Online Appendix for Castiglionesi, Feriozzi, and Lorenzoni (2017)

A. Integration and volatility with fixed y

With CRRA preferences, for a given level of liquidity y the interest rate as a function of the liquidity shock ω is

$$r = \frac{u'(c_1)}{u'(c_2)} - 1 = h(\omega)$$

where the function h is defined as follows

$$h(\omega) \equiv \max\{\left(\frac{1-\omega}{\omega}\right)^{-\gamma}\left(\frac{y}{R(1-y)}\right)^{-\gamma} - 1, 0\}$$

Lemma 6 *If $\gamma \geq 1$ the function h is convex in ω*

Proof. Notice that

$$\left(\frac{1-\omega}{\omega}\right)^{-\gamma} = \left(\frac{1}{1-\omega} - 1\right)^{\gamma}$$

is convex and so is the constant function. The maximum of two convex functions is convex. ■

What happens to the volatility of r if two regions integrate but keep the level of y unchanged? With integration every pair of realizations ω^A and ω^B in the two regions is replaced by

$$\Omega = \frac{1}{2}\omega^A + \frac{1}{2}\omega^B.$$

So, assuming a symmetric joint distribution of the shocks, going from integration to autarky is the same as having a mean preserving spread of the distribution of ω and the question we want to address in general is what happens to the moments of $h(\omega)$? The following proposition provides a sufficient condition on the curvature of h that ensures that the volatility of interest rates decreases with integration.

Proposition 12 *Suppose $\gamma \geq 1$ and the distribution of ω in autarky satisfies*

$$(h(\omega) - E[h(\Omega)])\, h''(\omega) + h'(\omega)^2 > 0$$

for all ω in the support of the autarky distribution. Then if the regions integrate and the choice of y is kept fixed, interest rate volatility decreases.

Proof. Define a parametric family of distributions for ω with CDF

$$F(\omega, \alpha) = \alpha F_{Aut}(\omega) + (1-\alpha) F_{Int}(\omega),$$

for $\alpha \in [0,1]$, where F_{Aut} is the distribution under autarky and F_{Int} is the distribution under integration. Let $[a,b]$ denote the support of the distribution of ω. Define the first and second moment of the interest rate distribution as functions of α

$$M_1(\alpha) = \int_a^b h(\omega) f(\omega, \alpha)\, d\omega,$$

$$M_2(\alpha) = \int_a^b (h(\omega))^2 f(\omega, \alpha) \, d\omega.$$

Integrating by parts and using the fact that all distributions $F(\omega, \alpha)$ have the same mean, gives the following two results

$$\frac{d(M_1(\alpha))^2}{d\alpha} = 2M_1(\alpha) \int_a^b h''(\omega) \left(\int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds \right) d\omega$$

and

$$M_2'(\alpha) = \int_a^b \left[2h(\omega) h''(\omega) + 2(h'(\omega))^2 \right] \left(\int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds \right) d\omega.$$

Since the variance of the interest rate is $M_2 - M_1^2$, the variance is increasing in α if

$$\int_a^b \left[[h(\omega) - M_1(\alpha)] h''(\omega) + (h'(\omega))^2 \right] \left(\int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds \right) d\omega > 0. \tag{26}$$

Since F_{Aut} is a mean preserving spread of F_{Int},we have

$$\int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds = \int_a^\omega (F_{Aut}(s) - F_{Int}(s)) \, ds > 0 \text{ for all } \omega.$$

Moreover $M_1(1) > M_1(\alpha)$, because h is a convex function when $\gamma \geq 1$. So the condition in statement of the proposition is sufficient to ensure that $[h(\omega) - M_1(\alpha)] h''(\omega) + (h'(\omega))^2 > 0$ for all ω in the support of the autarky distribution. This completes the argument for inequality (26) and completes the proof. ■

B. More on consumption skewness

To illustrate numerically when condition (12) is satisfied we assume again a CRRA utility with relative risk aversion γ. In all our examples, the condition holds when γ is above some cutoff, irrespective of the value of p. Table 1 shows this threshold for γ in some numerical examples. It is clear that for $R \leq 3$ a value of $\gamma > 3$ is sufficient to obtain negative skewness. Table 2 shows the standard deviation and skewness of consumption in various examples where $R = 1.4$, $\omega_L = 0.35$, and $\omega_H = 0.65$.

