Online Supplement to
“Impact of Electricity Pricing Policies on Renewable Energy Investments and Carbon Emissions”

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A Summary of Notation

Notation	Explanation	Notation	Explanation
$i \in \{n, d\}$	subscript for period	q_i	probability that $\tilde{q}_i = 1$ when $\tilde{q}_i$ follows a two-point distribution
$j \in \{\text{flat}, \text{peak}\}$	superscript for pricing policy	$g(\cdot)$	generation cost function
p_i	price of electricity in period i	C_1, C_2, C_3, C_4	cost paramaters in $g(\cdot)$
$D_i(p_i, p_{-i})$	demand of electricity in period i	v	variable cost of the new conventional source
a_i	market size in period i	e	emission intensity of the new conventional source
γ	own-price sensitivity of demand	$\alpha_r(\cdot), \alpha_c(\cdot)$	investment cost function of renewable and conventional source, respectively
δ	cross-price sensitivity of demand	β_r, β_c	unit investment cost of renewable and conventional source, respectively
$\tilde{q}_i$	intermittency factor in period i	ECE^j	expected carbon emissions under pricing policy j
$\xi_i(\cdot, \cdot)$	inverse demand function in period i	CS^j	consumer surplus under pricing policy j

B Proofs

In the following proofs, we suppress the arguments of the demand function for brevity whenever no confusion arises, i.e., we let $D_i^j = D_i^j(p_i, p_{-i})$ for $i \in \{n, d\}$ and $j \in \{\text{flat}, \text{peak}\}$. Also, to simplify notation, we drop the asterisk sign $(^*)$, e.g., we denote the optimal nighttime price under flat pricing as p_n^{flat} instead of $p_n^{\text{flat}^*}$.

Proof of Lemma 1. Under the two point distribution assumption for $\tilde{q}_i$, (4) can be written as:

$$\begin{aligned}\max_{k_r, k_c, p_n, p_d} \Pi(k_r, k_c, p_n, p_d) = \sum_{i \in \{n,d\}} \Big[& p_i D_i - (1 - q_i)\left(g\left((D_i - k_c)^+\right) + v \min(k_c, D_i)\right) \\ & - q_i \left(g\left((D_i - k_r - k_c)^+\right) + v \min\left((k_c, D_i - k_r)^+\right)\right)\Big] - \beta_c k_c - \beta_r k_r.\end{aligned}$$

The hessian of this function is negative definite so that the First Order Conditions (FOCs) are sufficient. We first show that under Assumption 1, $k_r + k_c$ is always less than D_n. Suppose otherwise that $k_r + k_c \geq D_n$. There are two cases: first, assume that $k_r + k_c > D_d$. Then, the FOC with respect to (wrt) k_r is $\mathcal{F}(k_r, k_c, p_n, p_d) = q_n v 1_{k_r \leq D_n} + q_d v 1_{k_r \leq D_d} - \beta_r$, where 1. is the indicator function. By Assumption 1, $\beta_r \geq v(q_n + q_d)$, so this case cannot appear in the optimal solution. Second, assume that $k_r + k_c \leq D_d$. Then, the FOC wrt k_r is $\mathcal{F}(k_r, k_c, p_n, p_d) = q_n v 1_{k_r \leq D_n} + q_d g^{'}(D_d - k_c - k_r) - \beta_r$. Note that this function can at most be $q_n v + q_d g^{'}(a_d - a_n) - \beta_r \leq 0$ as $D_d - D_n = a_d - a_n - (\gamma + \delta)(p_d - p_n)$. By Assumption 1, this case cannot appear in the optimal solution either. Thus, at optimality, $k_r + k_c \leq D_n$. Next, consider the FOCs wrt k_c, p_n, and p_d given sequentially by $\mathcal{G}(k_r, k_c, p_n, p_d)$, $\mathcal{H}(k_r, k_c, p_n, p_d)$, and $\mathcal{I}(k_r, k_c, p_n, p_d)$:

$$\begin{aligned}\mathcal{G}(k_r, k_c, p_n, p_d) &= (1 - q_n) g^{'}(D_n - k_c) + (1 - q_d) g^{'}(D_d - k_c) \\ &\quad + q_n g^{'}(D_n - k_c - k_r) + q_d g^{'}(D_d - k_c - k_r) - 2v - \beta_c \\ \mathcal{H}(k_r, k_c, p_n, p_d) &= a_n - 2\gamma p_n + 2\delta p_d + \gamma\left((1 - q_n) g^{'}(D_n - k_c) + q_n g^{'}(D_n - k_c - k_r)\right) \\ &\quad - \delta\left((1 - q_d) g^{'}(D_d - k_c) + q_d g^{'}(D_d - k_c - k_r)\right) \\ \mathcal{I}(k_r, k_c, p_n, p_d) &= a_d - 2\gamma p_d + 2\delta p_n - \delta\left((1 - q_n) g^{'}(D_n - k_c) + q_n g^{'}(D_n - k_c - k_r)\right) \\ &\quad + \gamma\left((1 - q_d) g^{'}(D_d - k_c) + q_d g^{'}(D_d - k_c - k_r)\right)\end{aligned}$$

