Online Supplement for “The Value of ‘Bespoke’: Demand Learning, Preference Learning, and Customer Behavior”

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This Online Supplement includes four sections. Online Supplement 1 provides the technical proofs to the propositions, theorems and corollaries in the paper. All the other sections include some supporting materials and extra analysis for the paper. Specifically, Online Appendix 2 includes supplemental analysis for §5 of the paper. in Online Supplement 3, we analyze a variant of the basic model in §3 and §4 by allowing the firm to price *after* seeing the demand realization. Online Supplement 4 shows that our model used in §5.2 of the paper can be operationalized and supported by the interpretation of product line design or assortment planning.

Online Supplement 1. Proofs

Proof of Proposition 1. Letting $u(\theta,p)\equiv w-p-|\theta|t=0$, we have $|\theta|=\frac{w-p}{t}$. Note $0\leq|\theta|\leq\frac{1}{2}$. The demand is $2\frac{w-p}{t}D$ when $w-\frac{1}{2}t\leq p\leq w$, is 0 when $p>w$, and is D when $p<w-\frac{1}{2}t$. Let p_0^*, q_0^* and π_0^* denote the optimal price, optimal production quantity and optimal profit of the traditional system, respectively. We must have $w-\frac{1}{2}t\leq p_0^*\leq w$ and $2\frac{w-p_0^*}{t}d_L\leq q_0^*\leq 2\frac{w-p_0^*}{t}d_H$. Note that for $w-\frac{1}{2}t\leq p\leq w$ and $2\frac{w-p}{t}d_L\leq q\leq 2\frac{w-p}{t}d_H$, the firm's profit $\pi(p,q)=p\min(2\frac{w-p}{t}d_H,q)-cq=(p-c)q$ when $D=d_H$, and $\pi(p,q)=p\min(2\frac{w-p}{t}d_L,q)-cq=2p\frac{w-p}{t}d_L-cq$ when $D=d_L$. By taking the expectation over D, we have $\pi(p,q)=p[\alpha q+2(1-\alpha)\frac{w-p}{t}d_L]-cq$. Notice that $\frac{\partial\pi(p,q)}{\partial q}=p\alpha-c$ and $\pi(p,q)$ is linear in q. For any p where $w-\frac{1}{2}t\leq p\leq w$, the optimal q is either $2\frac{w-p}{t}d_L$ or $2\frac{w-p}{t}d_H$.

(i) When $q=2\frac{w-p}{t}d_L$, we have $\pi(p,q)=2\frac{w-p}{t}d_L(p-c)$, which is maximized at $p=p_L=\frac{w+c}{2}$ by solving $\frac{d\pi}{dp}=0$, and the corresponding production quantity and profit are $q=q_L=\frac{d_L}{t}(w-c)$ and

$\pi(p_L, q_L) = \frac{d_L}{2t}(w-c)^2$. Note $w - \frac{1}{2}t \le p_L \le w$ due to $w \le c + t$ and $w > c$.

(ii) When $q = 2\frac{w-p}{t}d_H$, we have $\pi(p,q) = 2\frac{\mu_D}{t}(w-p)(p - \frac{d_H}{\mu_D}c)$. For $\frac{w}{c} \ge \frac{d_H}{\mu_D}$, we have that $2\frac{\mu_D}{t}(w-p)(p - \frac{d_H}{\mu_D}c)$ is maximized at $p = p_H = \frac{w + \frac{d_H}{\mu_D}c}{2}$ by solving $\frac{d\pi}{dp} = 0$, and the corresponding production quantity and profit are $q = q_H = \frac{d_H}{t}(w - \frac{d_H}{\mu_D}c)$ and $\pi(p_H, q_H) = \frac{\mu_D}{2t}(w - \frac{d_H}{\mu_D}c)^2$. Note that $w - \frac{1}{2}t \le p_L \le p_H \le w$ due to $d_H \ge \mu_D$ and $\frac{w}{c} \ge \frac{d_H}{\mu_D}$. For $\frac{w}{c} < \frac{d_H}{\mu_D}$, we know $2\frac{\mu_D}{t}(w-p)(p - \frac{d_H}{\mu_D}c) \le 0$ when $w - \frac{1}{2}t \le p \le w$.

Note that $\pi(p_L, q_L) \ge \pi(p_H, q_H)$ if and only if $\frac{w}{c} \le \frac{d_H - \sqrt{d_L\mu_D}}{\mu_D - \sqrt{d_L\mu_D}}$. Combining cases (i) and (ii), we have the following. If $\frac{w}{c} < \frac{d_H}{\mu_D}$, then $p_0^* = p_L = \frac{w+c}{2}$, $q_0^* = q_L = \frac{d_L}{t}(w-c)$, and $\pi_0^* = \pi(p_L, q_L) = \frac{d_L}{2t}(w-c)^2$. If $\frac{w}{c} \ge \frac{d_H}{\mu_D}$, then when $\frac{w}{c} \le \frac{d_H - \sqrt{d_L\mu_D}}{\mu_D - \sqrt{d_L\mu_D}}$, we have that $p_0^* = p_L = \frac{w+c}{2}$, $q_0^* = q_L = \frac{d_L}{t}(w-c)$, and $\pi_0^* = \pi(p_L, q_L) = \frac{d_L}{2t}(w-c)^2$; otherwise, we have that $p_0^* = p_H = \frac{w + \frac{d_H}{\mu_D}c}{2}$, $q_0^* = q_H = \frac{d_H}{t}(w - \frac{d_H}{\mu_D}c)$, and $\pi_0^* = \pi(p_H, q_H) = \frac{\mu_D}{2t}(w - \frac{d_H}{\mu_D}c)^2$. Because $\frac{d_H}{\mu_D} < \frac{d_H - \sqrt{d_L\mu_D}}{\mu_D - \sqrt{d_L\mu_D}}$, Proposition 1 follows. $\square$

Proof of Proposition 2. Because the firm determines its production quantity q after learning the realization of D, it always sets $q = 2\frac{w-p}{t}D$ for any price p, where $D = d_L$ or d_H depending on the demand realization. The profit can be expressed as $\pi(p) = 2\alpha\frac{w-p}{t}d_H(p-c) + 2(1-\alpha)\frac{w-p}{t}d_L(p-c) = 2\frac{\mu_D}{t}(p-c)(w-p)$, which is maximized at $p = p_L = \frac{w+c}{2}$, and the corresponding optimal profit is $\frac{\mu_D}{2t}(w-c)^2$. $\square$

Proof of Proposition 3. Because the firm can completely remove the preference-mismatch cost, it always sets the optimal price $p_p^* = w$ and all the customers would like to buy at this price. The firm's profit is $\pi(w,q) = w\mathbf{E}[\min(D,q)] - c_p q$. Similar to the analysis in Proposition 1, we know that the optimal production quantity q_p^* satisfies $d_L \le q_p^* \le d_H$. After taking the expectation over D, we have $\pi(w,q) = w[\alpha q + (1-\alpha)d_L] - c_p q$. Notice that $\frac{\partial \pi(w,q)}{\partial q} = w\alpha - c_p$ and $\pi(w,q)$ is linear in q. We know that if $w \ge \frac{c_p}{\alpha}$, then $q_p^* = d_H$ and the firm's optimal profit $\pi_P^* = w\mu_D - c_p d_H$; otherwise, $q_p^* = d_L$ and $\pi_P^* = (w - c_p)d_L$. $\square$

Proof of Proposition 4. In Bespoke system, the firm always charges the price w since it can completely remove the preference-mismatch, and it always sets the production quantity equal to the demand realization (d_H or d_L) since it can learn the demand realization before determining the quantity. Therefore, its optimal profit $\pi_{AP}^* = \alpha(w - c_p)d_H + (1-\alpha)(w - c_p)d_L = (w - c_p)\mu_D$. $\square$

Proof of Lemma 1. Recall $\kappa(\alpha) = \frac{d_H - \sqrt{d_L\mu_D}}{\mu_D - \sqrt{d_L\mu_D}}$. We can prove that $\kappa(\alpha)$ decreases in α by showing $\frac{d\kappa(\alpha)}{d\alpha} < 0$. Note $\mu_D = \alpha d_H + (1-\alpha)d_L$. We have $\mu_D - \sqrt{d_L\mu_D} = \alpha(d_H - \sqrt{d_L\mu_D}) + (1-\alpha)(d_L - \sqrt{d_L\mu_D}) \le \alpha(d_H - \sqrt{d_L\mu_D})$, and the result $\kappa(\alpha) \ge \frac{1}{\alpha}$ follows. Notice that $\sqrt{d_L\mu_D} \le \frac{d_L + \mu_D}{2}$ and $\kappa(\alpha) \ge 1$. We have $\kappa(\alpha) \le \frac{d_H - \frac{d_L+\mu_D}{2}}{\mu_D - \frac{d_L+\mu_D}{2}} = \frac{(d_H - d_L) + (d_H - \mu_D)}{\mu_D - d_L} = \frac{(d_H - d_L) + (1-\alpha)(d_H - d_L)}{\alpha(d_H - d_L)} = \frac{2-\alpha}{\alpha}$. $\square$

Proof of Theorem 1. Recall $\kappa(\alpha) = \frac{d_H - \sqrt{d_L\mu_D}}{\mu_D - \sqrt{d_L\mu_D}}$. We have shown that $\kappa(\alpha)$ is decreasing in α, and $\kappa(\frac{c}{w}) > \frac{w}{c}$. There exists $\alpha_0 > \frac{c}{w}$ such that $\kappa(\alpha_0) = \frac{w}{c}$. We discuss the following cases:

(i) Suppose $w < \min\left\{\frac{c_p}{\alpha}, \kappa(\alpha)c\right\}$. Then by Propositions 1-4 and simplifying, the sufficient and necessary condition $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ is equivalent to $c_p \leq w - \frac{(w-c)^2}{2t}$. Note that the condition $w < \min\left\{\frac{c_p}{\alpha}, \frac{d_H - \sqrt{d_L\mu_D}}{\mu_D - \sqrt{d_L\mu_D}}c\right\}$ is equivalent to $\alpha < \frac{c_p}{w}$ and $\frac{d_H-\sqrt{d_L\mu_D}}{\mu_D-\sqrt{d_L\mu_D}} > \frac{w}{c}$. Because $\kappa(\alpha) = \frac{d_H-\sqrt{d_L\mu_D}}{\mu_D-\sqrt{d_L\mu_D}} \geq \frac{1}{\alpha}$ and is decreasing in α, we have that $\frac{d_H-\sqrt{d_L\mu_D}}{\mu_D-\sqrt{d_L\mu_D}} > \frac{w}{c}$ holds if $\alpha < \frac{c}{w}$, and thus $w < \min\left\{\frac{c_p}{\alpha}, \frac{d_H-\sqrt{d_L\mu_D}}{\mu_D-\sqrt{d_L\mu_D}}c\right\}$ is equivalent to $\alpha < \min\{\frac{c_p}{w}, \alpha_0\}$, where $\kappa(\alpha_0) = \frac{w}{c}$. Hence, in this case, the condition for complements is $\alpha < \min\{\frac{c_p}{w}, \alpha_0\}$ and $c_p \leq w - \frac{(w-c)^2}{2t}$.

(ii) Suppose $\min\left\{\frac{c_p}{\alpha}, \kappa(\alpha)c\right\} \leq w \leq \max\left\{\frac{c_p}{\alpha}, \kappa(\alpha)c\right\}$. This condition is equivalent to $\min\left\{\frac{c_p}{w}, \alpha_0\right\} \leq \alpha \leq \max\left\{\frac{c_p}{w}, \alpha_0\right\}$. We discuss two subcases: (ii.a) If $\alpha_0 < \frac{c_p}{w}$, then the supposed condition becomes $\alpha_0 \leq \alpha \leq \frac{c_p}{w}$. In this subcase, we have $w \leq \frac{c_p}{\alpha}$and $w \geq \kappa(\alpha)c$. By Propositions 1-4 and simplifying, the sufficient and necessary condition $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ is equivalent to $c_p \leq w - \frac{c(1-\alpha)}{2t\alpha}\left[2w - c\left(1 + \frac{d_H}{\mu_D}\right)\right]$. (ii.b) If $\alpha_0 \geq \frac{c_p}{w}$, then the supposed condition becomes $\frac{c_p}{w} \leq \alpha \leq \alpha_0$. In this subcase, we have $w \geq \frac{c_p}{\alpha}$ and $w \leq \kappa(\alpha)c$. By Propositions 1-4 and simplifying, the sufficient and necessary condition $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ is equivalent to $\alpha \leq \frac{2tc_p}{2tc_p+(w-c)^2}$. Next, we show $\alpha \leq \frac{2tc_p}{2tc_p+(w-c)^2}$ always holds in this subcase. Because $\alpha \leq \alpha_0$ in this subcase and $\kappa(\alpha)$ decreases in α, it is sufficient to show $\kappa(\alpha_0) \geq \kappa\left(\frac{2tc_p}{2tc_p+(w-c)^2}\right)$. Note $\kappa(\alpha_0) = \frac{w}{c}$, $\kappa(\frac{2tc_p}{2tc_p+(w-c)^2}) \leq \frac{2-\frac{2tc_p}{2tc_p+(w-c)^2}}{\frac{2tc_p}{2tc_p+(w-c)^2}}$ (Lemma 1), and that $\frac{w}{c} \geq \frac{2-\frac{2tc_p}{2tc_p+(w-c)^2}}{\frac{2tc_p}{2tc_p+(w-c)^2}}$ is equivalent to $w \leq c + \frac{tc_p}{c}$ which is always true due to $c_p > c$ and $w \leq c+t$. Hence, the condition in this subcase is $\frac{c_p}{w} \leq \alpha \leq \alpha_0$.

