

Implications of Market Spillovers

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ONLINE APPENDIX

In this online appendix, we provide the proofs for the lemmas and propositions presented in the paper.

I) Proof of Lemma 1:

The subgame equilibrium prices for $3t/2 < v_2 < 2t$ are derived in the main text, with profits defined by equations (2)-(8). With these prices, the spillover-producing market is fully covered if $v_2 > 3t/2$. If $v_2 < t$, both firms are local monopolies and set prices of $v_2/2$, earning profits of $mv_2^2/4t$.

CLAIM: For $t < v_2 < 3t/2$, in the {sell, sell} subgame, each firm charges the highest price possible in the spillover-producing market that will cover half of the market, which is $v_2 - t/2$. PROOF: No firm can deviate to higher or lower prices profitably. The profit of charging $v_2 - t/2$ is $m(v_2 - t/2)/2$ for both firms. If firm A increases its price, demand becomes $m(v_2 - p_{A2})/t$. The profit function becomes $m(v_2 p_{A2} - p_{A2}^2)/t$, which is decreasing in p_{A2} for $p_{A2} \geq v_2 - t/2$. If firm A decreases its price, the demand will be derived from $v_2 - yt - p_{A2} = v_2 - (1-y)t - (v_2 - t/2)$ and profit would be $m(t/2 + v_2 - p_{A2})p_{A2}/t$, which is diminished by decreasing p_{A2} from $v_2 - t/2$. Thus, $p_{A2} = v_2 - t/2$ is the equilibrium price. Similar analysis can be used to show $p_{B2} = v_2 - t/2$. \square

We summarize the payoffs from each possibility in Tables A1, A2, and A3.

Table A1. Profit Outcomes if $c < 3t - (v_A - v_B)$

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$\text{If } \frac{3t}{2} < v_2 < 2t : \begin{cases} \pi_A^{SShighv_2} = ((v_A - v_B) / 3 + t)^2 / 2t + mt / 2 \\ \pi_B^{SShighv_2} = ((v_B - v_A) / 3 + t)^2 / 2t + mt / 2 \end{cases}$ $\text{If } t < v_2 < \frac{3t}{2} : \begin{cases} \pi_A^{SSmedv_2} = ((v_A - v_B) / 3 + t)^2 / 2t + m(v_2 - t / 2) / 2 \\ \pi_B^{SSmedv_2} = ((v_B - v_A) / 3 + t)^2 / 2t + m(v_2 - t / 2) / 2 \end{cases}$ $\text{If } 0 < v_2 < t : \begin{cases} \pi_A^{SSlowv_2} = ((v_A - v_B) / 3 + t)^2 / 2t + mv_2^2 / 4t \\ \pi_B^{SSlowv_2} = ((v_B - v_A) / 3 + t)^2 / 2t + mv_2^2 / 4t \end{cases}$	$\begin{cases} \pi_A^{SNlowc} = ((v_B - v_A + c) / 3 - t)^2 / 2t + mv_2^2 / 4t \\ \pi_B^{SNlowc} = ((v_B - v_A + c) / 3 + t)^2 / 2t \end{cases}$
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NSlowc} = ((v_A - v_B + c) / 3 + t)^2 / 2t \\ \pi_B^{NSlowc} = ((v_A - v_B + c) / 3 - t)^2 / 2t + mv_2^2 / 4t \end{cases}$	$\begin{cases} \pi_A^{NN} = ((v_A - v_B) / 3 + t)^2 / 2t \\ \pi_B^{NN} = ((v_B - v_A) / 3 + t)^2 / 2t \end{cases}$

Table A2. Profit Outcomes if $3t - (v_A - v_B) < c < 3t + (v_A - v_B)$

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	Same as Table A1	Same as Table A1
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NShighc} = v_A - v_B + c - t \\ \pi_B^{NShighc} = mv_2^2 / 4t \end{cases}$	Same as Table A1

Table A3. Profit Outcomes if $c > 3t + (v_A - v_B)$

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	Same as Table A1	$\begin{cases} \pi_A^{SNhighc} = mv_2^2 / 4t \\ \pi_B^{SNhighc} = v_B - v_A + c - t \end{cases}$
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NShighc} = v_A - v_B + c - t \\ \pi_B^{NShighc} = mv_2^2 / 4t \end{cases}$	Same as Table A1

If $c < 3t - (v_A - v_B)$, then all subgames have interior solutions and the equilibrium is found from Table A1. If $3t - (v_A - v_B) < c < 3t + (v_A - v_B)$, then the {not sell, sell} subgame reaches a corner solution, resulting in Table A2. Finally if $c > 3t + (v_A - v_B)$, then both subgames with asymmetric strategies have corner solutions, shown in Table A3.

We start by analyzing the profits in Table A1 when $3t/2 < v_2 < 2t$, denoting $K \doteq v_A - v_B$. Comparing profits, $\pi_A^{SShighv_2} < \pi_A^{NSlowc}$ if $c > c_{NS/SS-H} \equiv -K - 3t + \sqrt{9mt^2 + (K + 3t)^2}$. Recall our bound on c for Table A1 and note that $c_{NS/SS-H} < 3t - K$ iff $K < 3t(\sqrt{4-m} - 1)$. Therefore in Table A1, $\pi_A^{SShighv_2} < \pi_A^{NSlowc}$ if $K < 3t(\sqrt{4-m} - 1)$ and $c > c_{NS/SS-H}$. Using similar logic we find the cutoffs for each range of v_2 . For $t < v_2 < 3t/2$,

$$\pi_A^{SSmedv_2} < \pi_A^{NSlowc} \quad \text{if} \quad K < -3t + 3\sqrt{t((8+m)t/2 - mv_2)} \quad \text{and}$$

$$c > c_{NS/SS-M} = -3t - K + \sqrt{9mt(v_2 - t/2) + (3t + K)^2}.$$

Similarly for $v_2 < t$, $\pi_A^{SSlowv_2} < \pi_A^{NSlowc}$ if $K < -3t + 3\sqrt{4t^2 - mv_2^2/2}$ and $c > c_{NS/SS-L} = -3t - K + \sqrt{9mv_2^2/2 + (3t + K)^2}$.

