Appendix A

Proof of Proposition 1. First, we consider the situation when customers do showrooming. All customers who exercise the same option constitute a market segment. For Lows, the ex-ante utility from exercising the second option $(v - cx - p_o)$ is always higher than that of the third option $(v - \delta - cx - p_o)$. Therefore, they will always prefer option (b) over (c). For Highs, the ex-ante utilities from exercising options (b) and (c) are $(v - t - cx - p_o)$ and $(v - \delta - cx - p_o)$, respectively. We assume that $t > \Delta$. This assumption implies that option (c) dominates (b) for this customer set, thus ensuring that we study a market configuration where some customers prefer the online channel for both evaluation and purchase rather than doing showrooming. We also assume that c and v are large enough to enable the existence of equilibria in all the situations we study henceforth.

The ex-ante surplus from exercising option (a) is independent of x. However, it is decreasing in x for options (b) and (c). Hence, customers exercising options (b) or (c), would be positioned towards the left while those who prefer option (a) would be towards the right on the Hotelling line. This arrangement is depicted in Figures 5(A) and 5(B) for Lows and Highs, respectively. We focus on the situation when all customers get positive utilities, implying a competitive situation. In Figure 5(A), lines $\mathrm{P_L}$ and S depict customer utilities from exercising options (a) and (b), respectively. Similarly, in Figure 5(B), lines $\mathrm{P_H}$ and O depict customer utilities from exercising options (a) and (c), respectively.

Let the index of the Lows customer who is indifferent between choosing options (a) and (b) be x_1 $(x_1 = \frac{p_s - p_o}{c})$, and let that of the Highs customer who is indifferent between options (a) and (c) be x_2 $(x_2 = \frac{p_s - p_o + t - \delta}{c})$. Substituting these values in the profit functions, the profits of the BM store is $p_s\left[\lambda(1 - x_1) + (1 - \lambda)(1 - x_2)\right]$ and that of the online retailer is $p_o\left[\lambda x_1 + (1 - \lambda)x_2\right]$. Using their first order conditions, we find the equilibrium values of