Notice that $\alpha \leq \frac{2tc_p}{2tc_p+(w-c)^2}$ can be rewritten as $c_p \geq \frac{(w-c)^2}{2t}\frac{\alpha}{1-\alpha}$. Since $\alpha \geq \frac{c_p}{w}$ in this subcase, we have $c_p \geq \frac{(w-c)^2}{2t}\frac{\frac{c_p}{w}}{1-\frac{c_p}{w}}$, which is simplified to $c_p \leq w - \frac{(w-c)^2}{2t}$. Therefore, the conditions in this subcase can also be written as $\frac{c_p}{w} \leq \alpha \leq \alpha_0$, and $c_p \leq w - \frac{(w-c)^2}{2t}$. (iii) Suppose $w > \max\left\{\frac{c_p}{\alpha}, \kappa(\alpha)c\right\}$, then by Propositions 1-4 and simplifying, the sufficient and necessary condition $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ is equivalent to $w \leq \frac{c_p t}{c} + \frac{1}{2}c(1 + \frac{d_H}{\mu_D})$, which always holds due to $w \leq c+t$, $c_p > c$, and $d_H \geq \mu_D$. Hence, the condition in this case is equivalent to $\alpha > \max\left\{\frac{c_p}{w}, \alpha_0\right\}$. We are now ready to summarize all these cases together: If and only if one of the following two conditions holds, demand learning and preference learning are complements: (1) $\alpha \leq \alpha_0$ and $c_p \leq w - \frac{(w-c)^2}{2t}$; (2) $\alpha \geq \alpha_0$ and $c_p \leq \overline{C}_p(\alpha)$, where $\overline{C}_p(\alpha) = \max\left\{w - \frac{c(1-\alpha)}{2t\alpha}\left[2w - c\left(1 + \frac{d_H}{\mu_D}\right)\right], \alpha w\right\}$. Here (1) is from cases (i) and (ii.b), and (2) is from cases (ii.a) and (iii). □

Proof of Corollary 1. When $\alpha_0 \geq \frac{c_p}{w}$, the sufficient and necessary conditions can be written as $\alpha < \frac{c_p}{w}$ and $c_p \leq w - \frac{(w-c)^2}{2t}$in case (i), $\frac{c_p}{w} \leq \alpha \leq \alpha_0$ in case (ii) (i.e., case (ii.b)), and $\alpha > \alpha_0$ in case (iii). In order to get the result, we just need to show that $c_p \leq w - \frac{(w-c)^2}{2t}$ is redundant in case (i) when $\alpha_0 \geq \frac{c_p}{w}$. Because $c_P \leq \alpha_0 w$, it is sufficient to show $\alpha_0 \leq 1 - \frac{(w-c)^2}{2tw}$, equivalently,

$\kappa(\alpha_0) \geq \kappa\left(1 - \frac{(w-c)^2}{2tw}\right)$. Note $\kappa(\alpha_0) = \frac{w}{c}$, $\kappa(1 - \frac{(w-c)^2}{2tw}) \leq \frac{2-\left(1-\frac{(w-c)^2}{2tw}\right)}{1-\frac{(w-c)^2}{2tw}}$ (Lemma 1), and that $\frac{w}{c} \geq \frac{2-\left(1-\frac{(w-c)^2}{2tw}\right)}{1-\frac{(w-c)^2}{2tw}}$ is equivalent to $w(w-2t) \leq c^2$ which is always true due to $w \leq c+t$. Hence, when $\alpha_0 \geq \frac{c_p}{w}$, the sufficient and necessary condition in case (i) can be simplified to $\alpha < \frac{c_p}{w}$, and the desired result follows. □

Proof of Lemma 2. From equations (1)-(5) in the paper, we can express, in terms of demand learning accuracy I, the firm's posterior probabilities of the demand forecast after observing signal s as

$$P(D = d_H | s = d_H) = \beta_H = 1 - (1-I)(1-\alpha),$$
$$P(D = d_L | s = d_H) = 1 - \beta_H = (1-I)(1-\alpha),$$

and

$$P(D = d_L | s = d_L) = \beta_L = 1 - (1-I)\alpha,$$
$$P(D = d_H | s = d_L) = 1 - \beta_L = (1-I)\alpha.$$

Suppose that the firm observes demand signal $s = d_H$, then its updated demand $D(s = d_H) = d_H$ with probability $\alpha_H(I) \equiv \beta_H = 1 - (1-I)(1-\alpha)$ and $D(s = d_H) = d_L = 0$ with probability $1 - \alpha_H(I) = (1-I)(1-\alpha)$. Based on the analysis for the traditional system, we know that we need to discuss the following two cases: (1) If $\alpha_H(I) \leq c/w$, i.e., $I \leq 1 - \frac{w-c}{(1-\alpha)w}$, then it is optimal to produce zero quantity, which results in zero expected profit, i.e., $\pi^*_{s=d_H} = 0$. (2) If $\alpha_H(I) \geq c/w$, i.e., $I \geq 1 - \frac{w-c}{(1-\alpha)w}$, then the firm's optimal price $p^*_{s=d_H} = \frac{w + c/\alpha_H(I)}{2}$, optimal production quantity $q^*_{s=d_H} = \frac{d_H(w - \frac{c}{\alpha_H(I)})}{t}$, and optimal profit $\pi^*_{s=d_H} = \frac{\alpha_H(I) d_H}{2t}(w - \frac{c}{\alpha_H(I)})^2$.

Suppose that the firm observes demand signal $s = d_L$, then its updated demand $D(s = d_L) = d_H$ with probability $\alpha_L(I) \equiv 1 - \beta_L = (1-I)\alpha$ and $D(s = d_L) = d_L = 0$ with probability $1 - \alpha_L(I) = 1 - (1-I)\alpha$. Based on the analysis for the traditional system, we know that we need to discuss the following two cases: (1) If $\alpha_L(I) \leq c/w$, i.e., $I \geq \frac{\alpha w - c}{\alpha w}$, then it is optimal to produce zero quantity, which results in zero expected profit, i.e., $\pi^*_{s=d_L} = 0$. (2) If $\alpha_L(I) \geq c/w$, i.e., $I \leq \frac{\alpha w - c}{\alpha w}$, then the firm's optimal price $p^*_{s=d_L} = \frac{w + c/\alpha_L(I)}{2}$, optimal production quantity $q^*_{s=d_L} = \frac{d_H(w - \frac{c}{\alpha_L(I)})}{t}$, and optimal profit $\pi^*_{s=d_L} = \frac{\alpha_L(I) d_H}{2t}(w - \frac{c}{\alpha_L(I)})^2$.

To write the firm's expected profit before observing the demand signal s, we need to compare $I_1(\alpha) \equiv 1 - \frac{w-c}{(1-\alpha)w}$ and $I_2(\alpha) \equiv \frac{\alpha w - c}{\alpha w}$. It turns out $I_1(\alpha) \geq I_2(\alpha)$ is equivalent to $\alpha \leq c/w$.

Note that $\pi^*_A(I, \alpha) = \alpha \pi^*_{s=d_H} + (1-\alpha)\pi^*_{s=d_L}$. Then, we can compute this expected profit based on different combinations of the conditions on α and I. For example, if $\alpha \in [0, \frac{c}{w}]$, we have $I_1(\alpha) \geq 0 \geq I_2(\alpha)$. If we further assume $I \in [I_1(\alpha), 1]$, then $\pi^*_A(I, \alpha) = \frac{\alpha(1-(1-I)(1-\alpha))d_H}{2t}(w - \frac{c}{1-(1-I)(1-\alpha)})^2$. Similarly, we obtain the profit functions in other cases. □

Proof of Theorem 2. We first discuss all the possible cases.

Suppose $\alpha \le \frac{c}{w}$. We discuss the following two cases depending on the magnitude of I: (1) If $I \in [I_1(\alpha), 1]$, then $\pi_0^* + \pi_{AP}^* \ge \pi_A^* + \pi_p^*$ is equivalent to $0 + \alpha d_H(w - c_p) \ge \frac{\alpha\alpha_H(I)d_H}{2t}\left(w - \frac{c}{\alpha_H(I)}\right)^2 + 0$, which is simplified to $c_p \le w - \frac{1-(1-I)(1-\alpha)}{2t}\left(w - \frac{c}{1-(1-I)(1-\alpha)}\right)^2$. (2) If $I \in [0, I_1(\alpha)]$, then $\pi_0^* = \pi_A^* = \pi_p^* = 0$ and $\pi_{AP}^* = \alpha d_H(w - c_p)$. It is clear that it is always that demand learning and preference learning are complements in this case.

Suppose $\alpha \in [\frac{c}{w}, \frac{c_p}{w}]$. Under this assumption, we discuss the following cases depending on the magnitude of I: (1) If $I \in [I_2(\alpha), 1]$, then the condition for complements is $\frac{\alpha d_H}{2t}\left(w - \frac{c}{\alpha}\right)^2 + \alpha d_H\left(w - c_p\right) \ge \frac{\alpha\alpha_H(I)d_H}{2t}\left(w - \frac{c}{\alpha_H(I)}\right)^2$, which can be simplified to $c_p \le w - \frac{1-(1-I)(1-\alpha)}{2t}\left(w - \frac{c}{1-(1-I)(1-\alpha)}\right)^2 + \frac{1}{2t}\left(w - \frac{c}{\alpha}\right)^2$. (2) If $I \in [0, I_2(\alpha))$, then the condition for complements is $\frac{\alpha d_H}{2t}\left(w - \frac{c}{\alpha}\right)^2 + \alpha d_H\left(w - c_p\right) \ge \frac{\alpha\alpha_H(I)d_H}{2t}\left(w - \frac{c}{\alpha_H(I)}\right)^2 + \frac{(1-\alpha)\alpha_L(I)d_H}{2t}\left(w - \frac{c}{\alpha_L(I)}\right)^2$, which can be simplified to

$$\begin{aligned} c_p \le w &- \frac{1-(1-I)(1-\alpha)}{2t}\left(w - \frac{c}{1-(1-I)(1-\alpha)}\right)^2 \\ &+ \frac{1}{2t}\left(w - \frac{c}{\alpha}\right)^2 - \frac{(1-I)(1-\alpha)}{2t}\left(w - \frac{c}{(1-I)\alpha}\right)^2. \end{aligned}$$

Suppose $\alpha \ge \frac{c_p}{w}$. We discuss the following cases depending on the magnitude of I: (1) If $I \in [I_2(\alpha), 1]$, the condition for them being complements is $\frac{\alpha d_H}{2t}\left(w - \frac{c}{\alpha}\right)^2 + \alpha d_H\left(w - c_p\right) \ge \frac{\alpha\alpha_H(I)d_H}{2t}\left(w - \frac{c}{\alpha_H(I)}\right)^2 + (\alpha w - c_p)d_H$, which can be simplified to

$$c_p \ge \frac{\alpha(1-(1-I)(1-\alpha))}{2t(1-\alpha)}\left(w - \frac{c}{1-(1-I)(1-\alpha)}\right)^2 - \frac{\alpha}{2t(1-\alpha)}\left(w - \frac{c}{\alpha}\right)^2. \qquad \text{(A-1)}$$

Notice that this inequality holds with the *strict* inequality when $I = 1$ as shown in the basic model, since the RHS can be simplified to $\frac{c}{t}[w - \frac{1}{2}(1 + \frac{1}{\alpha})c] < \frac{c}{t}(w - c) \le \frac{c}{t}t = c \le c_p$. However, we can show that the RHS is a strictly increasing function at the point $I = 1$. To see this, denote $f(I) \equiv \frac{\alpha(1-(1-I)(1-\alpha))}{2t(1-\alpha)}\left(w - \frac{c}{1-(1-I)(1-\alpha)}\right)^2 - \frac{\alpha}{2t(1-\alpha)}\left(w - \frac{c}{\alpha}\right)^2$. Then, we have the first-order derivative $f'(I) = \frac{\alpha}{2t}\left[w^2 - \frac{c^2}{(1-(1-\alpha)(1-I))^2}\right]$. At $I = 1$, we have $f'(1) = \frac{\alpha}{2t}\left(w^2 - c^2\right) > 0$. Hence, there exists $\underline{I} \in (0, 1)$ such that the RHS is equal to c, i.e., $f(\underline{I}) < c \le c_p$. Hence, for $I \in [\max(\underline{I}, I_2(\alpha)), 1]$, this inequality always holds. (2) If $I \in [0, I_2(\alpha)]$, then the condition for them being complements is

$$\begin{aligned} \frac{\alpha d_H}{2t}\left(w - \frac{c}{\alpha}\right)^2 + \alpha d_H\left(w - c_p\right) \ge{}& \frac{\alpha\alpha_H(I)d_H}{2t}\left(w - \frac{c}{\alpha_H(I)}\right)^2 \\ &+ (1-\alpha)\frac{\alpha_L(I)d_H}{2t}\left(w - \frac{c}{\alpha_L(I)}\right)^2 + (\alpha w - c_p)d_H, \end{aligned}$$

which can be simplified to

$$\begin{aligned} c_p \ge{}& \frac{\alpha(1-(1-I)(1-\alpha))}{2t(1-\alpha)}\left(w - \frac{c}{1-(1-I)(1-\alpha)}\right)^2 - \frac{\alpha}{2t(1-\alpha)}\left(w - \frac{c}{\alpha}\right)^2 \\ &+ \frac{\alpha(1-I)\alpha}{2t}\left(w - \frac{c}{(1-I)\alpha}\right)^2. \end{aligned} \qquad \text{(A-2)}$$

Therefore, it becomes straightforward to check that the statement for a high I holds. This completes the proof. $\square$

Proof of Theorem 3. (1) To pin down the profit functions for the four systems, we need to discuss whether $c_p + t_p \geq \frac{c_p}{\alpha}$ holds or not.

Suppose $c_p + t_p \geq \frac{c_p}{\alpha}$. We have $\alpha \geq \frac{c_p}{c_p+t_p}$. Under this assumption, we first consider the subcase $w \leq \frac{c}{\alpha}$, i.e., $\alpha \leq \frac{c}{w}$. We have $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ equivalent to $0 + \frac{\alpha d_H}{2t_p}(w-c_p)^2 \geq 0 + \frac{\alpha d_H}{2t}(w-c)^2$. Simplification of this equation and substituting $t_p = t(1-J)$ yields $c_p(J) \leq w - \sqrt{1-J}(w-c)$.

Suppose $c_p + t_p \leq \frac{c_p}{\alpha}$. We have $\alpha \leq \frac{c_p}{c_p+t_p}$. Under this assumption, we have two subcases: (i) $c_p + t_p \leq \frac{c}{\alpha}$ and (ii) $c_p + t_p \geq \frac{c}{\alpha}$. We discuss each of these subcases. (i) Case (i.a): If $w \leq c_p + t_p \leq \frac{c}{\alpha}$, then the inequality $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ is equivalent to $0 + \frac{\alpha d_H}{2t_p}(w-c_p)^2 \geq 0 + \frac{\alpha d_H}{2t}(w-c)^2$. Simplification of this equation and substituting $t_p = t(1-J)$ yields $c_p(J) \leq w - \sqrt{1-J}(w-c)$. Case (i.b): If $c_p + t_p \leq w \leq \frac{c}{\alpha}$, then inequality $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ is equivalent to $0 + \alpha d_H(w - \frac{1}{2}t_p - c_p) \geq 0 + \frac{\alpha d_H}{2t}(w-c)^2$, simplifying which yields $c_p(J) \leq w - \frac{(w-c)^2}{2t} - \frac{1}{2}t(1-J)$. (ii) $c_p + t_p \geq \frac{c}{\alpha}$. If $\alpha \leq \frac{c}{w}$, we have $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ equivalent to $0 + \frac{\alpha d_H}{2t_p}(w-c_p)^2 \geq 0 + \frac{\alpha d_H}{2t}(w-c)^2$. Simplification of this equation and substituting $t_p = t(1-J)$ yields $c_p(J) \leq w - \sqrt{1-J}(w-c)$.

To summarize the result above, we have obtained the result in part (1) proved: If $\alpha \leq \frac{c}{w}$ and $c_p(J) \leq \min\left\{w - \frac{(w-c)^2}{2t} - \frac{1}{2}t(1-J), w - \sqrt{1-J}(w-c)\right\}$, demand learning and imperfect preference learning are complements.

