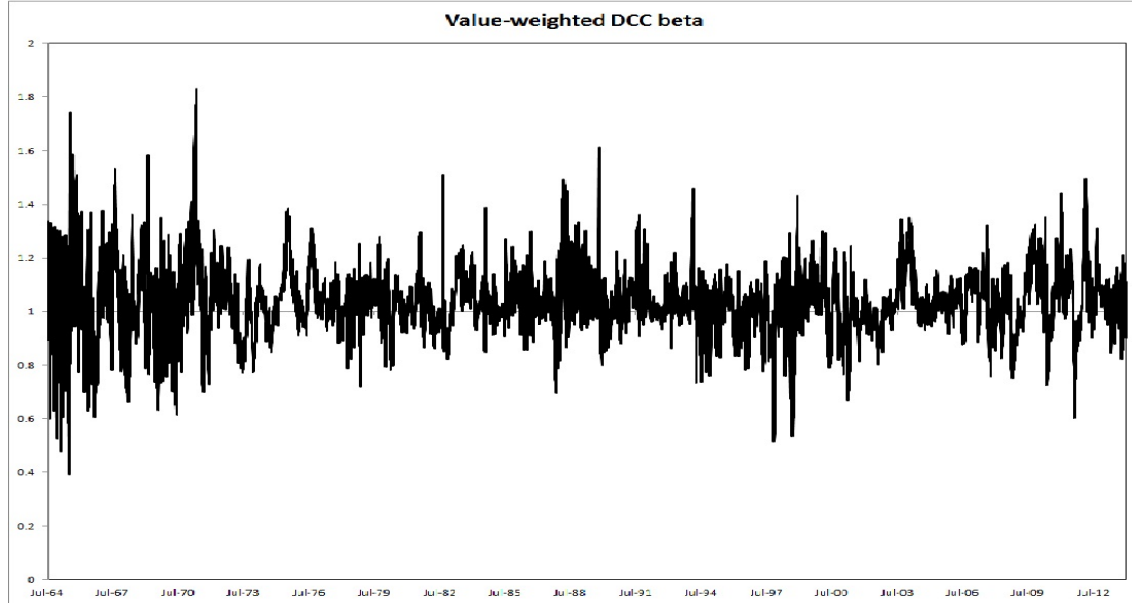


Figure A2. Value-weighted sum of daily DCC beta This figure depicts the value-weighted sum of daily DCC beta estimates of all stocks in our sample.

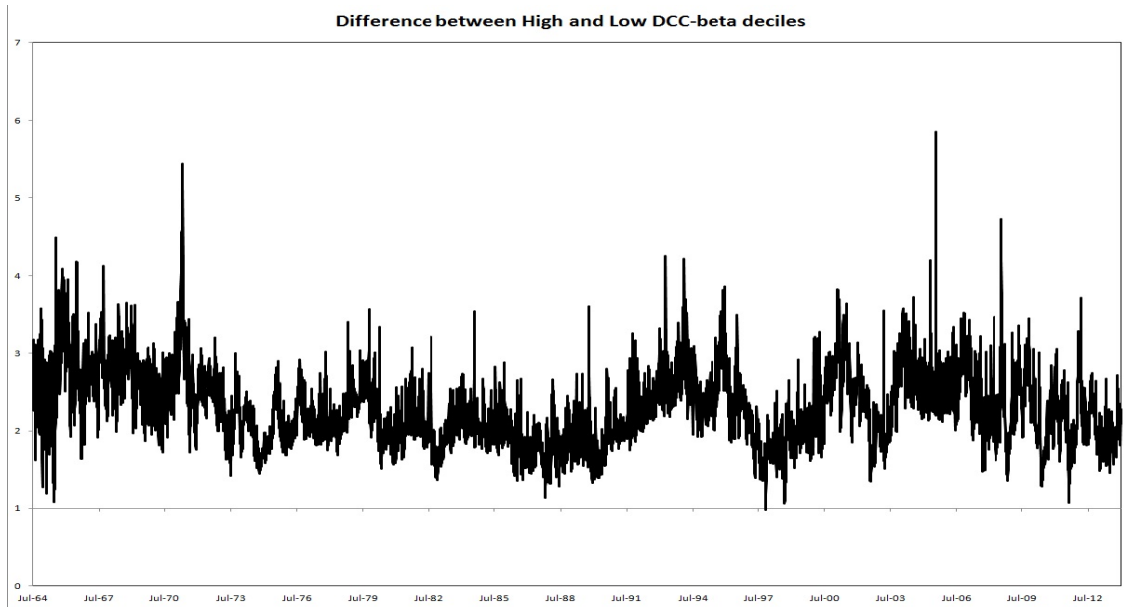


Figure A3. Difference in value-weighted DCC beta This figure depicts the difference in daily value-weighted DCC-beta of stocks in the highest and the lowest DCC-beta deciles.