Also, for high v_2 , $\pi_B^{SNlowc} > \pi_B^{SShighv_2}$ if $c > c_{SN/SS-H} = -3t + K + \sqrt{9mt^2 + (3t - K)^2}$ and $K < t(3 - \sqrt{3m})$. For $t < v_2 < 3t/2$, $\pi_B^{SNlowc} > \pi_B^{SSmedv_2}$ if $K < 3t - \sqrt{3mt(v_2 - t/2)}$ and $c > c_{SN/SS-M} = -3t + K + \sqrt{9mt(v_2 - t/2) + (3t - K)^2}$. Similarly for $v_2 < t$, $\pi_B^{SNlowc} > \pi_B^{SSlowv_2}$ if $K < 3t - (\sqrt{3m}/2)v_2$ and $c > c_{SN/SS-L} = -3t + K + \sqrt{9mv_2^2/2 + (3t - K)^2}$.

Also, $\pi_A^{SNlowc} > \pi_A^{NN}$ for $v_2 > v_{2SN/NN} \equiv \sqrt{2c(6t + 2K - c)/(9m)}$. Finally, $\pi_B^{NSlowc} > \pi_B^{NN}$ for $v_2 > v_{2NS/NN} \equiv \sqrt{2c(6t - 2K - c)/(9m)}$. Both $v_{2SN/NN}$ and $v_{2NS/NN}$ have real values in for $c < 3t - K$.

Next we analyze profits in Table A2, when $3t - K < c < 3t + K$. For high v_2 , $\pi_A^{SShighv_2} > \pi_A^{NShighc}$ if $K > 3t(\sqrt{4-m} - 1)$ and $3t - K < c < c_{NS/SS2-H} = (3+m)t/2 + K(K-12t)/18t$; otherwise $\pi_A^{SShighv_2} < \pi_A^{NShighc}$. For medium v_2 , $\pi_A^{SSmedv_2} > \pi_A^{NShighc}$ if $K > -3t + 3\sqrt{t((8+m)t/2 - mv_2)}$ and $3t - K < c < c_{NS/SS2-M} = (6-m)t/4 + mv_2/2 + K(K-12t)/18t$; otherwise $\pi_A^{SSmedv_2} < \pi_A^{NShighc}$. Similarly, for low v_2 , we have $\pi_A^{SSlowv_2} > \pi_A^{NShighc}$ if $K > -3t + 3\sqrt{4t^2 - mv_2^2/2}$ and $3t - K < c < c_{NS/SS2-L} = mv_2^2/4t + (3t - K)(1/2 - K/18t)$; otherwise $\pi_A^{SSlowv_2} < \pi_A^{NShighc}$. Comparing π_B^{SS} and π_B^{SN} is similar to Table A1, but with different conditions on K : $3t - K < c_{SN/SS-H} < 3t + K$ if

$K > t(3 - \sqrt{3m})$, $3t - K < c_{SN/SS-M} < 3t + K$ if $K > 3t - \sqrt{3mt(v_2 - t/2)}$, and $3t - K < c_{SN/SS-L} < 3t + K$ if $K > 3t - \sqrt{3m/2}v_2$. We also find $\pi_A^{SNlowc} > \pi_A^{NN}$ if $v_2 > v_{2SN/NN} \equiv \sqrt{2c(6t + 2K - c)/(9m)}$, and $\pi_B^{NShighc} > \pi_B^{NN}$ if $v_2 > \sqrt{2/m}(3t - K)/3$.

Finally, we analyze Table A3 when $c > 3t + K$. For all $0 < K < 3t$, we know that $\pi_B^{SNhighc} > \pi_B^{SNhighc} \Big|_{c \rightarrow 3t+K} = 2t$ and $\pi_B^{SSlowv_2}, \pi_B^{SSmedv_2}, \pi_B^{SShighv_2} < \pi_B^{SShighv_2} \Big|_{K \rightarrow 0} = (m+1)t/2 < t$, which means in this table we always have $\pi_B^{SNhighc} > \pi_B^{SS}$, where π_B^{SS} could be any of the three profits $\pi_B^{SSlowv_2}$, $\pi_B^{SSmedv_2}$, or $\pi_B^{SShighv_2}$. Also for $0 < K < 3t$, we know $c_{NS/SS-H}, c_{NS/SS-M}, c_{NS/SS-L} < 3t + K$, which means in Table 3 we always have $\pi_A^{NShighc} > \pi_A^{SS}$. The inequality $\pi_A^{NShighc} > \pi_A^{NN}$ holds if $v_2 > \sqrt{2/m}(3t + K)/3$, and $\pi_B^{NShighc} > \pi_B^{NN}$ holds if $v_2 > \sqrt{2/m}(3t - K)/3$.

We compare the condition for firm A not deviating from {sell, sell} with the condition for firm B not deviating to see which condition is stricter. Starting with high v_2 , we show that for $K > 0$, $c_{NS/SS-H} < c_{SN/SS-H}$; taking the derivatives of $c_{NS/SS-H}$ and $c_{SN/SS-H}$ with respect to K , we find $\partial c_{NS/SS-H} / \partial K = -1 + (K + 3t) / \sqrt{(K + 3t)^2 + 9mt^2} < 0$ and $\partial c_{SN/SS-H} / \partial K = 1 - (K - 3t) / \sqrt{(K - 3t)^2 + 9mt^2} > 0$. Also note that $c_{NS/SS-H} \Big|_{K=0} = c_{SN/SS-H} \Big|_{K=0}$. Thus, $c_{NS/SS-H}$ and $c_{SN/SS-H}$ have the same value at $K = 0$, but $c_{NS/SS-H}$ is decreasing in K , while $c_{SN/SS-H}$ is increasing. Thus for all $K > 0$, $c_{NS/SS-H} < c_{SN/SS-H}$. For $K > 3t(\sqrt{4-m} - 1)$, we have $c_{NS/SS2-H} < c_{NS/SS-H}$ which also implies $c_{NS/SS2-H} < c_{SN/SS-H}$. Using similar logic for other ranges of v_2 , we show that $c_{NS/SS-M} < c_{SN/SS-M}$ and for $K > -3t + 3\sqrt{t((8+m)t/2 - mv_2^2)}$ we have $c_{NS/SS2-M} < c_{NS/SS-M} < c_{SN/SS-M}$. Finally, $c_{NS/SS-L} < c_{SN/SS-L}$ and for $K > -3t + 3\sqrt{4t^2 - mv_2^2}/2$ we have $c_{NS/SS2-L} < c_{NS/SS-L} < c_{SN/SS-L}$.