(2) If $\alpha \leq \frac{c_p(J)}{c_p(J)+t(1-J)}$, we have $c_p + t_p \leq \frac{c_p}{\alpha}$. If we further have $\alpha > \frac{c}{w}$, we need to discuss the following subcases: (a) If $\frac{c}{\alpha} \leq w \leq \frac{c_p}{\alpha}$, we have $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ equivalent to $\frac{\alpha d_H}{2t}\left(w - \frac{c}{\alpha}\right)^2 + \alpha d_H(w - \frac{1}{2}t_p - c_p) \geq 0 + \frac{\alpha d_H}{2t}(w-c)^2$. Simplification of this equation and substituting $t_p = t(1-J)$ yields $c_p \leq w - \frac{c(1-\alpha)}{2t\alpha}\left[2w - c\left(1 + \frac{1}{\alpha}\right)\right] - \frac{1}{2}t(1-J)$. Note that in this case, we also have the condition $c_p \geq \alpha w \geq \alpha(w - t_p)$. (b) If $w \geq \frac{c_p}{\alpha} + t_p$, we have $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$ equivalent to $\frac{\alpha d_H}{2t}\left(w - \frac{c}{\alpha}\right)^2 + \alpha d_H(w - \frac{1}{2}t_p - c_p) \geq d_H(\alpha(w - \frac{1}{2}t_p) - c_p) + \frac{\alpha d_H}{2t}(w-c)^2$. Simplifying this inequality yields $c_p \geq \frac{c}{2t}(2w - \frac{1+\alpha}{\alpha}c)$. We claim that this inequality always holds for the following reasons. Since $c_p \geq c$, it is sufficient to show that $c \geq \frac{c}{2t}(2w - \frac{1+\alpha}{\alpha}c)$, which can be simplified to $2(w-t) \leq \frac{1+\alpha}{\alpha}c$. But $w - t \leq c$, and $\frac{1+\alpha}{\alpha} \geq 2$, we indeed have $2(w-t) \leq \frac{1+\alpha}{\alpha}c$. Hence, if $c_p \leq \alpha(w - t_p)$, we always have $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_p^*$. Combining case (a) and (b), we obtain the result. $\square$

Proof of Proposition 5. Suppose $K_{AP0} \leq K_{A0} + K_{P0}$, $\alpha \leq 1 - \frac{w-c}{(1-I_0)w}$ and $K_A \geq K_{A1}(\alpha)$. Note $1 - \frac{w-c}{(1-I_0)w} \leq \frac{c}{w} < \frac{c_p}{w}$ and $K_{AP0} - K_{A0} - K_{P0} \leq 0$. From Proposition 1, and Propositions A-3-A-5 (in Online Appendix 4), we have the following result: (1) If $c_p \geq w - (1 - J_{AP0})t$ and $K_{AP} > K_{AP1}(\alpha, c_p)$, then the complementary condition $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_P^*$ is equivalent to $\frac{\alpha d_H}{2t(1-J_{AP0})}(w-c_p)^2 \geq K_{AP0} - K_{A0} - K_{P0}$, which always holds. (2) If $c_p < w - (1 - J_{AP0})t$ and $K_{AP} > K_{AP2}(\alpha)$, then $\pi_0^* + \pi_{AP}^* \geq$

$\pi_A^* + \pi_P^*$ if and only if $\alpha d_H[w - \frac{1}{2}t(1 - J_{AP0}) - c_p] \geq K_{AP0} - K_{A0} - K_{P0}$, which always holds due to $c_p < w - (1 - J_{AP0})t$. (3) If $c_p \geq w - (1 - J_{AP0})t$ and $K_{AP} \leq K_{AP1}(\alpha, c_p)$, then $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_P^*$ if and only if $\alpha d_H(w - c_p) - K_{AP}(1 - J_{AP0}) \geq K_{AP0} - K_{A0} - K_{P0}$, which always holds due to $K_{AP} \leq K_{AP1}(\alpha, c_p)$ and $K_{AP1}(\alpha, c_p) = \frac{\alpha d_H(w-c_p)}{1-J_{AP0}}[1 - \frac{w-c_p}{2t(1-J_{AP0})}]$. (4) If $c_p < w - (1 - J_{AP0})t$ and $K_{AP} \leq K_{AP2}(\alpha)$, then $\pi_0^* + \pi_{AP}^* \geq \pi_A^* + \pi_P^*$ if and only if $\alpha d_H(w - c_p) - K_{AP}(1 - J_{AP0}) \geq K_{AP0} - K_{A0} - K_{P0}$, which always holds. This is because with $c_p < w - (1 - J_{AP0})t$, we have $w - c_p > (1 - J_{AP0})t$. With $K_{AP} \leq K_{AP2}(\alpha)$, $K_{AP2}(\alpha) = \frac{1}{2}\alpha d_H t$, we further have $\frac{1}{2}\alpha d_H t(w - c_p) > K_{AP}(1 - J_{AP0})t$ and thus $\alpha d_H(w - c_p) > K_{AP}(1 - J_{AP0})$.

From (1)-(4), we know that demand learning and preference learning are complements with $K_{AP0} \leq K_{A0} + K_{P0}$, $\alpha \leq 1 - \frac{w-c}{(1-I_0)w}$ and $K_A > K_{A1}(\alpha)$. Because $K_{A1}(\alpha) = \frac{\alpha d_H}{2t(1-I_0)}(w-c)^2$, we know that $K_A > K_{A1}(\alpha)$ is equivalent to $\alpha \leq \frac{2tK_A(1-I_0)}{d_H(w-c)^2}$. Note that $1 - \frac{w-c}{(1-I_0)w} \geq 0$ is equivalent to $I_0 \leq \frac{c}{w}$. Let $\tilde{\alpha} = \min\{1 - \frac{w-c}{(1-I_0)w}, \frac{2tK_A(1-I_0)}{d_H(w-c)^2}\}$ and the desired result follows. □

Proof of Lemma 3. We solve for sub-game perfect equilibria by analyzing the firm's decision in the second period first.

Because $c \geq rw \geq s$, the firm will never choose an initial inventory level higher than the first-period sales in high demand case. Therefore, the firm will have remaining inventory q_2 in the second period only if the demand is low. Given p_2, a customer located at θ buys if $rw - p_2 - \theta t \geq 0$ and $\theta \geq \theta_1$. Solving $rw - p_2 - \theta t = 0$, we have $\theta_2(p_2) = \frac{rw-p_2}{t}$.

Then the period 2 demand is $2d_L(\theta_2(p_2) - \theta_1)^+ = 2d_L\left(\frac{rw-p_2}{t} - \theta_1\right)^+$. And the firm's period 2 profit maximization problem is

$$\max_{p_2 \leq rw - \theta_1 t} \pi_2(p_2) = \min\left\{2d_L\left(\frac{rw - p_2}{t} - \theta_1\right), q_2\right\} p_2 + s\left[q_2 - 2d_L\left(\frac{rw - p_2}{t} - \theta_1\right)\right]^+.$$

Solving this optimization problem, we have: if $rw - \theta_1 t > s$, i.e., $\theta_1 < (rw - s)/t$, then

$$p_2(q_2, \theta_1) = \begin{cases} \frac{rw - t\theta_1 + s}{2} & if\, q_2 \geq \left(\frac{rw-s}{t} - \theta_1\right) d_L, \\ rw - t\theta_1 - \frac{tq_2}{2d_L} & if\, q_2 < \left(\frac{rw-s}{t} - \theta_1\right) d_L, \end{cases} \tag{A-3}$$

whereas if $rw - \theta_1 t \leq s$, i.e., $\theta_1 \geq (rw - s)/t$, the firm does not want to sell in the second period and all q_2 units are salvaged at s.

Substituting the optimal p_2 back, we have the optimal period-2 profit

$$\pi_2^*(q_2, \theta_1) = \begin{cases} sq_2 & if\, \theta_1 \geq \frac{rw-s}{t} \\ \frac{d_L(rw - t\theta_1 - s)^2}{2t} + sq_2 & if\, \frac{rw-s}{t} - \frac{q_2}{d_L} \leq \theta_1 \leq \frac{rw-s}{t}, \\ q_2\left(rw - t\theta_1 - \frac{tq_2}{2d_L}\right) & if\, \theta_1 < \frac{rw-s}{t} - \frac{q_2}{d_L}. \end{cases} \tag{A-4}$$

We now consider the first period. An individual consumer does not know the demand realization. The utility of purchasing in period 1 is $w - p_1 - \theta t$. If $\theta_1 \geq (rw - s)/t$, then the firm does not sell in the second period, and $\theta_1(p_1,\bar{p}_2)=\frac{w-p_1}{t}$. If $\theta_1<(rw-s)/t$, the expected utility of purchasing in period 2 is $(1-\alpha)(rw-\bar{p}_2-\theta t)$ where $\bar{p}_2$ is consumers rational expectation of the period 2 price when the demand is low. Though the optimal p_2 depends on θ_1, for any individual consumer, $\bar{p}_2$ does not depend on her own decision. Solving $w-p_1-\theta t=(1-\alpha)(rw-\bar{p}_2-\theta t)$, we have $\theta_1(p_1,\bar{p}_2)=\frac{w-p_1-(1-\alpha)(rw-\bar{p}_2)}{\alpha t}$. Therefore, customers' optimal purchasing decision in the first period is

$$\theta_1(p_1,\bar{p}_2)=\begin{cases}\frac{w-p_1-(1-\alpha)(rw-\bar{p}_2)}{\alpha t} & if\,\theta_1<(rw-s)/t,\\ \frac{w-p_1}{t} & if\,\theta_1\geq(rw-s)/t.\end{cases} \tag{A-5}$$

The first-period demand is $2D\theta_1(p_1,\bar{p}_2)$.

We next analyze the firm's stocking/inventory decision q in period 1. Note that $\theta_1(p_1,\bar{p}_2)$ does not change with respect to the firm's initial inventory decision. Hence, taking p_1 and θ_1 as given, the firm should choose q to maximize its expected profit. Because p_1 is always higher than c, given θ_1, the optimal inventory level should be higher than $2\theta_1 d_L$. In addition, as $c\geq rw\geq s$, the optimal inventory level should never exceed $2\theta_1 d_H$. When the demand is low, $q_2=q-2\theta_1 d_L$. So $q_2\geq\left(\frac{rw-s}{t}-\theta_1\right)d_L$ is equivalent to $q\geq\left(\frac{rw-s}{t}+\theta_1\right)d_L$. The firm's initial inventory decision is as follows:

$$\max_{2\theta_1 d_L\leq q\leq 2\theta_1 d_H}\alpha p_1 q+(1-\alpha)\left(2p_1\theta_1 d_L+\pi_2^*(q-2\theta_1 d_L,\theta_1)\right)-cq,$$

where $\pi_2^*(q_2,\theta_1)$ is given in equation (A-4).

If $\theta_1\geq\frac{rw-s}{t}$, the firm's optimal inventory problem is

$$\max_{2\theta_1 d_L\leq q\leq 2\theta_1 d_H}\alpha p_1 q+(1-\alpha)2p_1\theta_1 d_L-cq+(1-\alpha)s(q-2\theta_1 d_L).$$

The first order derivative is $\pi'(q)=\alpha p_1-c+(1-\alpha)s$. Hence, the optimal initial quantity is $q^*=2\theta_1 d_L$ if $p_1<\frac{c-(1-\alpha)s}{\alpha}$, and $q^*=2\theta_1 d_H$ if $p_1\geq\frac{c-(1-\alpha)s}{\alpha}$. In this case, the firm does not sell in the second period, and $\theta_1=\frac{w-p_1}{t}$. Substituting $\theta_1=\frac{w-p_1}{t}$ back to $\theta_1\geq\frac{rw-s}{t}$, we have $p_1\leq w-rw+s$. In this equilibrium, if $p_1<\frac{c-(1-\alpha)s}{\alpha}$,

$$\pi^1(p_1)=2(\alpha p_1-c)\frac{w-p_1}{t}d_L+(1-\alpha)2p_1\frac{w-p_1}{t}d_L,$$

and if $p_1\geq\frac{c-(1-\alpha)s}{\alpha}$,

$$\pi^2(p_1)=2(\alpha p_1-c)\frac{w-p_1}{t}d_H+(1-\alpha)2p_1\frac{w-p_1}{t}d_L+2(1-\alpha)s\frac{w-p_1}{t}(d_H-d_L).$$

If $\theta_1<\frac{rw-s}{t}$, the firm's optimal inventory/stocking problem is

$$\max_{2\theta_1 d_L\leq q\leq 2\theta_1 d_H}\pi(q;p_1,\theta_1), \tag{A-6}$$

where

$$\pi\left(q;p_1,\theta_1\right)=\begin{cases}\alpha p_1 q+(1-\alpha)\left(2p_1\theta_1 d_L+(q-2\theta_1 d_L)\left(rw-t\theta_1-\frac{t(q-2\theta_1 d_L)}{2d_L}\right)\right)-cq\\ \text{if } 2\theta_1 d_L\le q<\left(\frac{rw-s}{t}+\theta_1\right)d_L,\\ \alpha p_1 q+(1-\alpha)\left(2p_1\theta_1 d_L+\frac{d_L(rw-t\theta_1-s)^2}{2t}\right)-cq+(1-\alpha)s(q-2\theta_1 d_L)\\ \text{if } \left(\frac{rw-s}{t}+\theta_1\right)d_L\le q\le 2\theta_1 d_H.\end{cases}\tag{A-7}$$

When $2\theta_1 d_L\le q<\left(\frac{rw-s}{t}+\theta_1\right)d_L$, we have $\pi'(q)=\alpha p_1-c+(1-\alpha)\left(rw+t\theta_1-\frac{tq}{d_L}\right)$, $\pi''\left(q;p_1,\theta_1\right)=(1-\alpha)t/d_L<0$, and the solution to the first-order condition $\pi'\left(q;p_1,\theta_1\right)=0$ is $q^*=\left(\frac{rw}{t}+\theta_1\right)d_L+\frac{d_L}{(1-\alpha)t}\left(\alpha p_1-c\right)$.

When $\left(\frac{rw-s}{t}+\theta_1\right)d_L\le q\le 2\theta_1 d_H$, $\pi'(q)=\alpha p_1-c+(1-\alpha)s$. If $p_1<\frac{(\theta_1 t-rw)(1-\alpha)+c}{\alpha}<\frac{c-(1-\alpha)s}{\alpha}$, then $q^*\le\left(\frac{rw-s}{t}+\theta_1\right)d_L$ and $\pi'(q)\le 0$ for $\left(\frac{rw-s}{t}+\theta_1\right)d_L\le q\le 2\theta_1 d_H$. In addition, we can verify that $q^*<2d_L\theta_1$. Therefore, the optimal initial inventory is $q=2d_L\theta_1$ and the firm does not sell in period 2. Then, $\theta_1=\frac{w-p_1}{t}$. Hence, $\theta_1<\frac{rw-s}{t}$ is equivalent to $p_1>w-rw+s$. The firm's profit is $\pi^1\left(p_1\right)$. Substituting θ_1 back to $p_1<\frac{(\theta_1 t-rw)(1-\alpha)+c}{\alpha}$, we have $p_1<c+(1-\alpha)(1-r)w$.

