Additional Proofs and Experimental Details

EC.1. Additional Proofs

Proof of Proposition 2. First, note that Procedure 1 outputs an acyclic directed graph $\mathcal{T}^R = (\mathcal{S}^R, A)$ with n layers and for which any node has at most two outgoing arcs, each associated with a control $d \in \{0,1\}$. Thus, $\mathcal{T}^R$ can be perceived as the transition graph of some function $\tilde{f}'$ with domain $\tilde{D}'$. Specifically, there is a one-to-one correspondence between elements of $\tilde{D}'$ and the n-arc paths in $\mathcal{T}^R$.

We first show that $\tilde{D} \subseteq \tilde{D}'$. Recall that any $T \in \tilde{D}$ is associated with a state transition $s_1, \ldots, s_{n+1}$ in $\mathcal{S}$ that is governed by the control $\chi(T)$, where $\mathcal{R}(s_i, i, T) > -\infty$ for each $i = 1, \ldots, n+1$ according to Definition 1. Suppose, for the purpose of contradiction, that $T \notin \tilde{D}$, i.e., there is no path in $\mathcal{T}^R$ associated with T. At some stage i, the state s_i was replaced by $\oplus(s_i, s')$ for some $s' \in \mathcal{S}$ such that $\mathcal{R}(\oplus(s_i, s'), i, T) = -\infty$ (that is, the aggregate state leads to an infeasible state, which is removed from $\mathcal{T}^R$ by step 2(a)ii). However,

$$\mathcal{R}(\oplus(s_i, s'), i, T) \geq \max\{\mathcal{R}(s_i, i, T), \mathcal{R}(s', i, T)\} \geq \mathcal{R}(s_i, i, T) > -\infty,$$

a contradiction. Thus, $\oplus$ never includes state transition leading to infeasible states, and therefore $\tilde{D} \subseteq \tilde{D}'$.

We are left to prove that $\tilde{f}'(T) \geq \tilde{f}(T)$ for any $T \in \tilde{D}$, that is, the length of the path associated with T in $\mathcal{T}^R$ is an upper bound to its evaluation under $\tilde{f}$. Let $s_1^R, \ldots, s_{n+1}^R$ be the state transition associated with the element T in $\mathcal{T}^R$. We will show that

$$\sum_{j=1}^{i-1} w(s_j^R, j, \chi_j(T)) + \mathcal{R}(s_i^R, i, T) \geq \tilde{f}(T) \tag{EC.1}$$

for any $i \in \{1, \ldots, n\}$. If that is the case, fixing $i = n$ gives us the desired result.

The inequality (EC.1) follows by induction. As before, let $s_1, \ldots, s_{n+1}$ be the state transition associated with T in the original state space $\mathcal{S}$. For the basis case, $\mathcal{S}_1^R$ has a single state, $s_\emptyset$, and hence $s_1 = s_1^R$. For $i=2$, the inequality trivially holds for $s_2^R = s_2$. If otherwise $s_2^R = \oplus(s_2, s')$ for some $s' \in \mathcal{S}$, then

$$\begin{aligned} w(s_1^R, 1, \chi_1(T)) + \mathcal{R}(s_2^R, 2, T) &= w(s_1, 1, \chi_1(T)) + \mathcal{R}(\oplus(s_2, s'), 2, T) \\ &\geq w(s_1, 1, \chi_1(T)) + \max\{\mathcal{R}(s_2, 2, T), \mathcal{R}(s', 2, T)\} \\ &\geq w(s_1, 1, \chi_1(T)) + \mathcal{R}(s_2, 2, T) = \tilde{f}(T). \end{aligned}$$