C. Alternative shock distributions

Consider a version of the model of financial integration presented in Section 2 where regional liquidity shocks are identically distributed, continuous random variables with expected value μ. Let σ^2 denote their common variance and $\rho = cov(\omega^A, \omega^B)/\sigma^2$ their linear correlation. In autarky, the optimal level of liquidity in region $i = A, B$ maximizes $E[V(y, \omega^i)]$, where the expectation is taken with respect to the regional shock ω^i. Let $\Omega = (\omega^A + \omega^B)/2$ denote the average liquidity shock in the two-region economy. The argument behind Lemma 2 can be extended to show that,

Table 1: Cutoffs for γ

R	$\omega_L = 0.35$ $\omega_H = 0.65$	$\omega_L = 0.1$ $\omega_H = 0.9$
1.1	0.25	0.15
1.2	0.48	0.28
1.4	0.88	0.52
1.8	1.56	0.92
2.2	2.07	1.24
2.6	2.52	1.50
3.0	2.89	1.72

Table 2: Standard deviation and skewness of first-period consumption

p	$\gamma = 4$		$\gamma = 0.75$		$\gamma = 0.5$	
	Std Dev	Skew	Std Dev	Skew	Std Dev	Skew
0	0.036	0	0.204	0	0.275	0
0.1	0.040	-0.201	0.200	-0.032	0.262	0.091
0.2	0.045	-0.408	0.194	-0.040	0.248	0.192
0.3	0.050	-0.629	0.186	-0.028	0.233	0.306
0.4	0.058	-0.873	0.175	0.002	0.217	0.438
0.5	0.066	-0.155	0.162	0.053	0.199	0.596
0.6	0.077	-1.500	0.147	0.128	0.178	0.794
0.7	0.089	-1.960	0.129	0.237	0.155	1.062
0.8	0.080	-2.653	0.107	0.411	0.127	1.477
0.9	0.058	-4.023	0.076	0.763	0.090	2.334
1	0	0	0	0	0	0

under integration, the optimal level of liquidity in each region maximizes $E[V(y, \Omega)]$, where the expectation is taken with respect to Ω.

Using the law of iterated expectations, the difference between the marginal value of liquidity under integration and in autarky in region i can be written as

$$E\big[\partial V(y,\Omega)/\partial y - \partial V(y,\omega^i)/\partial y \mid \omega^i > \Omega\big] \Pr(\omega^i > \Omega) \\ + E\big[\partial V(y,\Omega)/\partial y - \partial V(y,\omega^i)/\partial y \mid \omega^i < \Omega\big] \Pr(\omega^i < \Omega).$$

Given the monotonicity properties of $\partial V(y, \omega)/\partial y$, the first term is non negative while the second is non positive. This expression captures in a more general setup the opposing effects of integration on banks' optimal liquidity holding that are discussed in Section 4 for the case of binary shocks.

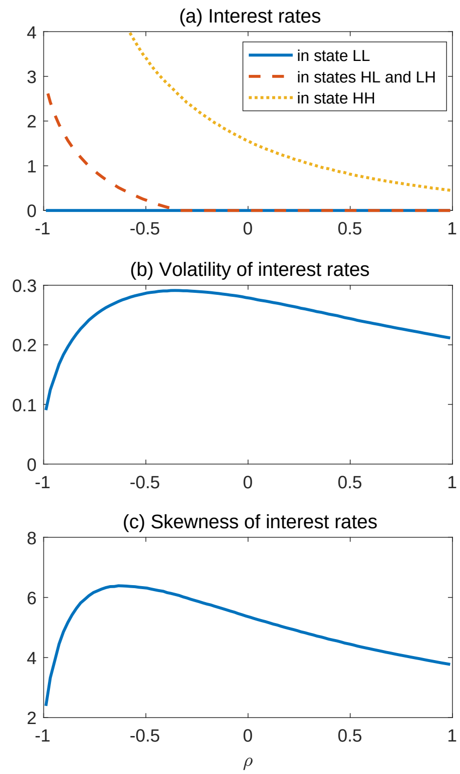

Figure 1: Interest rates.