If $\frac{(\theta_1 t-rw)(1-\alpha)+c}{\alpha}\le p_1<\frac{c-(1-\alpha)s}{\alpha}$, then $q^*\le\left(\frac{rw-s}{t}+\theta_1\right)d_L$ and $\pi'(q)\le 0$ for $\left(\frac{rw-s}{t}+\theta_1\right)d_L\le q\le 2\theta_1 d_H$. In addition, we can verify that $q^*>2d_L\theta_1$. In this case, the optimal inventory is $q=q^*$. Then $q_2=\left(\frac{rw}{t}-\theta_1\right)d_L+\frac{d_L}{(1-\alpha)t}\left(\alpha p_1-c\right)$ and $p_2=rw-t\theta_1-\frac{tq_2}{2d_L}=\frac{rw-t\theta_1}{2}-\frac{\alpha p_1-c}{2(1-\alpha)}$. Substituting $p_2=\frac{rw-t\theta_1}{2}-\frac{\alpha p_1-c}{2(1-\alpha)}$ into equation (A-5) and solving for θ_1, we have $\theta_1=\frac{2w-(1-\alpha)rw-(2+\alpha)p_1+c}{(1+\alpha)t}$. Then $\theta_1<\frac{rw-s}{t}$ is equivalent to $p_1>\frac{c+2(1-r)w+s(1+\alpha)}{2+\alpha}$, and $q=2d_L\frac{(1-\alpha)(1+\alpha r)w+\left(\alpha^2+\alpha-1\right)p_1-\alpha c}{(1-\alpha)(1+\alpha)t}$. Substituting θ_1 and q back to equation (A-7), we have the firm's profit as $\pi^3\left(p_1\right)$. Substituting θ_1 back to $p_1\ge\frac{(\theta_1 t-rw)(1-\alpha)+c}{\alpha}$, we have $p_1\ge c+(1-\alpha)(1-r)w$.

If $\alpha p_1-c+(1-\alpha)s>0$, i.e., $p_1>\frac{c-(1-\alpha)s}{\alpha}$, then $q^*>\left(\frac{rw-s}{t}+\theta_1\right)d_L$ and $\pi'(q)>0$ for $\left(\frac{rw-s}{t}+\theta_1\right)d_L\le q\le 2\theta_1 d_H$. In this case, the optimal inventory is $q=2\theta_1 d_H$. Then $q_2=2\theta_1\left(d_H-d_L\right)$ and $p_2=\frac{rw-t\theta_1+s}{2}$. Substituting $p_2=\frac{rw-t\theta_1+s}{2}$ into equation (A-5) and solve for θ_1 we have $\theta_1=\frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$. Then $q=2d_H\frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$. Substituting $\theta_1=\frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$ back to $\theta_1<\frac{rw-s}{t}$, we have $p_1>w-rw+s$. Substituting θ_1 and q back into equation (A-7), we have the firm's profit as $\pi^4\left(p_1\right)$.

Comparing the thresholds of p_1, we have two cases depending on the primitive parameters:

Case 1: When $\frac{c-(1-\alpha)s}{\alpha}\le w-rw+s$, then $\frac{2w-2rw+c+(1+\alpha)s}{2+\alpha}\ge\frac{c-(1-\alpha)s}{\alpha}$, and $c+(1-\alpha)(1-r)w\le w-rw+s$.

If $p_1<\frac{c-(1-\alpha)s}{\alpha}$, then because $p_1<\frac{c-(1-\alpha)s}{\alpha}\le\frac{2w-2rw-c+(1+\alpha)s}{2-\alpha}$ and $p_1<\frac{c-(1-\alpha)s}{\alpha}\le w-rw+s$, in equilibrium $\theta_1\ge\frac{rw-s}{t}$. Therefore, $\theta_1=\frac{w-p_1}{t}$ and $q=2\frac{w-p_1}{t}d_L$.

If $\frac{c-(1-\alpha)s}{\alpha}\le p_1<w-rw+s$, then because $\frac{c-(1-\alpha)s}{\alpha}\le\frac{2w-2rw-c+s(1+\alpha)}{2-\alpha}$, in equilibrium $\theta_1\ge\frac{rw-s}{t}$. Therefore, $\theta_1=\frac{w-p_1}{t}$ and $q=2\frac{w-p_1}{t}d_H$.

If $p_1 \geq w - rw + s$, then because $p_1 > \frac{c-(1-\alpha)s}{\alpha}$, in equilibrium $\theta_1 \leq \frac{rw-s}{t}$. Therefore, $\theta_1 = \frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$ and $q = 2d_H\frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$.

To summarize, in this case, the firm's profit as a function of p_1 is

$$\pi(p_1) = \begin{cases} \pi^1(p_1) & if\, p_1 < \frac{c-(1-\alpha)s}{\alpha}, \\ \pi^2(p_1) & if\, \frac{c-(1-\alpha)s}{\alpha} \leq p_1 < w - rw + s, \\ \pi^4(p_1) & if\, p_1 \geq w - rw + s. \end{cases} \tag{A-8}$$

The profit $\pi^4(p_1)$ can be obtained by substituting $q = 2d_H\frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$ and $\theta_1 = \frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$ back into equation (A-7) (the part with $\left(\frac{rw-s}{t}+\theta_1\right)d_L \leq q \leq 2\theta_1 d_H$).

Case 2: When $\frac{c-(1-\alpha)s}{\alpha} > w-rw+s$, then $c+(1-\alpha)(1-r)w > w-rw+s$, $c+(1-\alpha)(1-r)w > \frac{2w-2rw+c+(1+\alpha)s}{2+\alpha}$ and $c+(1-\alpha)(1-r)w < \frac{c-(1-\alpha)s}{\alpha}$.

If $p_1 < w-rw+s$, then because $p_1 < w-rw+s < \frac{c-(1-\alpha)s}{\alpha}$, in equilibrium $\theta_1 \geq \frac{rw-s}{t}$. Therefore, $\theta_1 = \frac{w-p_1}{t}$ and $q = 2\frac{w-p_1}{t}d_L$.

If $w-rw+s < p_1 < c+(1-\alpha)(1-r)w$, then in equilibrium $\theta_1 < \frac{rw-s}{t}$. Therefore, $\theta_1 = \frac{w-p_1}{t}$ and $q = 2\frac{w-p_1}{t}d_L$.

If $c+(1-\alpha)(1-r)w \leq p_1 < \frac{c-(1-\alpha)s}{\alpha}$, then because $p_1 \geq c+(1-\alpha)(1-r)w > \frac{2w-2rw+c+s(1+\alpha)}{2+\alpha}$, in equilibrium $\theta_1 < \frac{rw-s}{t}$. Therefore, $\theta_1 = \frac{2w-(1-\alpha)rw-(2+\alpha)p_1+c}{(1+\alpha)t}$ and $q = 2d_L\frac{(1-\alpha)(1+\alpha r)w+\left(\alpha^2+\alpha-1\right)p_1-\alpha c}{(1-\alpha)(1+\alpha)t}$.

If $p_1 \geq \frac{c-(1-\alpha)s}{\alpha}$, then $p_1 > w-rw+s$, and thus in equilibrium $\theta_1 \leq \frac{rw-s}{t}$. Therefore, $\theta_1 = \frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$ and $q = 2d_H\frac{2w-(1-\alpha)rw-2p_1+(1-\alpha)s}{(1+\alpha)t}$.

To summarize, in this case, the firm's profit as a function of p_1 is

$$\pi(p_1) = \begin{cases} \pi^1(p_1) & if\, p_1 < c+(1-\alpha)(1-r)w, \\ \pi^3(p_1) & if\, c+(1-\alpha)(1-r)w \leq p_1 < \frac{c-(1-\alpha)s}{\alpha}, \\ \pi^4(p_1) & if\, p_1 \geq \frac{c-(1-\alpha)s}{\alpha}. \end{cases} \tag{A-9}$$

The profit $\pi^3(p_1)$ can be obtained by substituting $q = 2d_L\frac{(1-\alpha)(1+\alpha r)w+\left(\alpha^2+\alpha-1\right)p_1-\alpha c}{(1-\alpha)(1+\alpha)t}$ and $\theta_1 = \frac{2w-(1-\alpha)rw-(2+\alpha)p_1+c}{(1+\alpha)t}$ back into equation (A-7) (the part with $2\theta_1 d_L \leq q < \left(\frac{rw-s}{t}+\theta_1\right)d_L$). □

Proof of Lemma 4. In the preference learning system, all customers obtain their ideal products since there is no preference mismatch. Thus, in the second period, the firm's optimal price is $p_2 = rw$ and customers buying in the second period will get zero utility. Therefore, the firm's optimal price in the first period is $p_1 = w$ and all customers want to buy in period 1. The only remaining decision is the initial inventory level. It is clear that the optimal inventory level is between d_L and d_H. The firm's expected profit is $\max_{d_L \leq q \leq d_H} \pi(q) = \alpha wq + (1-\alpha)(wd_L + s(q-d_L)) - c_p q$. Then, $\pi'(q) = \alpha w + (1-\alpha)s - c_p$. Hence, the results follow. □

Proof of Proposition 6. With $d_L = 0$, we can use Proposition 1 for the traditional system as: (i) if $\alpha \leq \frac{c}{w}$, the $\pi_0^* = 0$; (ii) if $\alpha > \frac{c}{w}$, the $\pi_0^* = \frac{\alpha d_H}{2t}(w - \frac{c}{\alpha})^2$. With $d_L = 0$, we can use Proposition 4 for the Bespoke system as $\pi^*_{AP} = \alpha d_H(w - c_p)$. Note $w \geq c + t(1-J)$ is equivalent to $J \geq 1 - \frac{w-c}{t}$. We discuss the complementary condition in twelve different parameter ranges: (1) Suppose $J \leq 1 - \frac{w-c}{t}$, $I \in [0, I_A(\alpha, c_p)]$ and $\alpha < \frac{c}{w}$. Then, by Propositions A-6 and A-7 (in Online Appendix 4) and the profit expressions for the traditional and the Bespoke systems listed above, the sufficient and necessary condition $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ can be written as $0 + \alpha d_H(w - c_p) \geq \frac{\alpha d_H}{2t(1-J)}(w-c)^2 + 0$, which can be simplified as $c_p \leq w - \frac{(w-c)^2}{2t(1-J)}$. (2) Suppose $J \leq 1 - \frac{w-c}{t}$, $I \in [0, I_A(\alpha, c_p)]$ and $\frac{c}{w} \leq \alpha < \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $c_p \leq w - \frac{1}{2t}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]$. (3) Suppose $J \leq 1 - \frac{w-c}{t}$, $I \in [0, I_B(\alpha, c_p)]$ and $\alpha \geq \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $c_p \geq \frac{\alpha}{2t(1-\alpha)}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]$. (4) Suppose $J \leq 1 - \frac{w-c}{t}$, $I \in [I_A(\alpha, c_p), 1]$ and $\alpha < \frac{c}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $J \leq 1 - \frac{(w-c)^2}{2tw(1-\alpha)(1-I)}$. (5) Suppose $J \leq 1 - \frac{w-c}{t}$, $I \in [I_A(\alpha, c_p), 1]$ and $\frac{c}{w} \leq \alpha < \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $J \leq 1 - \frac{(w-c)^2}{(w-\frac{c}{\alpha})^2 + 2tw(1-\alpha)(1-I)}$. (6) Suppose $J \leq 1 - \frac{w-c}{t}$, $I \in [I_B(\alpha, c_p), 1]$ and $\alpha \geq \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $J \leq 1 - \frac{(w-c)^2}{(w-\frac{c}{\alpha})^2 + 2tw(1-\alpha)(1-I)}$. (7) Suppose $J > 1 - \frac{w-c}{t}$, $I \in [0, I_A(\alpha, c_p)]$ and $\alpha < \frac{c}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $c_p \leq c + \frac{1}{2}t(1-J)$. (8) Suppose $J > 1 - \frac{w-c}{t}$, $I \in [0, I_A(\alpha, c_p)]$ and $\frac{c}{w} \leq \alpha < \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $c_p \leq \frac{1}{2t}(w - \frac{c}{\alpha})^2 + \frac{1}{2}t(1-J) + c$. (9) Suppose $J > 1 - \frac{w-c}{t}$, $I \in [0, I_B(\alpha, c_p)]$ and $\alpha \geq \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $c_p \geq \frac{\alpha}{1-\alpha}[w - c - \frac{1}{2}t(1-J) - \frac{1}{2t}(w - \frac{c}{\alpha})^2]$. (10) Suppose $J > 1 - \frac{w-c}{t}$, $I \in [I_A(\alpha, c_p), 1]$ and $\alpha < \frac{c}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $J \leq 1 - \frac{w-c-(1-I)(1-\alpha)w}{\frac{1}{2}t}$. (11) Suppose $J > 1 - \frac{w-c}{t}$, $I \in [I_A(\alpha, c_p), 1]$ and $\frac{c}{w} \leq \alpha < \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $J \leq 1 - \frac{w-c-\frac{1}{2t}(w-\frac{c}{\alpha})^2-(1-I)(1-\alpha)w}{\frac{1}{2}t}$. (12) Suppose $J > 1 - \frac{w-c}{t}$, $I \in [I_B(\alpha, c_p), 1]$ and $\alpha \geq \frac{c_p}{w}$. Then, $\pi^*_0 + \pi^*_{AP} \geq \pi^*_A + \pi^*_p$ if and only if $J \leq 1 - \frac{w-c-\frac{1}{2t}(w-\frac{c}{\alpha})^2-(1-I)(1-\alpha)w}{\frac{1}{2}t}$.

Now we investigate the particular case stated in this proposition. Suppose $I \in [0, I_B(\alpha, c_p)]$ and $J \in [0, 1 - \frac{w-c}{t}]$. From claim (3) analyzed above, the conditions are $\frac{\alpha}{2t(1-\alpha)}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2] \leq c_p < \alpha w$. Next, we prove that there exists $\hat{J}(\alpha) > 0$ such that when $J \leq \hat{J}(\alpha)$, $c_p \geq \frac{\alpha}{2t(1-\alpha)}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]$ always holds. Note that $\frac{\alpha}{2t(1-\alpha)}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]\Big|_{J=0} = \frac{\alpha}{2t(1-\alpha)}[(w-c)^2 - (w - \frac{c}{\alpha})^2]$. Because $c_p > \frac{\alpha}{2t(1-\alpha)}[(w-c)^2 - (w - \frac{c}{\alpha})^2]$ is equivalent to $w < \frac{c_p t}{c} + \frac{c}{2}(1 + \frac{1}{\alpha})$ which is always true due to $c_p > c$, $\alpha < 1$ and $w \leq c + t$, we know that $c_p > \frac{\alpha}{2t(1-\alpha)}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]\Big|_{J=0}$ always holds. Because $\frac{\alpha}{2t(1-\alpha)}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]$ is continuous and increasing at J, we can always find such an $\hat{J}(\alpha) > 0$. Furthermore, by solving $c_p \geq \frac{\alpha}{2t(1-\alpha)}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]$, we have $\hat{J}(\alpha) = 1 - \frac{(w-c)^2}{\frac{2tc_p(1-\alpha)}{\alpha} + (w-\frac{c}{\alpha})^2}$.

Suppose $I \in [0, \min\{I_A(\alpha, c_p), I_B(\alpha, c_p)\}]$ and $J \in [0, \min\{1 - \frac{w-c}{t}, 1 - \frac{(w-c)^2}{\frac{2tc_p(1-\alpha)}{\alpha} + (w-\frac{c}{\alpha})^2}\}]$. Then from claims (1)-(3) of above, demand learning and preference learning are complements if and only if one of the three conditions holds: (1) $\alpha \leq \frac{c}{w}$ and $c_p \leq w - \frac{(w-c)^2}{2t(1-J)}$, (2) $\frac{c}{w} < \alpha \leq \frac{c_p}{w}$ and $c_p \leq w - \frac{1}{2t}[\frac{(w-c)^2}{1-J} - (w - \frac{c}{\alpha})^2]$, and (3) $c_p < \alpha w$. Proposition 6 follows by combining conditions (2) and (3). $\square$

Online Supplement 2. Supplemental Materials for §5

In this section, we provide supplemental materials for §5: Extensions to the Basic Model.

