

Online Supplement for Buyer Intermediation in Supplier Finance

Tunay I. Tunca and Weiming Zhu

A Notation

Table A.1: Model Notation

w :	Unit wholesale price for the product
Q :	Retailer's order quantity
D :	Consumer demand
F, f :	c.d.f. and p.d.f. for D
F_R, f_R :	c.d.f. and p.d.f. for $w \cdot D$
p :	Unit retail price for the product
c_p :	Supplier's unit production cost
c_g :	Retailer's unit goodwill loss cost
c_e :	Retailer's unit processing and shipping cost for returned products
κ :	Percentage of production cost of expected goods sold yielded as the supplier's profit share
r_f :	Risk-free interest rate
δ_{bi} :	Bank's discount rate on the loan for BIF
B_0 :	Supplier's initial cash position at $t = 0$
B_1 :	Supplier's cash position before paying the bank loan at $t = 1$
L :	The supplier's selected loan amount
Π_r :	Retailer's expected net profit at time $t = 1$
Π_s :	Supplier's expected ending cash position at time $t = 1$
$\hat{\Pi}_s$:	Supplier's expected profit (net of opportunity cost) at time $t = 1$
a_l :	The defect probability of the supplier's product for a high quality supplier
a_h :	The defect probability of the supplier's product for a low quality supplier
a_n :	The return rate of non-defective products by the consumers
π_l :	Ex-ante probability of the supplier's product having a low defect rate
η_a :	Ex-ante overall defect rate of the supplier's product
ρ :	$(1 + r_f)/(1 - \eta_a)$

B Proofs of Propositions

Proof of Proposition 1: We will present the proof only for the commercial loan case. The proofs for the buyer intermediated financing scheme for this proposition will be similar and hence be omitted. By backwards induction, we first solve the bank's competitive interest rate setting problem for any given

positive $Q, w, l > 0$. Define

$$G(l, r_{cl}) = (1 - \eta_a)\mathbb{E}[\min\{l(1 + r_{cl}), (B_0 + l - c_p Q)(1 + r_f) + w \min\{Q, D\}\}] \\ + \eta_a \min\{l(1 + r_{cl}), (B_0 + l - c_p Q)(1 + r_f)\} - l(1 + r_f). \quad (\text{B.1})$$

By (3), the bank sets the interest rate r_{cl}^* by solving

$$G(l, r_{cl}) = 0. \quad (\text{B.2})$$

Notice that for any fixed $l \geq 0$, $G(l, r_{cl})$ is strictly increasing in r_{cl} for $l(1 + r_{cl}) < (B_0 + l - c_p Q)(1 + r_f) + wQ$, and equals $(B_0 - c_p Q)(1 + r_f) + (1 - \eta_a)w\mathbb{E}[\min\{Q, D\}]$ for $l(1 + r_{cl}) \geq (B_0 + l - c_p Q)(1 + r_f) + wQ$.

When $B_0 \geq c_p Q$, then $l(1 + r_f) \leq (B_0 + l - c_p Q)(1 + r_f)$, and hence $G(l, r_{cl})|_{r_{cl}=r_f} = 0$. That is, for any $B_0 \geq c_p Q$, $r_{cl}^* = r_f$ is a solution to (B.2), and since $l(1 + r_f) < (B_0 + l - c_p Q)(1 + r_f) + wQ$, $G(l, r_{cl})$ is strictly increasing in r_{cl} at $r_{cl} = r_f$ and is non-decreasing everywhere, it is the unique solution. When $B_0 < c_p Q$, plugging in $r_{cl} = r_f$, since $\eta_a > 0$, we obtain

$$G(l, r_f) = (1 - \eta_a)\mathbb{E}[\min\{l(1 + r_f), (B_0 + l - c_p Q)(1 + r_f) + w \min\{Q, D\}\}] \\ + \eta_a \min\{l(1 + r_f), (B_0 + l - c_p Q)(1 + r_f)\} - l(1 + r_f) \\ < (1 - \eta_a)l(1 + r_f) + \eta_a l(1 + r_f) - l(1 + r_f) = 0. \quad (\text{B.3})$$

It then follows that since $G(l, r_{cl})$ is strictly increasing for $l(1 + r_{cl}) < (B_0 + l - c_p Q)(1 + r_f) + wQ$, when $B_0 < c_p Q$, (B.2) will have a unique solution, r_{cl}^* in $(r_f, ((B_0 + l - c_p Q)(1 + r_f) + wQ)/l - 1)$, if and only if $(B_0 - c_p Q)(1 + r_f) + (1 - \eta_a)w\mathbb{E}[\min\{Q, D\}] > 0$. Otherwise, if $(B_0 - c_p Q)(1 + r_f) + (1 - \eta_a)w\mathbb{E}[\min\{Q, D\}] = 0$, then all $r_{cl} \geq ((B_0 + l - c_p Q)(1 + r_f) + wQ)/l - 1$ will be a solution, and if $(B_0 - c_p Q)(1 + r_f) + (1 - \eta_a)w\mathbb{E}[\min\{Q, D\}] < 0$, then there will be no solution.

To summarize, when $(B_0 - c_p Q)(1 + r_f) + (1 - \eta_a)w\mathbb{E}[\min\{Q, D\}] > 0$, we have

$$r_{cl}^* = \begin{cases} r_f, & \text{if } B_0 \geq c_p Q \\ r_{cl} \in (r_f, ((B_0 + l - c_p Q)(1 + r_f) + wQ)/l - 1) \text{ that solves (B.2)}, & \text{if } B_0 < c_p Q. \end{cases} \quad (\text{B.4})$$

Now, to solve (3), notice that, when the supplier determines the loan amount l , how much he chooses to borrow affects the bank's interest rate, as determined by the bank's competitive interest setting equation in (3), r_{cl}^* is a function of l . Further, when $(B_0 - c_p Q)(1 + r_f) + (1 - \eta_a)w\mathbb{E}[\min\{Q, D\}] > 0$, $G(l, r_{cl})$ has continuously differentiable partial derivatives in l and r_{cl} , which implies by the implicit function theorem that $r_{cl}^*(l)$ is continuously differentiable in l . Therefore, in this region, taking the total derivative of the

supplier's objective function as given in (2),

$$\frac{d\Pi_s^{cl}(l, r_{cl}^*(l))}{dl} = \frac{\partial\Pi_s^{cl}(l, r_{cl}^*(l))}{\partial l} + \frac{\partial\Pi_s^{cl}(l, r_{cl}^*(l))}{\partial r_{cl}^*} \frac{dr_{cl}^*(l)}{dl}. \quad (\text{B.5})$$

When $B_0 \geq c_p Q$, from (B.4) we have $r_{cl}^*(l) = r_f$ for all $l \geq 0$. By (2), we can then see that $\Pi_s^{cl}(l, r_{cl}(l))$ is independent of l , and hence $d\Pi_s^{cl}/dl = 0$. When $B_0 < c_p Q$, defining

$$D^* = \frac{l(r_{cl} - r_f) + (c_p Q - B_0)(1 + r_f)}{w}, \quad (\text{B.6})$$

and again applying the implicit function theorem to (B.1), we have

$$\frac{dr_{cl}^*(l)}{dl} = - \frac{\partial G(l, r_{cl})/\partial l}{\partial G(l, r_{cl})/\partial r_{cl}} \Big|_{r_{cl}=r_{cl}^*(l)} = - \frac{(1 - \eta_a)\bar{F}(D^*)(r_{cl}^*(l) - r_f)}{(1 - \eta_a)\bar{F}(D^*)l} = \frac{(r_f - r_{cl}^*(l))}{l}. \quad (\text{B.7})$$

In addition, from (2), we have

$$\frac{\partial\Pi_s^{cl}(l, r_{cl}^*(l))}{\partial l} = (1 - a_l)(r_f - r_{cl}^*(l))F(D^*), \quad \text{and} \quad \frac{\partial\Pi_s^{cl}(l, r_{cl}^*(l))}{\partial r_{cl}} = -(1 - a_l)lF(D^*). \quad (\text{B.8})$$

By plugging (B.7) and (B.8) into (B.5), we then have

$$\frac{d\Pi_s^{cl}(l, r_{cl}^*(l))}{dl} = (1 - a_l)(r_f - r_{cl}^*(l))F(D^*) - (1 - a_l)lF(D^*) \frac{(r_f - r_{cl}^*(l))}{l} = 0. \quad (\text{B.9})$$

That is, given the bank's response in setting the interest rate competitively, the supplier is indifferent about the loan amount he receives for $B_0 < c_p Q$ as well as for the case $B_0 \geq c_p Q$. Note that the supplier also has to satisfy the production budget constraint in (3), therefore, the loan borrowed has to be sufficient to cover the production cost, i.e., $l \geq (c_p Q - B_0)^+$. However, if the supplier borrows more than needed, he will invest the excess amount $l - (c_p Q - B_0)^+$ in the risk-free asset and pay it back to the bank without improving his objective function, i.e., the borrowing to lend the money will be a trivial action. Therefore, the only amount the supplier can borrow to cover its production costs without any trivial borrowing and lending is when $L = (c_p Q - B_0)^+$.