Now assume as an induction hypothesis that (EC.1) holds for any $2 \leq i < i^*$. Note that possibly $s_{i^*}^R \neq tr(s_{i^*-1}^R, i^*-1, \chi_{i^*-1}(T))$, since $s_{i^*}^R$ might be the result of consecutive state aggregations at stage i^*. That is, we can assume there exists states $s_1^\perp, s_2^\perp, \ldots, s_m^\perp$ for an odd number m such that $s_{j+2}^\perp = \oplus(s_j^\perp, s_{j+1}^\perp),\ j = 1, 3, 5, 7, \ldots, m-2$, where $s_1^\perp = tr(s_{i^*-1}^R, i^*-1, \chi_{i^*-1}(T))$ and $s_m^\perp = s_{i^*}^R$. From assumption (R-UB), we have

$$\mathcal{R}(s_m^\perp, i^*, T) \geq \mathcal{R}(s_{m-2}^\perp, i^*, T) \geq \mathcal{R}(s_{m-4}^\perp, i^*, T) \geq \cdots \geq \mathcal{R}(s_1^\perp, i^*, T),$$

and therefore

$$\begin{aligned} \mathcal{R}(s_{i^*}^R, i^*, T) = \mathcal{R}(s_m^\perp, i^*, T) \geq \mathcal{R}(s_1^\perp, i^*, T) &= \mathcal{R}(tr(s_{i^*-1}^R, i^*-1, \chi_{i^*-1}(T)), i^*, T) \\ &= \mathcal{R}(s_{i^*-1}^R, i^*-1, T) - w(s_{i^*-1}^R, i^*-1, \chi_{i^*-1}(T)). \end{aligned}$$

The last equality above follows from the definition of $\mathcal{R}$. Finally, developing inequality (EC.1), we obtain

$$\begin{aligned} \sum_{j=1}^{i^*-1} w(s_j^R, j, \chi_j(T)) + \mathcal{R}(s_{i^*}^R, i^*, T) &= \sum_{j=1}^{i^*-2} w(s_j^R, j, \chi_j(T)) + w(s_{i^*-1}^R, i^*-1, \chi_{i^*-1}(T)) + \mathcal{R}(s_{i^*}^R, i^*, T) \\ &\geq \sum_{j=1}^{i^*-2} w(s_j^R, j, \chi_j(T)) + \mathcal{R}(s_{i^*-1}^R, i^*-1, T) \geq \tilde{f}(T), \end{aligned}$$

where the last inequality follows from induction hypothesis. Q.E.D.

EC.2. Instance Generation Procedures

EC.2.1. Instance Generation Method for the Latent-Class Logit Assortment Problem

The instances are generated according to the methodology proposed by Méndez-Díaz et al. (2014). We consider two instance classes, as follows.

- *Type 1 problems:* There are $n = 500$ products and $\kappa = 200$ segments. The arrival rates λ_k are independently and randomly drawn from a continuous uniform $U(0,1)$ distribution. v_i^k are independently and randomly drawn from a discrete uniform $U[0,10]$ distribution and v_0^k from a discrete uniform $U[0,4]$ distribution. All prices w_i are drawn from a continuous uniform $U[100,Z]$ distribution, $Z \in \{150, 200, 250, 300, 350\}$, with 10 instances per Z.

The consideration sets C_k are generated so that $C_k \supset C_{k+1}$ for all odd k. This is done by letting C_k be a random subset of L of size 6, and C_{k+1} be defined by the 3 lowest-priced elements in C_k. As a result, customers in segment C_{k+1} are interested in the same products as those in segment C_k, but are more price sensitive. For each instance, the capacity c of the retail store is considered at $0.2, 0.5, 1$ in order to reflect a space availability of 20%, 50%, and 100%, respectively. It is noted here that, at capacity $c = 1$, it still might be beneficial for the retail outlet not to display all products because customers may opt for lower-priced items.

- *Type 2 problems*: There are $n = 500$ products and $L \in \{30, 50, 70, 100\}$ segments. A parameter m is defined to control the minimum cardinality of the intersection between C_k and C_{k+1}. In particular, 15 random products are chosen for C_1 and, for $k = 2, \ldots, \kappa$, C_k is constructed by taking the last k elements selected for C_{k-1} together with $(15 - m)$ additional randomly selected products. We consider $m \in \{3, 6, 9, 12\}$, and 10 instances per pair of L and m are generated. The values of λ_k, v_i^k, and v_0^k are generated as in the Type 1 instances. All prices w_i are drawn from a continuous uniform $U(100, 200)$ distribution. Again, for each instance, the capacity c is considered at $0.2, 0.5$, and 1.