Online Supplement 2.1. Analysis for §5.2. Imperfect Preference Learning

We first analyze the preference learning system. For analytical convenience, we also normalize d_L to zero in this section. We will abuse notation by writing $c_p = c_p(J)$ given that J is fixed. The proposition below provides the firm's optimal expected profit for an imperfect preference learning system.

PROPOSITION A-1. *In an imperfect preference learning system with the level of customization J:*

(i) If $w \in [\frac{c_p}{\alpha} + t(1-J), +\infty)$, the firm optimal expected profit $\pi_p^*(J,\alpha) = [\alpha(w - \frac{1}{2}t(1-J)) - c_p]d_H$.

(ii) If $w \in [\frac{c_p}{\alpha}, \frac{c_p}{\alpha} + t(1-J)]$, the firm optimal expected profit $\pi_p^*(J,\alpha) = \frac{\alpha d_H}{2t(1-J)}(w - \frac{c_p}{\alpha})^2$.

(iii) If $w \in [0, \frac{c_p}{\alpha}]$, the firm optimal expected profit $\pi_p^*(J,\alpha) = 0$.

Proof of Proposition A-1. To find the consumers who are indifferent to purchasing the product versus not purchasing located at $\theta(p)$ for a given price p, we let $u(\theta,p) \equiv w - p - |\theta|t_p = 0$. We have $|\theta(p)| = \frac{w-p}{t_p}$. If $p \in [w - \frac{1}{2}t_p, w]$, we have $|\theta(p)| \in [0, \frac{1}{2}]$ so that the number of consumers who purchase the product is $D_p(p) = \frac{2(w-p)D}{t_p}$. If $p \in [0, w - \frac{1}{2}t_p]$, all the consumers will purchase the product, i.e., $D_p(p) = D$. If $p > w$, $D_p(p) = 0$. The firm chooses its price p and production quantity q to maximize the expected profit $\pi(p,q) = pE\min(D_p(p), q) - c_p q$.

$$\pi(p,q) = \begin{cases} p\alpha\min(d_H, q) - c_p q, & \text{if } p \in [0, w - \frac{1}{2}t_p] \\ p\alpha\min(\frac{2(w-p)d_H}{t_p}, q) - c_p q, & \text{if } p \in [w - \frac{1}{2}t_p, w] \\ -c_p q, & \text{if } p \in [w, +\infty). \end{cases}$$

Let us discuss different cases depending on the range of price p.

For $p \in [0, w - \frac{1}{2}t_p]$, the optimal price has to be $w - \frac{1}{2}t_p$ since $\pi(p,q)$ strictly increases in p. Then, the optimal quantity $q^* = d_H$ if $\alpha(w - \frac{1}{2}t_p) \geq c_p$ and $q^* = 0$ if $\alpha(w - \frac{1}{2}t_p) < c_p$. Hence, in this region, the maximum expected profit $\pi(p,q) = [\alpha(w - \frac{1}{2}t_p) - c_p]d_H$ if $\alpha(w - \frac{1}{2}t_p) \geq c_p$ and $\pi(p,q) = 0$ if $\alpha(w - \frac{1}{2}t_p) < c_p$.

For $p \in [w - \frac{1}{2}t_p, w]$, we have $\pi(p,q) = p\alpha\min(\frac{2(w-p)d_H}{t_p}, q) - c_p q \leq (\alpha p - c_p)q$ and $q \leq \frac{2(w-p)d_H}{t_p}$. Therefore, $\pi(p,q) = (\alpha p - c_p)\frac{2(w-p)d_H}{t_p}$. First-order condition with respect to price p yields $p^* = \frac{w}{2} + \frac{c_p}{2\alpha}$. Then, we obtain the corresponding quantity $q^* = \frac{d_H}{t_p}(w - \frac{c_p}{\alpha})$. Finally, we need to make sure that the price is within this region, i.e., $w \geq p^* = \frac{w}{2} + \frac{c_p}{2\alpha} \geq w - \frac{1}{2}t_p$, which is simplified to $\frac{c_p}{\alpha} \leq w \leq \frac{c_p}{\alpha} + t_p$. Under this condition, the maximized expected profit $\pi(p^*,q^*) = \frac{\alpha d_H}{2t_p}(w - \frac{c_p}{\alpha})^2$. If this condition does not hold, we discuss two subcases: If $w > \frac{c_p}{\alpha} + t_p$, then the optimal price should be on the boundary, $p^* = w - \frac{1}{2}t_p$ and the optimal quantity $q^* = d_H$. The optimal expected profit is $\pi(p^*, q^*) = [\alpha(w - \frac{1}{2}t_p) - c_p]d_H$; If $w < \frac{c_p}{\alpha}$, then the optimal price $p^* = w$, optimal quantity $q^* = 0$, and expected profit is zero since no consumers purchase at this price.

To sum, the optimal expected profit is

$$\pi_p^* = \begin{cases} [\alpha(w - \frac{1}{2}t_p) - c_p]d_H, & \text{if } w \in [\frac{c_p}{\alpha} + t_p, +\infty) \\ \frac{\alpha d_H}{2t_p}(w - \frac{c_p}{\alpha})^2, & \text{if } w \in [\frac{c_p}{\alpha}, \frac{c_p}{\alpha} + t_p] \\ 0, & \text{if } w \in [0, \frac{c_p}{\alpha}]. \end{cases}$$

Since $t_p = t(1 - J)$, we can express the profit function in terms of J. $\square$

Next, we analyze the Bespoke system where preference learning is imperfect with the level of customization J and demand learning is perfect.

Proposition A-2. *In a Bespoke system of imperfect preference learning with the level of customization J and perfect demand learning:*
(i) If $w \geq c_p + t(1 - J)$, the firm optimal expected profit $\pi_{AP}^*(J, \alpha) = \alpha d_H(w - \frac{1}{2}t(1 - J) - c_p)$.
(ii) If $w \leq c_p + t(1 - J)$, the firm optimal expected profit $\pi_{AP}^*(J, \alpha) = \frac{\alpha d_H}{2t(1-J)}(w - c_p)^2$.

Proof of Proposition A-2. In this system, the firm can observe the demand realization D before making its quantity decision. Hence, the firm can perfectly match supply with demand. Suppose $D = d_H$, then the number of consumers who will purchase the product is

$$D_{AP}(d_H) = \begin{cases} d_H, & \text{if } p \in [0, w - \frac{1}{2}t_p] \\ \frac{2(w-p)d_H}{t_p}, & \text{if } p \in [w - \frac{1}{2}t_p, w] \\ 0, & \text{if } p \in [w, +\infty). \end{cases}$$

We then discuss under each of the three price ranges, what the optimal price should be. If $p \in [0, w - \frac{1}{2}t_p]$, the firm profit is $\pi_{d_H}(p, q) = (p - c_p)d_H$. The optimal price $p_{d_H}^* = w - \frac{1}{2}t_p$ and optimal quantity $q_{d_H}^* = d_H$ if $w - \frac{1}{2}t_p \geq c_p$. Hence, $\pi_{d_H}^* = (w - \frac{1}{2}t_p - c_p)d_H$ if $w \geq \frac{1}{2}t_p + c_p$.

If $p \in [w - \frac{1}{2}t_p, w]$, then $q_{d_H}^* = \frac{2(w-p)d_H}{t_p}$. The firm profit $\pi_{d_H}(p, q) = \frac{2(p-c_p)(w-p)d_H}{t_p}$. First-order condition with respect to price p yields $w - p - (p - c_p) = 0$. Hence, $p^* = \frac{w+c_p}{2}$. For it to be optimal, we require $p^* = \frac{w+c_p}{2} \in [w - \frac{1}{2}t_p, w]$, which is simplified to $w \in [c_p, c_p + t_p]$. Hence, under this condition, we have $p_{d_H}^* = \frac{w+c_p}{2}$, $q_{d_H}^* = \frac{d_H(w-c_p)}{t_p}$, and the optimal profit $\pi_{d_H}^* = \frac{d_H}{2t_p}(w - c_p)^2$. If this condition does not hold, $w \geq c_p + t_p$, we have $p_{d_H}^* = w - \frac{1}{2}t_p$, $q_{d_H}^* = d_H$, and the optimal profit $\pi_{d_H}^* = (w - \frac{1}{2}t_p - c_p)d_H$. The optimal expected profit is $\pi_{AP}^* = \alpha\pi_{d_H}^*$. Hence, we obtain

$$\pi_{AP}^* = \begin{cases} \alpha d_H(w - \frac{1}{2}t_p - c_p), & \text{if } w \geq c_p + t_p \\ \frac{\alpha d_H}{2t_p}(w - c_p)^2, & \text{if } w \leq c_p + t_p. \end{cases}$$

Since $t_p = t(1 - J)$, we can express the profit function in terms of J. $\square$

Online Supplement 2.2. Analysis for §5.3: Investment Costs and Endogenous Learning Levels

In this section, we provide the detailed analysis for the three systems in the linear-cost setting. The result for traditional system remains the same as in the basic model. Propositions A-3-A-5 below summarize the results for demand learning, preference learning and Bespoke systems, respectively.

We first introduce some notations for expositional convenience. Let $K_{A1}(\alpha) \equiv \frac{\alpha d_H}{2t(1-I_0)}(w-c)^2$, $K_{A2}(\alpha) \equiv \frac{\alpha d_H}{2t(1-I_0)}\{(w-c)^2 - [1-(1-I_0)(1-\alpha)][w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2\}$, and $K_{A3}(\alpha) \equiv \frac{\alpha d_H}{2t(1-I_0)}\{(w-c)^2 - [1-(1-I_0)(1-\alpha)][w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 - (1-I_0)(1-\alpha)[w - \frac{c}{(1-I_0)\alpha}]^2\}$.

Proposition A-3. *In a demand learning system with linear costs, we have:*
(i) The optimal learning level $I^* = I_0$, *if and only if one of the following three conditions is satisfied: (1)* $\alpha \le 1 - \frac{w-c}{(1-I_0)w}$ *and* $K_A > K_{A1}(\alpha)$, *(2)* $1 - \frac{w-c}{(1-I_0)w} < \alpha \le \frac{c}{(1-I_0)w}$ *and* $K_A > K_{A2}(\alpha)$, *(3)*$\frac{c}{(1-I_0)w} < \alpha \le 1$ *and* $K_A > K_{A3}(\alpha)$. *The corresponding optimal profits are* $\pi_A^* = -K_{A0}$, $\pi_A^* = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 - K_{A0}$ *and* $\pi_A^* = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 + \frac{\alpha(1-I_0)(1-\alpha)d_H}{2t}[w - \frac{c}{(1-I_0)\alpha}]^2 - K_{A0}$.
(ii) Otherwise, the optimal learning level $I^* = 1$, *and* $\pi_A^* = \frac{\alpha d_H}{2t}(w-c)^2 - K_A(1-I_0) - K_{A0}$.

Proof of Proposition A-3. We prove the result in two cases $\alpha \le \frac{c}{w}$ and $\alpha > \frac{c}{w}$.

Case 1: When $\alpha \le \frac{c}{w}$, from Proposition 5 and $CA(I) = K_{A0} + K_A(I - I_0)$, we have:

$$\pi_A(I) = \begin{cases} \pi_A^1(I) \equiv -K_{A0} - K_A(I - I_0) & \text{if } I \in [I_0, I_1(\alpha)] \\ \pi_A^2(I) \equiv \frac{\alpha[1-(1-I)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I)(1-\alpha)}]^2 - K_{A0} - K_A(I - I_0) & \text{if } I \in [\max\{I_0, I_1(\alpha)\}, 1] \end{cases},$$

where $I_1(\alpha) = 1 - \frac{w-c}{(1-\alpha)w}$. Because $\frac{d\pi_A^1(I)}{dI} = -K_A$, $\frac{d^2\pi_A^1(I)}{dI^2} = 0$, $\frac{d\pi_A^2(I)}{dI} = \frac{\alpha(1-\alpha)d_H}{2t}[w^2 - \frac{c^2}{[1-(1-I)(1-\alpha)]^2}] - K_A$, and $\frac{d^2\pi_A^2(I)}{dI^2} > 0$, we know that the optimal learning level I^* is either I_0 or 1. Note that $I_0 \le I_1(\alpha)$ is equivalent to $\alpha \le 1 - \frac{w-c}{(1-I_0)w}$ and that $1 - \frac{w-c}{(1-I_0)w} < \frac{c}{w}$ always holds. We have the following:

For $\alpha \le 1 - \frac{w-c}{(1-I_0)w}$,

$$\pi_A(I) = \begin{cases} \pi_A^1(I) & \text{if } I \in [I_0, I_1(\alpha)] \\ \pi_A^2(I) & \text{if } I \in [I_1(\alpha), 1] \end{cases}.$$

For $1 - \frac{w-c}{(1-I_0)w} < \alpha \le \frac{c}{w}$, $\pi_A(I) = \pi_A^2(I)$ for $I \in [I_0, 1]$.

For $\alpha \le 1 - \frac{w-c}{(1-I_0)w}$, by comparing $\pi_A(I_0) = -K_{A0}$ and $\pi_A(1) = \frac{\alpha d_H}{2t}(w-c)^2 - K_{A0} - K_A(1 - I_0)$, we know that if $K_A > K_{A1}$, then $I = I_0$ and $\pi_A^* = -K_{A0}$; otherwise, $I^* = 1$, and $\pi_A^* = \frac{\alpha d_H}{2t}(w-c)^2 - K_A(1-I_0) - K_{A0}$.

For $1 - \frac{w-c}{(1-I_0)w} < \alpha \le \frac{c}{w}$, by comparing $\pi_A(I_0) = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 - K_{A0}$ and $\pi_A(1) = \frac{\alpha d_H}{2t}(w-c)^2 - K_{A0} - K_A(1-I_0)$, we know that if $K_A > K_{A2}$, then $I^* = I_0$ and $\pi_A^* = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 - K_{A0}$; otherwise, $I^* = 1$, and $\pi_A^* = \frac{\alpha d_H}{2t}(w-c)^2 - K_A(1-I_0) - K_{A0}$.

Case 2: When $\alpha > \frac{c}{w}$, from Proposition 5 and $CA(I) = K_{A0} + K_A(I - I_0)$, we have:

$$\pi_A(I) = \begin{cases} \pi_A^3(I) \equiv \frac{\alpha[1-(1-I)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I)(1-\alpha)}]^2 + \frac{\alpha(1-I)(1-\alpha)d_H}{2t}[w - \frac{c}{(1-I)\alpha}]^2 - K_{A0} - K_A(I - I_0) & \text{if } I \in [I_0, I_2(\alpha)] \\ \pi_A^4(I) \equiv \frac{\alpha[1-(1-I)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I)(1-\alpha)}]^2 - K_{A0} - K_A(I - I_0) & \text{if } I \in [\max\{I_0, I_2(\alpha)\}, 1] \end{cases},$$

where $I_2(\alpha) = \frac{\alpha w - c}{\alpha w}$. Note that $\pi_A^4(I) = \pi_A^2(I)$ in case 1. Because $\frac{d\pi_A^3(I)}{dI} = \frac{\alpha(1-\alpha)d_H}{2t}c^2[\frac{1}{[(1-I)\alpha]^2} - \frac{1}{[1-(1-I)(1-\alpha)]^2}] - K_A$, $\frac{d^2\pi_A^3(I)}{dI^2} > 0$, $\frac{d\pi_A^4(I)}{dI} = \frac{\alpha(1-\alpha)d_H}{2t}[w^2 - \frac{c^2}{[1-(1-I)(1-\alpha)]^2}] - K_A$, and $\frac{d^2\pi_A^4(I)}{dI^2} > 0$, we know that the optimal learning level I^* is either I_0 or 1. Note that $I_0 \le I_2(\alpha)$ is equivalent to $\alpha \ge \frac{c}{(1-I_0)w}$ and that $\frac{c}{(1-I_0)w} > \frac{c}{w}$ always holds. We have:

For $\frac{c}{w} < \alpha \le \frac{c}{(1-I_0)w}$, $\pi_A(I) = \pi_A^4(I)$ for $I \in [I_0, 1]$.