In order to solve the retailer's optimization problem, (5), first consider the case $B_0 \geq c_p Q$. As we have shown above, in this case $L = 0$. Then the supplier's IR constraint in (5) becomes

$$(B_0 - c_p Q)(1 + r_f) + (1 - a_l)(wQ - w\mathbb{E}[(Q - D)^+]) \geq B_0(1 + r_f) + \kappa c_p \mathbb{E}[\min\{Q, D\}]. \quad (\text{B.10})$$

Notice that the left hand side of (B.10) is increasing in w . However, again from (5), we have

$$\frac{\partial\Pi_r^{cl}(Q, w)}{\partial w} = (1 - a_l)(\mathbb{E}[(Q - D)^+] - Q) < 0, \quad (\text{B.11})$$

which means that the retailer's profit is decreasing in w . Therefore, for any given $Q \geq 0$, in the optimal solution (B.10) must be binding. Thus, solving for w and plugging in $\Pi_r^d(Q, w)$, the retailer's profit function on $B_0 \geq c_p Q$ then is

$$\Pi_r^1(Q) \triangleq ((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p) \mathbb{E}[\min\{Q, D\}] - c_g \mathbb{E}[(D - Q)^+] - c_p Q(1 + r_f). \quad (\text{B.12})$$

Also note that,

$$\frac{d^2 \Pi_r^1(Q)}{dQ^2} = -((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g) f(Q) < 0, \quad (\text{B.13})$$

since $(1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e > 0$. Therefore $\Pi_r^1(Q)$ is concave and, by solving the first order condition, is maximized at

$$Q = \bar{Q}_0 = F^{-1} \left(1 - \frac{c_p(1 + r_f)}{(1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g} \right) \leq Q_{fb}^*. \quad (\text{B.14})$$

Next, suppose $B_0 < c_p Q$, then the supplier will borrow $L = c_p Q - B_0$ and his participation constraint in (5) will be

$$(1 - a_l)(w \mathbb{E}[\min\{Q, D\}] - \mathbb{E}[\min\{L(1 + r_{cl}), w \min\{Q, D\}\}]) \geq B_0(1 + r_f) + \kappa c_p \mathbb{E}[\min\{Q, D\}]. \quad (\text{B.15})$$

Further, from the bank's interest rate setting equation,

$$\frac{L(1 + r_f)}{(1 - \eta_a)} = \mathbb{E}[\min\{L(1 + r_{cl}), w \min\{Q, D\}\}]. \quad (\text{B.16})$$

Once again, since the retailer's objective function is decreasing in w , (B.15) must be binding in optimality. Therefore by plugging (B.15) and (B.16) in Π_r^d , the retailer's profit function on $B_0 < c_p Q$ is

$$\begin{aligned} \Pi_r^2(Q) \triangleq & ((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p) \mathbb{E}[\min\{Q, D\}] - c_g \mathbb{E}[(D - Q)^+] \\ & - (c_p Q - B_0) \frac{(1 + r_f)(1 - a_l)}{1 - \eta_a} - B_0(1 + r_f). \end{aligned} \quad (\text{B.17})$$

Further, $d^2 \Pi_r^2(Q)/dQ^2$ is also as given in (B.13) and is negative. That is, $\Pi_r^2(Q)$ is also concave, and by solving its first order condition, is maximized at

$$Q = \bar{Q}_{cl} \triangleq F^{-1} \left(1 - \frac{c_p(1 + r_f)(1 - a_l)}{(1 - \eta_a)((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g)} \right). \quad (\text{B.18})$$

In addition, plugging $L = c_p Q - B_0$ in the supplier's IR constraint in (B.15), since (B.15) is binding and

by (B.16) we obtain

$$(1 - a_l) \left(w \mathbb{E}[\min\{Q, D\}] - \frac{(c_p Q - B_0)(1 + r_f)}{1 - \eta_a} \right) = B_0(1 + r_f) + \kappa c_p \mathbb{E}[\min\{Q, D\}]. \quad (\text{B.19})$$

This means that $(B_0 - c_p Q)(1 + r_f) + w \mathbb{E}[\min\{Q, D\}] \geq 0$, i.e., the bank's competitive interest rate setting equation in (3) has a solution, confirming the feasibility of \bar{Q}_{cl} for (5). Note that

$$\begin{aligned} \bar{Q}_0 &= F^{-1} \left(1 - \frac{c_p(1 + r_f)}{(1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g} \right) \\ &> F^{-1} \left(1 - \frac{c_p(1 - a_l)(1 + r_f)}{(1 - \eta_a)((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g)} \right) = \bar{Q}_{cl}, \end{aligned} \quad (\text{B.20})$$

as stated in the proposition, since $a_l < \eta_a$. Finally,

$$\Pi_r^1 \left(\frac{B_0}{c_p} \right) = \Pi_r^2 \left(\frac{B_0}{c_p} \right) = B_0(1 + r_f). \quad (\text{B.21})$$

When $B_0/c_p > Q_{fb}^*$, $\bar{Q}_{cl} < Q_{fb}^* < B_0/c_p$, and since Π_r^2 is concave, Π_r^2 is then decreasing on $Q > B_0/c_p$. Further, again since $B_0/c_p > Q_{fb}^*$, the maximizer Q_{fb}^* of $\Pi_r^1(Q)$ is in $0 \leq Q < B_0/c_p$. Hence $\Pi_r^{cl}(Q_{fb}^*, w^*(Q_{fb}^*)) = \Pi_r^1(Q_{fb}^*) > \Pi_r^1(B_0/c_p) = \Pi_r^2(B_0/c_p) > \Pi_r^2(Q) = \Pi_r^{cl}(Q, w^*(Q))$ for any $Q > B_0/c_p$, where, for a given $Q \geq 0$, $w^*(Q)$ is the solution of (B.15). Therefore, we can conclude that if $B_0/c_p > Q_{fb}^*$, the retailer's optimal order quantity is Q_{fb}^* and $L = 0$, i.e., the supplier does not get any loan. The corresponding wholesale price can be obtained as given in (15) by solving (B.10) as binding. This proves part (i).

For part (ii), notice that when $\bar{Q}_{cl} < B_0/c_p < \bar{Q}_0$, since $\Pi_r^1(Q)$ is concave and maximized at \bar{Q}_0 , $\Pi_r^1(Q)$ is increasing on $0 \leq Q \leq B_0/c_p$, and attains its maximum on this interval at $Q = B_0/c_p$. On the other hand, since $\Pi_r^2(Q)$ is also concave and is maximized at \bar{Q}_{cl} , it is decreasing on $Q \geq B_0/c_p$. Since $\Pi_r^1(B_0/c_p) = \Pi_r^2(B_0/c_p)$, and since $\Pi_r^{cl}(Q, w^*(Q)) = \Pi_r^1(Q)$ for $0 \leq Q \leq B_0/c_p$, and $\Pi_r^{cl}(Q, w^*(Q)) = \Pi_r^2(Q)$ for $Q > B_0/c_p$ it follows that $\Pi_r^{cl}(Q, w^*(Q))$ is maximized at $Q_{cl}^* = B_0/c_p$, and the supplier again borrows $L = c_p Q_{cl}^* - B_0 = 0$. Once again plugging this value into the supplier's binding participation constraint, we find that w_{cl}^* satisfies (15). ■

Proof of Proposition 2: Suppose $B_0/c_p < \bar{Q}_{cl}$. Then, using the notation of the proof of Proposition 1, since $\bar{Q}_{cl} < Q_{fb}^*$, and by concavity of Π_r^1 , $\Pi_r^1(Q)$ is increasing on $0 \leq Q \leq B_0/c_p$ and attains its maximum at $Q = B_0/c_p$. On the other hand, $\Pi_r^2(Q)$ has its global maximizer \bar{Q}_{cl} in $B_0/c_p > \bar{Q}_{cl}$. Again since, Π_r^2 is concave and $\Pi_r^1(B_0/c_p) = \Pi_r^2(B_0/c_p)$, this means that $\Pi_r^2(Q)$ is increasing on $0 \leq Q \leq \bar{Q}_{cl}$ and decreasing for $Q \geq \bar{Q}_{cl}$, i.e., $\Pi_r^2(Q)$ is maximized at $Q_{cl}^* = \bar{Q}_{cl}$. Once again, plugging Q_{cl}^* into the supplier's binding participation constraint, we obtain w_{cl}^* as given in (15). Further, as we have shown in the proof of Proposition 1, the supplier's budget constraint is binding in (3), i.e., $L_{cl}^* = c_p \bar{Q}_{cl} - B_0 > 0$ as given in

