Comment on "Strategic Information Management Under Leakage in a Supply Chain"

Online Appendix

In this appendix, we present the detailed analysis that have been omitted in the main paper. Part I provides the details of how to solve constraints (1) and (2). Part II proves $\underline{q}^L_{(ip)\min}(q_{ip}) < \underline{q}^H_{(ip)\min}(q_{ip}) < q_{ip}$. Part III describes the undefeated equilibrium and the lexicographically maximum sequential equilibrium (LMSE).

Part I: Solving Constraints (1) and (2)

We first solve constraint (1) step by step : $(A_H - q_{ip} - \frac{\mu - q_{ip}}{2})q_{ip} \geq \max_{q_{iH} \leq \underline{q}_p}(A_H - q_{iH} - \frac{A_L - q_{iH}}{2})q_{iH}$, which is equivalent to solve:

$$(2A_H - \mu - q_{ip})q_{ip} \geq \max_{q_{iH} \leq \underline{q}_p}(2A_H - A_L - q_{iH})q_{iH}. \qquad \text{(A1)}$$

We examine the two cases for $\underline{q}_p$. Suppose $\underline{q}_p \geq \frac{2A_H - A_L}{2}$. Then, $\max_{q_{iH} \leq \underline{q}_p}(2A_H - A_L - q_{iH})q_{iH} = \frac{(2A_H - A_L)^2}{4} > \frac{(2A_H - \mu)^2}{4} \geq (2A_H - \mu - q_{ip})q_{ip}$, which contradicts (A1). Thus, we must have $\underline{q}_p < \frac{2A_H - A_L}{2}$. Given $\underline{q}_p < \frac{2A_H - A_L}{2}$, $\max_{q_{iH} \leq \underline{q}_p}(2A_H - A_L - q_{iH})q_{iH} = (2A_H - A_L - \underline{q}_p)\underline{q}_p$. That is, we need to solve $(2A_H - \mu - q_{ip})q_{ip} \geq (2A_H - A_L - \underline{q}_p)\underline{q}_p$, i.e., $\underline{q}_p^2 - (2A_H - A_L)\underline{q}_p + (2A_H - \mu - q_{ip})q_{ip} \geq 0$. Solving this inequality, we obtain $\underline{q}_p \leq \frac{2A_H - A_L - \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip})q_{ip}}}{2}$ or $\underline{q}_p \geq \frac{2A_H - A_L + \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip})q_{ip}}}{2}$. Combining with the condition $\underline{q}_p < \frac{2A_H - A_L}{2}$, we get $\underline{q}_p \leq \underline{q}^H_{(ip)\min}(q_{ip}) \equiv \frac{2A_H - A_L - \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip})q_{ip}}}{2}$ $(< \frac{2A_H - A_L}{2})$.

Next we solve constraint (2): $(A_L - q_{ip} - \frac{\mu - q_{ip}}{2})q_{ip} \geq \max_{q_{iL} \leq \underline{q}_p}(A_L - q_{iL} - \frac{A_L - q_{iL}}{2})q_{iL}$, which is equivalent to solve:

$$(2A_L - \mu - q_{ip})q_{ip} \geq \max_{q_{iL} \leq \underline{q}_p}(A_L - q_{iL})q_{iL}. \qquad \text{(A2)}$$

We examine the two cases for $\underline{q}_p$. Suppose $\underline{q}_p \geq \frac{A_L}{2}$. Then, $\max_{q_{iL} \leq \underline{q}_p}(A_L - q_{iL})q_{iL} = \frac{A_L{}^2}{4} > \frac{(2A_L - \mu)^2}{4} \geq (2A_L - \mu - q_{ip})q_{ip}$, which contradicts (A2). Thus, we must have $\underline{q}_p < \frac{A_L}{2}$. Given $\underline{q}_p < \frac{A_L}{2}$,

$\max_{q_{iL}\le \underline{q}_p}(A_L - q_{iL})q_{iL} = (A_L - \underline{q}_p)\underline{q}_p$. That is, we need to solve $\left(2A_L - \mu - q_{ip}\right)q_{ip} \ge (A_L - \underline{q}_p)\underline{q}_p$, i.e., $\underline{q}_p{}^2 - A_L\underline{q}_p + \left(2A_L - \mu - q_{ip}\right)q_{ip} \ge 0$. Solving this inequality, we obtain $\underline{q}_p \le \frac{A_L - \sqrt{A_L{}^2 - 4\left(2A_L - \mu - q_{ip}\right)q_{ip}}}{2}$ or $\underline{q}_p \ge \frac{A_L + \sqrt{A_L{}^2 - 4\left(2A_L - \mu - q_{ip}\right)q_{ip}}}{2}$. Combining with the condition $\underline{q}_p < \frac{A_L}{2}$, we get $\underline{q}_p \le \underline{q}^L_{(ip)\min}(q_{ip}) \equiv \frac{A_L - \sqrt{A_L{}^2 - 4\left(2A_L - \mu - q_{ip}\right)q_{ip}}}{2}$ $(< \frac{A_L}{2})$.