For $\frac{c}{(1-I_0)w} < \alpha \le 1$,

$$\pi_A(I) = \begin{cases} \pi_A^3(I) & \text{if } I \in [I_0, I_2(\alpha)] \\ \pi_A^4(I) & \text{if } I \in [I_2(\alpha), 1] \end{cases}.$$

For $\frac{c}{w} < \alpha \le \frac{c}{(1-I_0)w}$, by comparing $\pi_A(I_0) = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 - K_{A0}$ and $\pi_A(1) = \frac{\alpha d_H}{2t}(w-c)^2 - K_{A0} - K_A(1 - I_0)$, we know that if $K_A > K_{A2}$, then $I^* = I_0$ and $\pi_A^* = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 - K_{A0}$; otherwise, $I^* = 1$, and $\pi_A^* = \frac{\alpha d_H}{2t}(w-c)^2 - K_A(1-I_0) - K_{A0}$.

For $\frac{c}{(1-I_0)w} < \alpha \le 1$, by comparing $\pi_A(I_0) = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 + \frac{\alpha(1-I_0)(1-\alpha)d_H}{2t}[w - \frac{c}{(1-I_0)\alpha}]^2 - K_{A0}$ and $\pi_A(1) = \frac{\alpha d_H}{2t}(w-c)^2 - K_{A0} - K_A(1-I_0)$, we know that if $K_A > K_{A3}$, then $I^* = I_0$ and $\pi_A^* = \frac{\alpha[1-(1-I_0)(1-\alpha)]d_H}{2t}[w - \frac{c}{1-(1-I_0)(1-\alpha)}]^2 + \frac{\alpha(1-I_0)(1-\alpha)d_H}{2t}[w - \frac{c}{(1-I_0)\alpha}]^2 - K_{A0}$; otherwise, $I^* = 1$, and $\pi_A^* = \frac{\alpha d_H}{2t}(w-c)^2 - K_A(1-I_0) - K_{A0}$. □

Proposition A-3 shows that for any given α, the optimal demand learning level is high when the investment cost K_A is small, which is consistent with our intuition that the optimal learning accuracy level is higher when learning is less expensive. We also observe that demand learning level is high when α is intermediate. When α is small, the average demand αd_H is small, which implies a high learning cost per unit. When α is large, the demand uncertainty is low (the coefficient of variation is close to zero) and thus the firm has less incentive to learn the demand.

Let $K_{P1}(\alpha, c_p) \equiv \frac{d_H}{1-J_{P0}}[\alpha w - c_p - \frac{\alpha}{2t(1-J_{P0})}(w - \frac{c_p}{\alpha})^2]$ and $K_{P2}(\alpha) \equiv \frac{1}{2}\alpha d_H t$. In preference learning and Bespoke systems, we may abuse notation by writing $J_p = J$ and $J_{AP} = J$ whenever no confusion arises.

PROPOSITION A-4. *In a preference learning system with linear costs, we have:*
(i) The optimal learning level $J^* = J_{P0}$, *if and only if one of the following three conditions is satisfied: (1)* $\alpha \le \frac{c_p}{w}$, *(2)* $\frac{c_p}{w} < \alpha \le \frac{c_p}{w - t(1-J_{P0})}$ *and* $K_P > K_{P1}(\alpha, c_p)$, *(3)* $\alpha > \frac{c_p}{w - t(1-J_{P0})}$ *and* $K_P > K_{P2}(\alpha)$. *The corresponding optimal profits are* $\pi_P^* = -K_{P0}$, $\pi_P^* = \frac{\alpha d_H}{2t(1-J_{P0})}(w - \frac{c_p}{\alpha})^2 - K_{P0}$ *and* $\pi_P^* = \{\alpha[w - \frac{1}{2}t(1-J_{P0})] - c_p\}d_H - K_{P0}$.
(ii) Otherwise, the optimal learning level $J^* = 1$, *and* $\pi_P^* = (\alpha w - c_p)d_H - K_{P0} - K_P(1 - J_{P0})$.

Proof of Proposition A-4. From Proposition A-1, we have

$$\pi_P(J)=\begin{cases}\pi_P^1(J)\equiv -K_{P0}-K_P(J-J_{P0}) & \text{if } w<\frac{c_p}{\alpha}\\ \pi_P^2(J)\equiv \frac{\alpha d_H}{2t(1-J)}(w-\frac{c_p}{\alpha})^2-K_{P0}-K_P(J-J_{P0}) & \text{if } \frac{c_p}{\alpha}\le w\le \frac{c_p}{\alpha}+t(1-J)\\ \pi_P^3(J)\equiv \{\alpha[w-\frac{1}{2}t(1-J)]-c_p\}d_H-K_{P0}-K_P(J-J_{P0}) & \text{if } w>\frac{c_p}{\alpha}+t(1-J)\end{cases}.$$

Because $\frac{d\pi_P^1(J)}{dJ}=-K_P$, $\frac{d^2\pi_P^1(J)}{dJ^2}=0$, $\frac{d\pi_P^2(J)}{dJ}=\frac{\alpha d_H}{2t}\frac{(w-\frac{c_p}{\alpha})^2}{(1-J)^2}-K_P$, $\frac{d^2\pi_P^2(J)}{dJ^2}=\frac{\alpha d_H}{t}\frac{(w-\frac{c_p}{\alpha})^2}{(1-J)^3}>0$, $\frac{d\pi_P^3(J)}{dJ}=\frac{1}{2}\alpha t d_H-K_P$, and $\frac{d^2\pi_P^3(J)}{dJ^2}=0$, the optimal learning level J^* is either J_{P0} or 1 due to $\frac{d^2\pi_P^i(J)}{dJ^2}\ge 0$ $(i=1,2,3)$.

Note that $w\le \frac{c_p}{\alpha}+t(1-J)$ is equivalent to $J\le 1-\frac{\alpha w-c_p}{\alpha t}$ and that $1-\frac{\alpha w-c_p}{\alpha t}\le J_{P0}$ is equivalent to $\alpha\ge \frac{c_p}{w-t(1-J_{P0})}$. We have the following:

If $\alpha\le\frac{c_p}{w}$, then $\pi_P(J)=\pi_P^1(J)$ for $J_{P0}\le J\le 1$.

If $\frac{c_p}{w}<\alpha\le\frac{c_p}{w-t(1-J_{P0})}$, then

$$\pi_P(J)=\begin{cases}\pi_P^2(J) & \text{if } J_{P0}\le J\le 1-\frac{\alpha w-c_p}{\alpha t}\\ \pi_P^3(J) & \text{if } 1-\frac{\alpha w-c_p}{\alpha t}<J\le 1\end{cases}.$$

If $\alpha>\frac{c_p}{w-t(1-J_{P0})}$, then $\pi_P(J)=\pi_P^3(J)$ for $J_{P0}\le J\le 1$.

By comparing $\pi_P(J_{P0})$ and $\pi_P(1)$ in $\alpha\le\frac{c_p}{w}$, $\frac{c_p}{w}<\alpha\le\frac{c_p}{w-t(1-J_{P0})}$ and $\alpha>\frac{c_p}{w-t(1-J_{P0})}$, we have the desired results. □

Next, we present the result for a Bespoke system with linear costs. Let $K_{AP1}(\alpha,c_p)\equiv \frac{\alpha d_H(w-c_p)}{1-J_{AP0}}[1-\frac{w-c_p}{2t(1-J_{AP0})}]$ and $K_{AP2}(\alpha)\equiv\frac{1}{2}\alpha d_H t$.

Proposition A-5. *In a Bespoke system with linear costs, we have:*

(i) The optimal learning level $J^*=J_{AP0}$, if and only if one of the following two conditions is satisfied: (1) $c_p\ge w-(1-J_{AP0})t$ and $K_{AP}>K_{AP1}(\alpha,c_p)$, (2) $c_p<w-(1-J_{AP0})t$ and $K_{AP}>K_{AP2}(\alpha)$. The corresponding optimal profits are $\pi_{AP}^*=\frac{\alpha d_H}{2t(1-J_{AP0})}(w-c_p)^2-K_{AP0}$ and $\pi_{AP}^*=\alpha d_H[w-\frac{1}{2}t(1-J_{AP0})-c_p]-K_{AP0}$.

(ii) Otherwise, the optimal learning level $J^*=1$, and $\pi_{AP}^*=\alpha d_H(w-c_p)-K_{AP0}-K_{AP}(1-J_{AP0})$.

Proof of Proposition A-5. From Proposition A-2, we have

$$\pi_{AP}(J)=\begin{cases}\pi_{AP}^1(J)\equiv\frac{\alpha d_H}{2t(1-J)}(w-c_p)^2-K_{AP0}-K_{AP}(J-J_{AP0}) & \text{if } w\le c_p+t(1-J)\\ \pi_{AP}^2(J)\equiv\alpha d_H[w-\frac{1}{2}t(1-J)-c_p]-K_{AP0}-K_{AP}(J-J_{AP0}) & \text{if } w>c_p+t(1-J)\end{cases}.$$

Since $\frac{d\pi_{AP}^1(J)}{dJ}=\frac{\alpha d_H}{2t}\frac{(w-c_p)^2}{(1-J)^2}-K_{AP}$, $\frac{d^2\pi_{AP}^1(J)}{dJ^2}=\frac{\alpha d_H}{t}\frac{(w-c_p)^2}{(1-J)^3}>0$, $\frac{d\pi_{AP}^2(J)}{dJ}=\frac{1}{2}\alpha t d_H-K_{AP}$, and $\frac{d^2\pi_{AP}^2(J)}{dJ^2}=0$, the optimal learning level J^* is either J_{AP0} or 1 due to $\frac{d^2\pi_{AP}^i(J)}{dJ^2}\ge 0$ $(i=1,2)$.

Note that $w\le c_p+t(1-J)$ is equivalent to $J\le 1-\frac{w-c_p}{t}$ and that $J_{AP0}\le 1-\frac{w-c_p}{t}$ is equivalent to $c_p\ge w-(1-J_{AP0})t$. We have the following:

If $w \leq c_p + t(1 - J_{AP0})$, then

$$\pi_{AP}(J) = \begin{cases} \pi_{AP}^1(J) & \text{if } J_{AP0} \leq J \leq 1 - \frac{w-c_p}{t} \\ \pi_{AP}^2(J) & \text{if } 1 - \frac{w-c_p}{t} < J \leq 1 \end{cases}.$$

If $w > c_p + t(1 - J_{AP0})$, then $\pi_{AP}(J) = \pi_{AP}^2(J)$ for $J_{AP0} \leq J \leq 1$.

By comparing $\pi_{AP}(J_{AP0})$ and $\pi_{AP}(1)$ in $w \leq c_p + t(1 - J_{AP0})$ and $w > c_p + t(1 - J_{AP0})$, we have the desired result. □

Propositions A-4-A-5 show that the optimal preference learning accuracy levels in both preference and Bespoke systems tend to be high when the investment costs K_P and K_{AP} are small. We also observe that the firm tends to choose a high preference learning accuracy level when the expected demand is high (i.e., a large α and/or d_H) or when c_p is small. This is because a high expected demand implies a relatively low average investment cost per-unit product. A small c_p means a high margin and thus more incentive to learn the preference. Our numerical study also shows that when $K_P = K_{AP}$ and $J_{P0} = J_{AP0}$, the firm always chooses a higher preference learning accuracy level in the Bespoke system compared to the preference learning system. A higher J means a higher investment cost. The firm's learning cost may not pay off in the preference learning system when the realized demand is less than its production quantity, while the firm never faces this scenario in the Bespoke system because it can exactly match supply and demand.

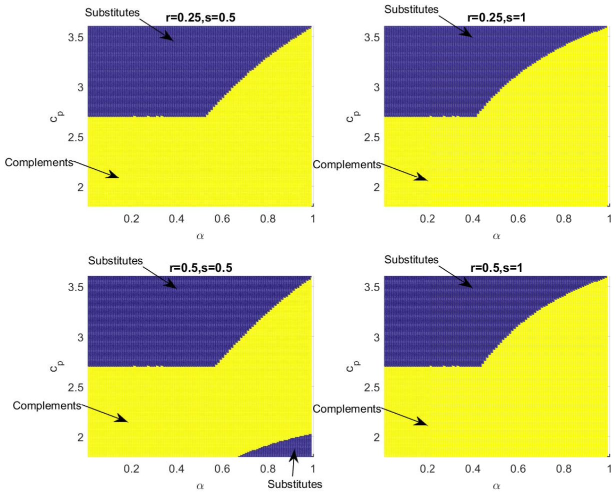
Based on the results for all the four systems, we are ready to investigate the interrelationship between demand learning and preference learning. We are able to provide all the necessary and sufficient conditions for demand learning and preference learning being complements. For brevity, they are not presented here but available from the authors.

Online Supplement 2.3. Analysis for §5.4: Impact of Salvage Value

In this section, we provide a representative numerical study to demonstrate the impact of salvage value s in the context of strategic customers and bargain-hunting consumers.

From our numerical analysis, we observe that as the salvage value s increases, demand learning and preference learning are more likely to be complementary regardless how strategic customers are; see Figure A-1 for representative examples. Since there is no leftover inventory in both demand learning and Bespoke systems (i.e., their profits are invariant with respect to s), this observation implies that the traditional system benefits more from high salvage value compared to the preference learning system. The explanation is as follows. With a higher salvage value, the firm tends to order a higher quantity in both traditional and preference learning systems. In the preference learning system, the firm has reached the highest quantity d_H (when c_p is small) and the highest price w it can charge even with zero salvage value, while it does not in the traditional system. This implies that the firm has more flexibility to adjust the price and order quantity to better leverage the benefit from a higher salvage value in the traditional system. Furthermore, with the presence of strategic behavior in the traditional system, a higher price in period 2 resulting from a higher salvage value reduces customers' waiting incentives and thus increases the firm's profit.

Figure A-1 **The Impact of Salvage Value** ($d_H = 60, d_L = 10, w = 3.6, t = 1.8, c = 1.8$)



Online Supplement 2.4. Analysis for §5.5: Correlation Between Demand and Preference Learnings

Proposition A-6 below summarizes the result for the demand learning system in which consumer preference is also imperfectly learned. We follow the framework in Section 5.2 to model the imperfect preference learning. Without losing managerial insights, we normalize d_L to zero for analytical tractability in this section.

Proposition A-6. *In a perfect demand learning system with imperfect preference learning of accuracy* J*:*

(i) If $w \geq c + t(1-J)$*, the firm's optimal expected profit* $\pi_A^* = \alpha d_H[w - \frac{1}{2}t(1-J) - c]$.