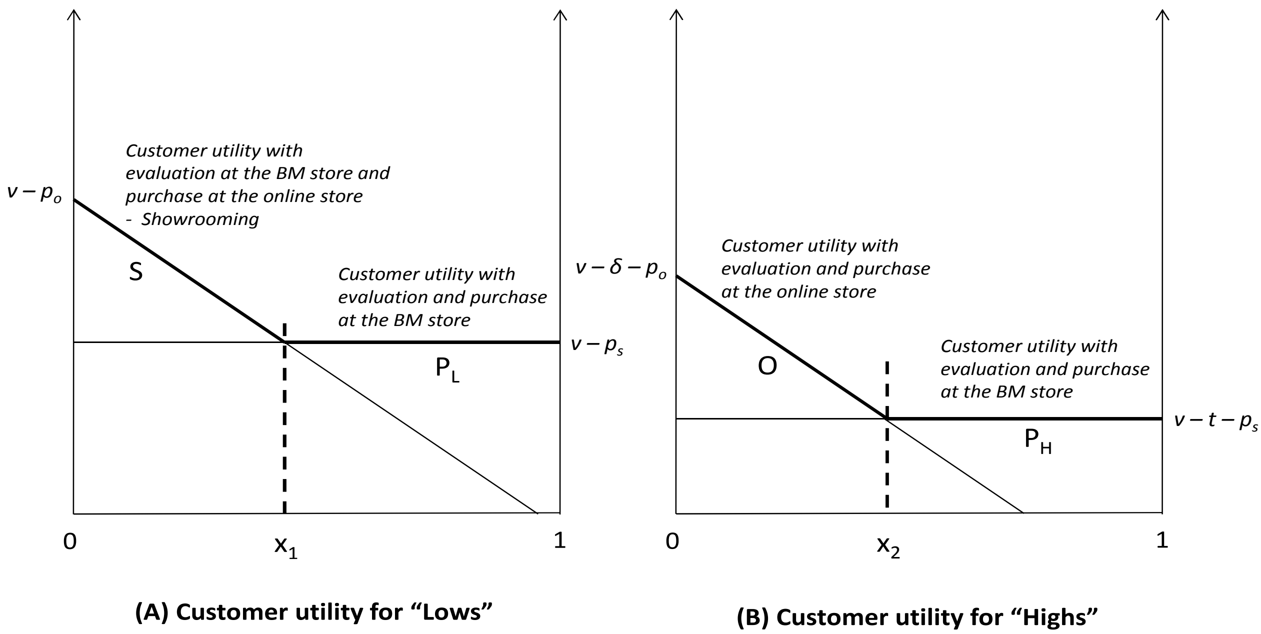

Figure 6 Customer Utilities in the Competitive Showrooming Equilibrium

the competitive showrooming equilibrium prices as $p_s^* = \frac{2c-(t-\delta)(1-\lambda)}{3}$ and $p_o^* = \frac{c+(t-\delta)(1-\lambda)}{3}$. For consistency of the equilibrium, we check for the conditions $p_s^* > 0$, $p_o^* > 0$, $p_s^* > p_o^*$, $0 < x_1^* < 1$, $0 < x_2^* < 1$, and $v - t - p_s^* > 0$. Further, to ensure the stability of this equilibrium, we verify that the retailers have no incentive to deviate from the equilibrium prices, even if such a deviation causes a discontinuous change in the market configuration. We find that $c > 2(t-\delta)$ and $v > 3t + 2c$ are sufficient to satisfy all the conditions needed for the equilibrium.

We now study the benchmark case where customers do not engage in showrooming. This case represents the situation where customers do not separate their evaluation and purchase decisions across the physical and the online channels. In other words, a customer can exercise only options (a), or (c). In a competitive situation, the markets of the two retailers must be contiguous, and therefore, we find the indifferent Lows customer, x_{b1}, by solving $(v - p_s = v - \delta - cx - p_o)$, and the indifferent Highs customer, x_{b2}, by solving

$(v-t-p_s = v-\delta-cx-p_o)$. Profit functions of the BM store and the online retailer can be written as $p_s\left[\lambda(1-x_{b1})+(1-\lambda)(1-x_{b2})\right]$ and $p_o\left[\lambda x_{b1}+(1-\lambda)x_{b2}\right]$, respectively, and the corresponding equilibrium prices of the BM store and the online retailer are $p_s^{b*}=\frac{2c+\delta-(1-\lambda)t}{3}$ and $p_o^{b*}=\frac{c-\delta+(1-\lambda)t}{3}$. We check for consistency as well as any incentives to deviate from this equilibrium.

Now, we substitute the optimal prices in the profit functions to obtain the following competitive showrooming equilibrium and benchmark equilibrium profits of the BM store.

$$\pi_s^* = \frac{((1-\lambda)(t-\delta)-2c)^2}{9c},$$

$$\pi_s^{b*} = \frac{(2c+\delta-(1-\lambda)t)^2}{9c}.$$

Comparing these two profits, we see that $\pi_s^* < \pi_s^{b*}$. ■

Appendix B

Proof of Proposition 2. After establishing the equilibrium under price matching commitment, we compare the BM store's profits with and without price matching commitment in the following two cases.

1. $0 < M < \bar{M}$:

We substitute the equilibrium prices in the profit function to get

$$\pi_s^{m*} = \frac{c^2(4-M)(1-M) - 2c(1-\lambda)(2-(6-M)M)(t-\delta) + (1-\lambda)^2(1-(7-M)M)(t-\delta)^2}{9c(1-M)}.$$

The second derivative of π_s^{m*} w.r.t. M is $\frac{2(1-\lambda)(5(1-\lambda)(t-\delta)-6c)(t-\delta)}{9c(M-1)^3} > 0$, implying that the equilibrium profit is convex in M. Also, $\pi_s^{m*} = \pi_s^*$ at $M = 0$ and $M = \hat{M}$.

If $\hat{M} > 0$, $\frac{\partial \pi_s^{m*}}{\partial M} = \frac{8c(1-\lambda)(t-\delta)-6(1-\lambda)^2(t-\delta)^2-c^2}{9c} < 0$ at $M = 0$ meaning $\frac{\partial \pi_s^{m*}}{\partial M} > 0$ at $M = \hat{M}$ due to convexity. Therefore, $\pi_s^{m*} > \pi_s^* \ \forall \ \hat{M} < M < \bar{M}$. Further, if $\hat{M} < 0$, $\pi_s^{m*} > \pi_s^* \ \forall \ 0 < M < \bar{M}$.