At optimality, $\mathcal{H}(k_r^*, k_c^*, p_n^*, p_d^*) + \mathcal{I}(k_r^*, k_c^*, p_n^*, p_d^*) = 0$ and incorporating the fact that $\mathcal{G}(k_r^*, k_c^*, p_n^*, p_d^*) = 0$, we observe that (5) holds true. This result implies that the sum of the nighttime and daytime demand is equal under both pricing policies as $D_n + D_d = a_n + a_d - (\gamma - \delta)(p_n + p_d)$.

Finally, we show that under Assumption 1 part (ii), $p_d^* \geq p_n^*$ and $p_d^* - p_n^* \leq \frac{a_d - a_n}{\gamma + \delta}$ under any pricing policy so that $D_d(p_d^*, p_n^*) - D_n(p_n^*, p_d^*) = a_d - a_n - (\gamma + \delta)(p_d^* - p_n^*) \geq 0$. This is because $a_d - a_n - (\gamma + \delta)(2(p_d^* - p_n^*) + 2v + \beta_c) \leq \mathcal{I}(k_r^*, k_c^*, p_n^*, p_d^*) - \mathcal{H}(k_r^*, k_c^*, p_n^*, p_d^*) = 0 \leq a_d - a_n - (\gamma + \delta)(2(p_d^* - p_n^*) - 2v - \beta_c)$, where we use that $\mathcal{G}(k_r^*, k_c^*, p_n^*, p_d^*) = 0$. Thus, under Assumption 1 part (ii), $0 \leq p_d^* - p_n^* \leq \frac{a_d - a_n + (\gamma+\delta)(2v + \beta_c)}{2(\gamma + \delta)}$, which is further lower than $\frac{a_d - a_n}{\gamma + \delta}$. This ensures that $D_d(p_d^*, p_n^*) \geq D_n(p_n^*, p_d^*)$. $\square$

Proof of Proposition 1. (i) First, note that the FOC wrt k_r is given as $\mathcal{F}(k_r, k_c, p_n, p_d) = q_n g'(D_n - k_r - k_c) + q_d g'(D_d - k_r - k_c) - \beta_r$. Considering the FOCs wrt k_r and wrt k_c given above, we observe that at optimality $(1 - q_n) g'(D_n - k_c) + (1 - q_d) g'(D_d - k_c) = 2v + \beta_c - \beta_r$. Thus, considering this identity under flat and peak pricing, we can show that

$$\begin{aligned}\frac{1 - q_n}{1 - q_d} &= \frac{g'\left(D_d^{\text{flat}} - k_c^{\text{flat}}\right) - g'\left(D_d^{\text{peak}} - k_c^{\text{peak}}\right)}{g'\left(D_n^{\text{peak}} - k_c^{\text{peak}}\right) - g'\left(D_n^{\text{flat}} - k_c^{\text{flat}}\right)} \\ &= \frac{\left(\tilde{D} - \left(k_c^{\text{flat}} - k_c^{\text{peak}}\right)\right)\left(C_1\left(2D_d^{\text{flat}} - \tilde{D} - k_c^{\text{flat}} - k_c^{\text{peak}}\right) + C_2\right)}{\left(\tilde{D} + \left(k_c^{\text{flat}} - k_c^{\text{peak}}\right)\right)\left(C_1\left(2D_n^{\text{flat}} + \tilde{D} - k_c^{\text{flat}} - k_c^{\text{peak}}\right) + C_2\right)} = R_c, \qquad \text{(B.1)}\end{aligned}$$

where D_i^j is the demand corresponding to the optimal prices in period $i \in \{n, d\}$ under pricing policy $j \in \{\text{flat}, \text{peak}\}$ and $\tilde{D}$ is the difference between optimal demand levels under flat and peak pricing in a period. Considering the FOC wrt k_r, we similarly observe that

$$\frac{q_n}{q_d} = \frac{\left(\tilde{D} - \left(k_c^{\text{flat}} - k_c^{\text{peak}}\right) - \left(k_r^{\text{flat}} - k_r^{\text{peak}}\right)\right)\left(C_1\left(2D_d^{\text{flat}} - \tilde{D} - k_c^{\text{flat}} - k_c^{\text{peak}} - k_r^{\text{flat}} - k_r^{\text{peak}}\right) + C_2\right)}{\left(\tilde{D} + \left(k_c^{\text{flat}} - k_c^{\text{peak}}\right) + \left(k_r^{\text{flat}} - k_r^{\text{peak}}\right)\right)\left(C_1\left(2D_n^{\text{flat}} + \tilde{D} - k_c^{\text{flat}} - k_c^{\text{peak}} - k_r^{\text{flat}} - k_r^{\text{peak}}\right) + C_2\right)} = R_k.$$