(ii) If $w < c + t(1-J)$*, the firm's optimal expected profit* $\pi_A^* = \frac{\alpha d_H}{2t(1-J)}(w-c)^2$.

Proof of Proposition A-6. In this system, the firm can observe the demand realization D before making its quantity decision. Hence, the firm can perfectly match supply with demand. Suppose $D = d_H$, then the number of consumers who will purchase the product is

$$D_A(d_H) = \begin{cases} d_H, & \text{if } p \in [0, w - \frac{1}{2}t_p] \\ \frac{2(w-p)d_H}{t_p}, & \text{if } p \in [w - \frac{1}{2}t_p, w] \\ 0, & \text{if } p \in [w, +\infty). \end{cases}$$

We then discuss under each of the three price ranges, what the optimal price should be. If $p \in [0, w - \frac{1}{2}t_p]$, the firm profit is $\pi_{d_H}(p,q) = (p-c)d_H$. The optimal price $p_{d_H}^* = w - \frac{1}{2}t_p$ and optimal quantity $q_{d_H}^* = d_H$ if $w - \frac{1}{2}t_p \geq c$. Hence, $\pi_{d_H}^* = (w - \frac{1}{2}t_p - c)d_H$ if $w \geq \frac{1}{2}t_p + c$.

If $p \in [w - \frac{1}{2}t_p, w]$, then $q_{d_H}^* = \frac{2(w-p)d_H}{t_p}$. The firm profit $\pi_{d_H}(p,q) = \frac{2(p-c)(w-p)d_H}{t_p}$. First-order condition with respect to price p yields $w - p - (p - c) = 0$. Hence, $p^* = \frac{w+c}{2}$. For it being optimal, we require $p^* = \frac{w+c}{2} \in [w - \frac{1}{2}t_p, w]$, which is simplified to $w \in [c, c + t_p]$. Hence, under this condition, we have $p_{d_H}^* = \frac{w+c}{2}$, $q_{d_H}^* = \frac{d_H(w-c)}{t_p}$, and the optimal profit $\pi_{d_H}^* = \frac{d_H}{2t_p}(w-c)^2$. If this condition does not hold, i.e., $w > c + t_p$, we have $p^*_{d_H} = w - \frac{1}{2}t_p$, $q^*_{d_H} = d_H$, and the optimal profit $\pi^*_{d_H} = (w - \frac{1}{2}t_p - c)d_H$. The optimal expected profit is $\pi^*_A = \alpha\pi^*_{d_H}$. Hence, we obtain

$$\pi^*_A = \begin{cases} \alpha d_H(w - \frac{1}{2}t_p - c), & \text{if } w \geq c + t_p \\ \frac{\alpha d_H}{2t_p}(w-c)^2, & \text{if } w \leq c + t_p. \end{cases}$$

Since $t_p = t(1-J)$, we can express the profit function in terms of J. □

It is expected that the firm's profit increases as the preference learning accuracy J increases.[1] Proposition A-7 below summarizes the result for the preference learning system in which demand is also imperfectly learned. We follow the framework in Section 5.1 to model the imperfect demand learning. For expositional convenience, we introduce the notations: $I_A(\alpha, c_p) \equiv 1 - \frac{w-c_p}{(1-\alpha)w}$ and $I_B(\alpha, c_p) \equiv 1 - \frac{c_p}{\alpha w}$.

Proposition A-7. *In a perfect preference learning system with imperfect demand learning of accuracy I:*
(i) If $\alpha \in [0, \frac{c_p}{w}]$ and $I \in [I_A(\alpha, c_p), 1]$, the firm's optimal expected profit $\pi^*_p = \alpha d_H\{w[1-(1-I)(1-\alpha)] - c_p\}$.
(ii) If $\alpha \in [0, \frac{c_p}{w}]$ and $I \in [0, I_A(\alpha, c_p))$, the firm's optimal expected profit $\pi^*_p = 0$.
(i) If $\alpha \in [\frac{c_p}{w}, 1]$ and $I \in [I_B(\alpha, c_p), 1]$, the firm's optimal expected profit $\pi^*_p = \alpha d_H\{w[1-(1-I)(1-\alpha)] - c_p\}$.
(i) If $\alpha \in [\frac{c_p}{w}, 1]$ and $I \in [0, I_B(\alpha, c_p))$, the firm's optimal expected profit $\pi^*_p = (\alpha w - c_p)d_H$.

Proof of Proposition A-7. With perfect preference learning, the firm also sets $p = w$ and all customers would like to buy at this price. The firm's expected profit can be expressed as $\pi_p(q) = w\mathbf{E}[\min(D, q)] - c_p q$.

Suppose that the firm observes demand signal $s = d_H$, then its updated demand $D(s = d_H) = d_H$ with probability $\alpha_H(I) \equiv 1 - (1-I)(1-\alpha)$ and $D(s = d_H) = d_L = 0$ with probability $1 - \alpha_H(I) = (1-I)(1-\alpha)$. So the profit $\pi_{s=d_H} = (w\alpha_H(I) - c_p)q$. When $I \geq I_A(\alpha, c_p)$ (i.e., $w\alpha_H(I) - c_p \geq 0$), $q^*_{s=d_H} = d_H$ and $\pi^*_{s=d_H} = (w\alpha_H(I) - c_p)d_H$; when $I < I_A(\alpha, c_p)$, $q^*_{s=d_H} = 0$ and $\pi^*_{s=d_H} = 0$.

Suppose that the firm observes demand signal $s = d_L$, then its updated demand $D(s = d_L) = d_H$ with probability $\alpha_L(I) \equiv (1-I)\alpha$ and $D(s = d_L) = d_L = 0$ with probability $1 - \alpha_L(I) = 1 - (1-I)\alpha$. So the profit $\pi_{s=d_L} = (w\alpha_L(I) - c_p)q$. When $I \leq I_B(\alpha, c_p)$ (i.e., $w\alpha_L(I) - c_p \geq 0$), $q^*_{s=d_L} = d_H$ and $\pi^*_{s=d_H} = (w\alpha_L(I) - c_p)d_H$; when $I > I_B(\alpha, c_p)$, $q^*_{s=d_L} = 0$ and $\pi^*_{s=d_L} = 0$.

To write the firm's expected profit before observing the demand signal s, we need to compare $I_A(\alpha, c_p) \equiv 1 - \frac{w-c_p}{(1-\alpha)w}$ and $I_B(\alpha, c_p) \equiv 1 - \frac{c_p}{\alpha w}$. It turns out $I_A(\alpha, c_p) \geq I_B(\alpha, c_p)$ is equivalent to $\alpha \leq c_p/w$.

Note that $\pi^*_p(I, a) = \alpha\pi^*_{s=d_H} + (1-\alpha)\pi^*_{s=d_L}$. Then, we can compute this expected profit based on different combinations of the conditions on α and I. For example, if $\alpha \in [0, \frac{c_p}{w}]$, we have $I_A(\alpha, c_p) \geq 0 \geq I_B(\alpha, c_p)$. If we further assume $I \in [I_A(\alpha, c_p), 1]$, then $\pi^*_p(I, \alpha) = \alpha d_H\{w[1-(1-I)(1-\alpha)] - c_p\}$. Similarly, we obtain the profit functions in other cases. □

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[1] One may argue that although the firm imperfectly learns the consumer preference without spending specific efforts in the demand learning system, the production cost might still increase above c. We keep the production cost unchanged as c in this section in order to isolate the impact of the learning-mechanism correlation.

Online Supplement 3. Responsive Pricing

In the basic model, we assumed that the firm determines the price before uncertain demand is realized or learned. This assumption is consistent with the extensive operations literature (see, e.g., Petruzzi and Dada 1999 and references therein). This fits the scenarios where the firm observes the demand after selling the product, or the firm needs to use the price for advertisement before observing the demand, as it is typically the case for many products.

However, in some settings, the firm may be able to set the price *after* the demand is realized or learned. In this section, we investigate how responsive/contingent pricing may affect our main result in the basic model. We denote q_0^* and π_0^* as the firm's optimal production quantity and optimal expected profit respectively. For the traditional system, the following proposition characterizes the equilibrium outcome.

PROPOSITION A-8. *In a traditional system with responsive pricing:*

(i) When $w \leq t$, *we have:*

(i.a) For $w \leq \frac{1}{\alpha(1-\frac{d_L}{d_H})}c$, $q_0^* = \frac{w-c}{t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$ *and* $\pi_0^* = \frac{(w-c)^2}{2t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$;

(i.b) For $w > \frac{1}{\alpha(1-\frac{d_L}{d_H})}c$, $q_0^* = \frac{d_H(w-\frac{c}{\alpha})}{t}$ *and* $\pi_0^* = \alpha\frac{d_H}{2t}(w-\frac{c}{\alpha})^2 + (1-\alpha)\frac{d_L}{2t}w^2$;

(i.c) For any given demand realization D *and production quantity* q, *the optimal price* $p(q,D) = \frac{w}{2}$ *if* $q \geq \frac{Dw}{t}$ *and* $p(q,D) = w - \frac{tq}{2D}$ *otherwise.*

(ii) When $w > t$, *let* $w_1 \equiv c + td_L(\frac{\alpha}{d_H} + \frac{1-\alpha}{d_L})$, $w_2 \equiv \frac{c}{\alpha} + \frac{d_L}{d_H}t$, *and we have:*

(ii.a) For $w \leq \min(w_1,w_2)$, $q_0^* = \frac{w-c}{t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$ *and* $\pi_0^* = \frac{(w-c)^2}{2t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$;

(ii.b) If $w_1 < w_2$, *then for* $w_1 < w < w_2$, $q_0^* = d_L$ *and* $\pi_0^* = (w-c)d_L - \frac{t}{2}d_L^2(\frac{\alpha}{d_H} + \frac{1-\alpha}{d_L})$;

(ii.c) If $w_1 > w_2$, *then for* $w_2 < w < w_1$, $\pi_0^* = \max(\frac{(w-c)^2}{2t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}, \frac{\alpha d_H}{2t}(w-\frac{c}{\alpha})^2 + (1-\alpha)(w-\frac{1}{2}t)d_L)$, *and the corresponding* $q_0^* = \frac{w-c}{t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$ *or* $q_0^* = \frac{d_H}{t}(w-\frac{c}{\alpha})$;

(ii.d) For $w > \max(w_1,w_2)$, $q_0^* = \frac{d_H}{t}(w-\frac{c}{\alpha})$ *and* $\pi_0^* = \frac{\alpha d_H}{2t}(w-\frac{c}{\alpha})^2 + (1-\alpha)(w-\frac{1}{2}t)d_L$;

(ii.e) For any given demand realization D *and production quantity* q, *the optimal price* $p(q,D) = w - \frac{1}{2}t$ *if* $q \geq D$, *and* $p(q,D) = w - \frac{tq}{2D}$ *otherwise.*

Proof of Proposition A-8. We work backwards: first solve for the optimal price $p(q,D)$ given production quantity q and demand realization D, and then solve for q_0^*. Clearly, for the optimal price p we have $w - \frac{1}{2}t \leq p \leq w$, and the demand is $2\frac{w-p}{t}D$. The firm profit can be expressed as $\pi(p;q,D) = p\min\{q, 2\frac{w-p}{t}D\} - cq$.

When $p \leq w - \frac{tq}{2D}$, we have $q \leq 2\frac{w-p}{t}D$ and $\pi(p;q,D) = (p-c)q$. In this case, $\pi(p;q,D)$ is maximized at $p(q,D) = w - \frac{tq}{2D}$ and the resulting profit is $\pi(q,D) = (w - \frac{tq}{2D} - c)q$. When $p > w - \frac{tq}{2D}$, we have $q > 2\frac{w-p}{t}D$ and $\pi(p;q,D) = \frac{2D}{t}p(w-p) - cq$. In this case, $\pi(p;q,D)$ is maximized at $p(q,D) = \frac{w}{2}$ and the resulting profit is $\pi(q,D) = \frac{Dw^2}{2t} - cq$. Note that $w - \frac{tq}{2D} \geq w - \frac{1}{2}t$ is equivalent to $D \geq q$, $\frac{w}{2} \geq w - \frac{1}{2}t$ is equivalent to $w \leq t$, $\frac{w}{2} \geq w - \frac{tq}{2D}$ is equivalent to $D \leq \frac{tq}{w}$, $\frac{tq}{w} > q$ is equivalent to $t > w$, $w - \frac{tq}{2D} < w$ and $\frac{w}{2} < w$

Next, we discuss $p(q,D)$ in different regions. (1) When $t > w$, we have $\frac{tq}{w} > q$. (1.a) For $D < q$, we have $w - \frac{tq}{2D} < w - \frac{1}{2}t < \frac{w}{2} < w$. Because $w - \frac{tq}{2D} < w - \frac{1}{2}t$, we know that for $w - \frac{1}{2}t \leq p \leq w$, we have $p > w - \frac{tq}{2D}$ and thus $\pi(p;q,D) = \frac{2D}{t}p(w-p) - cq$. The optimal price $p(q,D) = \frac{w}{2}$ and the resulting profit $\pi(q,D) = \frac{Dw^2}{2t} - cq$. (1.b) For $q \leq D \leq \frac{tq}{w}$, we have $w - \frac{1}{2}t < w - \frac{tq}{2D} < \frac{w}{2} < w$. Combining the optimal price $p(q,D)$ for $w - \frac{1}{2}t \leq p \leq$

$w-\frac{tq}{2D}$ and for $w-\frac{tq}{2D} \leq p \leq w$, we have that for the whole range $w-\frac{1}{2}t \leq p \leq w$, the optimal price $p(q,D)=\frac{w}{2}$ and the resulting profit $\pi(q,D)=\frac{Dw^2}{2t}-cq$. (1.c) For $D>\frac{tq}{w}$, we have $w-\frac{1}{2}t<\frac{w}{2}<w-\frac{tq}{2D}<w$. Combining the optimal price $p(q,D)$ for $w-\frac{1}{2}t \leq p \leq w-\frac{tq}{2D}$ and for $w-\frac{tq}{2D} \leq p \leq w$, we have that for the whole range for $w-\frac{1}{2}t \leq p \leq w$, the optimal price $p(q,D)=w-\frac{tq}{2D}$ and the resulting profit $\pi(q,D)=(w-\frac{tq}{2D}-c)q$.

