Summary of Concavity, Monotonicity and Differentiability Results used in the Paper

Here, we summarize the adapted results from existing literature on the concavity, monotonicity and differentiability results regarding the following parametric optimization problem:

$$V(t) = \max_{x\in S(t)} g(x,t), \quad S^*(t) = \arg\max_{x\in S(t)} g(x,t), \tag{EC.1}$$

where for simplicity we assume $g(x,t): \mathbb{R}^2 \to \mathbb{R}$ to be continuous, $S(t) \subset \mathbb{R}$ and $S^*(t)$ to be non-empty for any t. Relevant generalizations can be found in the reference provided after each of the corresponding result.

Theorem EC.1 **(Concavity)**. *Suppose that $g(x,t)$ is jointly concave in (x,t) and $S := \{(x,y) | x \in S(t), x \in \mathbb{R}\}$ is convex, then $V(t)$ is concave on $\mathbb{R}$.*

We refer the reader to Proposition 2.1.15 in Simchi-Levi et al. (2014) for a higher dimension generalization of Theorem EC.1.

Theorem EC.2 **(Monotonicity)**. *Suppose that $S(t)$ is increasing in t and $g(x,t)$ is supermodular in (x,t).*

(a) There exists $x(t) \in S(t)$ such that $x(t)$ is increasing in t.
(b) Suppose, in addition, $g(x,t)$ is continuously differentiable on $\mathbb{R}^2$, strictly supermodular and for any t there exists $x(t)$ in the interior of $S(t)$, then $x(t)$ is strictly increasing in t.

Part (a) of Theorem EC.2 is adapted from Theorem 2.2.8 in Simchi-Levi et al. (2014) while part (b) of Theorem EC.2 is proved in Edlin and Shannon (1998).

Theorem EC.3 **(Differentiability)**. *Suppose that $g(x,t)$ is continuously differentiable in t and $V(t)$ is concave. Furthermore, $S(t) = S$, i.e., the constraint set is independent of parameter t. Then $V(t)$ is also continuously differentiable and*

$$\frac{dV(t)}{dt} = g_t(x^*,t), \quad x^* \in S^*(t).$$

Theorem EC.3 can be easily proved using Theorem 1 in Milgrom and Segal (2002). We remark here that if $V(t)$ is only concave for a subset $T \subset \mathbb{R}$, then the differentiability result will hold only for $t \in T$.

Additional Numerical Results

Gain-Seeking Demand

When demand is gain-seeking, even in a stationary environment, the optimal price path can demonstrate very complicated cyclic patterns (see Hu et al. 2016). For the special case when Assumption 5 is satisfied, however, Hu et al. (2016) proves that in the long run a cyclic skimming pricing strategy is optimal, i.e., within a price cycle the prices decrease over time. In the following, we check through some examples whether the insights derived by Hu et al. (2016) still hold in a non-stationary environment. In Figure EC.1, we have fixed $b = 40, a = 10, \eta^+ = 6, \eta^- = 0, \alpha = 0, r_1 = 0$ and examine how the strength of seasonality effect ρ affects the optimal price path. First observe from the upper left panel in Figure EC.1 that without seasonality effect, the optimal price path is of high-low pattern, which is a special case of the cyclic skimming pricing strategy and confirms the analytical result in Hu et al. (2016). With seasonality, one would naturally expect that, similar to the loss-averse case (see Figure 3), the optimal prices would tend to fluctuate more violently over time. Figure EC.1 shows that this is indeed the case, but more interestingly it also shows that the pattern of the optimal price path changes as well. Specifically, when the seasonality effect increases, one may no longer find a price cycle in which the prices decrease within the cycle.

