Appendix of "Sequential Search with Refinement: Model and Application with Click-stream Data"

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A1. Robustness Tests of An Alternative Information Structure

One assumption we have pertaining to consumers' information structure is that they know the slot- and refinement-specific attributes distribution for each search. Correspondingly, in our estimation we use the empirical distribution of attributes from the data that are slot and refinement specific. The consumers use these distributions to form their expectations about the return of search. It is possible, however, that consumers are agnostic about the attributes distribution. We consider two robustness tests below to check the assumption.

1. We re-estimate the model under an alternative assumption: While the consumer still knows the attributes distribution, it is no longer slot and refinement-specific. Instead, we assume that the distribution is common across all slot and refinement methods. Accordingly, all searches, including those without refinement, share the same attributes distribution. Under the original assumption, expected returns and search cost both vary depending on slot and refinement decisions. In comparison, under this new assumption, all searches have the same *gross* expected utility. It is the search cost difference that solely explains search decisions (recall that slot enters the search cost but not the utility). We find that the model fit worsen as indicated by the hit rates as described in Section 5.2.2. We find the hit rates change from 0.83 and 0.68 under the original assumption to 0.53 and 0.44 (no hit rates comparison for refinement tools because the distribution is no longer refinement-specific). In other words, the model fits the data better under the original assumption. The coefficients of slot position (Equation 15) have the same sign but becomes insignificant. This may be because the slot position alone cannot sufficiently explain the variation of search decisions observed in the data.

2. In the second test, we assume that the attribute distribution is slot- and refinement-specific. However, in contrast to the original assumption where changing slot and refinement affects the *joint* distribution of attributes, it only affects the "refined attribute." For example, if a consumer uses "sort by consumer ratings," the distribution of consumer ratings changes but the remaining attributes are still drawn from their respective non-refined distributions. In this case, search decisions are determined by both expected utility and search cost but consumers have less knowledge about non-refined attributes. We find that the model fit deteriorates as measured by hit rates. The measures drop to 0.60, 0.45, and 0.67.

Through these two tests, we conclude that the original assumption is more consistent with the data. Note that these tests are only necessary conditions for the assumption's validity.

A2. Robustness Test of the Definition of Search

We consider a new robustness test to evaluate the current definition of a search. We define a search as a click-through (Section 2). One concern is that some attributes may be more transparent than the remaining before the click-through. To evaluate the current specification, we consider the following test: When a consumer makes her decision about search, instead of using the current distribution $P_j(x_{ij})$ where all x_{ij} are unknown before a click, we use $P_j(x_{ij}^{unobserved}|x_{ij}^{displayed})$. This implies that the consumer knows some attributes even before the clicks. And the remaining attributes still require a click-through to resolve the uncertainty. Among the attributes that enter the utility specification (Equation 1), we set $x_{ij}^{displayed}$ as Price, Star Rating, Consumer Rating, and Promotion Flag. The remaining attributes are $x_{ij}^{unobserved}$, including Hotel Chain and Distance to City Center. These two attributes are more obscure without click-through. At the time of data collection by the focal website, "Hotel Chain" and "Distance to City Center" were not shown on the results list at all.

The model fits across the two specifications are quite similar as measured using hit rates: 0.83, 0.68, and 0.72 under the original assumption vs. 0.82, 0.69, and 0.73 under the alternative assumption. While both describe the data reasonably well, we decide to keep the current specification because it is slightly more reasonable for the following reasons:

1. When the consumer browses, hotels on the next page or outside the screen viewing area remain unknown to the consumer.

2. Even if the hotel shows on the screen, the consumer may not register the attribute information displayed before she decides to "explore" through the link.

However, it is crucial to use proper definition of "search" depending on the context. It is important to consider robustness checks when applying the model in practice.

A3. Robustness Tests of "Search with Learning"

In our model, we assume that there is no learning during search and consumers do not update their knowledge about attributes distribution. To test this assumption, we consider the following two robustness tests.

- In our data, we have 146 frequent users who were automatically logged into their accounts upon arrival at the website. We surmise that these users are more knowledgable about attributes distribution than infrequent users. We estimate the model separately using these two groups and the estimates on attributes are statistically equivalent.

- In our data, we have 171 consumers searching for hotels in the America (Cancun and Manhattan) and 324 consumers searching for hotels in the Europe (Budapest and Paris). We expect that the first set of consumers have better knowledge about the marketplace because the data provider is a US company and the majority of its consumers live in the US. We estimate the model separately using the two groups of consumers and compare the estimates. The estimates are statistically equivalent.

We consider these tests as evidence supporting our assumption. However, note that they are only necessary conditions for the validity of the assumption. We call for future research to advance the literature of search with learning.

