Online Appendices

Appendix 1: Forward Simulation

The objective of this simulation exercise is to calculate the continuation values $EV_R(Z)$, and $EV_j(Z),\ j=1,2,...,J$ in the Bellman equations of the retailer and each manufacturer for a given cost structure $(\alpha_{0j},\alpha,\sigma_j)$, and policy function parameters (θ_j^R,θ_j^M) in the numerical search routine. We simulate numerous paths. For each simulated path, we first choose Z_0 from the state space. We then run the following simulation routine:

1. Given Z_0 and the assumed parametric policy functions calculate retail and wholesale policies $P_0(Z_0)$ and $W_0(Z_0)$. Then, calculate demand by using the predictive aggregate brand market share function $MS_0(Z_0,P_0)$.
2. Given P_0, W_0 and MS_0, calculate $\pi_0{}^R = \sum_j (P_{0j}-W_{0j})MS_{0j}$ and $\pi_0{}^j = (W_{0j}-C_j-\nu_{0j})MS_{0j}$, *j=1,2,…,J.* Then calculate installed customer base in the next period $S_1(Z_0,P_0)$.
3. Given S_1, draw the rest of the observed and unobserved states from their empirical distributions, and label the full set of states as Z_1. Given Z_1 repeat steps 1,2.
4. Repeat step 3 for T times until that $\rho^T \approx 0$.

Taking discounted sum of profits calculated for each of the T periods, and averaging over all simulation paths gives us the set of approximated values for each channel member $V_R(Z_0)$, and $V_j(Z_0),\ j=1,2,...,J$. We then regress these values on the initial state space Z_0 to get approximated value functions for any arbitrary state combination.

Appendix 2: Bootstrapping Procedure

We use the following bootstrapping procedure to calculate the standard errors of our supply side estimates:

1. We draw Θ^{Ds}, $s = 1, 2, \ldots, ns$, from the asymptotic normal distribution of the demand model parameter estimates, $N(\hat{\Theta}^D, \hat{\Sigma}^D)$, where $\hat{\Theta}^D$ stands for the estimated demand parameters, and $\hat{\Sigma}^D$ stands for the estimated covariance matrix of the estimated demand parameters (which accounts for the estimation error in the predictive aggregate market share function).
2. We obtain bootstrapped retail prices and installed customer bases, (P_t^s, S_t^s, $s = 1, 2, \ldots, ns$), by drawing independent, random samples, with replacement, from the original data.
3. We re-estimate the cost parameters of the structural econometric model of dynamic channel pricing for each bootstrapped draw of the original data (from Step 2 above), while generating the evolution of states, *S*, as well as the market share function, *MS*, based on each bootstrapped draw of the estimated demand model parameters (from Step 1 above).
4. Using the estimated cost parameters from Step 3 above, across all bootstrapped draws, we calculate the standard errors associated with those estimates.

Appendix 3: Monte Carlo Simulation to Test Our Proposed Estimation Algorithm

In this simulation exercise, we study the pricing behavior of two competing manufacturers and one common retailer. For demand parameters, we assume similar levels for intrinsic preferences, price sensitivity and switching cost compared to our demand estimates. For supply side parameters, we assume four different sets of manufacturing cost specifications (Scenario 1: symmetric costs, large cost shocks; Scenario 2: small asymmetric costs and medium cost shocks, Scenario 3: small asymmetric costs and small cost shocks; Scenario 4: large asymmetric costs and very large cost shocks). Given the assumed demand parameters, four each cost scenario, we use the NFXP algorithm (in Appendix 3) to calculate the dynamic policy functions of the retailer (under the repeated Stackelberg assumption for the vertical interactions) and duopolist manufacturers (under the repeated Bertrand assumption for the horizontal interactions). Next, we use these optimal policy functions to simulate data sets (one set for each cost scenario) with the size of 100. After that, we use our proposed algorithm to estimate the assumed scenario specific cost parameters back. We repeat this procedure 100 times, and calculate the mean and the standard deviation of the corresponding estimates.