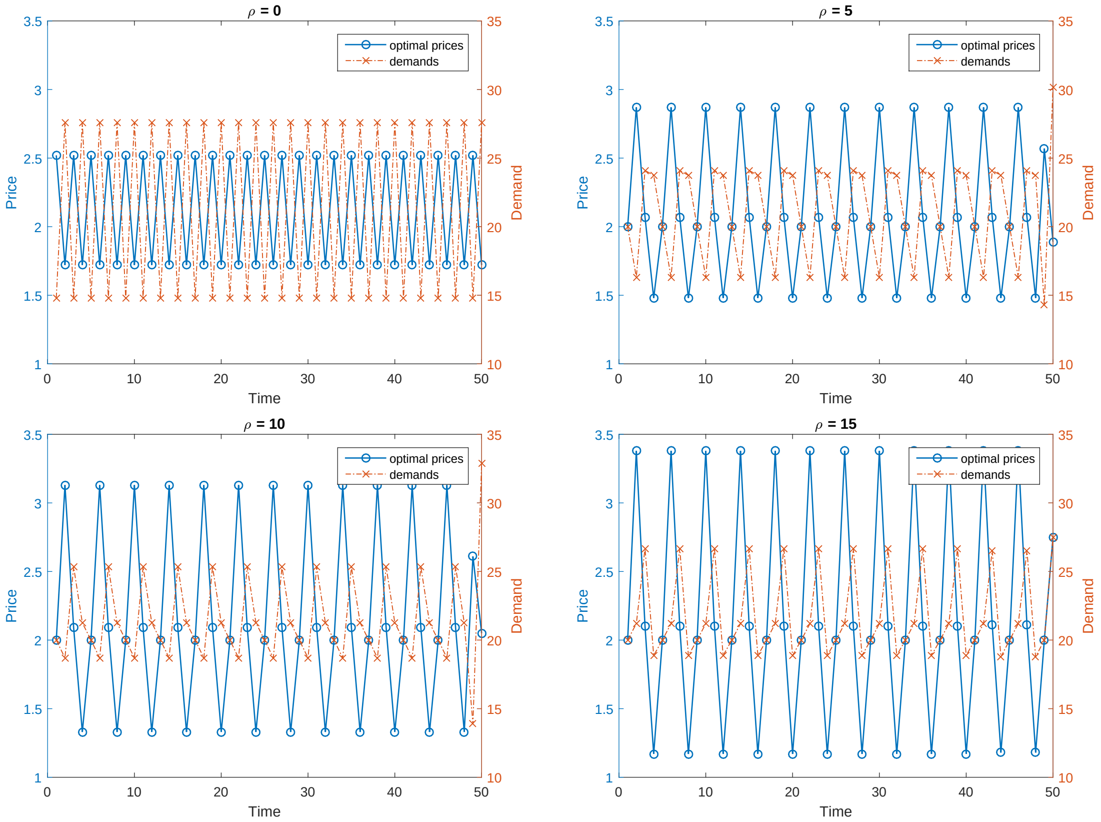

Figure EC.1 **Optimal price path and corresponding demands at different** ρ

Multiplicative Form of Seasonality

For multiplicative form of seasonality, we use the following model

$$D_t(r,p) = (1 + \varrho \sin(\phi + \omega t))D(r,p),$$

where $\varrho \in [0,1]$ is the amplitude of the seasonality effect.

Note that in this case, all the parameters in the demand function become non-stationary. Assumption 4 (the equivalent condition in this case is $\eta^-_{t+1} - \eta^+_t \le 2a_t - 2\alpha a_{t+1}$) then translates to the following set of conditions on problem parameters:

$$(1+\varrho)\eta^- - \eta^+ \le 2a - 2\alpha(1+\varrho)a,$$
$$\eta^- - (1-\varrho)\eta^+ \le 2(1-\varrho)a - 2\alpha a.$$

Many parameters we designed for the additive case, however, will no longer satisfy the above conditions. Thus, in the case of multiplicative seasonality, we fix $T = 50$, $b = 40$, $\phi = -\frac{\pi}{2}$, $\omega = \frac{\pi}{2}$ and consider different combinations of the rest of the parameters with $\varrho \in \{0.1, 0.2, 0.3, 0.4\}$, $\Delta \in \{2,4,6,8\}$, $\eta^+ \in \{0,5,10\}$, $a \in \{20,25,30\}$, $\alpha \in \{0,0.1,0.2,0.3\}$ and $r_1 \in \{0, b/a\}$, which amount to a total of 1,152 problem instances. For each problem instance, we let $L_t = 0$ and $U_t = b/a$ for $1 \le t \le T$.

Our findings for the price path is similar to the additive case—as ϱ increases and Δ decreases, the prices tend to cycle. See Figure EC.2 for the effect of ϱ, where we have fixed $b = 40, a = 20, \eta^+ = 5, \Delta = 2, \alpha = 0.3, r_1 = 0$.