(16). Finally, after plugging L_{cl}^* into the bank's interest setting equation (B.16), we have

$$\frac{L_{cl}^*(1+r_f)}{(1-\eta_a)} = \mathbb{E}[\min\{L_{cl}^*(1+r_{cl}), w \min\{Q, D\}\}] \quad (\text{B.22})$$

$$= \int_0^{L_{cl}^*(1+r_{cl})} z f_R(z) dz + L_{cl}^*(1+r_{cl}) \bar{F}_R(L_{cl}^*(1+r_{cl})), \quad (\text{B.23})$$

where the second equality follows from the fact that $\min\{L_{cl}^*(1+r_{cl}), w \min\{Q, D\}\} = L_{cl}^*(1+r_{cl})$ when $wD \geq L_{cl}^*(1+r_{cl})$. From (B.23), we obtain r_{cl}^* as given in (17). This completes the proof. ■

Proof of Proposition 3: First, on $Q < B_0/c_p$, the retailer's objective function is again $\Pi_r^1(Q)$ as given in the proof of Proposition 1. For $Q \geq B_0/c_p$, we first derive the retailer's optimal δ_{bi} . We start by examining the supplier's borrowing behavior. Now, from (6),

$$\frac{\partial \Pi_s^{bi}(Q, w, \delta_{bi}, l)}{\partial l} = (1+r_f)(1-\delta_{bi}) - (1-a_l), \quad (\text{B.24})$$

and the supplier's profit is non-increasing in l if and only if

$$\delta_{bi} \geq 1 - \frac{1-a_l}{1+r_f}. \quad (\text{B.25})$$

For any $\delta_{bi} \geq 0$ for which (B.25) is not satisfied, supplier will choose to obtain as high a loan as possible, while the retailer's goal is inducing the supplier to borrow no more than the amount needed to cover production, $(c_p Q - B_0)^+$ as stated in (7). Therefore, the retailer sets $\delta_{bi} \geq 1 - (1-a_l)/(1+r_f)$ and the supplier will borrow the exact amount to cover his production costs, i.e., $L = (c_p Q - B_0)^+ / (1-\delta_{bi})$. Plugging into the retailer's objective in (7) and by (4), we obtain

$$\Pi_r^{bi}(Q, w, \delta_{bi}) = \mathbb{E}[\{(1-a_l)((1-a_n)p - a_n c_e - w) - a_l c_e\} \min\{Q, D\} - c_g(D-Q)^+] - \frac{a_l(c_p Q - B_0)}{1-\delta_{bi}}. \quad (\text{B.26})$$

For a given solution (Q, w, δ_{bi}) , suppose (B.25) is not binding. Then by (B.26), decreasing δ_{bi} increases $\Pi_r^{bi}(Q, w, \delta_{bi})$, while still preventing the supplier from borrowing over $(c_p Q - B_0)/(1-\delta_{bi})$. Further, again plugging in (3),

$$\Pi_s^{bi}(Q, w, \delta_{bi}, l) = (1-a_l)(w \mathbb{E} \min\{Q, D\} - \frac{a_l(c_p Q - B_0)}{1-\delta_{bi}}) \quad (\text{B.27})$$

increases. Finally,

$$(1+r_f)(1-\delta_{bi}^*) < (1+r_f) \frac{1-a_l}{1+r_f} = 1-a_l < 1. \quad (\text{B.28})$$

That is, by decreasing δ_{bi} , the supplier's IR constraint and the bank's non-negative profit constraint in (7) will still be satisfied. Therefore, in the optimal solution (B.25) must be binding, i.e., $\delta_{bi}^* = 1 - (1-a_l)/(1+r_f)$

and $L_{bi}^* = (c_p Q - B_0)(1 + r_f)/(1 - a_l)$. Plugging into the supplier's IR constraint, we have

$$w\mathbb{E}[\min\{Q, D\}] - \frac{(c_p Q - B_0)(1 + r_f)}{1 - a_l} \geq B_0 \frac{(1 + r_f)}{1 - a_l} + \kappa c_p \mathbb{E}[\min\{Q, D\}]. \quad (\text{B.29})$$

As the left hand side of (B.29) is increasing in w , and the retailer's objective function in (7) is decreasing in w , (B.29) must also bind in optimality. Solving for w , for any given $Q > B_0/c_p$, we obtain

$$w^*(Q) = \frac{c_p Q(1 + r_f)}{(1 - a_l)\mathbb{E}[\min\{Q, D\}]} + \frac{\kappa c_p}{1 - a_l}. \quad (\text{B.30})$$

Notice that by (B.30),

$$L_{bi}^* = \frac{(c_p Q - B_0)(1 + r_f)}{1 - a_l} \leq \frac{c_p Q(1 + r_f)}{1 - a_l} \leq \frac{c_p Q(1 + r_f)}{(1 - a_l)\mathbb{E}[\min\{Q, D\}]} \cdot Q \leq w^*(Q)Q. \quad (\text{B.31})$$

That is, L_{bi}^* is feasible for the supplier's problem (6). Finally, plugging (B.30) in the retailer's objective we obtain

$$\begin{aligned} \Pi_r^{bi}(Q, w^*(Q), \delta_{bi}) = \Pi_r^3(Q) &\triangleq \mathbb{E}[\left((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p\right) \min\{Q, D\} - c_g(D - Q)^+] \\ &\quad - (c_p Q - B_0) \frac{(1 + r_f)}{(1 - a_l)} - B_0(1 + r_f), \end{aligned} \quad (\text{B.32})$$

on $Q > B_0/c_p$, which is again concave in Q , and has a unique maximum at

$$Q_{bi}^* = \bar{Q}_{bi} = F^{-1}\left(1 - \frac{c_p(1 + r_f)}{(1 - a_l)((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g}\right) < Q_{fb}^*. \quad (\text{B.33})$$

Finally, again notice that as in the proof of Proposition 1, $\Pi_r^1(B_0/c_p) = \Pi_r^3(B_0/c_p)$. Given these, the rest of the proof proceeds in the similar fashion as in the proofs of Proposition 2 and is skipped. ■

Proof of Proposition 4: To see part (i), first, notice that by (17), we have

$$\begin{aligned} L_{cl}^* \frac{1 + r_f}{1 - \eta_a} &= \int_0^{L_{cl}^*(1 + r_{cl})} z f_R(z) dz + L_{cl}^*(1 + r_{cl}) \bar{F}_R(L_{cl}^*(1 + r_{cl})) \\ &< L_{cl}^*(1 + r_{cl}^*) F_R(L_{cl}^*(1 + r_{cl}^*)) + L_{cl}^*(1 + r_{cl}^*) \bar{F}_R(L_{cl}^*(1 + r_{cl}^*)) = L_{cl}^*(1 + r_{cl}^*), \end{aligned} \quad (\text{B.34})$$

which implies $1 + r_{cl}^* > (1 + r_f)/(1 - \eta_a)$. Since $1 + r_{bi} = 1/(1 - \delta_{bi})$, by (18) we then have

$$1 + r_f < \frac{1 + r_f}{1 - a_l} = 1 + r_{bi} < \frac{1 + r_f}{1 - \eta_a} < 1 + r_{cl}^*, \quad (\text{B.35})$$

since $0 < a_l < \eta_a$. It follows that $r_f < r_{bi}^* < r_{cl}^*$.

For part (ii), first, for the commercial loan case, when the supplier is borrowing a positive amount, by

(14),

$$F(Q_{cl}^*) = \int_0^{Q_{cl}^*} f(D)dD = 1 - \frac{c_p(1+r_f)(1-a_l)}{(1-\eta_a)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)} > 0, \quad (\text{B.36})$$

and the right hand side of (B.36) is independent of $Var[D]$. Now, as $Var[D] \rightarrow 0$, for any $D \neq \mathbb{E}[D]$, $f(D) \rightarrow 0$. Therefore, $\lim_{Var[D] \rightarrow 0} Q_{cl}^* = \mathbb{E}[D]$. Similarly $\lim_{Var[D] \rightarrow 0} Q_{bi}^* = \mathbb{E}[D]$ also follows. Further, as $Var[D] \rightarrow 0$, $D \xrightarrow{p} \mathbb{E}[D]$ as well, which implies that for $\varphi = cl, bi$,

$$\lim_{Var[D] \rightarrow 0} \{Q_{\varphi}^* - \mathbb{E}[(Q_{\varphi}^* - D)^+]\} = \lim_{Var[D] \rightarrow 0} \mathbb{E}[\min\{Q_{\varphi}^*, D\}] = \mathbb{E}[D]. \quad (\text{B.37})$$

Plugging into (15) and (20), we then have

$$\begin{aligned} \lim_{Var[D] \rightarrow 0} (w_{cl}^* - w_{bi}^*) &= \frac{c_p \mathbb{E}[D]^{\frac{(1+r_f)}{1-\eta_a}} - B_0(1+r_f)(\frac{1}{1-\eta_a} - \frac{1}{1-a_l})}{\mathbb{E}[D]} - \frac{c_p(1+r_f)}{1-a_l} \\ &= \frac{(1+r_f)}{\mathbb{E}[D]} \left(\frac{1}{1-\eta_a} - \frac{1}{1-a_l} \right) (c_p \mathbb{E}[D] - B_0), \end{aligned} \quad (\text{B.38})$$

Notice that since $L_{cl}^* > 0$, if and only if $c_p Q_{cl}^* < B_0$. Since $\lim_{Var[D] \rightarrow 0} Q_{cl}^* = \mathbb{E}[D]$, if $c_p \mathbb{E}[D] < B_0$, there exists $\vartheta > 0$, such that for all $Var[D] < \vartheta$, $L_{cl}^* = 0$. Therefore, there exists $\vartheta > 0$ such that, if $Var[D] < \vartheta$, then $c_p \mathbb{E}[D] > B_0$ must hold, which, by (B.38) and since $\eta_a > a_l$, implies $w_{cl}^* > w_{bi}^*$. This proves part (ii).