A4. Estimates of a Model without Refinement Decisions

In this Appendix we re-estimate the model where the refinement component is dropped. The model estimation is similar. However, we treat the attributes distribution of all searches the same independent of the refinement method applied. In particular, recall that when considering how consumers form their expected return of a search, we use the empirical distribution of attributes. To generate the empirical distribution in this estimation, we pool together the hotel lists of all consumers within the same city. We rank the hotels according to different refinement methods. For a given slot position, we use the observed attributes across all refinement methods at that slot to form the empirical distribution and use it in estimation. This empirical distribution is slot-specific but not refinement-specific as it is aggregated across all refinement methods.

Table 8 shows the results. In comparison to model estimates in the paper, Table 6, we can see that nearly all estimates have larger standard errors. Potentially this is due to that the model cannot fully account for the noise in the data when we ignore refinement decisions. The model fit as measured by the hit rates of search and purchase also drops to 0.62 and 0.49, in comparison to 0.83 and 0.68 of the proposed model.

[Insert Table 8 About Here]

Table 8: Estimates without Refinement Decisions*

	Mean Parameters (Std. Err.)	Heterogeneity (Std. Err.)
Search Cost		
Constant	**2.87 (1.04)**	**2.13 (0.59)**
Time Constraint (Days)	**-0.06 (0.02)**	**0.10 (0.03)**
Slot	**0.02 (0.01)**	**0.06 (0.04)**
Utility		
Average Daily Price (normalized)	-1	**5.17 (1.72)**
Budapest	**99.11 (40.05)**	**26.82 (3.72)**
Cancun	**90.04 (61.33)**	**20.12 (7.31)**
Manhattan	**108.90 (22.10)**	**30.17 (8.34)**
Paris	**112.06 (29.89)**	**15.04 (3.88)**
Star Rating	**37.43 (18.11)**	**23.62 (4.00)**
Consumer Rating greater than 4.5	**110.25 (27.71)**	**16.02 (8.22)**
Consumer Rating between 4 and 4.5	**80.74 (32.41)**	**19.46 (3.62)**
Distance to City Center (kilometers)	-7.01 (20.61)	**82.15 (11.48)**
Hotel Chain	**37.94 (10.92)**	**12.31 (1.11)**
Promotion Flag	**53.27 (22.67)**	8.52 (3.22)

Note: *Bold fonts indicate estimates significant at the 95% level.

A5. Details of How to Simulate Consumer Search/Refinement/Purchase Behavior

The simulation of a consumer's search/refinement and purchase involves the following steps.

1. We first infer a consumer's heterogenous preference and search cost parameters. For a given consumer, we observe which options (slot+refinement) are searched and which product is purchased. Accordingly, we can infer the heterogenous preference and search cost parameters of that consumer. To start, we first make $R = 100$ sets of draws from the model estimates (mean estimates and heterogeneity estimates from Table 6). Denote a given set of parameter draws as Δ_r. For consumer i, the posterior distribution

of Δ_r can be approximated as

$$\widehat{P}(\Delta_r|Data_i) = \frac{L_i^{purchase} L_i^{search}(Data_i|\Delta_r)}{\sum_{m=1}^{R=100} L_i^{purchase} L_i^{search}(Data_i|\Delta_m)}$$

where $L_i^{purchase} L_i^{search}(Data_i|\Delta_r)$ is the likelihood (Equation 7 and Equation 8 in the paper).

2. For a given set of Δ_r, we can evaluate the reservation values of each search option. We then draw 100 sets of normally distributed random shocks for each hotel. For a consumer, given Δ_r and a set of hotel random shocks, we simulate the choices of consumer i using the optimal search strategy of Weitzman (1979), i.e., which options (slots and refinement methods) will be searched and which hotel will be purchased.

3. The search and purchase activities can then be used to compute the measure of interest such as hit rates, the utility of the final purchase, etc. Note that across the 100 sets of parameters ($\Delta_r, r = 1, 2, ..., 100$), we will have 100 sets of simulation results for consumer i and 100 sets of measures of interest. We need to integrate these 100 sets of measures over the consumer's posterior parameters distribution. We hence use the $\widehat{P}(\Delta_r|Data_i)$ calibrated in Step 1 above to compute the weighted average across the 100 measures.

4. To compute the confidence interval of a particular measure of interest, e.g., the average number of searches across consumers, we use bootstrapping approach by repeating the simulation 200 times.

A6. A Simple Simulation: the Effect of Refinement on Consumer Decisions

To demonstrate the effect of refinement on consumer decisions, we consider the following simulation to show that more likely it is the lowered search cost has increased the number of searches.

Recall that in Section 5.3, "Educating Consumers about the Default Ranking Rule," we find that the average number of searches of consumers is 1.63 when there is no refinement and consumers know the default ranking attributes distribution. In that case, there are only J options.