MANUFACTURER DUOPOLY WITH RETAILER

	Scenario 1	**Scenario 2**	**Scenario 3**	**Scenario 4**
$C_{True,Coke}$	0.50	0.50	0.55	0.60
$C_{Est,Coke}$	0.5001 (0.0062)	0.5000 (0.0038)	0.5503 (0.0016)	0.5999 (0.0098)
$C_{True,Pepsi}$	0.50	0.55	0.50	0.40
$C_{Est,Pepsi}$	0.4988 (0.0075)	0.5496 (0.0037)	0.4995 (0.0028)	0.3986 (0.0096)
$v_{True,Coke}$	0.08	0.05	0.02	0.12
$v_{Est,Coke}$	0.0786 (0.0041)	0.0493 (0.0021)	0.0195 (0.0010)	0.1192 (0.0071)
$v_{True,Pepsi}$	0.08	0.04	0.03	0.10
$v_{Est,Pepsi}$	0.0795 (0.0036)	0.0397 (0.0018)	0.0298 (0.0009)	0.0992 (0.0058)

Appendix 4: Multi-Agent NFXP Algorithms for Counterfactual Studies

Appendix 4.1: Dynamic Channel Pricing

In this numerical exercise, given the demand and supply parameters, we calculate the optimal dynamic policies of the retailer as well as manufacturers Coke and Pepsi. Each period, the retailer chooses two policies ($P_j, j=1,2$), and each manufacturer chooses one policy, i.e., Coke chooses W_1 and Pepsi chooses W_2 respectively. Here is the algorithm:

1. Start with $P_1^0, P_2^0, W_1^0, W_2^0$:

   1.1. Given W_1^0, W_2^0, get the optimal dynamic response of the retailer by running the subroutine in the Appendix 4.1.2.

   1.2. In order to get $\partial P / \partial W_0$, run the subroutine in the Appendix 4.1.3.

   1.3. Given $\partial P / \partial W_0$, find W_1^1, W_2^1 as follows

      1.3.1. Given W_2^0, get $W_1^{0,1}$ by running the subroutine in the Appendix 4.1.5.

      1.3.2. Given $W_1^{0,1}$, get $W_2^{0,1}$ by running the subroutine in the Appendix 4.1.7.

      1.3.3. Repeat 1.3.1-1.3.2 until $\left\| W^{0,n} - W^{0,n-1} \right\| \approx 0$.

      1.3.4. Set $W^1 = W^{0,n}$

   1.4. Repeat 1.1-1.4 until $\left\| W^n - W^{n-1} \right\| \approx 0$, $\left\| P^n - P^{n-1} \right\| \approx 0$.

   1.5. Set $W^* = W^n$, $P^* = P^n$

Appendix 4.1.1: Subroutine Retailer Optimality

The objective of this subroutine is to find the best response of the retailer P_1^*, P_2^* to a given set of actions of Coke and Pepsi, W_1, W_2 under a given expected continuation value in the retailer's Bellman equation $EV_R(Z)$. In other words, the objective is given by

$$(P_1, P_2) = \arg\max\left\{(P_1 - W_1)MS_1 + (P_2 - W_2)MS_2 + \rho EV_R(Z' \mid Z, P)\right\}$$

where MS_j is the market share for product *j=1,2*. In order to find optimal P_1^*, P_2^*

1. Start with P_1^0, P_2^0. Given P_1^0, P_2^0 calculate the following:

$$\frac{\partial V_R(Z)}{\partial P_1} = MS_1 + (P_1 - W_1)MS_{11} + (P_2 - W_2)MS_{21} + \rho EV_{R1}(Z')$$
$$\frac{\partial V_R(Z)}{\partial P_2} = MS_2 + (P_1 - W_1)MS_{12} + (P_2 - W_2)MS_{22} + \rho EV_{R2}(Z')$$

where $MS_{jk} = \partial MS_j / \partial P_k$, *j,k=1,2,* and $EV_{Rj} = (\partial EV_R(Z' \mid Z, P) / \partial Z')(\partial Z' / \partial P_j),\ j = 1,2$