This proves that the {sell, sell} equilibrium exists only for $c < c'$. Let $r \in \{L, M, H\}$ represent the region to which v_2 belongs, such that $r = L$ requires $0 < v_2 < t$, $r = M$ requires $t < v_2 < 3t/2$, and $r = H$

requires $3t/2 < v_2 < 2t$. The definition of c' is such that for v_2 belonging to the region $r \in \{L, M, H\}$,

$$c' = c_{NS/SS-r} \quad \text{if } K < K_r^*, \quad \text{and} \quad c' = c_{NS/SS2-r} \quad \text{if } K > K_r^*, \quad \text{where } K_L^* = -3t + 3\sqrt{4t^2 - mv_2^2/2},$$

$$K_M^* = -3t + 3\sqrt{t((8+m)t/2 - mv_2^2)}, \quad \text{and} \quad K_H^* = 3t(\sqrt{4-m} - 1).$$

For the {not sell, not sell} equilibrium, we compare the $\pi_A^{NN} > \pi_A^{SN}$ and $\pi_B^{NN} > \pi_B^{NS}$ conditions. For $c < 3t - K$, we have $v_{2NS/NN} < v_{2SN/NN}$. For $3t - K < c < 3t + K$, the minimum of $v_{2SN/NN}$ occurs at $c = 3t - K$ and is equal to $\sqrt{2(3t - K)(3t + 3K)/(9m)}$ which is greater than $\sqrt{2(3t - K)/(3\sqrt{m})}$. Thus, we have $v_{2SN/NN} > \sqrt{2(3t - K)/(3\sqrt{m})}$. Finally, for $c > 3t + K$ the threshold for $\pi_B^{NN} > \pi_B^{NS}$, $v_2 = \sqrt{2/m}(3t - K)/3$, is less than the threshold for $\pi_A^{NN} > \pi_A^{SN}$, $v_2 = \sqrt{2/m}(3t + K)/3$. Thus the {not sell, not sell} equilibrium exists iff $v_2 < v'_2$, where $v'_2 = v_{2NS/NN}$ for $c < 3t - K$ and $v'_2 = \sqrt{2/m}(3t - K)/3$ for $c > 3t - K$. *Q.E.D.*

II) Proof of Proposition 1:

Based on the proof of Lemma 1, the {not sell, sell} equilibrium exists for $c > c'$ and $v_2 > v'_2$. Also the {sell, not sell} equilibrium exists for $c > c''$ and $v_2 > v''_2$. Let v_2 belong to the region $r \in \{L, M, H\}$. We define $c'' = c_{SN/SS-r}$, $v''_2 = v_{2SN/NN}$ for $c < 3t + K$, and $v''_2 = \sqrt{2/m}(3t + K)/3$ for $c > 3t + K$. We showed $v'_2 < v''_2$ and thus the region for $\pi_A^{SN} > \pi_A^{NN}$ is a subset of the region for $\pi_B^{NS} > \pi_B^{NN}$. We also showed $c' < c''$ and thus the region for $\pi_B^{SN} > \pi_B^{SS}$ is a subset of the region for $\pi_A^{NS} > \pi_A^{SS}$. Thus, the {not sell, sell} equilibrium is unique for $c' < c < c''$ and $v'_2 < v_2 < v''_2$. This proves Proposition 1(a).

Next we prove the risk-dominance of the {not sell, sell} equilibrium over the {sell, not sell} equilibrium. The condition for the risk-dominance of the {not sell, sell} equilibrium is $RD = (\pi_A^{NS} - \pi_A^{SS})(\pi_B^{NS} - \pi_B^{NN}) - (\pi_A^{SN} - \pi_A^{NN})(\pi_B^{SN} - \pi_B^{SS}) > 0$. For low v_2 , there are three tables to consider. For Table A1, $RD = 2c^3K/81t^2 > 0$. For Table A2, $\partial(RD)/\partial v_2 = mv_2(c - 3t + K)/6t$ is positive, meaning RD is minimized with respect to v_2 at $v_2 = 0$. $\partial(RD)|_{v_2 \rightarrow 0}/\partial K$ is continuous in K and never

zero for $0 < K < 3t$, which means $RD|_{v_2 \rightarrow 0}$ is monotonic for $0 < K < 3t$. Thus, the minimum of $RD|_{v_2 \rightarrow 0}$ with respect to K is at one of the corners of the region $0 \leq K \leq 3t$. Since $RD|_{v_2 \rightarrow 0, K \rightarrow 0} > 0$ and $RD|_{v_2 \rightarrow 0, K \rightarrow 3t} > 0$, the minimum of $RD|_{v_2 \rightarrow 0}$ is positive, thus $RD|_{v_2 \rightarrow 0} > 0$ and $RD > 0$. For Table A3, $RD = (6(2c - 5t)t + 3mv_2^2 - 2K^2)K / 18t > 0$ for $c > \hat{c} = (-3mv_2^2 + 30t^2 + 2K^2) / 12t$, where $\hat{c} < 3t + K$ for all $K < 3t$. Using similar logic, we show that for higher values of v_2 , $RD > 0$. *Q.E.D.*

Proof for when Firm A is Superior in Both Markets:

We assume firm A's spillover-producing product value is $v_2 + K_2$, while firm B's is v_2 . We show solutions for $v_2 < t - K_2 / 2$, when both firms can have local monopolies in the spillover-producing market. Table A4 shows profits when c is low enough.