Next, we append the default list with $J * (R - 1)$ hotels by randomly sampling hotels from the default list. We also assume that consumers cannot refine. As a result, a consumer faces $J * R$ options.[40] We further assume that consumers know the distribution of attributes for each slot. Using the estimates of preference and search cost, we simulate the searches and choices of consumers. We find that the average number of searches is 1.67, with a 95% confidence interval of (1.56, 1.77).

Furthermore, we calculate the correlation between the choices for these two scenarios across all consumers. We find that the correlation between choices are fairly high, reaching 0.93 for searches and 0.97 for purchases. Because there are only J options overlap across the two scenarios (the default list), the high correlations imply that consumers mainly focus their searches and purchases among the default list.

This exercise shows that even the number of options has increased from J to $J * R$, the number of searches does not increase significantly. Consumers' choices are still concentrating among the top J options (the default list). In comparison, as shown in Section 5.3, when consumers can refine results, the number of searches averages at 2.10. Accordingly, the increased number of searches is more likely to be a result of lowered search cost, caused by the introduction of refinement.

[40] In the actual estimation, the number of options a consumer faces is less than $J * R$ because some refinement methods involve filtering. Furthermore, the consumer knows the number of products under a filtering option (e.g., 4-star hotels and above) because the website shows that number on its webpages before the consumer applies that filter.

A7. A Simple Simulation: the Effect of Slot-Refinement Specific Distribution on Consumer Search Outcome

In our model, the distribution of attributes is slot-refinement specific. Refinement not just "reshuffles" the order of hotels presented to the consumer, but also changes the search cost and reservation value of each hotel. This is because (1) the attributes distribution of each slot has changed, and (2) the search cost to discover a hotel has changed due to slot position difference. Such a specification can nicely describe the effect of refinement on consumer search behavior and outcome. To demonstrate, we consider the following simulation.

Suppose the utility of a consumer is $u = 100 - p$, where p is the price of a hotel, i.i.d. with a normal distribution with the mean as \$50 and the standard deviation of \$5. There are 10 slots and the search cost of slot m is $cost = 5 + 0.1 * m$. We consider the following two scenarios:

In the first scenario (Table 9), 10 hotels are randomly allocated across the 10 slots and a consumer cannot refine. The distribution of price on each slot is therefore the normal distribution $N(50, 5^2)$. Based on this distribution, we may calculate the reservation value for each slot. According to Weitzman's optimal search strategy, the consumer will obtain a net utility level of \$40.27.

In the second scenario (Table 10), the consumer can sort the hotels by price ascendingly. Note that the price of each hotel still follows normal distribution $N(50, 5^2)$. However, the distribution at *each slot* has changed due to the refinement. For example, the first slot now has a distribution of $N(42.23, 2.79^2)$.[41] Based on the distribution of each slot, we re-calculate the reservation values. Now the consumer will obtain a higher net utility of 52.95.

As this simulation demonstrates, the slot-refinement specific distribution can flexibly capture consumer's refinement during search and its consequence on the search outcome.

[41] We obtain this distribution by drawing 1000 sets of 10 hotels, sorting by prices for each set, and then allocate them to the 10 slots. We can then approximate the empirical distribution of each slot.

Table 9: Ten Hotels without Refinement

Slot	Search Cost	Price	Mean	S.D.	Reservation	Net Utility if Chosen
1	5.1	54.63	50	5	45.38	40.27
2	5.2	59.11	50	5	45.26	30.59
3	5.3	41.95	50	5	45.14	42.45
4	5.4	48.57	50	5	45.02	30.43
5	5.5	48.29	50	5	44.90	25.21
6	5.6	51.83	50	5	44.78	16.07
7	5.7	43.36	50	5	44.67	18.84
8	5.8	62.06	50	5	44.55	-5.66
9	5.9	50.32	50	5	44.43	0.18
10	6.0	57.73	50	5	44.32	-13.23

Table 10: Ten Hotels without Refinement

Slot	Search Cost	Price	Mean	S.D.	Reservation	Net Utility if Chosen
1	5.1	41.95	42.23	2.79	52.70	52.95
2	5.2	43.36	44.87	2.28	49.94	46.34
3	5.3	48.29	46.63	2.12	48.07	36.11
4	5.4	48.57	48.08	2.03	46.53	30.43
5	5.5	50.32	49.32	2.00	45.18	23.18
6	5.6	51.83	50.55	1.99	43.86	16.07
7	5.7	54.63	51.83	2.02	42.47	7.57
8	5.8	57.73	53.27	2.10	40.93	-1.33
9	5.9	59.11	54.88	2.36	39.22	-8.61
10	6.0	62.06	57.55	2.87	36.47	-17.56