Notice that Ω has the same expected value as the regional shocks, but its variance is $(1+\rho)\sigma^2/2$. This variance can be taken as a measure of aggregate liquidity uncertainty and declines as ρ approaches -1. In the extreme situation where $\rho = -1$, the average shock Ω is equal to μ with probability 1, in which case consumers can be completely insured against liquidity shocks. If instead $\rho = 1$, the regional shocks are identical (i.e., $\omega^A = \omega^B$) with probability 1, and integration offers no coinsurance possibility.[1] In this case, Ω is distributed as the regional shocks, which means that banks's behavior in case of financial integration is the same as in autarky.

We now turn to numerical examples in which the continuous joint distribution of regional liquidity shocks is parametrized as follows. Let (U^A, U^B) be two random variables distributed according to a bivariate Gaussian copula. Define

$$(X^A, X^B) = (H^{-1}(U^A, \alpha, \beta), H^{-1}(U^B, \alpha, \beta)),$$

where $H^{-1}(\cdot, \alpha, \beta)$ is the inverse of the cumulative distribution function of a beta random variable with shape parameters α and β. It follows that both X^A and X^B are identically-distributed, beta

[1]To see this, notice that a linear correlation of 1 means that there exists two real numbers a and b such that $b > 0$ and $\omega^A = a + b\omega^B$. However, ω^A and ω^B are identically distributed with, in particular, identical mean and variance. Hence, $\mu = a + b\mu$ and $\sigma^2 = b^2\sigma^2$, which imply that $a = 0$ and $b = 1$.

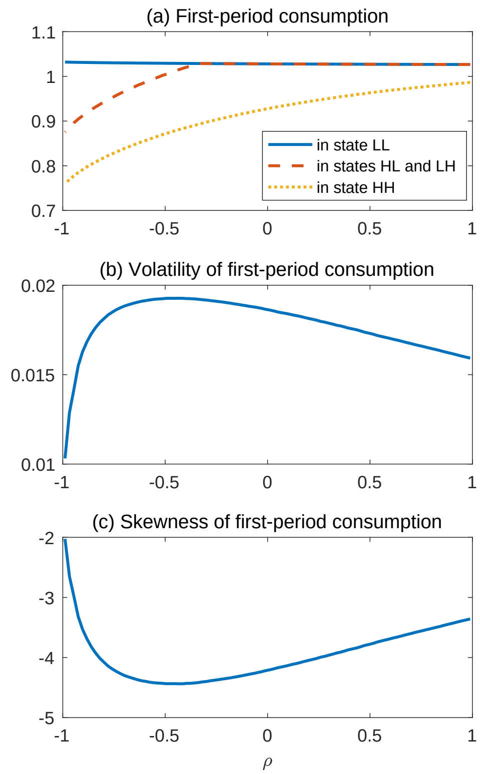

Figure 2: First-period consumption.

random variables. Let $\rho = corr(X^A, X^B)$ denote their linear correlation. The liquidity shock in region i is then defined as $\omega^i = \omega_L + (\omega_H - \omega_L)X^i$, with $0 \leq \omega_L < \omega_H \leq 1$. In this way shocks are restricted to the range $[\omega_L, \omega_H]$ and maintain a linear correlation of ρ. The numerical example assumes $\alpha = \beta = 1.1$, $\omega_L = 0.2$ and $\omega_H = 0.5$. This implies that, in each region, the liquidity shock has a symmetric distribution with an inverted-U shape on the support $[0.2, 0.5]$, a mean of 0.35 and a standard deviation of 0.084. The utility function is a CRRA with relative risk aversion of 5 and, finally, the return on the illiquid asset is $R = 1.05$.[2]