Table A4. Profit Outcomes if Firm A is Superior in the Spillover-Producing Market by K_2

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$\begin{cases} \pi_A^{SSK_2} = ((v_A - v_B) / 3 + t)^2 / 2t + m(v_2 + K_2)^2 / 4t \\ \pi_B^{SSK_2} = ((v_B - v_A) / 3 + t)^2 / 2t + mv_2^2 / 4t \end{cases}$	$\begin{cases} \pi_A^{SNK_2} = ((v_B - v_A + c) / 3 - t)^2 / 2t + m(v_2 + K_2)^2 / 4t \\ \pi_B^{SNK_2} = ((v_B - v_A + c) / 3 + t)^2 / 2t \end{cases}$
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NSK_2} = ((v_A - v_B + c) / 3 + t)^2 / 2t \\ \pi_B^{NSK_2} = ((v_A - v_B + c) / 3 - t)^2 / 2t + mv_2^2 / 4t \end{cases}$	$\begin{cases} \pi_A^{NNK_2} = ((v_A - v_B) / 3 + t)^2 / 2t \\ \pi_B^{NNK_2} = ((v_B - v_A) / 3 + t)^2 / 2t \end{cases}$

We find $\pi_A^{SSK_2} > \pi_A^{NSK_2}$ for $c < c_{NS/SSK_2} = -3t - K + \sqrt{9m(v_2 + K_2)^2 / 2 + (3t + K)^2}$. Also, $\pi_B^{SSK_2} > \pi_B^{SNK_2}$ for $c < c_{SN/SSK_2} = -3t + K + \sqrt{9mv_2^2 / 2 + (3t - K)^2}$. We find $c_{NS/SSK_2} < c_{SN/SSK_2}$ for $K_2 < K_2^* = \sqrt{v_2^2 + 8K \left(-3t + K + \sqrt{9mv_2^2 / 2 + (3t - K)^2} \right) / 9m - v_2}$. Similarly from Table A4 we know $\pi_A^{NNK_2} > \pi_A^{SNK_2}$ for $c > c_{SN/NNK_2} = 3t + K - \sqrt{(3t + K)^2 - 9m(K_2 + v_2)^2 / 2}$. Also $\pi_B^{NNK_2} > \pi_B^{NSK_2}$ for $c > c_{NS/NNK_2} = 3t - K - \sqrt{(3t - K)^2 - 9mv_2^2 / 2}$. We find that $c_{NS/NNK_2} > c_{SN/NNK_2}$ for $K_2 < K_2^{**} = \sqrt{v_2^2 + 8K \left(3t - K - \sqrt{(3t - K)^2 - 9mv_2^2 / 2} \right) / 9m - v_2}$. For $K > 0$, we have $K_2^* < K_2^{**}$. Thus as

long as $K_2 < K_2^*$, the {sell, not sell} equilibrium is a subset of the {not sell, sell} equilibrium. When

$$K_2 = K, \quad \text{for} \quad K > \hat{K} = (2(8-9m)v_2 - 4(6t + \sqrt{36t^2 + 48tv_2 + 2(-8+9m)v_2^2})) / (-16+9m) \quad \text{we have}$$

$$K_2^* > K. \quad \text{We show } \hat{K} < 3t \text{ for } m < 32v_2^2 / (9(3t + 2v_2)^2). \quad \text{Q.E.D.}$$

III) Proof of Proposition 2:

If no negative market spillover exists, then the two markets are independent. The firms' profits would be the same as in the {sell, sell} subgame shown in Table A1. We denote these profits π_A^Z and π_B^Z . In the {not sell, sell} equilibrium we have $\pi_A^Z = \pi_A^{SS} < \pi_A^{NS}$. In Table A1, for $3t/2 < v_2 < 2t$, we have

$$\pi_B^{Zhighv_2} < \pi_B^{NSlowc} \quad \text{if} \quad v_2 > v_{2NS/Zhigh} = \sqrt{2(9mt^2 + 6ct - c^2 - 2cK) / (9m)}. \quad \text{For } t < v_2 < 3t/2, \text{ we have}$$

$$\pi_B^{Zmedv_2} < \pi_B^{NSlowc} \quad \text{if} \quad v_2 > v_{2NS/ZMed} = t + \sqrt{2c(6t - c - 2K) / (9m)}. \quad \text{For } v_2 < t, \pi_B^{Zlowv_2} > \pi_B^{NSlowc}. \quad \text{We find}$$

$$3t/2 < v_{2NS/Zhigh} < 2t \text{ requires } c(6t - 2K - c) / (9t^2) < m < 8c(6t - 2K - c) / (9t^2). \quad \text{Also } t < v_{2NS/ZMed} < 3t/2$$

requires $m > 8c(6t - 2K - c) / (9t^2)$. We know $m > 8c(6t - 2K - c) / (9t^2)$ results in $v_{2NS/Zhigh} > v_{2NS/ZMed}$.

Thus, $\pi_B^Z < \pi_B^{NS}$ for $v_2 > \text{Min}\{v_{2NS/Zhigh}, v_{2NS/ZMed}\}$ and $m > c(6t - 2K - c) / (9t^2)$. Similarly, for tables A2

and A3, we find $\pi_B^Z < \pi_B^{NS}$ if $v_2 > \text{Min}\{\sqrt{2(9(1+m)t^2 - K(6t - K)) / (9m)}, t + \sqrt{2(3t - K) / (3\sqrt{m})}\}$ and

$m > (3t - K)^2 / (9t^2)$. Thus,

$$\hat{v}_2 = \begin{cases} \text{Min}\{\sqrt{2(9mt^2 + 6ct - c^2 - 2cK) / (9m)}, t + \sqrt{2c(6t - c - 2K) / (9m)}\} & \text{if } c < 3t - K \\ \text{Min}\{\sqrt{2(9(1+m)t^2 - K(6t - K)) / (9m)}, t + \sqrt{2(3t - K) / (3\sqrt{m})}\} & \text{if } c \geq 3t - K \end{cases} \quad \text{and}$$

$$\hat{m} = \begin{cases} c(6t - 2K - c) / (9t^2) & \text{if } c < 3t - K \\ (3t - K)^2 / (9t^2) & \text{if } c \geq 3t - K \end{cases}. \quad \text{Q.E.D.}$$

Proof for Symmetric Firms:

The analysis is similar to our proof for Lemma 1, with $K = 0$. If $c < 3t$, there's an interior solution and if

