Supplementary Materials:

Proof of the Logic by Mathematical Induction

Hit-and-go Formula

$$\text{Summed Utility} = \sum_{i=1}^{q} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right]$$

Miss-and-go Formula

$$\text{Summed Utility} = p^q + \sum_{i=1}^{q} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right]$$

Let S(n) be the statement that the summed utility for the hit-and-go formula is larger than the summed utility for the miss-and-go formula:

$$\sum_{i=1}^{q} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \left\{p^q + \sum_{i=1}^{q} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right]\right\} \geq 0$$

For $q = 1$, both the hit-and-go and miss-and-go formulas give a summed utility = p; that is, S(1) is true.

If we assume S(k) to be true, that is,

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \left\{p^k + \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right]\right\} \geq 0$$

then

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - p^k \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - p^k + p^{k+1} - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - p^k(1-p) - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - \frac{k+1}{k+1} p^k(1-p) - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - \frac{kp^k(1-p) + p^k(1-p)}{k+1} - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - \frac{kp^k(1-p)}{k+1} - \frac{p^k(1-p)}{k+1} - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \frac{p^k(1-p)}{k+1} - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - \frac{kp^k(1-p)}{k+1} - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \frac{p^k(1-p)}{k+1} - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - \frac{kp^k(1-p)}{k+1} - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] + \frac{p(1-p)^k}{k+1} - \frac{p(1-p)^k}{k+1} - \frac{p^k(1-p)}{k+1} - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - \frac{kp^k(1-p)}{k+1} - p^{k+1} \geq 0$$

$$\sum_{i=1}^{k} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] + \frac{p(1-p)^k}{k+1} - \sum_{i=1}^{k} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - \frac{kp^k(1-p)}{k+1} - p^{k+1} - \frac{p(1-p)^k}{k+1} - \frac{p^k(1-p)}{k+1} \geq 0$$

$$\sum_{i=1}^{k+1} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k+1} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - p^{k+1} - \frac{p(1-p)^k}{k+1} - \frac{p^k(1-p)}{k+1} \geq 0$$

$$\sum_{i=1}^{k+1} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k+1} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - p^{k+1} - \frac{p(1-p)}{k+1}\left[(1-p)^{k-1} + p^{k-1})\right] \geq 0$$

$$\sum_{i=1}^{k+1} \frac{1}{i} \times \left[(1-p)^{(i-1)} \times p\right] - \sum_{i=1}^{k+1} \frac{i-1}{i} \times \left[p^{(i-1)} \times (1-p)\right] - p^{k+1} \geq 0 \because \frac{p(1-p)}{k+1}\left[(1-p)^{k-1} + p^{k-1}\right) \geq 0$$

Therefore, H(k+1) is also true.

By mathematical induction, H(q) is true for all integers q ≥ 1.