Combining results (1.a)-(1.c), we have the claim (i.c) in Proposition A-8 as

$$p(q,D)=\begin{cases} w-\frac{tq}{2D} & \text{with } \pi(q,D)=(w-\frac{tq}{2D}-c)q \quad \text{if } q \leq \frac{Dw}{t}, \\ \frac{w}{2} & \text{with } \pi(q,D)=\frac{Dw^2}{2t}-cq \qquad \text{if } q>\frac{Dw}{t}. \end{cases} \tag{A-10}$$

Finally, we solve for the optimal quantity q_0^*. Note D is either d_H with probability α, or d_L with probability $1-\alpha$. We discuss three possible regions: $q \leq \frac{d_L w}{t}$, $\frac{d_L w}{t}<q \leq \frac{d_H w}{t}$ and $q>\frac{d_H w}{t}$. Based on equation (A-10), we compute $\pi(q)=\mathbf{E}[\pi(q,D)]$ by taking the expectation of $\pi(q,D)$ over D:

$$\pi(q)=\begin{cases} \pi_1(q)=(w-c)q-\frac{tq^2}{2}(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L}) & \text{if } q \leq \frac{d_L w}{t}, \\ \pi_2(q)=\alpha(w-\frac{tq}{2d_H})q+(1-\alpha)\frac{d_L w^2}{2t}-cq & \text{if } \frac{d_L w}{t}<q \leq \frac{d_H w}{t}, \\ \pi_3(q)=\frac{w^2}{2t}\mu_D-cq & \text{if } q>\frac{d_H w}{t}. \end{cases}$$

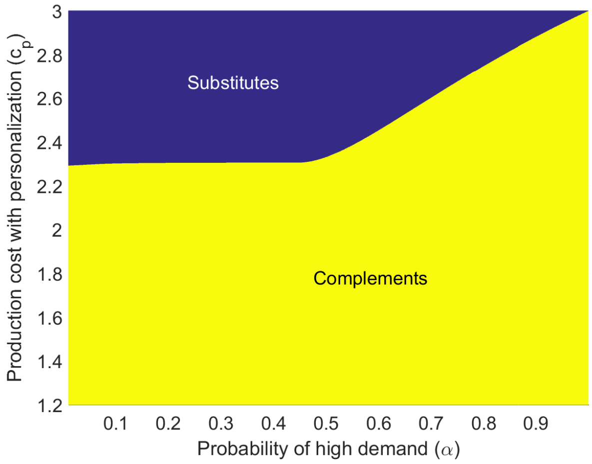
Notice that $\pi_1(q)$ is concave and maximized at $q=\frac{w-c}{t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$, $\pi_2(q)$ is concave and maximized at $q=\frac{d_H(w-\frac{c}{\alpha})}{t}$, and $\pi_3(q)$ decreases in q. Because $\frac{w-c}{t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})} \leq \frac{d_L w}{t}$ is equivalent to $w \leq \frac{1}{\alpha(1-\frac{d_L}{d_H})}c$, $\frac{d_H(w-\frac{c}{\alpha})}{t}>\frac{d_L w}{t}$ is equivalent to $w>\frac{1}{\alpha(1-\frac{d_L}{d_H})}c$, and $\frac{d_H(w-\frac{c}{\alpha})}{t} \leq \frac{d_H w}{t}$ is always true, we have the following result. If $w \leq \frac{1}{\alpha(1-\frac{d_L}{d_H})}c$, then $q_0^*=\frac{w-c}{t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$. By substituting $q_0^*=\frac{w-c}{t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$ into $\pi_1(q)$, we have $\pi_0^*=\frac{(w-c)^2}{2t(\frac{\alpha}{d_H}+\frac{1-\alpha}{d_L})}$. If $w>\frac{1}{\alpha(1-\frac{d_L}{d_H})}c$, then $q_0^*=\frac{d_H(w-\frac{c}{\alpha})}{t}$. By substituting $q_0^*=\frac{d_H(w-\frac{c}{\alpha})}{t}$ into $\pi_2(q)$, we have $\pi_0^*=\alpha\frac{d_H}{2t}(w-\frac{c}{\alpha})^2+(1-\alpha)\frac{d_L}{2t}w^2$. We have finished the proof for claim (i) $w \leq t$ in Proposition A-8. In a similar way, we can derive the result for claim (ii) $w>t$. □

With responsive pricing, the profit in traditional system should be higher than that in the basic model without responsive pricing. The price influences the number of customers who want to buy the product. With responsive pricing, the firm can set the price based on the realized demand to better match the number of customers who want to buy the product with the production quantity and thus improves its profit. Other than the traditional system, the optimal strategy and profit do not change in the other three systems: Regardless of the demand realization, the firm always sets $p=w$ in preference learning and Bespoke systems, and always sets $p=\frac{w+c}{2}$ in demand learning system. The equilibrium result for the demand learning system as stated in Proposition A-9 below is the same as that in the basic model (but the proof is slightly different due to responsive pricing).

PROPOSITION A-9. *In a demand learning system with responsive pricing: The firm's optimal price* $p_A^*=\frac{w+c}{2}$*, optimal production quantity* $q_A^*(D)=\frac{w-c}{t}D$ *for a demand realization* D*, and optimal profit* $\pi_A^*=\frac{\mu_D}{2t}(w-c)^2$.

Proof of Proposition A-9. The firm determines its price p and production quantity q after learning the realization of D. Obviously, $w-\frac{1}{2}t \leq p \leq w$, and in this case the number of customers who want to buy the

Figure A-2 **The Interrelationship of Demand Learning and Preference Learning with Responsive Pricing ($d_H = 60, d_L = 10, w = 3.0, t = 1.9, c = 1.2$)**



product is $2\frac{w-p}{t}D$. The profit can be expressed as $\pi(p,q;D) = p\min\{q, 2\frac{w-p}{t}D\} - cq$. It is easy to show that the firm always sets $q = 2\frac{w-p}{t}D$ for any p and D. Then, the profit can be expressed as

$$\begin{aligned}\pi(p;D) &= p\min\{2\frac{w-p}{t}D, 2\frac{w-p}{t}D\} - c * 2\frac{w-p}{t}D\\ &= 2\frac{D}{t}(w-p)(p-c)\end{aligned}$$

which is maximized at $p = \frac{w+c}{2}$, and the corresponding optimal quantity is $q_A^*(D) = 2\frac{w-\frac{w+c}{2}}{t}D = \frac{w-c}{t}D$. The optimal profit given D is $\frac{D}{2t}(w-c)^2$ and the expected optimal profit is $\frac{\mu_D}{2t}(w-c)^2$. $\square$

We expect that demand learning and preference learning are more likely to be complementary. Numerical results confirm our intuition, as shown in Figure A-2 where we use the same set of parameters as in Figure 1 for the basic model. Furthermore, consistent with our basic model, demand learning and preference learning are complementary when the cost of customization is low and the probability of having high demand is large.

Online Supplement 4. Connection to Product Line Design

The model setup follows the basic model in the main body of the paper. Suppose customers/consumers are uniformly distributed on a Hotelling line $[-1/2, 1/2]$. There is a monopolist firm selling n horizontally differentiated products, which are positioned at location θ_i, $i \in \{1, 2, ..., n\}$ on the Hotelling line, with corresponding prices p_i, $i \in \{1, 2, ..., n\}$. Without loss of generality, we assume that $\theta_1 \le \theta_2 \le ... \le \theta_n$. Consumers get utility w from the product, and incur the travelling cost and pay the price. The per unit travelling cost is denoted as t. If a consumer at location θ buys product i, her net utility is $w - |\theta - \theta_i|t - p_i$. A consumer buys the product that gives her the highest non-negative net utility. Consumers may substitute for another product (only if the product also gives her a non-negative net utility) if her favorable product is out of stock. The firm's per unit production cost is c.

For any given number of products n, the firm needs to determine both prices and stocking levels for all the products.

Lemma A-1. *In any optimal strategy, all products are sold at the same price, no consumer gets strictly positive utility from multiple products, and consumers on the boundaries (-1/2 and 1/2) get non-positive utility from purchasing.*

Proof of Lemma A-1. Define the market coverage of a product as the consumers who get non-negative utilities from purchasing this product. Clearly, the market coverage of any product is contiguous in that if customers at both locations buy product i, then any customer located between these two locations buys product i.

First, there is no product coverage overlapping in any optimal strategy. Otherwise, there are some customers who get positive utilities from multiple products. Then the firm can increase the product price or shift the product location to increase profit. To see this, suppose product i and j have overlapping coverage, then if one product (without loss of generality, assuming it is i) only overlaps on one side (with the other one), then we can increase p_i and shift the position of product i away from product j to increase the profit. If both products has coverage overlapping on both sides, then we can repeat the argument and show that all products have overlapping on both sides, and customers at both ends get strictly positive utility from purchasing. Then, we can increase all prices by the same amount without changing demands for all products, which will increase the total profit. Therefore, in any optimal strategy, there should be no product coverage overlapping.

Second, at the optimal strategy, customers at both ends (-1/2 and 1/2) get non-positive utility from purchasing. Suppose not, then a customer at one end (without loss of generality, assuming location 0) at strictly positive utility from purchasing product 1 (product 1 is closest to location 0), then the firm can increase p_1 to $p_1 + t\epsilon$ (where ϵ is an sufficiently small amount) and increase θ_1 to $\theta_1 + \epsilon$. Then, the demands are the same and the profit is higher.

Given there is no coverage overlapping and customers at both ends get non-positive utility from purchasing, L_i (define as $L_i = 2(w - p_i)/t$) is the coverage of product i and DL_i is the demand for product i. Then, the inventory problem for product i is

$$\max_{q_i} \left(\alpha \min\left\{d_H L_i, q_i\right\} p_i + (1-\alpha)\min\left\{d_L L_i, q_i\right\} p_i - c q_i\right).$$

Solving this newsvendor problem we have the optimal inventory

$$q_i = \begin{cases} d_H L_i & if\, p_i \geq \frac{c}{\alpha} \\ d_L L_i & if\, p_i < \frac{c}{\alpha}. \end{cases}$$

Substituting q_i back, we have the optimal profit

$$\pi_i(p_i) = \begin{cases} \left(\alpha d_H + (1-\alpha) d_L\right) \frac{2(w-p_i)}{t} p_i - d_H \frac{2(w-p_i)}{t} c & if\, p_i \geq \frac{c}{\alpha} \\ d_L \frac{2(w-p_i)}{t}(p_i - c) & if\, c \leq p_i < \frac{c}{\alpha}. \end{cases}$$

Third, all products must have the same price. Suppose not, then there exist two adjacent products with different prices. That is, there exists a product i, such that $p_i \neq p_{i+1}$. Without loss of generality, assume $p_i > p_{i+1}$. Then $L_i < L_{i+1}$ and the total coverage is $L_i + L_{i+1} = 2(2w - p_i - p_{i+1})/t$. As long as we keep the total price $p_i + p_{i+1}$ the same, we can keep the same total coverage by the two products $L_i + L_{i+1}$ (we may need to shift the locations) and do not affect other products. We can verify that $\pi_i(p_i)$ is concave in the two domains $[c, \frac{c}{\alpha})$ and $[c/\alpha, \infty)$, so if $p_i > p_{i+1} \geq c/\alpha$ or $p_{i+1} < p_i < c/\alpha$, we can replace p_i and p_{i+1} with their average $(p_i + p_{i+1})/2$ and relocate the two products so that the total coverage remains the same and we get a higher profit.

If $p_i > c/\alpha$ and $p_{i+1} < c/\alpha$, then $\pi'(p_{i+1}) = \frac{2d_L}{t}(w + c - 2p_{i+1})$, and $\pi'(p_i) = \frac{2\mu_D}{t}(w - 2p_i) + \frac{2d_H}{t}c$. We show that $\pi'(p_{i+1}) = \pi'(p_i) \leq 0$ can never be the case if $p_i > c/\alpha$ and $p_{i+1} < c/\alpha$.

Solving $\pi'(p_{i+1}) = \pi'(p_i)$ gives us $p_{i+1} = \frac{d_H - d_L}{2d_L}(\alpha w + c) - \frac{\mu_D}{d_L}p_i$.

Simplifying $\pi'(p_i) \leq 0$, we have $p_i \geq \frac{\mu_D w + d_H c}{2\mu_D}$. Then $p_{i+1} = \frac{d_H - d_L}{2d_L}(\alpha w + c) - \frac{\mu_D}{d_L}p_i \leq \frac{d_H - d_L}{2d_L}(\alpha w + c) - \frac{\mu_D w + d_H c}{2d_L} = \frac{w-c}{2}$. Therefore, $\pi'(p_{i+1}) = \frac{2d_L}{t}(w + c - 2p_{i+1}) \geq \frac{2d_L}{t}(w + c - (w - c)) = \frac{4d_L c}{t} > 0$, which contradicts with $\pi'(p_{i+1}) \leq 0$. Hence, $\pi'(p_{i+1}) = \pi'(p_i) \leq 0$ cannot be the case given $p_i > c/\alpha$ and $p_{i+1} < c/\alpha$. Then, there remain two possibilities: $\pi'(p_{i+1}) \neq \pi'(p_i)$ or $\pi'(p_{i+1}) = \pi'(p_i) > 0$. We next consider these two cases one by one.

Case 1: $\pi'(p_{i+1}) \neq \pi'(p_i)$.

If $\pi'(p_{i+1}) > \pi'(p_i)$, we can increase p_{i+1} by ϵ and reduce p_i by ϵ to increase the profits from the two products without affecting other products. Similarly, if $\pi'(p_{i+1}) < \pi'(p_i)$, we can reduce p_{i+1} by ϵ and increase p_i by ϵ to increase the profits from the two products without affecting other products.

Case 2: $\pi'(p_{i+1}) = \pi'(p_i) > 0$.

We can then increase both p_i and p_{i+1} to increase the profits from the two products. With the higher prices, the coverage of the two products will be smaller, and thus won't affect other products.

Therefore, in any optimal strategy, it must be that all prices are equal. Then all products should have equal coverage with no overlapping (and thus equal demand), and their optimal inventory levels are the same. $\square$

Then we only need to decide one price for all products, and the total inventory level for all products. Because there should be no overlapping, each product covers at most $1/n$ on the Hotelling line. Therefore, solving $2(w - p_i)/t \leq 1/n$, the optimal price must satisfy $p_i \geq w - t/2n$. The next proposition shows that at the optimal strategy, offering n products is equivalent to reducing the travelling cost to t/n.

PROPOSITION A-10. *Given any fixed c and w, denote the total profit $f(p,n,t)$ as a function of price p, number of products n, per-unit traveling cost t under the optimal product positioning and inventory strategy. Then, we have $f(p,n,t) = f(p,1,\frac{t}{n})$.*

Proof of Proposition A-10. In the optimal strategy, all n products have the same price and the same stocking level. Denote the optimal total stocking level given price p as $q(p)$. Since all products should have the same stocking level, the optimal stocking level for each product is $q(p)/n$. The demand for each product is $2D(w-p)/t$. Then, the total expected profit from all products is

$$f(p,n,t) = nE\left[\min\left\{\frac{2D(w-p)}{t}, \frac{q(p)}{n}\right\}\right]p - cq(p)$$

$$= E\left[\min\left\{\frac{2D\,(w-p)}{t/n}, q\,(p)\right\}\right] p - cq\,(p)\,.$$

Substituting $n = 1$ and $t = t/n$ we have

$$f(p,1,\frac{t}{n}) = E\left[\min\left\{\frac{2D\,(w-p)}{t/n}, q\,(p)\right\}\right] p - cq\,(p)\,.$$

Clearly, $f(p,n,t) = f(p,1,\frac{t}{n})$. $\square$

Mapping the number of products n **to the level of customization** J**.** Proposition A-10 allows us to establish a one-to-one mapping of the number of products n in this section to the level of customization/personalization J in the main body of the paper.

When the firm offers n products, the reduced travelling cost is $t_p = t/n$. Then the level of customization is $J = 1 - t_p/t = 1 - 1/n$. In the extreme, when $n = 1$, the level of customization is zero; whereas as n goes to infinity, the level of customization approaches one.

Due to this equivalence, choosing the number of products n is the same as determining the level of customization J. Therefore, the product line design interpretation justifies our stylized model of using J for preference learning and personalization.