$c > 3t$, we get a corner solution in both asymmetric subgames. When $K = 0$, we find that $v_2'|_{K \rightarrow 0} = v_2''|_{K \rightarrow 0}$

and $c'|_{K \rightarrow 0} = c''|_{K \rightarrow 0}$. Thus, for $v_2 > v'_2|_{K \rightarrow 0}$ and $c > c'|_{K \rightarrow 0}$, {not sell, sell} equilibrium and {sell, not sell} equilibrium both exist. For $c < 3t$, we have $\pi_B^Z < \pi_B^{NS}$ if $v_2 > \text{Min}\{\sqrt{2(9mt^2 + 6ct - c^2) / (9m)}, t + \sqrt{2c(6t - c) / (9m)}\}$ and $m > c(6t - c) / (9t^2)$. For $c > 3t$, for all $0 < m < 1$ we have $\pi_B^Z > \pi_B^{NS}$. Also, in the {not sell, sell} equilibrium, $\pi_A^Z < \pi_A^{NS}$. *Q.E.D.*

Proof for Total Industry Profits:

We find the region where the market spillover increases the sum of the two firms' profits. We show the proof for Table A1. The proof for Tables A2 and A3 follows similar steps. We have

$$(\pi_A^{Zhighv_2} + \pi_B^{Zhighv_2}) < (\pi_A^{NSlowc} + \pi_B^{NSlowc}) \text{ for } c > c_{T1} = \sqrt{4(K^2 + 9mt^2) - 9mv_2^2} / 2 - K.$$

$$(\pi_A^{Zmedv_2} + \pi_B^{Zmedv_2}) < (\pi_A^{NSlowc} + \pi_B^{NSlowc}) \text{ for } c > c_{T2} = \sqrt{4K^2 - 18mt^2 + 9mv_2(4t - v_2)} / 2 - K, \text{ and}$$

$$(\pi_A^{Zlowv_2} + \pi_B^{Zlowv_2}) < (\pi_A^{NSlowc} + \pi_B^{NSlowc}) \text{ for } c > c_{T3} = \sqrt{4K^2 + 9mv_2^2} / 2 - K.$$

Thus, the market spillover increases total profits in the {not sell, sell} equilibrium for $c > \text{Max}\{c_T, c'\}$, where

$$c_T = \begin{cases} c_{T1} & \text{if } 3t/2 < v_2 < 2t \\ c_{T2} & \text{if } t < v_2 < 3t/2 \\ c_{T3} & \text{if } 0 < v_2 < t \end{cases} . \text{ Q.E.D.}$$

IV) Proof of Proposition 3:

In Table A1, $\pi_A^{NS} < \pi_B^{NS}$ holds true if $v_2 > 2\sqrt{2/3m}\sqrt{ct + Kt}$, which guarantees $v_2 > v_{2NS/NN}$ and thus satisfies the {not sell, sell} equilibrium condition. Also, $2\sqrt{2/3m}\sqrt{ct + Kt}$ is below $v_2 = 2t$ if $c < 3mt/2 - K$, which can be satisfied in Table A1, since $3mt/2 - K$ is bigger than $c_{NS/SS-H}$ for $K < 3(\sqrt{4+m} - 2)t$. Thus the region where $\pi_A^{NS} < \pi_B^{NS}$ can satisfy the conditions $c > c'$ and $v_2 > v'_2$ required for the {not sell, sell} equilibrium in Table A1. Considering Tables A2 and A3, $\pi_A^{NS} < \pi_B^{NS}$ requires $v_2 > 2\sqrt{t/m}\sqrt{c-t+K}$. The threshold $2\sqrt{t/m}\sqrt{c-t+K}$ is larger than $v_2 = 2t$ for all $c > 3t - K$. Thus, only Table A1 can result in $\pi_A^{NS} < \pi_B^{NS}$.

Assuming firm A can costlessly reduce v_A to $\tilde{v}_A < v_A$, $\tilde{K} = v_B - \tilde{v}_A$ must become large enough such that $c > \tilde{c}_{NS/SS-H} \equiv -\tilde{K} - 3t + \sqrt{9mt^2 + (\tilde{K} + 3t)^2}$, in order to cause firm B to leave the spillover-producing market. Thus, we must have $\tilde{v}_A < v_B - (9mt^2 / 2c - c / 2 - 3t)$, which is not possible for $v_B < 9mt^2 / 2c - c / 2 - 3t$. *Q.E.D.*

V) Proof of Lemma 2:

The firms' profits in the four subgames are shown in the table below.

Table A5. Profits with a positive spillover (assuming $K \leq 3t - c$)

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$\text{If } \frac{3t}{2} < v_2 < 2t : \begin{cases} \pi_A^{SShighv_2} = (K/3+t)^2 / 2t + mt / 2 - O \\ \pi_B^{SShighv_2} = (t-K/3)^2 / 2t + mt / 2 - O \end{cases}$ $\text{If } t < v_2 < \frac{3t}{2} : \begin{cases} \pi_A^{SSmedv_2} = (K/3+t)^2 / 2t + m(v_2-t/2) / 2 - O \\ \pi_B^{SSmedv_2} = (t-K/3)^2 / 2t + m(v_2-t/2) / 2 - O \end{cases}$ $\text{If } 0 < v_2 < t : \begin{cases} \pi_A^{SSlowv_2} = (K/3+t)^2 / 2t + mv_2^2 / 4t - O \\ \pi_B^{SSlowv_2} = (t-K/3)^2 / 2t + mv_2^2 / 4t - O \end{cases}$	$\pi_A^{SN} = ((K+c)/3+t)^2 / 2t + mv_2^2 / 4t - O$ $\pi_B^{SN} = ((K+c)/3-t)^2 / 2t$
A: Not Sell Spillover-Producing Product	$\pi_A^{NS} = ((K-c)/3+t)^2 / 2t$ $\pi_B^{NS} = ((c-K)/3+t)^2 / 2t + mv_2^2 / 4t - O$	$\pi_A^{NN} = (K/3+t)^2 / 2t$ $\pi_B^{NN} = (t-K/3)^2 / 2t$

Comparing the profits in Table A5, for $c > c_{NS/SS-H} \equiv 3t + K - \sqrt{(K+3t)^2 + 9t(mt-2O)}$ we find $\pi_A^{SShighv_2} > \pi_A^{NS}$. Also, $\pi_A^{SSmedv_2} > \pi_A^{NS}$ if $v_2 > \tilde{v}_{NS-M} = (9t(4O + (m-2)t) - 2K(K+6t)) / (18mt)$ and $c > c_{NS/SS-M} = 3t + K - \sqrt{(K+3t)^2 - 9t(4O + mt - 2mv_2)} / 2$. Finally, we have $\pi_A^{SSlowv_2} > \pi_A^{NS}$ if $v_2 > \tilde{v}_{NS-L} = \sqrt{2/m} \sqrt{9t(2O-t) - K(K+6t)} / 3$ and $c > c_{NS/SS-L} = 3t + K - \sqrt{(K+3t)^2 + 9(mv_2^2 - 4Ot)} / 2$.