Part II: Proof of $\underline{q}^L_{(ip)\min}(q_{ip}) < \underline{q}^H_{(ip)\min}(q_{ip}) < q_{ip}$

Here we prove $\underline{q}^L_{(ip)\min}(q_{ip}) < \underline{q}^H_{(ip)\min}(q_{ip}) < q_{ip}$. First, we show $\underline{q}^L_{(ip)\min}(q_{ip}) < \underline{q}^H_{(ip)\min}(q_{ip})$. Substituting the expressions for $\underline{q}^L_{(ip)\min}(q_{ip})$ and $\underline{q}^H_{(ip)\min}(q_{ip})$, we obtain

$$\underline{q}^L_{(ip)\min}(q_{ip}) - \underline{q}^H_{(ip)\min}(q_{ip}) = \frac{\sqrt{(2A_H - A_L)^2 - 4\left(2A_H - \mu - q_{ip}\right)q_{ip}} - 2(A_H - A_L) - \sqrt{A_L{}^2 - 4\left(2A_L - \mu - q_{ip}\right)q_{ip}}}{2} = \frac{G_1 - G_2}{2},$$

where $G_1 \equiv \sqrt{(2A_H - A_L)^2 - 4\left(2A_H - \mu - q_{ip}\right)q_{ip}}$ and $G_2 \equiv 2(A_H - A_L) + \sqrt{A_L{}^2 - 4\left(2A_L - \mu - q_{ip}\right)q_{ip}}$. Note that

$$G_1{}^2 - G_2{}^2 = 4(A_H - A_L)\left(A_L - 2q_{ip} - \sqrt{A_L{}^2 - 4\left(2A_L - \mu - q_{ip}\right)q_{ip}}\right) = 4(A_H - A_L)(g_1 - g_2),$$

where $g_1 \equiv A_L - 2q_{ip}$ and $g_2 \equiv \sqrt{A_L{}^2 - 4\left(2A_L - \mu - q_{ip}\right)q_{ip}}$. Note that

$$g_1{}^2 - g_2{}^2 = 4(A_L - \mu)q_{ip} < 0.$$

Since $g_2 > 0$, we know $g_1 < g_2$, which implies $G_1 < G_2$. Thus we have $\underline{q}^L_{(ip)\min}(q_{ip}) < \underline{q}^H_{(ip)\min}(q_{ip})$.

Next we show $\underline{q}^H_{(ip)\min}(q_{ip}) < q_{ip}$. Substituting the expression for $\underline{q}^H_{(ip)\min}(q_{ip})$, we obtain

$$\underline{q}^H_{(ip)\min}(q_{ip}) - q_{ip} = \frac{2A_H - A_L - 2q_{ip} - \sqrt{(2A_H - A_L)^2 - 4\left(2A_H - \mu - q_{ip}\right)q_{ip}}}{2} = \frac{F_1 - F_2}{2},$$

where $F_1 \equiv 2A_H - A_L - 2q_{ip}$ and $F_2 \equiv \sqrt{(2A_H - A_L)^2 - 4\left(2A_H - \mu - q_{ip}\right)q_{ip}}$. Note that

$$F_1{}^2 - F_2{}^2 = 4(A_L - \mu)q_{ip} < 0.$$

Since $F_2 > 0$, we obtain $F_1 < F_2$, which implies $\underline{q}^H_{(ip)\min}(q_{ip}) < q_{ip}$. In summary, we conclude $\underline{q}^L_{(ip)\min}(q_{ip}) < \underline{q}^H_{(ip)\min}(q_{ip}) < q_{ip}$.

Part III: Undefeated Equilibrium and LMSE

The *undefeated equilibrium* and *LMSE* concepts have been commonly used as an alternative equilibrium selection criterion. Mailath et al. (1993) define these outcome-based refinement concepts of undefeated equilibrium and LMSE, which are introduced to avoid the limitations of other refinement techniques such as the Intuitive Criterion (which for example may eliminate all equilibria or fail to select a unique equilibrium). Essentially, for a general class of signaling games (including AG's setting), both undefeated equilibrium and the LMSE correspond to the same equilibrium outcome. Mailath et al. (1993) summarize and explain the advantages of these refinement concepts (Pareto-optimality, uniqueness and existence for a very general class of signaling games). Below we provide the definition of LMSE applied to AG's setting.

Definition *(Lexicographically Maximum Sequential Equilibrium) In a signaling game G, we denote the set of types by $\{H, L\}$, the i-type player's payoff by $\pi_i(\cdot)$, and the set of pure-strategy perfect Bayesian equilibria by $PBE(G)$. The strategy profile $\sigma' \in PBE(G)$ lexicographically dominates (L-dominates) $\sigma \in PBE(G)$ if $\pi_L(\sigma') > \pi_L(\sigma)$, or $\pi_L(\sigma') = \pi_L(\sigma)$ and $\pi_H(\sigma') > \pi_H(\sigma)$. The strategy profile $\sigma \in PBE(G)$ is an LMSE if there does not exist $\sigma' \in PBE(G)$ that L-dominates σ.*

Essentially, LMSE selects the most efficient (profitable) outcome from the perspective of the type of the informed player that has the most incentive to reveal his or her true identity. In AG's setting, the LMSE outcome is the PBNE outcome that the low-type incumbent finds most profitable among all PBNE. The rationale for this refinement is fairly intuitive. Note that the high-type incumbent has an incentive to mimic the low-type incumbent but not vice versa, i.e., the low-type incumbent is the one with an incentive to reveal its identity. So, when both separating and pooling PBNE exist, if the low-type incumbent makes a higher profit under the pooling outcome than under the separating outcome whereas the high-type incumbent makes a higher profit under the separating outcome than under the pooling outcome, then the pooling PBNE would be selected, since the separating equilibrium can be realized only by adopting an "unreasonable" belief system.

If LMSE refinement is adopted in AG's setting, the similar unique composite equilibrium will be achieved, which leads to the same qualitative results as in AG's original paper. The only difference is that the low threshold ($\underline{q}_p$) in the entrant's belief system under the pooling equilibrium should be $\underline{q}_p = \min[\underline{q}^L_{(ip)\min}(q_{ip}), q^H_{(ip)\min}]$, not $\underline{q}_p = q^H_{(ip)\min}$ as specified in AG. This is because AG missed some incentive compatibility (IC) constraints in their analysis for the pooling equilibrium.