Suppose $q_n/q_d \leq 1$, then $(1 - q_n)/(1 - q_d) \geq 1$ so that R_c is greater than R_k. Thus, $k_r^{\text{flat}} \geq k_r^{\text{peak}}$ because otherwise that would imply that R_c is less than R_k. (ii) We note that $k_r^{\text{peak}} \geq k_r^{\text{flat}}$ if $\mathcal{F}\left(k_r^{\text{flat}}, k_c^{\text{peak}}, p_n^{\text{peak}}, p_d^{\text{peak}}\right) \geq \mathcal{F}\left(k_r^{\text{flat}}, k_c^{\text{flat}}, p_n^{\text{flat}}, p_d^{\text{flat}}\right) = 0$, where all arguments of the FOC wrt k_r are their respective optimal values. This is because $\mathcal{F}(k_r, k_c, p_n, p_d)$ is a decreasing function in k_r. We also note that $\mathcal{F}\left(k_r^{\text{flat}}, k_p^{\text{peak}}, p_n^{\text{peak}}, p_d^{\text{peak}}\right) \geq \mathcal{F}\left(k_r^{\text{flat}}, k_c^{\text{flat}}, p_n^{\text{flat}}, p_d^{\text{flat}}\right)$ if and only if

$$\frac{q_n}{q_d} \geq \underline{R} = \frac{g'\left(D_d^{\text{flat}} - k_c^{\text{flat}} - k_r^{\text{flat}}\right) - g'\left(D_d^{\text{peak}} - k_c^{\text{peak}} - k_r^{\text{flat}}\right)}{g'\left(D_n^{\text{peak}} - k_c^{\text{peak}} - k_r^{\text{flat}}\right) - g'\left(D_n^{\text{flat}} - k_c^{\text{flat}} - k_r^{\text{flat}}\right)}.$$

Next, we show that R_1 given in (7) is such that $R_1 \geq \underline{R}$ so that $q_n/q_d \geq R_1 > 1$ implies that

$k_r^{\text{peak}} \geq k_r^{\text{flat}}$. Consider the following bound on $\underline{R}$

$$\underline{R} = \frac{\left(\tilde{D} - \left(k_c^{\text{flat}} - k_c^{\text{peak}}\right)\right)\left(C_1\left(2D_d^{\text{flat}} - \tilde{D} - k_c^{\text{flat}} - k_c^{\text{peak}} - 2k_r^{\text{flat}}\right) + C_2\right)}{\left(\tilde{D} + \left(k_c^{\text{flat}} - k_c^{\text{peak}}\right)\right)\left(C_1\left(2D_n^{\text{flat}} + \tilde{D} - k_c^{\text{flat}} - k_c^{\text{peak}} - 2k_r^{\text{flat}}\right) + C_2\right)} \leq \frac{2C_1\left(D_d^{\text{flat}} - \left(k_c^{\text{flat}} + k_r^{\text{flat}}\right)\right) + C_2}{2C_1\left(D_n^{\text{flat}} - \left(k_c^{\text{flat}} + k_r^{\text{flat}}\right)\right) + C_2},$$

let the term on the right hand side be denoted by $\bar{R}$, where the inequality is due to the fact that $\tilde{D} \geq 0$ and $k_c^{\text{flat}} \geq k_c^{\text{peak}}$ assuming $q_n \geq q_d$ as shown in Proposition 2. At this point, $\underline{R}$ is bounded by $\bar{R}$ that involves the optimal values of the decision variables. Next, we will obtain an upper bound for $\bar{R}$ by assuming that $k_c^{\text{flat}} + k_r^{\text{flat}} \leq D_n^{\text{flat}} - \epsilon$ for some $\epsilon \geq 0$. Define $R(\epsilon)$ as

$$R(\epsilon) = \frac{2C_1\left(a_d - a_n + \epsilon\right) + C_2}{2C_1\epsilon + C_2} \geq \bar{R},$$