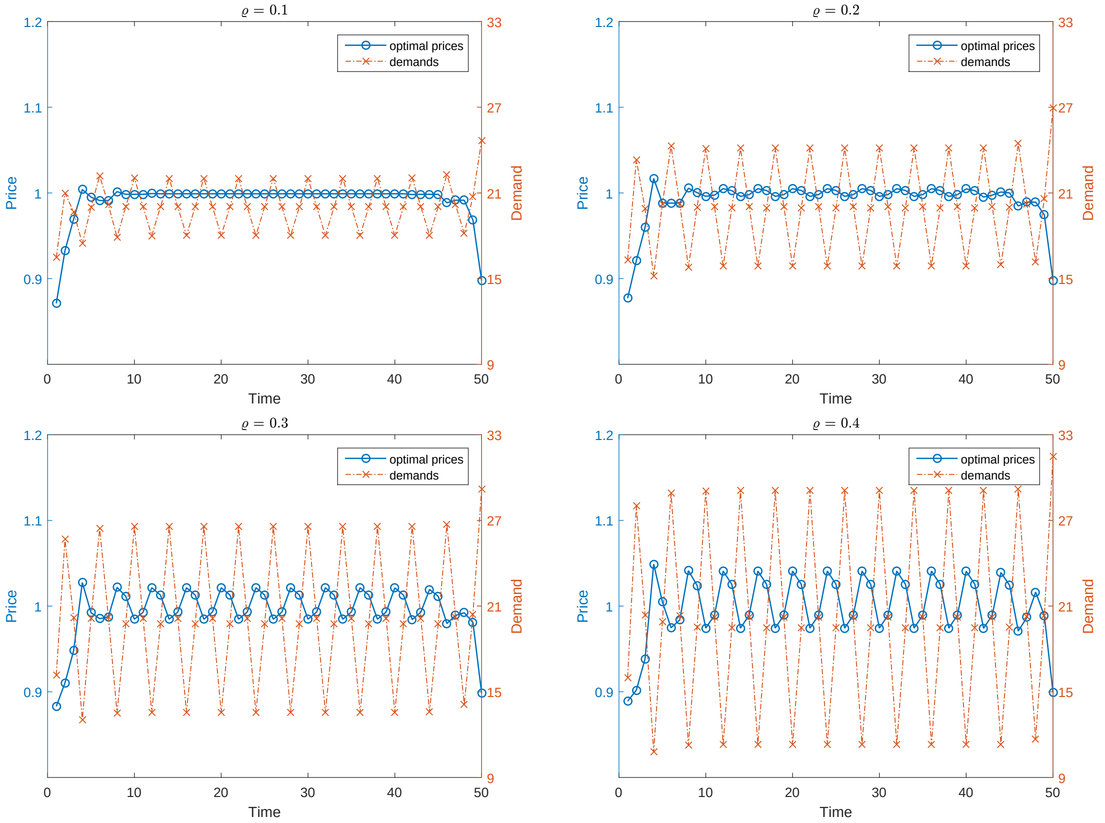

Figure EC.2 **Optimal price path and corresponding demands at different ϱ**

The general performance of IR and IS, on the other hand, is quite different from the additive case. From Table EC.1, one can see that both IR and IS perform surprisingly well, with the worst case percentage error remain in 1%. Intuitively speaking, this could be due to the fact that the multiplicative seasonality is not changing the relative magnitude of demand parameters, which in turn will not provide additional value in reference price effect or seasonality effect. The sensitivity of the performance regarding to ϱ and Δ is the same as the additive case. That is, as the loss-aversion effect (Δ) increases, the performance of IR decreases while the performance of IS increases. As the seasonality effect (ϱ) increases, the performance of both IR and IS deteriorates.

Table EC.1 **Average and worst percentage loss for different ϱ and Δ**

		$\varrho=0.1$		$\varrho=0.2$		$\varrho=0.3$		$\varrho=0.4$	
		IR	IS	IR	IS	IR	IS	IR	IS
$\Delta=2$	Avg	0.07	0.01	0.08	0.02	0.13	0.06	0.21	0.14
	Max	0.25	0.09	0.31	0.15	0.45	0.31	0.72	0.57
$\Delta=4$	Avg	0.08	0.01	0.08	0.01	0.11	0.04	0.17	0.10
	Max	0.30	0.09	0.32	0.11	0.42	0.20	0.62	0.42
$\Delta=6$	Avg	0.09	0.01	0.09	0.01	0.11	0.03	0.15	0.07
	Max	0.35	0.09	0.38	0.10	0.44	0.14	0.60	0.30
$\Delta=8$	Avg	0.10	0.01	0.11	0.01	0.12	0.02	0.15	0.06
	Max	0.41	0.09	0.44	0.10	0.49	0.11	0.61	0.22

The table that compares the heuristic with our exact algorithm in case of multiplicative seasonality is omitted here since even the sub-optimal pricing strategies like IR and IS already perform well enough.

References

Edlin, A.S., C. Shannon. 1998. Strict monotonicity in comparative statics. *J. Econom. Theory* **81**(1): 201–219.