In order to facilitate the comparison of this example with that presented in the case of binary shocks, in Figure 1 we call "low" a liquidity shock that is equal to the mean ($\omega = 0.35$), and we call "high" a shock that is 1.5 standard deviations above the mean ($\omega = 0.474$). Panel (a)

[2]The numerical example is obtained as follows. First, we generate a large number of random draws (U^A, U^B) from a bivariate Gaussian copula distribution with assigned linear correlation. We then apply the transformation $(X^A, X^B) = (F^{-1}(U^A, \alpha, \beta), F^{-1}(U^B, \alpha, \beta))$ and compute $\rho = corr(X^A, X^B)$. The simulated liquidity shocks (ω^A, ω^B) and their average Ω are then obtained as described in the text. Based on the simulated realizations of Ω we can finally estimate its probability distribution, which is then used to solve numerically for the level of liquidity that maximizes $E[V(y, \Omega)]$. Other quantities, such as interest rates and consumption levels, are obtained starting from the calculated optimal level of liquidity.

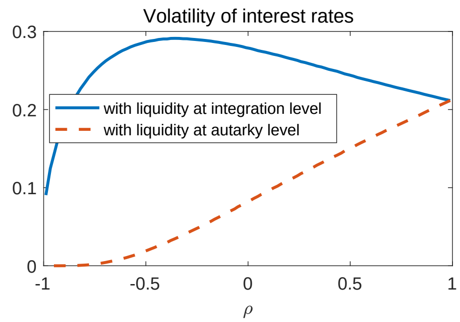

Figure 3: Decomposing the volatility of interest rates.

of Figure 1 plots the interest rate r in three alternative cases: i) both regions are hit by low liquidity shocks (LL); ii) the liquidity shock is high in one region and low in the other (LH, HL); and iii) both regional shocks are high (HH). It is useful to recall that the scope for integration increases as ρ moves away from 1 and towards -1, at which point aggregate liquidity uncertainty disappears. When $\rho = 1$ the two regions have identical liquidity shocks and the (local) interest rate in autarky is the one for case HH or LL, depending on whether the local shock is high or low. In Panel (a) of Figure 1 we can recognize the contrasting effects of financial integration. Consider for example what happens upon integration when $\rho = 0$. If a region has high liquidity needs, the interest rate under financial integration is lower than in autarky if in the other region liquidity needs are low (stabilizing effect). However, if also the other region has high liquidity needs, the interest rate is larger under integration than in autarky (destabilizing effect).[3] Panels (b) and (c) respectively plot the standard deviation and the skewness of interest rates for different values of ρ. The picture that emerges is very similar to that in Figure 5: due to its contrasting effects, financial integration can make interest rates and consumption levels more volatile and more skewed also in the case of continuous shocks. This happens for a relatively small value of R and a large value of γ in the example in Figure 1. However, provided that ρ is close enough to 1 and other parameters are held constant, it is sufficient that R does not exceed 1.3 or γ is above 0.8 for integration to increase the volatility of interest rates. On the other hand, provided again that ρ is close enough to one and given other parameters, the skewness of interest rates increases with integration for all value of R below 4.5 and for essentially any value of $\gamma > 0$. Figure 2 makes a similar point for consumption, whose standard deviation and (negative) skewness also increase under integration for values of ρ that are close enough to 1.

Finally, Figure 3 decomposes the effects of integration on interest rate volatility in two steps. First, the dashed red line shows what would happen if liquidity did not change upon integration. Interest rate volatility would necessarily decline in this case because, with fixed liquidity, only the stabilizing effect of integration would be at work. Second, the solid blue line shows what

[3]These effects could also be highlighted with alternative choices of the high and low shocks, as long as the high shock is large enough to ensure that the liquidity constraint is binding when it hits both regions.

happens when banks adjust their liquidity holdings. In this case, the endogenous reduction in banks' liquidity introduces a destabilizing effect of integration, which results in larger interest rate spikes in case of economy-wide liquidity shocks, and can ultimately increase the volatility of interest rate.