where we substitute $k_c^{\text{flat}} + k_r^{\text{flat}}$ with $D_n^{\text{flat}} - \epsilon$ in $\bar{R}$ and use the property that $D_d^{\text{flat}} - D_n^{\text{flat}} = a_d - a_n$. If $q_n/q_d \geq R(\epsilon)$, then $k_r^{\text{peak}} \geq k_r^{\text{flat}}$. Note that $R(\epsilon)$ decreases in ϵ and in order to characterize a greater portion of the q_n/q_d space, ϵ should be set to its maximum value. By examining the FOC wrt k_r, we observe that the assumption that $k_c^{\text{flat}} + k_r^{\text{flat}} \leq D_n^{\text{flat}} - \epsilon$ holds if $q_n g'\left(D_n^{\text{flat}} - \left(D_n^{\text{flat}} - \epsilon\right)\right) + q_d g'\left(D_d^{\text{flat}} - \left(D_n^{\text{flat}} - \epsilon\right)\right) \leq \beta_r$. This inequality is valid if $\max\left(q_n, q_d\right) g'\left(a_d - a_n + 2\epsilon\right) \leq \beta_r$, where we use the fact that $g'(\cdot)$ is a convex function. Thus, we observe that

$$\epsilon \leq \epsilon_{\max} = \frac{f\left(\beta_r/q_n\right) - \left(a_d - a_n\right)}{2},$$

where $f(\cdot)$ denotes the inverse of the derivative of the generation cost function, i.e., $f(\cdot) = \left(g'\right)^{-1}(\cdot)$, and we continue to assume that $q_n \geq q_d$. We note that $\epsilon_{\max}$ is positive by Assumption 1, and plugging it into $R(\epsilon)$, we observe that $R\left(\epsilon_{\max}\right) = R_1$ so that $q_n/q_d \geq R_1 \geq 1$ implies $k_r^{\text{peak}} \geq k_r^{\text{flat}}$. $\square$

Proof of Proposition 2. (i) Consider the relationship between FOCs wrt k_c given in (B.1). If $\left(1 - q_n\right)/\left(1 - q_d\right) \leq 1$, then $k_c^{\text{flat}} \geq k_c^{\text{peak}}$; otherwise R_c is greater than 1 contradicting the fact that $\left(1 - q_n\right)/\left(1 - q_d\right) \leq 1$. (ii) This part of the proof follows similar steps to the second part of the proof of Proposition 1 and hence is omitted. $\square$

Proof of Proposition 3. Based on (10), $ECE^{\text{flat}} - ECE^{\text{peak}} = -2\left(\left(1 - e\right)\left(k_r^{\text{flat}} + k_c^{\text{flat}} - k_r^{\text{peak}} - k_c^{\text{peak}}\right) + \left(e - \bar{e}\right)\left(k_r^{\text{flat}} - k_r^{\text{peak}}\right)\right)$, where $e \geq \bar{e}$ as we assume so. (i) Note that $ECE^{\text{flat}} \leq ECE^{\text{peak}}$ if $k_r^{\text{flat}} \geq k_r^{\text{peak}}$ and $k_r^{\text{flat}} + k_c^{\text{flat}} \geq k_r^{\text{peak}} + k_c^{\text{peak}}$. The first condition holds due to Proposition 1 as long as $q_n/q_d \leq 1$. Thus, to prove this part of the proposition, we need to show that $k_r^{\text{flat}} + k_c^{\text{flat}} \geq k_r^{\text{peak}} + k_c^{\text{peak}}$. To show this, consider the FOCs wrt k_r under flat and peak pricing by letting $k_r^j + k_c^j = k^j$ for $j \in \{\text{flat}, \text{peak}\}$ and observe that:

$$\frac{q_n}{q_d} = \frac{g^{'}\left(D_d^{\text{flat}} - k^{\text{flat}}\right) - g^{'}\left(D_d^{\text{peak}} - k^{\text{peak}}\right)}{g^{'}\left(D_n^{\text{peak}} - k^{\text{peak}}\right) - g^{'}\left(D_n^{\text{flat}} - k^{\text{flat}}\right)} = \frac{\left(\tilde{D} - \left(k^{\text{flat}} - k^{\text{peak}}\right)\right)\left(C_1\left(2D_d^{\text{flat}} - \tilde{D} - k^{\text{flat}} - k^{\text{peak}}\right) + C_2\right)}{\left(\tilde{D} + \left(k^{\text{flat}} - k^{\text{peak}}\right)\right)\left(C_1\left(2D_n^{\text{flat}} + \tilde{D} - k^{\text{flat}} - k^{\text{peak}}\right) + C_2\right)}.$$