Considering firm B, we find $\pi_B^{SShighv_2} > \pi_B^{SN}$ for $c > c_{SN/SS-H} \equiv 3t - K - \sqrt{(3t-K)^2 + 9t(mt-2O)}$. Also, we have $\pi_B^{SSmedv_2} > \pi_B^{SN}$ if $v_2 > \tilde{v}_{SN-M} = (2K(-K+6t) + 9t(4O + (m-2)t)) / (18mt)$ and

$c > c_{SN/SS-M} = 3t - K - \sqrt{(-K + 3t)^2 - 9t(4O + mt - 2mv_2)}/2$. Finally, we have $\pi_B^{SSlowv_2} > \pi_B^{SN}$ if

$$v_2 > \tilde{v}_{SN-L} = \sqrt{2/m} \sqrt{9(2O-t)t + K(6t-K)}/3 \text{ and } c > c_{SN/SS-L} = 3t - K - \sqrt{(-K + 3t)^2 + 9(mv_2^2 - 4Ot)}/2.$$

We prove $\pi_B^{SS} > \pi_B^{SN}$ guarantees $\pi_A^{SS} > \pi_A^{NS}$. We know $c_{NS/SS-r}|_{K \rightarrow 0} = c_{SN/SS-r}|_{K \rightarrow 0}$ for $r \in \{L, M, H\}$.

We also have $\frac{\partial c_{NS/SS-r}}{\partial K} < \frac{\partial c_{SN/SS-r}}{\partial K}$ for $r \in \{L, M, H\}$, when $K > 0$. Thus, for $K > 0$, we have

$c_{NS/SS-r} < c_{SN/SS-r}$. We also have $\tilde{v}_{SN-M} > \tilde{v}_{NS-M}$ and $\tilde{v}_{SN-L} > \tilde{v}_{NS-L}$. Thus, for $c > c'_+ = c_{SN/SS-r}$ and

$v_2 > \tilde{v} = \tilde{v}_{SN-r}$, we have $\pi_B^{SS} > \pi_B^{SN}$ and $\pi_A^{SS} > \pi_A^{NS}$, and therefore {sell, sell} is an equilibrium.

Considering the profits when neither firm sells the spillover-producing product, we find $\pi_A^{NN} > \pi_A^{SN}$

for $v_2 < v_{2SN/NN} = \sqrt{2/m} \sqrt{-2cK - c^2 - 6ct + 18Ot}/3$. Similarly, we find $\pi_B^{NN} > \pi_B^{NS}$ for

$v_2 < v_{2NS/NN} = \sqrt{2/m} \sqrt{2cK - c^2 - 6ct + 18Ot}/3$. We have $v_{2SN/NN} < v_{2NS/NN}$. Thus $\pi_A^{NN} > \pi_A^{SN}$ guarantees

$\pi_B^{NN} > \pi_B^{NS}$ and {not sell, not sell} is an equilibrium for $v_2 < \dot{v}_2 = v_{2SN/NN}$. *Q.E.D.*

VI) Proof of Proposition 4:

Based on the proof of Lemma 2, assuming $v_2 > \tilde{v}$, the {sell, not sell} equilibrium exists for $c < c'_+ = c_{SN/SS-r}$

and $v_2 > \dot{v}_2 = v_{2SN/NN}$. Also the {not sell, sell} equilibrium exists for $c < c''_+ = c_{NS/SS-r}$ and $v_2 > \ddot{v}_2 = v_{2NS/NN}$.

We showed $\dot{v}_2 < \ddot{v}_2$ and thus the region for $\pi_B^{NS} > \pi_B^{NN}$ is a subset of the region for $\pi_A^{SN} > \pi_A^{NN}$. We also

showed $c'_+ > c''_+$ and thus the region for $\pi_A^{NS} > \pi_A^{SS}$ is a subset of the region for $\pi_B^{SN} > \pi_B^{SS}$. Thus, the

{sell, not sell} equilibrium is unique for $c''_+ < c < c'_+$ and $\dot{v}_2 < v_2 < \ddot{v}_2$.

The condition for the {sell, not sell} equilibrium risk-dominating the {not sell, sell} equilibrium is

$$RD_{SN} = (\pi_A^{SN} - \pi_A^{NN})(\pi_B^{SN} - \pi_B^{SS}) - (\pi_A^{NS} - \pi_A^{SS})(\pi_B^{NS} - \pi_B^{NN}) > 0. \text{ For } 3t/2 < v_2 < 2t, \text{ we find}$$

$$RD_{SN} = cK(4c^2 - 18mt^2 + 9mv_2^2)/162t^2. \text{ Since } 3t/2 < v_2 < 2t, \text{ we have } RD_{SN} > 0 \text{ and thus the \{sell, not}$$

sell} equilibrium is risk-dominant. The proof for low and medium values of v_2 follows similar steps. *Q.E.D.*

VII) Proof of Proposition 5:

If no negative market spillover exists, then the firms' profits for $O > mv_2^2 / 4t$ would be the same as in the {not sell, not sell} subgame shown in Table A5. We denote these profits π_A^Z and π_B^Z . In the {sell, not sell} equilibrium, we have $\pi_A^Z = \pi_A^{NN} < \pi_A^{SN}$. Considering the inferior firm, we have $\pi_B^{SN} = (3t - K - c)^2 / 18t < \pi_B^Z = (3t - K)^2 / 18t$. In the {sell, sell} equilibrium, we have $\pi_A^Z > \pi_A^{SS}$ and $\pi_B^Z > \pi_B^{SS}$, since $O > mv_2^2 / 4t$. *Q.E.D.*

Proof for Total Industry Profits: We find the region where the market spillover increases the sum of the two firms' profits. From Table A5, We find $(\pi_A^{NN} + \pi_B^{NN}) < (\pi_A^{SN} + \pi_B^{SN})$ for $c > c'_T = \sqrt{K^2 + 9Ot - 9mv_2^2 / 4} - K$. Thus, the positive market spillover increases total profits in the {sell, not sell} equilibrium for $c > c'_T$. *Q.E.D.*

VIII) Proof of Lemma 3

Firms' profits from each of the subgames are presented in Table A6.

