Supplementary Appendix for "Capacity Sharing between Competitors"

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Market-Level Capacity Shortage ($k \leq 1/2$):

We now analyze the basic model when total capacity is less than total demand. Note first that, the firms are *de facto* local monopoly and charging r is the unique equilibrium, if and only if the higher-priced firm's demand $\alpha + s$ is higher than its endowed capacity k, i.e., $k < \frac{(1-\alpha)^2}{2-3\alpha}$. This means that capacity sharing can be a relevant issue if and only if $k > \frac{(1-\alpha)^2}{2-3\alpha}$ such that the higher-priced firm's residual capacity, $k - (\alpha + s)$, is positive. That is, it is meaningful to focus on $\frac{(1-\alpha)^2}{2-3\alpha} < k \leq 1/2$.

Consider the benchmark when capacity sharing is infeasible. In any pure-strategy equilibrium, the firms must charge the same price p and, given $k \leq 1/2$, earn a profit $\pi(p) = pk$. No firm would like to charge a lower price. If a firm charges the optimal deviating price r, its profit would be equal to $r(\alpha + s)$. Therefore, when $\frac{(1-\alpha)^2}{2-3\alpha} < k \leq 1/2$, both firms charging any price p that is between $r(\alpha + s)/k$ and r can constitute an equilibrium outcome. However, this continuum of pure-strategy equilibria are unstable (i.e., not robust to perturbations). That is, if a firm deviates to a price that is slightly lower than the candidate equilibrium price p, it is optimal for the rival firm to match this deviating price, leading to divergence away from the focal point p. As a result, we will focus on the unique mixed-strategy equilibrium, which is characterized in Section 3.1. The firms' equilibrium profit continues to be $\Pi_o = r(\alpha + s)$.

Note that, given $k \leq 1/2$, the higher-priced firm's residual capacity cannot completely satisfy the demand from the rationed loyals any more. This means that the maximum amount of capacity to share is $\tilde{w} \equiv k - (\alpha + s)$, rather than w. We can follow Section 3.2 and Section 3.3 to solve the games with capacity sharing, by replacing w with $\tilde{w}$. Consider first the ex ante contracting scheme. We can similarly show that, given any capacity transfer price $\lambda < r$, the lower bound of price support under the second-stage mixed-strategy equilibrium is $L_a = \frac{r(\alpha+s)+2\lambda\tilde{w}}{k+\tilde{w}} > \lambda$, and the firms' equilibrium subgame profit is $\Pi(\lambda) = r(\alpha + s) + \lambda\tilde{w}$, which are both increasing in λ.[1] So in the first stage the firms would set the capacity transfer price $\lambda^* = r - \epsilon$, where $\epsilon > 0$ is sufficiently small, and earn an equilibrium profit $\Pi_a \approx rk$. Similar to the case when $1/2 < k < 1 - \alpha$, in

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[1] If $\lambda = r$, capacity sharing is ex post undesirable (for the lower-priced firm), and the equilibrium is the same as that under the benchmark, yielding a lower profit (i.e., $r(\alpha + s)$) than that by setting $\lambda^* = r - \epsilon$.

equilibrium capacity sharing happens with probability one. In addition, the prosect of capacity sharing can soften price competition and improve firm profitability, i.e., $L_a > L_o$ and $\Pi_a > \Pi_o$.

The solution for the ex post contracting scheme is similar to that in the basic model. In particular, capacity sharing always takes place under the mixed-strategy equilibrium of price competition, which yields the lower bound of price support $L_p = r\left(\frac{\alpha+s}{k+\tilde{w}/2}\right)^{2/3}$ and the firms' equilibrium profit $\Pi_p = r(\alpha+s)\left(\frac{\alpha+s}{k+\tilde{w}/2}\right)^{-1/3}$. It can be verified that $\Pi_p > \Pi_o$.