Note that $q_n/q_d \leq 1$ implies that $k^{\text{flat}} \geq k^{\text{peak}}$, otherwise, the right hand side would be greater than 1, contradicting that $q_n/q_d \leq 1$. (ii) To prove this part, we first note that $ECE^{\text{flat}} \geq ECE^{\text{peak}}$ if $k_r^{\text{flat}} \leq k_r^{\text{peak}}$ and $k_r^{\text{flat}} + k_c^{\text{flat}} \leq k_r^{\text{peak}} + k_c^{\text{peak}}$. The first condition is given by Proposition 1 and the proof of the second condition follows similar steps to the part (ii) of the proof of Proposition 1 and hence is omitted. (iii) See Figure 2 for an example. $\square$

Proof of Proposition 4. Based on the definition of consumer surplus given in (13),

$$CS^{\text{flat}} - CS^{\text{peak}} = \Delta\left(a_d - a_n - (\gamma + \delta)\,\Delta\right),$$

where Δ denotes the absolute difference in price levels between flat and peak pricing in a period. (Note that, by Lemma 1, this difference is the same for the nighttime and daytime.) Thus, consumer surplus is higher under flat pricing if and only if $a_d - a_n \geq (\gamma + \delta)\Delta$. This holds because, as we show in the proof of Lemma 1, under Assumption 1, $(a_d - a_n)/(\gamma + \delta) \geq p_d^{\text{peak}} - p_n^{\text{peak}} = 2\Delta$. $\square$

Proof of Proposition 5. (i) Let (k_r, k_c, p_n, p_d) be the simultaneous solutions of the FOCs wrt k_r, k_c, p_n, and p_d, where the FOCs are given sequentially as $\mathcal{F}(k_r, k_c, p_n, p_d; \beta_r) = 0$, $\mathcal{G}(k_r, k_c, p_n, p_d; \beta_r) = 0$, $\mathcal{H}(k_r, k_c, p_n, p_d; \beta_r) = 0$, and $\mathcal{I}(k_r, k_c, p_n, p_d; \beta_r) = 0$. We show that $dk_r/d\beta_r$ is negative, $dk_c/d\beta_r$ is positive, and $dECE/d\beta_r$ is also positive if $e \geq \bar{e}$. By using implicit differentiation and Cramer's Rule, we can show that

$$\frac{dk_r}{d\beta_r} = \frac{\begin{vmatrix} -\frac{\partial \mathcal{F}}{\partial \beta_r} & \frac{\partial \mathcal{F}}{\partial k_c} & \frac{\partial \mathcal{F}}{\partial p_n} & \frac{\partial \mathcal{F}}{\partial p_d} \\ -\frac{\partial \mathcal{G}}{\partial \beta_r} & \frac{\partial \mathcal{G}}{\partial k_c} & \frac{\partial \mathcal{G}}{\partial p_n} & \frac{\partial \mathcal{G}}{\partial p_d} \\ -\frac{\partial \mathcal{H}}{\partial \beta_r} & \frac{\partial \mathcal{H}}{\partial k_c} & \frac{\partial \mathcal{H}}{\partial p_n} & \frac{\partial \mathcal{H}}{\partial p_d} \\ -\frac{\partial \mathcal{I}}{\partial \beta_r} & \frac{\partial \mathcal{I}}{\partial k_c} & \frac{\partial \mathcal{I}}{\partial p_n} & \frac{\partial \mathcal{I}}{\partial p_d} \end{vmatrix}}{H},$$

where $|\cdot|$ denotes the determinant operator and $H > 0$ is the determinant of the Hessian matrix. The numerator can be shown to be negative, hence, $dk_r/d\beta_r$ is negative. Similar steps prove that $dk_c/d\beta_r$ is positive, and $dECE/d\beta_r$ is also positive if $e \geq \bar{e}$. On the other hand, if $e < \bar{e}$, Figure 4 provides an example in which $dECE/d\beta_r$ is negative. (ii) This part is proved analogously to the previous part and omitted for brevity. $\square$

Proof of Proposition 6. This proof is similar to that of Proposition 5. ☐

Proof of Proposition 7. (i) It can be shown that the Hessian of (19) is negative definite so that the FOCs are sufficient. (ii) We first note that $k_{DG}^{\text{flat}} - k_{DG}^{\text{peak}} = \left(q_n\Delta_n - q_d\Delta_d\right)/\beta_{DG}$, where Δ_i is the absolute value of the difference between price levels of flat and peak pricing in period $i \in \{n, d\}$. We first show that if $q_n \geq q_d$, then $q_n\Delta_n \geq q_d\Delta_d$, which, in turn, implies that $k_{DG}^{\text{flat}} \geq k_{DG}^{\text{peak}}$. By using the FOCs wrt k_r, k_c, p_n, and p_d, we prove that