To show part (iii), we start by comparing the equilibrium order quantities. When $L_{cl}^*, L_{bi}^* > 0$, by Propositions 2 and 3 and equations (11) and (12), $Q_{cl}^* < Q_{bi}^*$ if and only if

$$\begin{aligned} F^{-1} \left(1 - \frac{c_p(1-a_l)(1+r_f)}{(1-\eta_a)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g)} \right) \\ < F^{-1} \left(1 - \frac{c_p(1+r_f)}{(1-a_l)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e - \kappa c_p + c_g)} \right). \end{aligned} \quad (\text{B.39})$$

Since F^{-1} is monotonically non-decreasing, (B.39) is satisfied if and only if $\eta_a > 1 - (1-a_l)^2$.

Since the supplier's IR constraint is binding, the supplier's ending cash position is $\Pi_s = B_0(1+r_f) + \kappa c_p \mathbb{E}[\min\{Q^*, D\}]$, and is non-decreasing in Q^* . Therefore, $\Pi_s^{cl} < \Pi_s^{bi}$ if and only if $Q_{cl}^* < Q_{bi}^*$, which, as we showed above, holds if and only if $\eta_a > 1 - (1-a_l)^2$. To see the retailer profit comparison, by (B.17) and (B.32), we have

$$\Pi_r^{bi}(Q) - \Pi_r^{cl}(Q) = \Pi_r^3(Q) - \Pi_r^2(Q) = (c_p Q - B_0)(1+r_f) \left(\frac{1-a_l}{1-\eta_a} - \frac{1}{1-a_l} \right). \quad (\text{B.40})$$

If $\eta_a > 1 - (1-a_l)^2$, then as we have shown above, $Q_{bi}^* > Q_{cl}^*$, and by concavity of Π_r^2 and Π_r^3 , we have $\Pi_r^3(Q_{bi}^*) > \Pi_r^3(Q_{cl}^*) > \Pi_r^2(Q_{cl}^*)$. The case for $\eta_a \leq 1 - (1-a_l)^2$ follows symmetrically. Therefore, we obtain that $\Pi_{bi}^* > \Pi_{cl}^*$ if and only if $\eta_a > 1 - (1-a_l)^2$ as stated in the proposition.

Finally, the percentage of the production costs the supplier borrows under the commercial loan is $(c_p Q_{cl}^* - B_0)/c_p Q_{cl}^* = 1 - B_0/c_p Q_{cl}^*$. Similarly, the percentage of the production costs he borrows under the commercial loan is $1 - B_0/c_p Q_{bi}^*$. Since $Q_{bi}^* > Q_{cl}^*$ if and only if $\eta_a > 1 - (1 - a_l)^2$, it follows that $1 - B_0/c_p Q_{bi}^* > 1 - B_0/c_p Q_{cl}^*$ if and only if $\eta_a > 1 - (1 - a_l)^2$. This completes the proof. ■

C Model Formulation and Propositions for Suppliers with Multiple SKUs

In this section, we present the multi-SKU versions of our model formulations and propositions from Section 3. The model description is identical to the single-SKU case as described in Section 3.1, except that the supplier now has M SKUs with each one having its distinct demand D_i with c.d.f. F_i , and p.d.f. f_i for product i , $1 \leq i \leq M$. Given the supplier has M SKUs, define $\mathbf{Q} = (Q_1, \dots, Q_M)$ as the vector of SKU order quantities, $\mathbf{w} = (w_1, \dots, w_M)$ as the vector of SKU wholesale prices, and c_{pi} as the unit production cost for SKU i , $1 \leq i \leq M$. For notational purposes, also define $\mathbf{0}$ as the M -vector of zeros. The retailer again yields a certain percentage, κ , of production cost of the goods sold to the supplier as net surplus from the transaction. The generalizations to the supplier with multiple SKUs is straight-forward, the changes to the formulations and propositions are minimal and the results are essentially unchanged. We provide the multi-SKU versions of the formulations and the propositions on the equilibrium outcomes here as developed in Sections 3.2.1, 3.2.2 and 3.4 for the reader's convenience. We will skip the proofs of the propositions because they are essentially identical to the single-SKU case.

C.1 Commercial Loan

Extending the formulation given in Section 3.2.1, the supplier's $t = 1$ budget can be written as

$$B_1 = \begin{cases} (B_0 + l - \sum_{i=1}^M c_{pi} Q_i)(1 + r_f) + \sum_{i=1}^M w_i \min\{Q_i, D_i\} & \text{if the product is not defective;} \\ (B_0 + l - \sum_{i=1}^M c_{pi} Q_i)(1 + r_f) & \text{if the product is defective.} \end{cases} \quad (\text{C.1})$$

and his expected total profit is

$$\begin{aligned} \Pi_s^{cl}(\mathbf{Q}, \mathbf{w}, l) &= \mathbb{E}[(B_1 - l(1 + r_{cl}))^+] \\ &= (1 - a_l) \mathbb{E}[(B_0 + l - \sum_{i=1}^M c_{pi} Q_i)(1 + r_f) + \sum_{i=1}^M w_i \min\{Q_i, D_i\} - l(1 + r_{cl})^+] \\ &\quad + a_l \mathbb{E}[(B_0 + l - \sum_{i=1}^M c_{pi} Q_i)(1 + r_f) - l(1 + r_{cl})^+]. \end{aligned} \quad (\text{C.2})$$

and $r_{cl}^* > r_f$ is the unique solution for r_{cl} to the equation

$$\rho L_{cl}^* = \int_0^{L_{cl}^*(1+r_{cl})} z f_R(z) dz + L_{cl}^*(1+r_{cl})(1 - F_R(L_{cl}^*(1+r_{cl}))). \quad (\text{C.10})$$

C.2 Buyer Intermediated Financing

Utilizing the same notation defined in Section C.1, and extending the expressions given in Section 3.2.2 to the multiple SKU case, we have

$$\begin{aligned} \max_l \Pi_s^{bi}(\mathbf{Q}, \mathbf{w}, \delta_{bi}, l) &= \max_l \sum_{i=1}^M \{(B_0 + l(1 - \delta_{bi}) - c_{pi}Q_i)(1 + r_f) + (1 - a_l)(w_i \mathbb{E}[\min\{Q_i, D_i\}] - l)\} \\ \text{s.t. } 0 &\leq l \leq \sum_{i=1}^M w_i Q_i, \\ B_0 + l(1 - \delta_{bi}) - \sum_{i=1}^M c_{pi}Q_i &\geq 0, \text{ (Supplier's production budget constraint)} \end{aligned} \quad (\text{C.11})$$

and the retailer's optimization problem for the BIF case becomes

$$\begin{aligned} \max_{\mathbf{Q}, \mathbf{w} \geq \mathbf{0}, \delta_{bi} \in (0,1)} \Pi_r^{bi}(\mathbf{Q}, \mathbf{w}, \delta_{bi}) &= \Pi_r^{cl}(\mathbf{Q}, \mathbf{w}) - a_l L \\ \text{s.t. } \Pi_s^{bi}(\mathbf{Q}, \mathbf{w}, \delta_{bi}, L) &\geq B_0(1 + r_f) + \kappa \sum_{i=1}^M c_{pi} \mathbb{E} \min\{Q_i, D_i\}, \text{ (IR)} \\ (1 + r_f)(1 - \delta_{bi}) &\leq 1, \text{ (The bank's non-negative profit constraint)} \\ L(1 - \delta_{bi}) &\leq \left(\sum_{i=1}^M c_{pi}Q_i - B_0\right)^+, \text{ (The constraint to limit} \\ &\quad \text{supplier overborrowing)} \end{aligned}$$

and where L solves the supplier's optimization problem for (Q, w, δ_{bi}) as given in (C.11). **(IC)**

(C.12)

Parallel to Section 3.4, for SKU i , $1 \leq i \leq M$, define

$$\bar{Q}_{bii} = F_i^{-1} \left(1 - \frac{c_{pi}(1 + r_f)}{(1 - a_l)((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e - \kappa c_{pi} + c_g)} \right). \quad (\text{C.13})$$

Then, again given that for each product i the retailer yields the supplier the fraction κ of the expected production costs of the sold goods, $c_{pi}E[\min\{Q_i, D_i\}]$, Proposition 3 in Section 3.4 can be extended as follows.