It is evident that $\Pi_a - \Pi_p > 0$ for any $\frac{(1-\alpha)^2}{2-3\alpha} < k \leq 1/2$. This generalizes the result in Proposition 5: due to the commitment effect of the capacity transfer price, ex ante contracting is more profitable than ex post contracting if the product market is not competitive.

Endogenous Capacity:

We consider the following setup for endogenous capacity. The demand structure is the same as that in the basic model. Before the firms engage in price competition, they simultaneously decide how much capacity to accumulate. To investigate the firms' strategic incentive for capacity choice, we assume that the cost of capacity is negligible. Let the firms' capacity be k_A and k_B, respectively. Define $s_i \equiv (1-2\alpha)(1-k_i/(1-\alpha))$ as the size of the rationed switchers for firm $i = A, B$. Note first that no firm can benefit from having a capacity higher than its maximum possible demand, $1-\alpha$. On the other hand, a firm can strictly benefit from increasing its capacity up to the level of its guaranteed demand. As a result, in equilibrium we must have $\alpha + s_B \leq k_A \leq 1-\alpha$ and $\alpha + s_A \leq k_B \leq 1-\alpha$, respectively.

The characterization of the price competition equilibrium is similar to that in Section 4.1. In particular, the equilibrium lower bound of price support for both firms is $L = \max\left\{\frac{r(\alpha+s_B)}{k_A}, \frac{r(\alpha+s_A)}{k_B}\right\}$, and the firms' equilibrium subgame profits are given by $\Pi_i = Lk_i$, $i = A, B$.

Now consider the firms' capacity choice. Let us look at firm A, taking firm B's capacity k_B as given. Note that $\Pi_A = \max\left\{\frac{r(\alpha+s_B)}{k_A}, \frac{r(\alpha+s_A)}{k_B}\right\} k_A$. When $k_B \neq \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$, the optimal capacity for firm A is $k_A = \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$; when $k_B = \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$, firm A is indifferent between any $k_A \in [\alpha + s_B, 1-\alpha]$. Therefore, in equilibrium we must have $k_A^* = \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$, $k_B^* = \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$, or both. The only stable equilibrium is $k_A^* = k_B^* = \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$. The other equilibria are not stable. Consider, for example, $k_A^* = \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$, $k_B^* \neq \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$. If firm A deviates a little bit from this prescribed equilibrium, it would be optimal for firm B to deviate by choosing $k_B = \min\left\{\frac{(1-\alpha)^2}{2-4\alpha}, 1-\alpha\right\}$. Note also that $\frac{(1-\alpha)^2}{2-4\alpha} > 1/2$ for any $\alpha > 0$. This means that the equilibrium capacity choices are such that there is market-level overcapacity, as assumed in the basic model.

Endogenous Timing in Capacity Transfer and the Rationed Buyers' Behavior:

In the basic model we assume that, if stockout occurs for the lower-priced firm, capacity sharing can happen only after the rationed switchers resort to the higher-priced firm for purchase. We now investigate the robustness of this assumption. Note first that this issue is irrelevant for the benchmark: when capacity sharing is infeasible, the rationed switchers have no choice but to buy from the higher-priced firm. So the equilibrium in Section 3.1 does not change.

Consider the following enriched setup when capacity sharing is possible. Consider the players' moves after the firms set their prices (i.e., p_A and p_B). We extend the subgame in Stage 2.3 and Stage 2.4 of Table 1, or that in Stage 1.3 and Stage 2 of Table 2, by replacing it with a multiple-period repeated game that endogenizes the timing of the players' behavior. In particular, in each of $N > 1$ periods, the rationed loyals choose whether to take the outside option of exiting the market, and the rationed switchers decide whether to purchase from the higher-priced firm (which always dominates the outside option). Similarly, the firms determine whether and how much to share their capacity. The capacity transfer price λ is committed under the ex ante contracting scheme, whereas the expected capacity transfer price under ex post contracting would be the same as that in the basic model (i.e., $p_l/2$). Recall that capacity sharing is voluntary between the firms. Any shared capacity will be sold to the rationed buyers at p_l, using the random allocation rule whenever applicable. If a rationed buyer neither makes any purchase nor exits the market, he/she will incur a per-period waiting cost of c and stay in the market in the next period. The (remaining) players will then repeat the same interaction in the next period until period N.