$$\frac{\beta_{DG}}{2}\left(a_n + a_d + (\gamma - \delta)\left(2v + \beta_c\right) + \beta_r\left(\frac{q_n + q_d}{\beta_{DG}}\right)\right) = \beta_{DG}\left(\gamma - \delta\right)\left(p_n + p_d\right) + \left(q_n + q_d\right)\left(q_n p_n + q_d p_d\right),$$

where the left hand side consists only of problem parameters. By using this relationship, we note that

$$\frac{-\Delta_n + \Delta_d}{q_n\Delta_n - q_d\Delta_d} = \frac{q_n + q_d}{\beta_{DG}\left(\gamma - \delta\right)}. \tag{B.2}$$

We now consider that $q_n \geq q_d$ and assume $\Delta_n \geq \Delta_d$ to prove by contradiction. In this case, the left hand side is negative in (B.2), which contradicts the fact that the right hand side is positive. Hence, $q_n \geq q_d$ implies $\Delta_d \geq \Delta_n$. Accordingly, $q_n\Delta_n \geq q_d\Delta_d$ to ensure that the left hand side of (B.2) is positive. Thus, $q_n \geq q_d$ implies that $q_n\Delta_n \geq q_d\Delta_d$, and hence $k_{DG}^{\text{flat}} \geq k_{DG}^{\text{peak}}$. Next, consider that $q_d \geq q_n$. In this case, $\Delta_n \geq \Delta_d$ as otherwise there would be a contradiction in (B.2) as the left hand side would be negative whereas the right hand side is positive. Hence, $q_d \geq q_n$ implies that $\Delta_n \geq \Delta_d$, which, further implies $q_d\Delta_d \geq q_n\Delta_n$. Thus, if $q_d \geq q_n$, then $k_{DG}^{\text{peak}} \geq k_{DG}^{\text{flat}}$. ☐

Proof of Proposition 8. The Hessian of the utility firm's profit maximization problem given in (4) is negative definite, and hence the FOCs are sufficient. Under Assumption 2, it can be shown that $k_r + k_c \leq D_n^j$ for $j \in \{\text{flat}, \text{peak}\}$. Furthermore, solving the FOCs simultaneously, we observe that $k_r^{\text{flat}} - k_r^{\text{peak}} = 2\tilde{D}\left(E\left[\tilde{q}_d\right] - E\left[\tilde{q}_n\right]\right)/A$, $k_c^{\text{flat}} - k_c^{\text{peak}} = \tilde{D}\left(-E\left[\tilde{q}_d\right]^2 + E\left[\tilde{q}_n\right]^2\right)/A$, and $ECE^{\text{flat}} - ECE^{\text{peak}} = 2\tilde{D}e\left(-E\left[\tilde{q}_d\right]^2 + E\left[\tilde{q}_n\right]^2\right)/A$, where $\tilde{D}$ is the difference between optimal demand levels under flat and peak pricing in a period and $A = Var\left[\tilde{q}_n\right] + Var\left[\tilde{q}_d\right] + E\left[\tilde{q}_n^2\right] + E\left[\tilde{q}_d^2\right] - 2E\left[\tilde{q}_n\right]E\left[\tilde{q}_d\right] \geq 0$. Hence, $k_r^{\text{flat}} \geq k_r^{\text{peak}}$, $k_c^{\text{peak}} \geq k_c^{\text{flat}}$ and $ECE^{\text{peak}} \geq ECE^{\text{flat}}$ if $E\left[\tilde{q}_d\right] \geq E\left[\tilde{q}_n\right]$, otherwise, all three inequalities are reversed. ☐

C Analysis of $\pi(k_r, k_j)$

We note that the benefit function is given as:

$$\begin{aligned}\pi^j(k_r,k_c) &= \Pi^j(k_r,k_c) - \Pi^j(0,0) = [G^j(0,0) - G^j(k_r,k_c)] + [\alpha_r(0) - \alpha_r(k_r)] + [\alpha_c(0) - \alpha_c(k_c)].\\ &= l_r^j k_r + m_r^j\sqrt{k_r} + l_c^j k_c + m_c^j \sqrt{k_c} - \beta_r k_r - \beta_c k_c,\end{aligned}$$