By rearranging, we can get P_1^1, P_2^1 as follows:

$$P_1^1 = W_1 - \left[MS_1 + (P_2 - W_2)MS_{21} + \rho EV_{R1}(Z' \mid Z, P)\right]\left[MS_{11} + MS_{21}\right]^{-1}$$
$$P_2^1 = W_2 - \left[MS_2 + (P_1 - W_1)MS_{12} + \rho EV_{R2}(Z' \mid Z, P)\right]\left[MS_{12} + MS_{22}\right]^{-1}$$

2. Given P_1^1, P_2^1, repeat step 1, to get P_1^2, P_2^2.

3. Repeat step 2 to update P_1, P_2 until an iteration *n* such that $\left\|P^n - P^{n-1}\right\| \approx 0$.

4. Set $P_1^* = P_1^n, P_2^* = P_2^n$.

Appendix 4.1.2: Subroutine Dynamic Retailer Response

The objective of this subroutine is to find the dynamic best response of the retailer to the actions of Coke and Pepsi, $W_1,\ W_2$. Here is the subroutine:

1. Start with $EV_R^0(Z) = 0$: the expected continuation value in the retailer's Bellman equation is zero.

   a. Get P_1^{0*}, P_2^{0*} under $EV_R^0(Z) = 0$ by using the subroutine in the Appendix 4.1.1. Given P_1^{0*}, P_2^{0*} calculate the following Bellman equation over the state space *Z.*

   $$V_R^1(Z) = (P_1^{0*} - W_1)MS_1 + (P_2^{0*} - W_2)MS_2 + \rho EV_R(Z' \mid Z, P^{0*})$$

Then, interpolate $EV_R^1(Z)$ for any arbitrary Z by using the calculated $V_R^1(Z)$ over the chosen state space *Z.*

b. Given $EV_R^1(Z)$, get P_1^{1*}, P_2^{1*} by using the subroutine in the Appendix 4.1.1. Calculate the Bellman equation in (a) under P_1^{1*}, P_2^{1*}. Update the expected continuation value to $EV_R^2(Z)$.

c. Repeat (b) until an iteration n such that $\left\| P^{n*} - P^{n-1*} \right\| \approx 0$.

d. Set $P_1^* = P_1^{n*}, P_2^* = P_2^{n*}$.

Appendix 4.1.3: Subroutine Retailer Best Response

The objective of this subroutine is to find the responses of the retailer to manufacturer's actions, namely $\partial P / \partial W$. In order to do that, we will repeat the subroutine in the Appendix 4.1.2 under the following set of actions of Coke and Pepsi:

$$(W_1 + h, W_2), (W_1, W_2 + h), (W_1 - h, W_2), (W_1, W_2 - h)$$

Then, with a small *h>0*, we can get the related derivatives numerically as follows:

$$\frac{\partial P_j}{\partial W_k} = \lim_{h \to 0} \frac{P_j^*(W_k + h, W_{-k}) - P_j^*(W_k - h, W_{-k})}{2h}, \; j,k=1,2$$

Appendix 4.1.4: Subroutine Coke's Optimality

The objective of this subroutine is to get the optimal response of Coke W_1^* to Pepsi's action W_2 under the retailer's response $\partial P / \partial W$ and the given expected continuation value of Coke $EV_1(Z)$. Here is the subroutine:

1. Start with W_1^0. Calculate the following Bellman equation

$$V_1(Z) = (W_1 - mc_1) MS_1 + \rho EV_1(Z' \mid Z, P)$$

where mc_1 is the marginal cost of Coke. If we take the derivative of the above Bellman equation with respect to W_1^0, we get the following:

$$\frac{\partial V_1(Z)}{\partial W_1} = MS_1 + MR_1 \frac{\partial MS_1}{\partial W_1} + \rho \frac{\partial EV_1(Z'|Z,P)}{\partial W_1}$$

where

$$MR_1 = (W_1 - mc_1)$$
$$\frac{\partial MS_1}{\partial W_1} = \frac{\partial MS_1}{\partial P_1}\frac{\partial P_1}{\partial W_1} + \frac{\partial MS_1}{\partial P_2}\frac{\partial P_2}{\partial W_1}$$
$$\frac{\partial EV_1(Z'|Z,P)}{\partial W_1} = \frac{\partial EV_1(Z'|Z,P)}{\partial S'}\frac{\partial S'}{\partial P_1}\frac{\partial P_1}{\partial W_1} + \frac{\partial EV_1(Z'|Z,P)}{\partial S'}\frac{\partial S'}{\partial P_2}\frac{\partial P_2}{\partial W_1}$$