Table A6. Profits with both positive and negative spillovers (assuming and $3t / 2 < v_2 < 2t$)

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$\text{If } \frac{3t}{2} < v_2 < 2t : \begin{cases} \pi_A^{SShighv_2} = (K / 3 + t)^2 / 2t + mt / 2 - O \\ \pi_B^{SShighv_2} = (t - K / 3)^2 / 2t + mt / 2 - O \end{cases}$ $\text{If } t < v_2 < \frac{3t}{2} : \begin{cases} \pi_A^{SSmedv_2} = (K / 3 + t)^2 / 2t + m(v_2 - t / 2) / 2 - O \\ \pi_B^{SSmedv_2} = (t - K / 3)^2 / 2t + m(v_2 - t / 2) / 2 - O \end{cases}$ $\text{If } 0 < v_2 < t : \begin{cases} \pi_A^{SSlowv_2} = (K / 3 + t)^2 / 2t + mv_2^2 / 4t - O \\ \pi_B^{SSlowv_2} = (t - K / 3)^2 / 2t + mv_2^2 / 4t - O \end{cases}$	$\pi_A^{SN} = \alpha((K + c_{pos}) / 3 + t)^2 / 2t + (1 - \alpha)((K - c_{neg}) / 3 + t)^2 / 2t + mv_2^2 / 4t - O$ $\pi_B^{SN} = \alpha((K + c_{pos}) / 3 - t)^2 / 2t + (1 - \alpha)((K - c_{neg}) / 3 - t)^2 / 2t$
A: Not Sell Spillover-Producing Product	$\pi_A^{NS} = \alpha((K - c_{pos}) / 3 + t)^2 / 2t + (1 - \alpha)((K + c_{neg}) / 3 + t)^2 / 2t$ $\pi_B^{NS} = \alpha((-K + c_{pos}) / 3 + t)^2 / 2t + (1 - \alpha)((-K - c_{neg}) / 3 + t)^2 / 2t + mv_2^2 / 4t - O$	$\pi_A^{NN} = (K / 3 + t)^2 / 2t$ $\pi_B^{NN} = (-K / 3 + t)^2 / 2t$

Comparing the profits from Table A6, we find the conditions for each equilibrium. We show the proof for $3t/2 < v_2 < 2t$. The proof for lower v_2 includes similar steps. We have $\pi_A^{SShighv_2} > \pi_A^{NS}$ for $O < O_{IA} = (2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(-c_{pos} + c_{neg} + 6t) - c_{neg}^2 - 6c_{neg}t + 9mt^2) / 18t$. Also for $O < O_{IB} = (-2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(-c_{pos} + c_{neg} + 6t) - c_{neg}^2 - 6c_{neg}t + 9mt^2) / 18t$, we have $\pi_B^{SShighv_2} > \pi_B^{SN}$. Thus, the {sell, sell} equilibrium exists for $O < O_l = \text{Min}\{O_{IA}, O_{IB}\}$. Note that for $\alpha < c_{neg} / (c_{pos} + c_{neg})$, we have $O_{IA} < O_{IB}$; otherwise, we have $O_{IA} > O_{IB}$.

Next, we find the condition for the {not sell, not sell} equilibrium. We have $\pi_A^{NN} > \pi_A^{SN}$ for $O > O_{hA} = (2(2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(c_{pos} - c_{neg} + 6t) + c_{neg}(c_{neg} - 6t)) + 9mv_2^2) / 36t$. Also, for $O > O_{hB} = (2(-2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(c_{pos} - c_{neg} + 6t) + c_{neg}(c_{neg} - 6t)) + 9mv_2^2) / 36t$, we have $\pi_B^{NN} > \pi_B^{NS}$. Thus, the {not sell, not sell} equilibrium exists for $O > O_h = \text{Max}\{O_{hA}, O_{hB}\}$. Note that for $\alpha < c_{neg} / (c_{pos} + c_{neg})$, we have $O_{hA} < O_{hB}$; otherwise, we have $O_{hA} > O_{hB}$.

The {not sell, sell} equilibrium exists for $O_{IA} < O < O_{hB}$ and the {sell, not sell} equilibrium exists for $O_{IB} < O < O_{hA}$. Thus, both equilibria exist when $\text{Max}\{O_{IA}, O_{IB}\} < O < \text{Min}\{O_{hA}, O_{hB}\}$. For this region, we find the risk dominant equilibrium. The condition for the risk dominance of the {sell, not sell} equilibrium

is $RD_{SN} = (\pi_A^{SN} - \pi_A^{NN})(\pi_B^{SN} - \pi_B^{SS}) - (\pi_A^{NS} - \pi_A^{SS})(\pi_B^{NS} - \pi_B^{NN}) > 0$. We find

$RD_{SN} = K(\alpha c_{pos} - (1 - \alpha)c_{neg})(4\alpha c_{pos}^2 + 4(1 - \alpha)c_{neg}^2 - 18mt^2 + 9mv_2^2) / 162t^2$. For $v_2 > 3t/2$, we have

$RD_{SN} > 0$ if and only if $\alpha > c_{neg} / (c_{pos} + c_{neg})$. Similarly, we find $RD_{NS} = -RD_{SN} > 0$ if and only if

$\alpha < c_{neg} / (c_{pos} + c_{neg})$. *Q.E.D.*

IX) Proof for the Model of Market Spillover as a Change in Purchase Quantity Per Consumer:

We show the proof of analysis for high v_2 , $v_2 > 3t/2$.