where we substitute (17) for $[G^j(0,0) - G^j(k_r,k_c)]$, $\beta_r k_r$ for $\alpha_r(k_r)$, and $\beta_c k_c$ for $\alpha_c(k_c)$. We evaluate (17) for a renewable energy investment level up to 20,000MW, and a conventional energy investment level up to 5,000 MW. This difference is to account for the intermittency of renewables so that the actual generation from both sources would be approximately equal at these investment levels. We estimate l_r^j, m_r^j, l_c^j, and m_c^j parameters with our case study for $j \in \{\text{flat}, \text{peak}\}$. The estimated values are presented in Tables 4 and 5, and Figure 6 plots this net benefit function for the estimated parameters and cost parameters described below. It is straightforward to show that $\pi^j(k_r,k_c)$ is jointly concave in k_r and k_c. The optimal investment levels are given as $k_r^j = \left(\frac{m_r^j}{2(\beta_r - l_r^j)}\right)^2$ and $k_c^j = \left(\frac{m_c^j}{2(\beta_c - l_c^j)}\right)^2$ as long as $\beta_r \geq l_r^j$ and $\beta_c \geq l_c^j$ under pricing policy $j \in \{\text{flat}, \text{peak}\}$. We report these optimal investment levels in Tables 2 and 3.

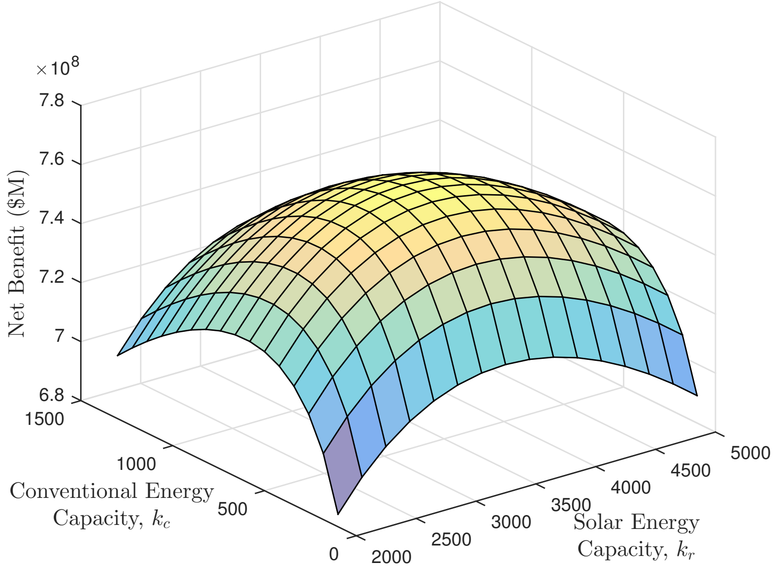

Figure 6: Net Benefit Function, $\pi(k_r, k_c)$, for Solar Energy under Flat Pricing

	Response Level	m_c^j	m_r^j	l_c^j	l_r^j
Flat Pricing	N/A	10,120,131	21,908,656	889,002	1,436,892
Peak Pricing	Low (5%)	19,003,649	20,161,371	786,002	1,400,913
	Medium (10%)	17,504,077	23,126,617	838,970	1,323,337
	High (15%)	18,117,049	30,909,338	857,373	1,199,025

Table 4: Parameter Estimates for Solar Energy

	Response Level	m_c^j	m_r^j	l_c^j	l_r^j
Flat Pricing	N/A	4,323,298	50,161,699	551,308	1,152,838
Peak Pricing	Low (5%)	9,706,227	54,680,383	490,897	1,095,233
	Medium (10%)	13,735,100	42,493,471	473,952	1,152,989
	High (15%)	13,081,120	30,448,889	490,256	1,214,981

Table 5: Parameter Estimates for Wind Energy

D Estimation of Cost Parameters

In this section, we describe the estimation procedure of investment and generation cost parameters, i.e., β_r, β_c, and v for various electricity sources. We use cost estimates from the Transparent Cost Database (TCDB, `http://en.openei.org/apps/TCDB`), which is designed by National Renewable Energy Laboratory (NREL) to track publications that estimate cost parameters for renewable and conventional energy investments. In particular, to find β_r and β_c, we divide the overnight capital cost reported in TCDB with the useful economic lifetime of a source. Specifically, for wind energy, TCDB reports the median overnight capital cost as $\$1,570,000$/MW. The useful economic life for wind is estimated by NREL as 20 years although it is also reported that the lifetime is shorter (Gordon 2012). Thus, we consider wind lifetime as 15–20 years with an average value of 17.5 years. Hence, $\beta_r = \$1.57 \times 10^6/\text{MW}/17.5\text{years} = \$249.2/\text{MW}$ per day for wind energy. For solar energy, the cost estimates vary significantly between different studies and we consider the median of the most recent (from the year 2014) cost estimates for solar energy given as $\$1,625,000$/MW. According to NREL[8], solar panels last 25–40 years and we consider the lifetime of solar panels as 32.5 years (=(25+40)/2), hence $\beta_r = \$138.9/\text{MW}$ per day. For coal energy, TCDB estimates useful life as 60 years so that we set $\beta_c = \$91.2/\text{MW}$ per day. For nuclear and natural gas (combustion turbine) sources, TCDB estimates suggest that $\beta_c = \$161.1/\text{MW}$ per day and $\beta_c = \$61.2/\text{MW}$ per day. We take nuclear and natural gas lifetime as 60 and 30 years, respectively (see EIA 2014a and

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[8] See `http://www.nrel.gov/analysis/tech_footprint.html`

TCDB). The generation cost data for estimating v is taken from Energy Information Administration (EIA 2014b) for conventional sources.