2. $\bar{M} < M < 1$: Using the equilibrium prices, we get $x^*_{\lambda M} = 0$, $x^*_{\lambda(1-M)} = 0$, and $x^*_{(1-\lambda)M} = x^*_{(1-\lambda)(1-M)} = \frac{t-\delta}{c} < 1$. The equilibrium BM store profit is given by:

$$\pi_s^{m*} = \frac{(1-\lambda)(t-\delta)(c-(1-\lambda)(t-\delta))}{c(1-M)}.$$

Comparing this profit with the BM store's profit in the competitive showrooming equilibrium, we have

$$\pi_s^{m*} - \pi_s^* = \frac{(c(1-\lambda)(13-4M)(t-\delta) - 4c^2(1-M) - (1-\lambda)^2(10-M)(t-\delta)^2}{9c(1-M)}.$$

We verify that $\pi_s^{m*} > \pi_s^* \ \forall \ \bar{M} < M < 1$. ■

Proof of Proposition 3. We find the threshold values of M in a way similar to the proof above. Comparing the thresholds derived here with the threshold values of M derived in proposition 2, we arrive at the results outlined in this proposition.

Appendix C

Lows Customers Prefer to Visit Store

From Figure 2 in the main body of the paper, we note that four different outcomes are possible for a Lows customer if she decides to visit the BM store:

A. Best-fit product is store brand and customer purchases this product at BM store; Best-fit product is not store brand and customer purchases this product at BM store.

B. Best-fit product is store brand and customer purchases this product at BM store; Best-fit product is not store brand and customer purchases this product at online retailer

C. Best-fit product is store brand and customer purchases different product at online retailer; Best-fit product is not store brand and customer purchases this product at BM store

D. Best-fit product is store brand and customer purchases different product at online retailer; Best-fit product is not store brand and customer purchases this product at online retailer

We now show that the expected utility a customer gets in each case is greater than the utility she gets by evaluating and purchasing online, i.e., $v-\Delta+(1-a)p\Delta-cx-p_o$.

A) Given the customers' choices, the following conditions must hold: $v-p_a>v-\Delta-cx-p_o$ and $v-p_s>v-cx-p_o$. Therefore, expected utility of the customer is given by $a(v-p_a)+(1-a)(v-p_s)$. Using the two inequalities, minimum value of the expected utility is, $a(v-p_o-cx-\Delta)+(1-a)(v-p_o-cx)=v-p_o-cx-a\Delta\geq v-\Delta+(1-a)p\Delta-cx-p_o$ as $p<1$.

B) Given the customers' choices, the following conditions must hold: $v-p_a>v-\Delta-cx-p_o$ and $v-p_s<v-cx-p_o$. Expected utility in this case is equal to $a(v-p_a)+(1-a)(v-cx-p_o)=v-ap_a-(1-a)cx-(1-a)p_o$. For $v-ap_a-(1-a)cx-(1-a)p_o\geq v-\Delta+$

$(1-a)p\Delta - cx - p_o$, we require $p_a \leq p_o + \frac{\Delta(1-(1-a)p)}{a} + cx$. form the first inequality, we have $p_a < p_o + cx + \Delta$. Therefore, it will be sufficient to show that $\frac{\Delta(1-(1-a)p)}{a} \geq \Delta$, which is true as $p < 1$.

C) Given the customers' choices, the following conditions must hold: $v - p_a < v - \Delta - cx - p_o$ and $v - p_s > v - cx - p_o$. Expected utility in this case is equal to $a(v - \Delta - cx - p_o) + (1-a)(v - p_s) = v - a\Delta - acx - ap_o - (1-a)p_s$. For $v - a\Delta - acx - ap_o - (1-a)p_s \geq v - \Delta + (1-a)p\Delta - cx - p_o$, we require $p_s \leq p_o + cx + (1-p)\Delta$. This condition holds because we have $p_s < p_o + cx$ from the second inequality, and $p < 1$.

D) Given the customers' choices, the following conditions must hold: $v - p_a < v - \Delta - cx - p_o$ and $v - p_s < v - cx - p_o$. Therefore, expected utility of the customer is given by $a(v - \Delta - cx - p_o) + (1-a)(v - cx - p_o) = v - cx - p_o - a\Delta$. But, $v - cx - p_o - a\Delta \geq v - \Delta + (1-a)p\Delta - cx - p_o$ as $p < 1$.

Therefore, a Lows customer never evaluates and purchases at the online retailer. Rather, she visits the BM store and exercises one of her options there.

Appendix D

Proof of Proposition 4.