The lower-priced firm is willing to purchase from the higher-priced firm as much as to fulfill its residual demand, if and only if the transfer price λ is not higher than p_l. In contrast, given $p_h > p_l \geq \lambda$, the higher-priced firm desires to sell to the rationed switchers directly at its own price p_h, rather than at the inter-firm capacity sharing price λ. This means that the higher-priced firm has an incentive to delay the transfer of capacity to the lower-priced firm, in order to push the rationed switchers to give up waiting and to buy from it directly. However, the potential downside of delaying capacity transfer is that the rationed loyals may choose to exit the market.

We start with analyzing the rationed buyers' incentives of waiting. The surplus of purchasing from the higher-priced firm is $r - p_h$ for the rationed switchers, which is higher than the utility of the outside option for the rationed loyals (i.e., 0). Nevertheless, for *all* rationed buyers, the prospect of waiting is to gain the opportunity to purchase from the lower-priced firm, which would result in the surplus of $r - p_l$. Therefore, the rationed loyals' incentive to exit the market is always lower than that of the rationed switchers to buy from the higher-priced firm. In other words, the rationed loyals would always prefer to wait longer than the rationed switchers.

As a result, in any period $t = 1, \ldots, N-1$, the higher-priced firm would delay capacity sharing until the rationed switchers buy from it directly, without inducing the rationed loyals to exit the market. In the last period, because there is no future period, the firms will choose to share capacity to completely clear any residual demand. Then neither the rationed loyals exit nor the rationed switchers buy from the higher-priced firm in the last period. So the rationed buyers can indeed purchase from the lower-priced firm, if and only if they choose to wait until the last period.

We can then solve for this enriched game. When $p_l < \lambda$, capacity sharing would never happen, and so the rationed switchers would purchase from the higher-priced firm immediately in the first period. When $p_l \geq \lambda$, the rationed switchers would choose to purchase from the higher-priced firm immediately, if and only if the surplus, $r - p_h$, is higher than the expected surplus of waiting, $r - p_l - Nc$. Therefore, if $Nc > r$, for any $p_l \leq p_h \leq r$, in equilibrium the rationed switchers would always resort to the higher-priced firm for purchase before capacity sharing occurs, as presumed in the basic model. Conversely, if c is small enough, the rationed buyers would be sufficiently patient and thus the higher-priced firm would prefer to share capacity immediately in the first period.

Rationed Switchers Waiting for Capacity Sharing:

We now examine the robustness of the results in the basic model, when the waiting cost c is sufficiently small. The rationed switchers would stay with the lower-priced firm in case capacity sharing is profitable, where the amount of shared capacity would be $w + s$. The game can be similarly solved as in the basic model. Consider first price competition under the ex ant contracting scheme. When the lower bound of price support is not lower than the capacity transfer price (i.e., $L \geq \lambda$), analogous to Equation (2), the firms' expected profit of setting price p becomes:

$$\begin{aligned}\pi(p) &= \textstyle\int_L^p \left[p\alpha + \lambda(w+s)\right] f(p')dp' + \int_p^r \left[pk + (p-\lambda)(w+s)\right] f(p')dp' \\ &= pF(p)(2\alpha - 1) + 2\lambda(w+s)F(p) + p(1-\alpha) - \lambda(w+s).\end{aligned}$$

The equilibrium pricing strategy can be obtained by solving the differential equation $\pi'(p) = 0$ and using the boundary condition $F(r) = 1$. We can then impose the condition $F(L) = 0$ to solve for the lower bound of price support $L = \frac{r\alpha + 2\lambda(w+s)}{1-\alpha}$. It follows that the condition $L \geq \lambda$ is equivalent to $\lambda \leq \frac{r\alpha}{2k-1+\alpha}$. The firms' equilibrium (conditional) profit in this case is $\Pi(\lambda) = r\alpha + \lambda(w+s)$. It is straightforward that $\frac{d\Pi(\lambda)}{d\lambda} = w + s > 0$. Thus, $\Pi(\lambda)$ increases in λ for $\lambda \leq \frac{r\alpha}{2k-1+\alpha}$.