Proposition C.2 *For buyer intermediated financing, if $B_0 < \sum_{i=1}^M c_{pi} \bar{Q}_{bii}$, then in equilibrium, the sup-*

plier borrows up to the level to cover his production costs. Further,

$$\delta_{bi}^* = 1 - \frac{1 - a_l}{1 + r_f}, \quad (\text{C.14})$$

$$Q_{bii}^* = \bar{Q}_{bii}, \quad (\text{C.15})$$

$$w_{bii}^* = \frac{c_{pi} Q_{bii}^* (1 + r_f)}{(1 - a_l)(Q_{bii}^* - \mathbb{E}[(Q_{bii}^* - D)^+])} + \frac{\kappa c_{pi}}{1 - a_l}, \quad (\text{C.16})$$

$$L_{bi}^* = \left(\sum_{i=1}^M c_{pi} Q_{bii}^* - B_0 \right) \frac{1 + r_f}{1 - a_l}. \quad (\text{C.17})$$

D Expectation Maximization (EM) Method Based Demand Estimation

Assuming log-normality for demand, define $\theta_j = (a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2)$ as the parameter vector to be estimated for industry j , where σ_j^2 is the variance of the error terms in (22). Since the demand data only reveals the realized sales, censored in the sense that unmet demand is lost and unobserved, the estimation needs to be appropriately adjusted. To this end, we employ an Expectation Maximization (EM) iterative regression method to account for the unobserved component of demand. Without loss of generality, let n_j be the sample size for industry segment j , in which the first m_j demand entries are not binding with the respective order quantity and the rest $n_j - m_j$ demand entries are binding. Further, let D_{ij} denote the real demand, and let $D_{obs(ij)}$ denote the observed demand documented in the data. Note that for each industry segment j and for $1 \leq i \leq m_j$, $D_{ij} = D_{obs(ij)}$ will hold. Then for each industry j , and any given parameter vectors θ_j and θ' , define the conditional log-likelihood function

$$\begin{aligned} \mathcal{L}(\theta_j; \theta', D_{ij}) = & -\frac{n_j}{2} \log(2\pi\sigma_j^2) - \frac{1}{2} \sum_{i=1}^{m_j} \frac{(\log(D_{ij}) - (a_j + b_j \log(p_{ij}) + \gamma_j I_{ij\{t=2013\}} + \lambda_j \log(p_{ij}) I_{ij\{t=2013\}}))^2}{\sigma_j^2} \\ & - \frac{1}{2} \sum_{i=m_j+1}^{n_j} \frac{(\mathbb{E}[\log(D_{ij})|\theta', D_{obs(ij)}] - (a_j + b_j \log(p_{ij}) + \gamma_j I_{ij\{t=2013\}} + \lambda_j \log(p_{ij}) I_{ij\{t=2013\}}))^2}{\sigma_j^2}. \end{aligned} \quad (\text{D.1})$$

The estimation starts with initial value θ_j^0 , for each iteration k with parameter vector θ_j^k , proceeds by finding $\theta_j^{(k+1)}$ such that

$$\theta_j^{(k+1)} = \arg \max_{\theta} \mathcal{L}(\theta; \theta_j^k, D_{ij}), \quad (\text{D.2})$$

replacing θ_j^k with $\theta_j^{(k+1)}$, and continuing until convergence.

In order to maximize $\mathcal{L}(\theta; \theta', D_{ij})$ at iteration k , writing the first order conditions for $a_j^{(k+1)}$ and $b_j^{(k+1)}$, we have

$$\partial \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \beta_j^{(k)}, D_{obs(ij)}] / \partial a_j = 0, \quad (\text{D.3})$$

$$\partial \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \beta_j^{(k)}, D_{obs(ij)}] / \partial b_j = 0, \quad (\text{D.4})$$

$$\partial \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}] / \partial \gamma_j = 0, \quad (\text{D.5})$$

and

$$\partial \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}] / \partial \lambda_j = 0. \quad (\text{D.6})$$

By, (D.1), (D.3)-(D.6), we obtain

$$\mathbf{T} \boldsymbol{\beta}_j^{(k)} = \mathbf{D}_j, \quad (\text{D.7})$$

where

$$\mathbf{T} = \begin{pmatrix} n_j & \sum_{i=1}^{n_j} \log(p_{ij}) & \sum_{i=1}^{n_j} I_{ij\{t=13\}} & \sum_{i=1}^{n_j} \log(p_{ij}) I_{ij\{t=13\}} \\ \sum_{i=1}^{n_j} \log(p_{ij}) & \sum_{i=1}^{n_j} (\log(p_{ij}))^2 & \sum_{i=1}^{n_j} \log(p_{ij}) I_{ij\{t=13\}} & \sum_{i=1}^{n_j} (\log(p_{ij}))^2 I_{ij\{t=13\}} \\ \sum_{i=1}^{n_j} I_{ij\{t=13\}} & \sum_{i=1}^{n_j} \log(p_{ij}) I_{ij\{t=13\}} & \sum_{i=1}^{n_j} I_{ij\{t=13\}} & \sum_{i=1}^{n_j} \log(p_{ij}) I_{ij\{t=13\}} \\ \sum_{i=1}^{n_j} \log(p_{ij}) I_{ij\{t=13\}} & \sum_{i=1}^{n_j} (\log(p_{ij}))^2 I_{ij\{t=13\}} & \sum_{i=1}^{n_j} \log(p_{ij}) I_{ij\{t=13\}} & \sum_{i=1}^{n_j} (\log(p_{ij}))^2 I_{ij\{t=13\}} \end{pmatrix}, \quad (\text{D.8})$$

$$\boldsymbol{\beta}_j^{(k)} = \begin{pmatrix} a_j^{(k+1)} \\ b_j^{(k+1)} \\ \gamma_j^{(k+1)} \\ \lambda_j^{(k+1)} \end{pmatrix}, \quad (\text{D.9})$$

and

$$\mathbf{D}_j = \begin{pmatrix} \sum_{i=1}^{m_j} \log(D_{ij}) + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}] \\ \sum_{i=1}^{m_j} \log(D_{ij}) \log(p_{ij}) + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij}) \log(p_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}] \\ \sum_{i=1}^{m_j} \log(D_{ij}) I_{ij\{t=13\}} + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij}) I_{ij\{t=13\}} | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}] \\ \sum_{i=1}^{m_j} \log(D_{ij}) \log(p_{ij}) I_{ij\{t=13\}} + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij}) \log(p_{ij}) I_{ij\{t=13\}} | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}] \end{pmatrix}. \quad (\text{D.10})$$

Since $I_{ij\{t=13\}}$'s are not all 1's, \mathbf{T} has full rank and is invertible. Also notice that since

$$\frac{\partial^2 \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}]}{\partial a_j^2} = -\frac{n_j}{\sigma_j^2} < 0, \quad (\text{D.11})$$

$$\frac{\partial^2 \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}]}{\partial b_j^2} = -\sum_{i=1}^{n_j} \frac{(\log(p_{ij}))^2}{\sigma_j^2} < 0, \quad (\text{D.12})$$

$$\frac{\partial^2 \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}]}{\partial \gamma_j^2} = -\sum_{i=1}^{n_j} \frac{I_{ij\{t=13\}}}{\sigma_j^2} < 0, \quad (\text{D.13})$$

and

$$\frac{\partial^2 \mathbb{E}[\mathcal{L}(a_j, b_j, \gamma_j, \lambda_j, \sigma_j^2; \theta'_k, D_{ij}) | \boldsymbol{\beta}_j^{(k)}, D_{obs(ij)}]}{\partial \lambda_j^2} = -\sum_{i=1}^{n_j} \frac{(\log(p_{ij}))^2 I_{ij\{t=13\}}}{\sigma_j^2} < 0, \quad (\text{D.14})$$