Equilibrium with Known Brand

When BM store keeps exclusive product of a known brand, profit functions of the BM store and the online store are given by following.

$$\pi_s^k = \lambda a(1-x_{La})p_a + \lambda(1-a)(1-x_{L(1-a)})p_s + a(1-\lambda)(1-x_{Ha})p_a + (1-\lambda)(1-a)(1-x_{H(1-a)})p_s$$

$$\pi_o^k = (\lambda a x_{La} + \lambda(1-a)x_{L(1-a)} + (1-\lambda)a x_{Ha} + (1-\lambda)(1-a)x_{H(1-a)})p_o$$

Solving the first order conditions with respect to p_a, p_s, and p_o, we get the equilibrium prices as $p_a^* = \frac{4c+\Delta(2+(1-a)(\lambda+(1-\lambda)p)-2(1-\lambda)t}{6}$, $p_s^* = \frac{4c+\Delta(2-(2+a)\lambda(1-p)-(2+a)p)-2(1-\lambda)t}{6}$, and $p_o^* = \frac{c+\Delta(\lambda(1-a)-1+(1-a)(1-\lambda)p)+(1-\lambda)t}{3}$. One can see that $p_o^* < p_s^* < p_a^* < p_s^* + \Delta$. Substituting these prices into the profit functions, we get the equilibrium profits as

$$\pi_s^{k*} = \frac{1}{36c}(16c^2 + \Delta^2(4+(1-a)(4+5a)\lambda^2(1-p)^2 + (1-a)p((4+5a)p-8)+$$
$$2(1-a)(1-p)\lambda(p(4+5a)-4)) + 8\Delta(1-\lambda)((1-a)(p+(1-p)\lambda)-1)t + 4(1-\lambda)^2t^2+$$
$$16c(\Delta(1-(1-a)(p-\lambda(1-p))) - (1-\lambda)t))$$
$$\pi_o^{k*} = \frac{(c+\Delta((1-a)(\lambda+(1-\lambda)p)-1)+(1-\lambda)t)^2}{9c}.$$

To compare the BM store's profit with known brand product exclusivity with its profit under competitive showrooming equilibrium, we analyze the behavior of π_s^{k*} w.r.t a. First, we notice that the profit is concave in a as $\frac{\partial^2\pi_s^{k*}}{\partial a^2} = -\frac{5\Delta^2(\lambda(1-p)+p)^2}{18c} < 0$. As $\pi_s^{k*} = \pi_s^*$ at $a=0$, we are interested in the sign of $\frac{\partial\pi_s^{k*}}{\partial a} = \frac{\Delta(\lambda(1-p)+p)(16c+\Delta(8+\lambda-10a\lambda-(10a-1)(1-\lambda)p)-8(1-\lambda)t)}{36c}$ at $a=0$. If the minimum value of the big term in the parenthesis is positive at $a=0$, profit will be greater for at least some $a>0$. As it is a positive function of c, we check its value after substituting the parameter's minimum value. After substituting $c = 2(t-(1-p)\Delta)$, the term becomes increasing in t. Therefore, we substitute $t = (1-p)\Delta$, which gives us $9\Delta(\lambda(1-p)+p) > 0$. Now, we see whether profit is higher $\forall\ a \in (0,1)$. To check this, we see the sign of $\pi_s^{k*} - \pi_s^*$ at $a = 1$. If it is positive then the profit with exclusivity is higher for the entire range of a. We have $\pi_s^{k*} - \pi_s^* = \frac{\Delta(\lambda(1-p)+p)(4c+\Delta(2-\lambda-(1-\lambda)p)-2(1-\lambda)t)}{9c}$ at $a = 1$. Minimum value of the big term in the parenthesis is $\Delta(\lambda(1-p)+p) > 0$. Therefore, BM store always does better by keeping exclusive product of a known brand.

Equilibrium with Store Brand

When BM store keeps exclusive product of a store brand, profit functions of the BM store and the online store are given by following.

$$\pi_s^u = \lambda a(1-x_a)p_a + \lambda(1-a)(1-x_{(1-a)})p_s + (1-\lambda)a(1-x_{H1})p_a +$$
$$(1-\lambda)(1-a)(1-x_{H1})p_s,$$
$$\pi_o^u = (\lambda(ax_a + (1-a)x_{(1-a)}) + (1-\lambda)x_{H1})p_o.$$