EC.2.2. Assignment Problem in Queuing Systems

In this section we describe the instance generation procedure for the experiments presented in Section 10.2. Specifically, instances are created by means of an auxiliary layered directed graph representing the workflow process (in this case, an open Jackson network). The graph contains two special nodes, r and t, representing the first and last task required by all jobs, respectively, so that any job arriving to the system first enters through r and leaves through t. All other tasks are arranged in l layers $M_1, \ldots, M_l$ with m tasks in each layer, so that the total number of tasks is $n = m \cdot l + 2$.

For every pair of nodes i, i', we define the probability that a job may require the processing of tasks i and i', in this order, as $P[i, i']$. In particular, we assume that jobs only transition between tasks in consecutive layers, i.e. $P[i, i'] > 0$ for all i, i' such that $i \in M_j$ and $i' \in M_{j+1}$ for some j, and $P[i, i'] = 0$ otherwise. Moreover, if $i \in M_j$, $j \leq l$, we also consider $\sum_{i' \in M_{j+1}} P[i, i'] = 1$. Define now a vector $a \in \mathbb{R}^n$ where a_i produces the arrival rate of jobs requiring task i. Since all jobs arrive at r, we can assume $a_r > 0$ and $a_i = 0$ for all other tasks $i \neq r$. Let I be the identity matrix. Given the transition matrix P and assuming $(I - P)$ is nonsingular, it follows from standard queuing theory that the net arrival rate λ of the tasks is given by

$$\lambda = (I - P)^{-T} a.$$

The specific values of each parameter used to generate the instances are as follows. $m = l \in \{4, 5, 6\}$ with the number of employees given by $\kappa = 4 \cdot l$. The arrival rate a_r is taken to be fixed at 0.01 for instances with $l = 4, 6$. For $l = 5$, the generated instances had $a_r \in \{0.06, 0.08, 0.1, \ldots, 0.22\}$. The waiting reward C_W is 2 for all instances and the reward c_i for each employee i is taken independently and uniformly at random from $\{5, 10, 15, 20, 25, 30, 35\}$. The service rates $\mu_{i,j}$ are independently drawn from the following discrete distribution:

$$\mu_{i,j} = \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 0.2 \cdot a_r^2 \cdot k_l \cdot k_n & \text{with probability } \frac{1}{8} \\ 0.4 \cdot a_r^2 \cdot k_l \cdot k_n & \text{with probability } \frac{1}{8} \\ 0.6 \cdot a_r^2 \cdot k_l \cdot k_n & \text{with probability } \frac{1}{8} \\ 0.8 \cdot a_r^2 \cdot k_l \cdot k_n & \text{with probability } \frac{1}{8}, \end{cases}$$

so that in expectation each job can be done by half of the employees, and that the service rates are proportional to both the number of jobs in the system and the square of the arrival rate.

The transition probabilities are generated as follows. For each task $i \in M_1$ (the layer following the entrance task), $P[i, i']$ is drawn uniformly at random from $\{0.2, 0.4, 0.6, 0.8\}$. For each task $i \in M_l$ (the layer proceeding the exit task), $P_{i',t} = 1$. For the remainder of tasks, consider task $i \in M_j$ for

$j = 1, \ldots, l-1$. A subset S of M_{j+1} is first selected uniformly at random, and the probabilities $P[i, i']$ are picked uniformly at random from the $\{0.2, 0.4, 0.6, 0.8\}$ if $i' \in S$, and $P[i, i'] = 0$ otherwise.

In total, 5 instances are generated per configuration (l, a_r). In all of our experiments, we observed that the presence of conflict sets would only make instances proportionally easier for all methodologies, and hence all results are based on instances without conflicts.