Table D.1: EM Estimation Outcomes for Demand Distributions

	a_j^*	b_j^*	γ_j^*	λ_j^*	σ_j^*
Auto parts					
Low price (< 4000)	6.0336*** (0.3579)	-0.2553*** (0.0744)	0.9728* (0.4735)	-0.0801 (0.0982)	1.6541*** (0.0855)
High price (\geq 4000)	24.7847*** (5.1655)	-2.4659*** (0.5785)	3.2880* (6.7084)	0.4407* (0.7544)	2.1451*** (0.1043)
Baby and Pregnancy products					
Low price (< 100)	9.7771*** (0.7162)	-1.2966 *** (0.1903)	-0.4645* (0.9865)	-0.1158** (0.2613)	2.3181*** (0.0823)
High price (\geq 100)	15.5908*** (2.3076)	-2.1387*** (0.4504)	-0.0149 (3.1813)	-0.0923 (0.6210)	2.3105*** (0.1140)
Clothing					
Low price (< 2000)	5.0132*** (0.1267)	-0.4929*** (0.0197)	0.8983*** (0.1523)	-0.1252*** (0.0240)	2.3181*** (0.1925)
High price (\geq 2000)	1.9367*** (0.5367)	-0.1308* (0.0659)	2.9595*** (0.6850)	-0.3577*** (0.0839)	2.3105*** (0.1127)
Coffee	7.6920*** (1.0052)	-0.7333* (0.2195)	-5.3482*** (1.1817)	1.1535*** (0.2664)	1.4579*** (0.1425)
Computer accessories	7.8873*** (0.6854)	-0.7627*** (0.1329)	-0.8993 (0.7868)	0.2727 (0.1523)	1.9800*** (0.1458)
Cosmetics	10.5626*** (0.2278)	-1.2404*** (0.0461)	-0.4943 (0.3025)	0.1527* (0.0610)	2.101*** (0.1127)
Electronic products					
Low price (< 3000)	8.0947*** (0.1173)	-0.6849*** (0.0211)	-1.4557*** (0.1591)	0.2791*** (0.0294)	2.3181*** (0.0951)
High price (\geq 3000)	14.0369*** (1.8028)	-1.2923*** (0.2110)	-4.5299 (2.6094)	0.5794 (0.3033)	2.3105*** (0.1848)
Home improvement					
Low price (< 200)	7.9058* (3.1342)	-0.6519 (0.6502)	-4.4432 (3.2002)	0.9834 (0.6672)	2.3181*** (0.2473)
High price (\geq 200)	7.0094*** (0.2723)	-0.5038*** (0.0614)	2.9257* (0.4024)	-0.5484** (0.0883)	2.3105*** (0.0432)
Household appliances					
Low price (< 2000)	4.3951*** (0.2723)	0.0855 (0.0614)	1.1970** (0.4024)	-0.3001*** (0.0883)	1.6541*** (0.0432)
High price (\geq 2000)	24.7847*** (4.8854)	-2.4659*** (0.5851)	3.2880* (7.2306)	0.4407* (0.8671)	2.1451*** (0.0747)
Sporting goods	5.5446*** (0.2456)	-0.5508*** (0.0474)	-1.2029*** (0.3107)	0.2157*** (0.0596)	1.8085*** (0.0699)
Staple goods	7.7621*** (0.2061)	-0.4900*** (0.0425)	0.7810** (0.2724)	-0.2423*** (0.0560)	2.3641*** (0.2570)
Wine	7.9349*** (0.2781)	-0.6520*** (0.0615)	-0.8733*** (0.3262)	-0.0148 (0.0718)	2.0104*** (0.1146)

p<0.01 ***, p<0.05 **, p<0.1 *. Numbers in brackets are the corresponding standard errors for the parameters.

the expected log likelihood function is concave in a_j , b_j , λ_j and γ_j , and hence the first order conditions are sufficient for optimality.

Finally, $\sigma_j^{2(k+1)}$ can be derived by a simpler approach. Since the normal distribution falls into the exponential family, the conditional expectations of the moments can be directly substituted for the moments that occur in the expressions obtained for the complete-data maximum likelihood estimators to perform

the next iteration. That is, we can replace the sample moments in $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n_j} (\log(D_{ij}))^2}{n_j} - \left(\frac{\sum_{i=1}^{n_j} \log(D_{ij})}{n_j} \right)^2$ by their conditional expectations and obtain $\sigma_j^{2(k+1)}$. It follows that

$$\sigma_j^{2(k+1)} = \frac{\sum_{i=1}^{m_j} (\log(D_{ij}))^2 + \mathbb{E}[\sum_{i=m_j+1}^{n_j} (\log(D_{ij}))^2 | \beta_j^{(k)}, D_{obs(ij)}]}{n_j} - \left(\frac{\sum_{j=i}^{m_j} (\log D_{ij}) + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij}) | \beta_j^{(k)}, D_{obs(ij)}]}{n_j} \right)^2. \quad (\text{D.15})$$

The estimation results are given in Table D.1.

E Equivalence of NLS and Maximum Likelihood Estimation with Normally Distributed Error Terms

In this section, we show that the NLS estimation given in equation (24) in Section 4.2.2 is equivalent to Maximum Likelihood Estimation under the assumption of normality of errors. Suppose ϵ_{ij} are i.i.d. normally distributed with mean 0 and standard deviation σ_0 . The likelihood function can then be written as

$$L(\xi_j^{(12)}, \xi_j^{(13)}, c_g, \rho; 0, \sigma_0) = \left(\frac{1}{2\pi\sigma_0^2} \right)^n \exp\left\{ -\frac{1}{2\sigma_0^2} \sum_{j=1}^N \sum_{i=1}^{M_j^{(12)}} \left(Q_{ij}^{(12)} - \bar{Q}_{ij}^{cl} \right)^2 - \frac{1}{2\sigma_0^2} \sum_{j=1}^N \sum_{i=1}^{M_j^{(13)}} \left(Q_{ij}^{(13)} - \bar{Q}_{ij}^{bi} \right)^2 \right\}, \quad (\text{E.1})$$

which implies that the log-likelihood function is

$$l(\xi_j^{(12)}, \xi_j^{(13)}, c_g, \rho; 0, \sigma_0) = -n \ln(2\pi) - n \ln(\sigma_0^2) - \frac{1}{2\sigma_0^2} \left(\sum_{j=1}^N \sum_{i=1}^{M_j^{(12)}} \left(Q_{ij}^{(12)} - \bar{Q}_{ij}^{cl} \right)^2 + \sum_{j=1}^N \sum_{i=1}^{M_j^{(13)}} \left(Q_{ij}^{(13)} - \bar{Q}_{ij}^{bi} \right)^2 \right). \quad (\text{E.2})$$

For each $\sigma_0 > 0$, the maximization of (E.2) is equivalent to minimizing the squared differences between the actual quantity ordered and the theoretically predicted order quantity given in the final term. That is, this Maximum Likelihood Estimation is equivalent to the Nonlinear Least Squares estimation given in (24).

F Identification of Parameters for the Structural Estimation

Our goal is to obtain the moment equations for our structural Non-Linear Least Square estimation given in (24), and show that, given the variation of the data, the joint solution to them is identifiable. Define

$$\Omega_{ij} = (1 - a_l)((1 - a_n)p_{ij} - a_n c_e) - a_l c_e - \kappa c_p, \quad (\text{F.1})$$

$$\nu_{ij}^{(12)} = F_{(\mu_{ij}^{(12)}(1+\xi_j^{(12)}), \sigma_{ij}^{(12)})}^{-1} \left(1 - \frac{c_{p(ij)}^{(12)} \rho}{(1 - a_l)((1 - a_n)p_{ij}^{(12)} - a_n c_e) - a_l c_e - \kappa c_p + c_g} \right), \quad (\text{F.2})$$

$$\nu_{ij}^{(13)} = F_{(\mu_{ij}^{(13)}(1+\xi_j^{(13)}), \sigma_{ij}^{(13)})}^{-1} \left(1 - \frac{c_{p(ij)}^{(13)}(1 + r_f)/(1 - a_l)}{(1 - a_l)((1 - a_n)p_{ij}^{(13)} - a_n c_e) - a_l c_e - \kappa c_p + c_g} \right), \quad (\text{F.3})$$

$$\phi_{ij}^{(12)}(x_{ij}) = \frac{\log(x_{ij}^{(12)}) - \mu_{ij}^{(12)}(1 + \xi_j^{(12)})}{\sigma_{ij}^{(12)}}, \quad (\text{F.4})$$

and

$$\phi_{ij}^{(13)}(x_{ij}) = \frac{\log(x_{ij}^{(13)}) - \mu_{ij}^{(13)}(1 + \xi_j^{(13)})}{\sigma_{ij}^{(13)}}. \quad (\text{F.5})$$