When $L \leq \lambda$, similar to Equation (3), the expected profit by setting the price $p \geq \lambda$ becomes:

$$\begin{aligned}\pi(p) &= \textstyle\int_L^\lambda p(\alpha+s)f_1(p')dp' + \int_\lambda^p \left[p\alpha + \lambda(w+s)\right] f_2(p')dp' + \int_p^r \left[pk + (p-\lambda)(w+s)\right] f_2(p')dp' \\ &= pF_2(p)(2\alpha-1) + 2\lambda(w+s)F_2(p) + p(1-\alpha) - \left[1 + F_1(\lambda)\right]\lambda(w+s) + F_1(\lambda)ps.\end{aligned}$$

Note that the expected profit for $p \leq \lambda$ remains the same as in Equation (4). We can then follow the same procedure to solve for the equilibrium. The equilibrium lower bound of price support is $L = \lambda + \frac{\lambda(k-\alpha-s)}{k}\frac{r\alpha-\lambda(2k-1+\alpha)}{rs+\lambda(2k-1-s)}$, which implies that $L \leq \lambda$ is equivalent to $\lambda \geq \frac{r\alpha}{2k-1+\alpha}$. The firms' equilibrium (conditional) profit in this case is $\Pi(\lambda) = Lk = r(\alpha + s)\frac{r(k-\alpha)-\lambda(1-\alpha-k)}{rs+\lambda(2k-1-s)}$. It can be verified that there exists a $\lambda^o < r$ such that $\Pi(\lambda)$ increases in λ for $\lambda < \lambda^o$ and decreases for $\lambda > \lambda^o$.

Therefore, similar to that in the basic model, overall the firms' profit $\Pi(\lambda)$ first increases and then decreases in λ, and the equilibrium capacity transfer price is given by $\lambda^* = \max\left\{\frac{r\alpha}{2k-1+\alpha}, \lambda^o\right\}$.

Consider then the ex post contracting scheme. Similar to Equation (5), the firms' expected profit of charging p is

$$\pi(p) \quad = \textstyle\int_L^p \left[p\alpha + p'(w+s)/2\right] f(p')dp' + \int_p^r (pk + p(w+s)/2) f(p')dp'.$$

By applying the equilibrium condition $\pi'(p) = 0$ and the boundary condition $F(r) = 1$, we can solve for the equilibrium pricing strategy. The lower bound of equilibrium price support is $L_p = r\left(\frac{\alpha}{k+(w+s)/2}\right)^{\frac{k-\alpha}{k+(w+s)/2-\alpha}}$ and the firms' equilibrium profit is $\Pi_p = r[k+(w+s)/2]\left(\frac{\alpha}{k+(w+s)/2}\right)^{\frac{k-\alpha}{k+(w+s)/2-\alpha}}$.

It can be checked that the equilibrium profit under the ex post contracting scheme can be lower than that under the benchmark without capacity sharing. In addition, the equilibrium profit under either contracting scheme is lower than that in the alternative scenario when the rationed switchers buy from the higher-priced firm before capacity sharing. Intuitively, knowing that the higher-priced firm would satisfy the demand of the rationed switchers at a lower margin (i.e., λ versus p_h), price competition will be more intense. This suggests that, if the firms can pre-determine (i.e., before price competition) the timing of capacity transfer, they would have an incentive to commit not to share capacity before the rationed switchers buy from the higher-priced firm.