Then, W_1^* becomes

$$W_1^* = mc_1 - \left[MS_1 + \rho \frac{\partial EV_1(Z'|Z,P)}{\partial W_1}\right]\left[\frac{\partial MS_1}{\partial W_1}\right]^{-1}$$

Then, set $W_1^1 = W_1^*$.

2. Repeat (1) with W_1^1 , and from the optimality condition above get W_1^2.

3. Repeat (2) until an iteration n such that $\left\|W_1^n - W_1^{n-1}\right\| \approx 0$.

4. Set $W_1^* = W_1^n$.

Appendix 4.1.5: Subroutine Dynamic Coke Response

The objective of this subroutine is to find the dynamic best response of Coke W_1 to Pepsi's action W_2 under the retailer's best response $\partial P / \partial W$. Here is the subroutine:

1. Start with $EV_1(Z) = 0$, i.e., the continuation value in Coke's Bellman equation is zero.

    a. Get W_1^{0*} under $EV_1(Z) = 0$, by using the subroutine in the Appendix 4.1.4. Calculate Coke's Bellman equation under W_1^{0*}, then interpolate the expected continuation value $EV_1^1(Z)$.

b. Given $EV_1^1(Z)$, get W_1^{1*} by using the subroutine in the Appendix 4.1.4. Calculate Coke's Bellman equation under W_1^{1*}. Update the expected continuation value via interpolation $EV_1^2(Z)$.

c. Repeat (b) until an iteration *n* such that $\left\|W_1^{n*} - W_1^{n-1*}\right\| \approx 0$.

d. Set $W_1^* = W_1^{n*}$.

Appendix 4.1.6: Subroutine Pepsi's Optimality

The objective of this subroutine is to get the optimal response of Pepsi W_2 to Coke's action W_1 under the retailer's response $\partial P / \partial W$ and given expected continuation value of Pepsi $EV_2(Z)$. The way this subroutine works is very similar to the subroutine in the Appendix 4.1.4. Here is the subroutine:

1. Start with W_2^0. Calculate the following Bellman equation

   $$V_2(Z) = (W_2 - mc_2)MS_2 + \rho EV_2(Z' \mid Z, P)$$

   where mc_2 is the marginal cost of Pepsi. Similar to the subroutine in the Appendix 4.1.4, we take the derivative of the above Bellman equation with respect to Pepsi's action. Then, we set W_2^1 to the optimal policy coming from the first-order condition.
2. Repeat (1) with W_2^1, and from the optimality of Pepsi, get W_2^2.
3. Repeat (2) until an iteration *n* such that $\left\|W_2^n - W_2^{n-1}\right\| \approx 0$.
4. Set $W_2^* = W_2^n$.

Appendix 4.1.7: Subroutine Dynamic Pepsi Response

The objective of this subroutine is to find the dynamic best response of Pepsi W_2 to Coke's action W_1 under the retailer's best response $\partial P / \partial W$. Here is the subroutine:

1. Start with $EV_2(Z) = 0$: the continuation value in Pepsi's Bellman equation is zero.

a. Get W_2^{0*} under $EV_2(Z) = 0$ for the entire state space chosen Z by using the subroutine in the Appendix 4.1.6. Given W_2^{0*} calculate the Bellman equation of Pepsi, and interpolate the calculated expected continuation value $EV_2^1(Z)$.

b. Given $EV_2^1(Z)$, get W_2^{1*} for the entire state space chosen Z by using the subroutine in the Appendix 4.1.6. Calculate the Bellman equation of Pepsi under W_2^{1*}, and interpolate the expected continuation value $EV_2^2(Z)$.

c. Repeat (b) until an iteration n such that $\left\| W_2^{n*} - W_2^{n-1*} \right\| \approx 0$

d. Set $W_2^* = W_2^{n*}$.