In the case study, we focus on the entire lifetime of investments rather than a representative day. Thus, we use the above overnight capital cost estimates directly and account for the differences in lifetimes of energy sources while considering investment costs. Specifically, when solar energy and coal investments are considered simultaneously, we divide β_c by a factor of 1.8 (=60/32.5) and set $\beta_c = 1,094,444 (= 1,970,000/1.8)$. On the other hand, when wind and coal energy investments are considered, we divide β_c by a factor of 3.4(=60/17.5) and set $\beta_c = 579,412$.

E Consumer Surplus and Utility

We first note that, in general, computing the line integral given in (13) is prone to the path dependency problem. Specifically, this integral may depend on the particular path on the (z_n, z_d) plane, with the starting point of $(0,0)$ and the ending point of (z_n^{j*}, z_d^{j*}). In our setting, this integral is path independent, i.e., computing it along any path that starts and ends at $(0,0)$ and (z_n^{j*}, z_d^{j*}) yields the same result. This is because the cross partial derivatives of the nighttime and daytime demand functions are equal to each other, i.e., $\frac{\partial^2 D_n(p_n,p_d)}{\partial p_n \partial p_d} = \frac{\partial^2 D_d(p_d,p_n)}{\partial p_d \partial p_n} = \delta$ (see Vives 2001 p.86 for a detailed discussion on the path dependency issue).

We next introduce the underlying utility maximization problem that is behind the linear demand model we use. Then, by using this utility function, we show that the utility of consumers is higher under flat pricing compared to peak pricing.

First, consider the following utility maximization problem subject to the budget constraint:

$$\begin{aligned} &\max_{z_n, z_d} U(z_n, z_d) = m + \zeta_n z_n + \zeta_d z_d - \frac{\eta}{2}\left(z_n^2 + 2\lambda z_n z_d + z_d^2\right) \\ &\text{s.t. } p_n z_n + p_d z_d + m \leq I, \end{aligned}$$

where a representative agent maximizes its utility over its consumption levels z_n and z_d in the nighttime and daytime periods, respectively. In this formulation, ζ_i, η, and λ represent parameters of the utility function, m is an outside good and I is the budget level. Similar utility maximization problems are considered in the literature (c.f., Shapley and Shubik 1969, Singh and Vives 1984, and Ledvina and Sircar 2012).

Solving this optimization problem, we observe that the optimal consumption level z_i^* in period $i \in \{n, d\}$ as a function of price levels p_i and p_{-i} is given as:

$$z_i^*\left(p_i, p_{-i}\right) = \frac{\zeta_i - \lambda\zeta_{-i}}{\eta\left(1-\lambda^2\right)} - \frac{p_i}{\eta\left(1-\lambda^2\right)} + \frac{\lambda p_{-i}}{\eta\left(1-\lambda^2\right)}, \qquad i \in \{n, d\}.$$

Notice that letting $\frac{\zeta_i - \lambda\zeta_{-i}}{\eta(1-\lambda^2)} \equiv a_i$, $\frac{1}{\eta(1-\lambda^2)} \equiv \gamma$, $\frac{\lambda}{\eta(1-\lambda^2)} \equiv \delta$, the above demand function is equivalent to our demand model. Hence, we characterize the underlying utility formulation behind our linear demand function as: $U\left(z_n, z_d\right) = m + \zeta_n z_n + \zeta_d z_d - \frac{\eta}{2}\left(z_n^2 + 2\lambda z_n z_d + z_d^2\right)$. We next compare the utility of consumers under flat and peak pricing. Note that $U\left(z_n^{\text{flat}*}, z_d^{\text{flat}*}\right) - \left(z_n^{\text{peak}*}, z_d^{\text{peak}*}\right) = \left(\delta + \gamma\right)\Delta^2 \geq 0$, where Δ is the absolute difference between the price under flat pricing and the nighttime or daytime price under peak pricing. (By Lemma 1, both of these differences are equal to each other.) Thus, this result suggests that the utility of the consumers is higher under flat pricing, confirming the result given in Proposition 4.

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