Taking the derivative of the objective in (24) with respect to each parameter to be estimated, namely c_g , ρ , and for each industry j , $1 \leq j \leq N$, $\xi_j^{(12)}$ and $\xi_j^{(13)}$, we respectively obtain the following sample moment equations:

$$\begin{aligned} \sum_{j=1}^N \sum_{i=1}^{M_j^{(12)}} \nu_{ij}^{(12)} \sigma_{ij}^{(12)} e^{\frac{1}{2}(\phi_{ij}^{(12)}(\nu_{ij}^{(12)}))^2} \frac{c_{p(ij)}^{(12)} \rho}{(\Omega_{ij} + c_g)^2} (Q_{ij}^{(12)} - \nu_{ij}^{(12)}) \\ + \sum_{j=1}^N \sum_{i=1}^{M_j^{(13)}} \nu_{ij}^{(13)} \sigma_{ij}^{(13)} e^{\frac{1}{2}(\phi_{ij}^{(13)}(\nu_{ij}^{(13)}))^2} \frac{c_{p(ij)}^{(13)}(1 + r_f)}{(1 - a_l)(\Omega_{ij} + c_g)^2} (Q_{ij}^{(13)} - \nu_{ij}^{(13)}) = 0, \end{aligned} \quad (\text{F.6})$$

$$\sum_{j=1}^N \sum_{i=1}^{M_j^{(12)}} \nu_{ij}^{(12)} \sigma_{ij}^{(12)} e^{\frac{1}{2}(\phi_{ij}^{(12)}(\nu_{ij}^{(12)}))^2} \frac{c_{p(ij)}^{(12)}}{\Omega_{ij} + c_g} (Q_{ij}^{(12)} - \nu_{ij}^{(12)}) = 0, \quad (\text{F.7})$$

$$\begin{aligned} \sum_{i=1}^{M_j^{(12)}} \left\{ \nu_{ij}^{(12)} \mu_{ij}^{(12)} e^{\frac{1}{2}(\phi_{ij}^{(12)}(\nu_{ij}^{(12)}))^2} (Q_{ij}^{(12)} - \nu_{ij}^{(12)}) \right. \\ \left. \left(\int_0^{Q_{ij}^{(12)}} \phi_{ij}^{(12)}(D) \frac{e^{-\frac{1}{2}(\phi_{ij}^{(12)}(D))^2}}{\sqrt{2}} dD + 2e^{-(\phi_{ij}^{(12)}(Q_{ij}^{(12)}))^2} \phi_{ij}^{(12)}(Q_{ij}^{(12)}) \right) \right\} = 0, \forall j \in \{1, \dots, N\}, \end{aligned} \quad (\text{F.8})$$

and

$$\sum_{i=1}^{M_j^{(13)}} \left\{ \nu_{ij}^{(13)} \mu_{ij}^{(13)} e^{\frac{1}{2}(\phi_{ij}^{(13)}(\nu_{ij}^{(13)}))^2} (Q_{ij}^{(13)} - \nu_{ij}^{(13)}) \cdot \left(\int_0^{Q_{ij}^{(13)}} \phi_{ij}^{(13)}(D) \frac{e^{-\frac{1}{2}(\phi_{ij}^{(13)}(D))^2}}{\sqrt{2}} dD + 2e^{-(\phi_{ij}^{(13)}(Q_{ij}^{(13)}))^2} \phi_{ij}^{(13)}(Q_{ij}^{(13)}) \right) \right\} = 0, \forall j \in \{1, \dots, N\}. \quad (\text{F.9})$$

Note that, given the variation among the contract quantities and prices (Q_{ij} and p_{ij}) of the 7098 total SKU's in our estimation (as demonstrated in Table F.1 below), none of the $2N + 2$ equations in (F.6)-(F.9) can structurally be written as a perfect algebraic combination of a subset of the others. Therefore, for any solution to (F.6)-(F.9) in our estimation, r_{cl} , c_g , $\{\xi_j^{(12)}\}$ and $\{\xi_j^{(13)}\}$ are identifiable.

Table F.1: Summary Statistics for Order Quantity, Sales, Retail and Wholesale Price Observations
2012

	Order Quantity			Sales			Price			Wholesale Price		
	Mean	Median	Std.dev	Mean	Median	Std.dev	Mean	Median	Std.dev	Mean	Median	Std.dev
Auto Parts	1819.54	317.50	10797.13	1397.34	210.00	7398.04	133.91	61.00	299.92	108.75	47.00	246.29
Baby and Preg.	3517.31	1158.00	6297.05	1685.39	486.00	3218.86	75.77	48.00	72.84	65.94	42.00	62.53
Clothing	1182.52	30.00	9498.33	512.39	14.00	2819.15	1062.83	471.00	1274.77	902.66	372.50	1100.91
Coffee	525.05	196.00	889.35	269.70	104.50	507.60	164.23	97.00	161.12	127.19	77.00	118.88
Computer Acc.	6538.74	246.00	29266.85	5274.76	158.00	25216.32	219.49	77.00	273.25	200.62	67.00	253.76
Cosmetics	4904.61	285.00	14605.00	3911.05	156.00	11842.53	177.55	99.00	199.59	157.11	90.00	171.84
Electronics	5470.08	270.00	27639.64	4546.08	187.00	24059.43	320.88	97.00	874.84	275.08	79.00	766.9
Home Improv.	539.65	92.00	1145.82	417.85	66.00	928.03	294.28	278.00	205.51	240.52	232.00	164.06
Household App.	1896.67	216.00	6554.77	1529.03	112.50	6083.50	619.64	98.00	1450.03	571.34	77.50	1399.54
Sporting Goods	646.01	46.00	2750.00	578.44	20.00	2562.61	438.40	110.00	1364.72	352.28	89.00	1090.29
Staple Goods	4586.76	1204.00	9583.81	3309.2	742.00	7851.99	158.20	79.00	242.29	141.36	72.00	205.52
Wine	1080.20	300.00	2395.17	657.54	112.00	1654.05	334.47	149.00	704.20	239.97	115.00	408.85

	Order Quantity			Sales			Price			Wholesale Price		
	Mean	Median	Std.dev	Mean	Median	Std.dev	Mean	Median	Std.dev	Mean	Median	Std.dev
Auto Parts	3635.60	488.50	14593.92	3055.61	395.00	12242.69	130.15	60.00	309.88	105.95	45.00	247.82
Baby and Preg.	4512.83	1093.00	9152.56	2994.72	750.00	6549.37	74.35	46.00	72.16	65.33	42.00	62.16
Clothing	1099.60	88.50	6150.71	878.11	73.50	4384.67	1018.63	447.00	1247.93	885.94	372.50	1078.46
Coffee	450.75	221.50	682.19	344.84	158.50	594.11	146.09	90.00	147.93	126.39	81.00	116.81
Computer Acc.	9731.64	1050.50	35528.31	8653.73	981.50	31186.06	182.26	69.00	233.09	164.06	59.00	213.72
Cosmetics	7632.79	982.00	19551.02	6218.55	834.00	15755.42	171.49	78.00	209.90	155.70	72.00	185.05
Electronics	8573.63	485.00	45494.09	7604.34	403.00	43058.52	293.90	87.00	800.83	259.60	77.00	721.04
Home Improv.	2162.78	207.00	4512.52	1640.10	143.50	3609.96	268.48	226.00	199.98	227.78	198.50	164.66
Household App.	3275.87	265.00	10421.53	2795.02	143.50	9740.09	549.76	86.00	1340.72	513.32	83.00	1274.29
Sporting Goods	683.73	80.00	2382.54	597.07	69.00	2042.60	414.08	104.00	1335.89	339.56	79.00	1073.12
Staple Goods	8237.40	1734.50	20549.81	6896.12	1348.50	17935.60	151.91	80.00	223.14	140.78	75.50	198.56
Wine	4062.91	1630.50	7885.89	2441.74	868.00	4650.45	289.27	133.00	468.91	243.43	114.50	413.07

G Empirical Determination of Supplier Loan Access in 2012

In this section we give a detailed description of the procedure for empirical determination of the suppliers who could not secure a loan in 2012, mentioned in Section 4.2.2. This procedure is based on the equilibrium commercial loan interest rate calculation given in equation (C.10) in Section C of this online supplement and studied for the single-product case in the proof of Proposition 1 (as stated in Section C, the extension of the proof to the multi-product case is straight-forward). To explain this in more detail, for a supplier with M SKUs, if $B_0 \leq \sum_{i=1}^M c_{pi}Q_i$, then depending on the magnitude of the supplier's expected revenues, $\mathbb{E}\left[\sum_{i=1}^M w_i \min\{Q_i, D_i\}\right]$, there exists either a unique solution $r_{cl}^* > r_f$, or no solution to equation (C.10) (similar to the single-product version in the proof of Proposition 1). For the equilibrium loan amount $L = \sum_{i=1}^M c_{pi}Q_i - B_0$ as given in Proposition C.1, we can demonstrate the formation of equation (C.10) graphically as in the Figure G.1 below. Panel (a) illustrates the right and left hand sides of equation (C.10) as r_{cl} varies for the case where there is a solution. The intersection point of the two is the equilibrium competitive interest rate, r_{cl}^* set by the bank that makes the bank break even in expectation (please see the proof of Proposition 1 for details on the derivation, existence, and uniqueness of r_{cl}^* for the single SKU case). The intuition for this is, when the supplier's revenues are sufficiently high in expectation, i.e., if $\mathbb{E}[\sum_{i=1}^M w_i \min\{Q_i, D_i\}]$ is sufficiently high, then there is a high enough $r_{cl}^* > r_f$ that can make the bank break even between lending $\sum_{i=1}^M c_{pi}Q_i - B_0$ to the supplier and investing the same amount in the risk-free asset at the rate r_f . $G(\sum_{i=1}^M c_{pi}Q_i - B_0, r_{cl}^*) = 0$, where parallel to the proof of Proposition 1, G is defined as

$$G(l, r_{cl}) = (1 - \eta_a)\mathbb{E}[\min\{l(1 + r_{cl}), \sum_{i=1}^M ((B_0 + l - c_{pi}Q_i)(1 + r_f) + w_i \min\{Q_i, D_i\})\}] \\ + \eta_a \min\{l(1 + r_{cl}), \sum_{i=1}^M ((B_0 + l - c_{pi}Q_i)(1 + r_f))\} - l(1 + r_f), \quad (\text{G.1})$$

which implies

$$G(\sum_{i=1}^M c_{pi}Q_i - B_0, r_{cl}) = (1 - \eta_a)\mathbb{E}[\min\{l(1 + r_{cl}), \sum_{i=1}^M (w_i \min\{Q_i, D_i\})\}]. \quad (\text{G.2})$$