When the market spillover is negative: Comparing profits from Table 3, we find $\pi_A^{SS} > \pi_A^{NS}$ for

$g < g_{NS/SS} = (3t + K)^2 / ((3t + K)^2 - 9mt^2)$. Also, we find $\pi_B^{SS} > \pi_B^{SN}$ for

$g < g_{SN/SS} = (3t - K)^2 / ((3t - K)^2 - 9mt^2)$. For all positive values of $g_{NS/SS}$ and $g_{SN/SS}$, we have $g_{SN/SS} > g_{NS/SS}$. Similarly, we find $\pi_A^{SN} > \pi_A^{NN}$ for $v_2 > v_{SN-} = \sqrt{2 - 2/g} (K + 3t) / 3\sqrt{m}$. Also, $\pi_B^{NS} > \pi_B^{NN}$ for $v_2 > v_{NS-} = \sqrt{2 - 2/g} (-K + 3t) / 3\sqrt{m}$. We have $v_{NS-} < v_{SN-}$. Thus, the {sell, not sell} equilibrium is a subset of the {not sell, sell} equilibrium region. When both equilibria exist, the condition for the risk-dominance of the {not sell, sell} equilibrium is $RD_{NS} = (g - 1)m(v_2^2 - 2t^2)(v_A - v_B) / (6gt) > 0$. Thus, for $v_2 > 3t / 2$, the {not sell, sell} equilibrium is risk dominant.

When no market spillover exists, we have $\pi_A^Z = (K / 3 + t)^2 / 2t + mt / 2$ and $\pi_B^Z = (-K / 3 + t)^2 / 2t + mt / 2$. Thus, $\pi_A^{NS} = \pi_A^Z - mt / 2 < \pi_A^Z$. Finally, comparing the two firms' profits, $\pi_B^{NS} > \pi_A^{NS}$ for $g < (3t - K)^2 / ((3t + K)^2 - 9mv_2^2 / 2)$. Considering the necessary condition $g > g_{NS/SS}$, we find $v_2 > \sqrt{8Kt(3t + K)^2 / 3m + 2t^2(3t - K)^2 / (3t + K)}$ must also hold. *Q.E.D.*

When the market spillover is positive: For $g > g_{NS/SS} = 1 + 9t(2O - mt) / (K + 3t)^2$, we have $\pi_A^{SS} > \pi_A^{NS}$. Also, $\pi_B^{SS} > \pi_B^{SN}$ for $g > g_{SN/SS} = 1 + 9t(2O - mt) / (3t - K)^2$. Thus, we have $g_{SN/SS} > g_{NS/SS}$. We find $\pi_A^{SN} > \pi_A^{NN}$ for $v_2 > v_{SN+} = \sqrt{2/m} \sqrt{9t(-gt + 2O + t) - (g - 1)K(K + 6t)} / 3$. Also, $\pi_B^{NS} > \pi_B^{NN}$ for $v_2 > v_{NS+} = \sqrt{2/m} \sqrt{(g - 1)K(-K + 6t) + 9t(-gt + 2O + t)} / 3$. Since $g > 1$, we have $v_{NS+} > v_{SN+}$. Thus, the {not sell, sell} equilibrium is a subset of the {sell, not sell} equilibrium region. For the region where both equilibria exist, the condition for the risk dominance of the {sell, not sell} equilibrium is $RD_{SN} = \frac{(g - 1)m(v_2^2 - 2t^2)K}{6t} > 0$, which holds for $v_2 > 3t / 2$.

Without any market spillover, neither firm sells the spillover-producing product and we have $\pi_A^Z = (3t + K)^2 / 18t$ and $\pi_B^Z = (3t - K)^2 / 18t$. We have $\pi_A^{SS} > \pi_A^Z$ for $g > g_{NS/SS}$. Also, $\pi_B^{SS} > \pi_B^Z$ for $g > g_{SN/SS}$. Thus, in the region for the {sell, sell} equilibrium, where $g > g_{SN/SS} > g_{NS/SS}$, firms earn higher profits with a positive market spillover than they would have earned without a market spillover. *Q.E.D.*

X) Ratings-Based Conjoint Studies:

We designed two ratings-based conjoint studies (see Schindler (2011), pp. 56-62) where we varied prices and whether the seller also sold unhealthy goods. In study 1, we studied the effect of a pharmacy selling tobacco on the subjects’ willingness to pay for unrelated products from that pharmacy. In this study, 91 participants on Amazon Mechanical Turk were told that “Pharmacy A sells cigarettes and other tobacco products in addition to medical drugs” and “Pharmacy B sells medical drugs, but does NOT sell tobacco products.” They were then asked to state their likelihood of purchasing travel immunization consulting from each of the pharmacies at the prices of \$10 and \$12 on a scale of 0 to 10. We regressed the ratings of likelihood to purchase on price and a dummy variable for the presence of tobacco. The results are shown in Table A7.

Table A7. Regression Results for Study 1

	Coefficients	Standard Error
Price	-0.59*	0.14
Tobacco Sale (dummy)	-2.62*	0.29
Intercept	14.05*	1.59

* *p-value < 0.001*

Table A8. Regression Results for Study 2

	Coefficients	Standard Error
Price	-1.84*	0.12
Confectionery Sale (dummy)	-1.11*	0.20
Intercept	13.73*	0.51

* *p-value < 0.001*

As described in Schindler (2011), the negative value of selling tobacco is calculated by

$$\frac{|\text{coefficient of dummy variable}|}{|\text{coefficient of price}|}$$

This gives the estimate of a negative market spillover of \$4.41.

Study 2 analyzed the effect of a grocery store selling confectionery at checkout lines on subjects’ willingness to pay for unrelated products at that store. In this study, 101 Amazon Mechanical Turk participants were told that “Grocery store A sells candy, chocolates, and other sugar-filled treats at their checkout lines” and “Grocery store B has removed candy, chocolates, and other sugar-filled treats from their checkout lines, replacing them instead with nuts, dried fruit, trail mixes, water, and other healthy snacks.” Then, they were asked to state their likelihood of purchasing a healthy salad from each store’s salad bar at the prices of \$3, \$4, and \$5 on a scale of 0 to 10. The results of the regression are shown in Table A8. The estimated negative value of selling confectionery is \$0.61.