Solving the first order conditions with respect to p_a, p_s, and p_o, we get the equilibrium prices as $p_a^* = \frac{4c+\Delta(5-3a-2\lambda(1-a)-2(1-\lambda)(1-a)p)-2(1-\lambda)t}{6}$, $p_s^* = \frac{4c+\Delta(2-3a-2\lambda(1-a)-2(1-\lambda)(1-a)p)-2(1-\lambda)t}{6}$, and $p_o^* = \frac{c+\Delta(\lambda(1-a)-1+(1-a)(1-\lambda)p)+(1-\lambda)t}{3}$. One can see that $p_o^* < p_s^* < p_a^* < p_s^* + \Delta$. Substituting these prices into the profit functions, we get the equilibrium profits as

$$\pi_s^{u*} = \frac{1}{36c}(16c^2 + \Delta^2(4(1-a)^2\lambda^2(1-p)^2 + 4(1-(1-a)p)^2 + (1-a)\lambda(a(9-8(1-p)p) - 8(1-p)^2))$$
$$+ 8\Delta(1-\lambda)((1-a)(1-p)\lambda - 1 + p(1-a))t + 4(1-\lambda)^2t^2 +$$
$$16c(\Delta(1-(1-a)(p-\lambda(1-p))) - (1-\lambda)t))$$
$$\pi_o^{u*} = \frac{(c+\Delta((1-a)(\lambda+(1-\lambda)p)-1) + (1-\lambda)t)^2}{9c}.$$

We prove that the profit of the BM store with store brand exclusivity is greater than its profit under competitive showrooming equilibrium by proceeding similarly to the case of known brand exclusivity discussed above.

Comparing the two product exclusivity strategies

Finally, we compare the profit of BM store in the case of store brand product exclusivity with its profit in the case of known brand exclusivity. We have $\pi_s^{k*} - \pi_s^{u*} = \frac{(1-a)(1-\lambda)a\Delta^2(p^2-\lambda(1-p)^2)}{4c}$. This is positive when $p > \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}$.

Discussion of the assumption on prices

In the analysis of the proof of proposition 4, we made the assumption that $p_s - \Delta < p_a < p_s + \Delta$ while writing the profit functions of the retailers. We then showed that a unique equilibrium satisfying these conditions exists.

Let us now consider the possibility of alternate equilibria, those where the ordering of the prices does not follow the assumption we made. To see this, let's take the left inequality and assume that the directionality is opposite, i.e., say $p_s > p_a + \Delta$. This situation would mean that no customer ever purchases non-exclusive products from the store. Consequently, the store has no incentive to stock the non-exclusive products. Of course, if the store does not stock the non-exclusive products at all, it is not possible for the customers to visit the store to evaluate the products. In other words, showrooming behavior is completely ruled out. Obviously, such a situation is not consistent with reality (because BM stores do stock items that are available online). This situation is really an extreme one where the store exists only to sell the merchandise of the manufacturer. Now, let's take the right end of the inequality and assume that the directionality is opposite, i.e., say $p_a > p_s + \Delta$. This situation would mean that no customers ever purchase exclusive products from the store. In this situation, the store would not stock exclusive items at all, implying that exclusive products have no role in the equilibrium. This situation when there are no exclusive products at the store is the competitive showrooming equilibrium, one that we already include in our analysis.

To summarize, it appears that there are two other possible equilibria other than the one we already considered in the context of product exclusivity. However, one of these is too extreme where the store sells only the exclusive products, and the other is nothing but an equilibrium we already study, i.e., the competitive showrooming equilibrium.

Proof of Proposition 5. The proof of this proposition follows in a way similar to that for Proposition 4.

Appendix E: Summary of Notation

Notation	Explanation
p	Probability with which a customer correctly selects her best fit product by evaluating only at the online retailer
v	Utility that a customer gets from her best fit product
$v-\Delta$	Utility that a customer gets from a product not her best fit, where $\Delta > 0$
$v-\delta$	Ex-ante expected utility a customer gets from a purchase after online evaluation only, where $\delta = (1-p)\Delta$
λ	Fraction of customers whose cost of visiting BM store is 0
t	Cost incurred in visiting the BM store by $1-\lambda$ fraction of customers
x	Measure of a customer's cost of purchasing online
c	Scaling factor using which a customer's cost of purchasing online is cx
M	Fraction of customers who seek price matching at the BM store
a	Probability with which a customer's best fit product will be the exclusive product
p_s	Price set by the BM store for the non-exclusive product
p_o	Price set by the online store
p_m	Price set by the online store and matched by BM store under price matching
p_a	Price set by the BM store for the exclusive product