Notice that $G(L, r_{cl})$ becomes flat beyond a certain r_{cl} value, $\bar{r}_{cl} = (\sum_{i=1}^M w_i Q_i)/L - 1$ (follows from the multi-product extension of equation (B.4) with $L = \sum_{i=1}^M c_{pi}Q_i - B_0$) reflecting the fact that the expected loan payment the supplier can make is limited by the total cash position he will have after collecting

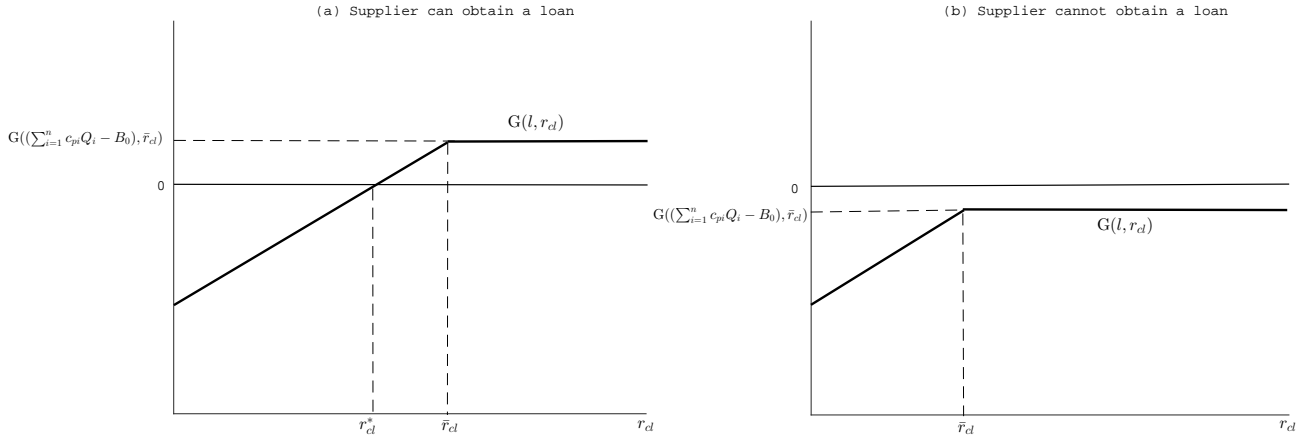


Figure G.1: The illustration of the cases for the existence of a commercial loan interest rate that can make the bank break even.

revenues, which is

$$\frac{G\left(\sum_{i=1}^M c_{pi} Q_i - B_0, \bar{r}_{cl}\right)}{1 - \eta_a} = \mathbb{E} \left[\sum_{i=1}^M w_i \min\{Q_i, D_i\} \right] - \left(\sum_{i=1}^M c_{pi} Q_i - B_0 \right) (1 + r_f).$$

This upper limit determines whether the supplier can obtain a loan from the bank or not. Panel (b) illustrates a case, where the supplier fails to secure a loan. In this case, the supplier's expected revenues, and hence his cash position before he pays off the loan, is lower than the bank's risk-free return from the loan, i.e., $G\left(\sum_{i=1}^M c_{pi} Q_i - B_0, \bar{r}_{cl}\right) < 0$. This means that no matter how high the bank sets the commercial loan interest rate, the expected amount the supplier pays back will be lower than the bank's cash position if it invests in the risk-free asset in the end. Hence, the bank would choose not to give a loan to the supplier.

Now, when we perform the estimation given in equation (24), we do not have the information on whether a given supplier in fact managed to receive a commercial loan in 2012. Rather, the objective in (24) assumes that all included suppliers received a commercial loan. Once we perform the estimation with this assumption, if the estimated parameters are consistent with the model, when we plug in the estimated parameters for each supplier for 2012, each supplier's commercial loan interest equation, $G\left(\sum_{i=1}^M c_{pi} Q_i - B_0, r_{cl}\right) = 0$, as shown in its open version in equation (26), would have a solution as demonstrated above. That is, the estimation needs to be checked with the model for consistency on the assumption that all suppliers included in the estimation would indeed obtain a loan under the estimated parameters as assumed in the estimation's objective function. If there are certain suppliers for whom, equation (26) is not satisfied, then these suppliers should not have been included in the estimation that assumes each included supplier secured a commercial loan in 2012, and estimated parameters will not be consistent with the model. Therefore,

Table H.1: Results for the Logistic Regression for Propensity Score Matching

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.1674	0.2119	0.79	0.4312
Auto parts	-0.0797	0.1242	-0.64	0.5224
Baby/Preg products	-0.0747	0.1176	-0.64	0.5265
Ceramics	-0.0492	0.2669	-0.18	0.8542
Clothing	-0.1473	0.0943	-1.56	0.1211
Coffee	-0.0494	0.2669	-0.19	0.8535
Computer accessories	-0.1844	0.1222	-1.51	0.1341
Cosmetics	-0.0156	0.0932	-0.17	0.8675
Electronic Products	-0.1752**	0.0757	-2.32	0.0224
Home improvement	-0.2252	0.1900	-1.19	0.2384
Household appliances	-0.2048*	0.1087	-1.88	0.0621
Sports	-0.2052*	0.1169	-1.76	0.0820
Yiwu	0.8815***	0.2039	4.32	0.0000
Fuoshan	0.9597***	0.2349	4.09	0.0001
Guangzhou	0.3367	0.2403	1.40	0.1638
Nanjing	0.9996**	0.3288	3.04	0.0029
Wuhan	0.2461	0.2626	0.94	0.3506
Shanghai	0.9697***	0.2130	4.55	0.0000
Shenzhen	0.9755***	0.2533	3.85	0.0002
Beijing	0.9096**	0.3463	2.63	0.0098
Jiaxing	0.1233	0.2265	0.54	0.5871
Sales Volume	0.0001	0.0002	0.60	0.5466

*Pseudo R*² : 0.4697, *N* = 133
p<0.01: ***, p<0.05: **, p<0.1: *

we iteratively take these suppliers out and re-estimate until for all included suppliers, equation (26) has a solution r_{cl}^* . This is essentially an extra step, which must be checked to ensure model consistency. In our particular estimation, this turns out to be a procedure that converges rather quickly. After the first estimation, we find that with the estimated parameters, for 8 out of 114 suppliers there is no solution to equation (26). Taking these suppliers out, re-performing the estimation, and rechecking equation (26) for each supplier shows that in this second iteration, with the estimated parameters, all suppliers are capable of securing a loan, and the estimation is consistent with the model.

H Propensity Score Matching Analysis for Control Group Selection

In this section, we present the Propensity Score Matching analysis to select the suppliers in the control groups that are used in testing our hypotheses in Section 4.3. To make sure that the suppliers in Treatment (BIF) Group, and the Control Groups 1 and 2 are comparable when we study the impact of BIF, we perform a logistic regression to assess a propensity score for each supplier, measuring their similarity to the suppliers in the treatment group. We use industry segment, location (the city the supplier is located in), and sales

volume as covariates in the logistic regression, as these variables are highly correlated with the strength of the suppliers in China and also affect how JD assesses supplier reliability. In our data set, we have 12 industry segments and 10 cities in which the suppliers are located. The regression equation is

$$P(BIF = 1|X) = \frac{1}{1 + e^{-\theta X}} = \frac{1}{1 + e^{-(\beta_0 + \sum_2^{12} \beta_i I_{\{Industry=i\}} + \sum_2^{10} \gamma_i I_{\{City=i\}} + \nu \cdot Sales)}} \quad (\text{H.1})$$

In (H.1), the base industry segment is Staple Goods and the base location is Zhuhai, and the sales are given in Millions of CNY. The regression outcome is given in Table H.1. Using this regression outcome, we can calculate the propensity score for each supplier in the control groups. This yields to a clear separation between the low propensity suppliers with scores on the range $[0, 0.399]$ and high propensity suppliers with scores on the range $[0.805, 0.965]$. We use the suppliers in the latter range to include in our final control and treatment groups to test Hypotheses 2 and 3, improving the match between the treatment and control groups.