Online Appendix

Appendix A: Numerical Examples

We construct two instances of the recycling network (RN) based on the EPR implementation in Washington state in the US and the Japanese implementation of the Home Appliance Recycling Law (HARL). We analyze whether and when these two instances satisfy the design-reinforcing condition.

Example 2 (EPR implementation in the Washington State). To construct an instance of the RN model in this paper, we first need to benchmark producers' access to recycling capacity if they were to operate independently. An example in practice where producers have made an effort to establish independent recycling capacities is the following: Washington state's e-waste law mandates all producers selling in the state to either participate in a plan run by the Washington Materials Management & Financing Authority (WMMFA), or establish and operate their own "independent plans" (subject to regulatory approval) to fulfill their collection and recycling obligations. Two such independent plans were submitted in the past: One by HP who operates a large scale in-house recycling facility in northern California, the other by a group of TV manufacturers (Panasonic, Sharp and Toshiba, which we refer to as "PST" below), who would work with processors through contractual arrangements. In this case, the question is whether there exists a cost allocation that incentivizes these producers to take part in a collective system that works with all available processors while guaranteeing at least as high design incentives as if they were to operate their independent plans.

To address this question, we adopt the RN framework and model the two proposed independent plans by two representative producer nodes A and B. In addition, note that if HP and PST operate their independent plans, the remaining producers can still participate in the plan run by WMMFA. This motivates us to model a third producer node C in the RN instance that represents all other producers whose products are recycled by the processors contracted by the WMMFA.

Capacities: The processing capacity at HP's recycling facility in California is estimated to be 18 million lbs per year[5]. However, given that there are three states on the west coast that have e-waste laws (CA, OR and WA), we do not have solid data on how much of that capacity would be used to recycle the return volume from Washington state. Similarly, we do not have solid capacity data from the contractors that PST works with. Hence, we shall denote these capacity parameters as $k_{r(A)}$ and $k_{r(B)}$ in the instance and later discuss the range of these parameters under which the design-reinforcing condition is met. For producer C, it has access to the total capacity contracted by WMMFA in Washington, which we assume to be the same as the total volume processed in the plan run by WMMFA in 2015 rounded up to the nearest million (i.e., 43 million lbs).

Return volumes: We calculate the return volume of the three representative producers based on the Washington operating data in 2015, including the total volume processed (42,585,847 lbs[6]) and each producer's return share[7]. We summarize the results in Table 4.

[5] `http://www.hp.com/hpinfo/globalcitizenship/environment/commitment/hp_environmental_history.pdf` (page 8)

[6] `http://www.ecy.wa.gov/programs/swfa/eproductrecycle/docs/2015WMMFAAnnualReport.pdf`

[7] `http://www.ecy.wa.gov/programs/swfa/eproductrecycle/docs/Final2015ReturnSharebyManufacturer.pdf`

Producer	Annual capacity in an individual system (million lbs)	Return share	Annual return volume (million lbs)	Average cost (cents/lb)
A	$k_{r(A)}$	5.52%	2.35	4.43
B	$k_{r(B)}$	21.1%	8.98	4.55
C	43	73.38%	31.25	8.73

Table 4 Summary of the RN instance based on the Washington state EPR implementation

Processing technology efficiency: To obtain a reasonable estimate of the overall processing technology efficiency at the processors, we adopt the cost structure proposed by Gui et al. (2016)[8]. Based on a minimum recycling rate of 90% and a 7:3 volume ratio between TVs/monitors and IT products[8], we can calculate the average unit recycling cost incurred at each processor node (see Table 4).

The RN instance illustrated in Table 4 satisfies the design-reinforcing condition identified in Theorem 1 (under which the marginal contribution based cost allocation ensures group incentive compatibility and a superior design outcome in the collective system) if

$$k_{r(A)} + k_{r(B)} < 31.25 \tag{1}$$

This condition translates to a total capacity for producers A and B that is less than 2.75 times of their total volume, which is realistic considering practice. For example, the condition is met if HP shares its capacity at the California recycling facility equally among the three states on the west coast that have e-waste legislation (i.e., $k_{r(A)} = 18/3 = 6$), and the capacity level at the contractors that PST works with equals twice PST's return volume.

Moreover, it can be further calculated that when $k_{r(A)} < 8.98$ and $k_{r(A)} + k_{r(B)} < 31.25$ both hold, the RN instance also satisfies the design-reinforcing condition identified in Proposition 1 under which the collective system leads to a superior design outcome given the dual-based cost allocation. This condition is also realistic considering that $k_{r(A)} = 6$ when HP equally shares the recycling capacity among the states of CA, OR and WA, which by itself is a high estimate as CA typically has a larger e-waste volume than OR and WA. Recall that our main analysis in the paper assumes that product recyclability and process technology efficiency are substitutes. Hence, the above discussion indicates that it is plausible for a collective implementation in the Washington state to enhance incentives for design-for-recyclability attributes that substitute processing technology efficiency (e.g., using less screws, modularity design, using less toxic materials).

Example 3 (Japanese HARL). The Japanese HARL implementation is known for incentivizing design-for-recyclability improvements that are tailored for the recycling technology used at the processors. Below we map HARL implementation data to our model and show the design-reinforcing condition is satisfied when product recyclability and processing technology efficiency are complements.

Capacities: A reasonable way to model producers' access to recycling capacities when operating independently is to assume that they will use their in-house facilities if there are sufficient capacities to process their own volumes. The Japanese HARL implementation provides solid information in this regard. In particular, its "Group B" collective system is mainly operated based on recycling plants built by joint ventures among

[8] L. Gui, A. Atasu, Ö. Ergun, L. B. Toktay (2016) Efficient Implementation of Collective Extended Producer Responsibility Legislation. *Management Science* 62(4):1098-1123. (Table 2)

producers, of which the capacity and the shareholder information is publicly available. We summarize this information in Table 5, where the major shareholder is defined as one with a significantly higher share than the rest of the shareholders[9]. Observing that for some recycling plants, there are two producers with similar shares that are much higher than the other shareholders[10], we list both producers as major shareholders in that case.

Recycling plant	Annual capacity (million units)	Major shareholder(s)
Kansai Recycling Systems Co., Ltd.	0.79 [11]	Sharp and Mitsubishi [12]
Hokkaido Eco Recycling Systems Co., Ltd.	1.39 [13]	Hitachi and Mitsubishi [13]
East Japan Recycling Systems Co., Ltd.	0.6 [14]	Mitsubishi [14]
Kantou Eco Recycle Co., Ltd.	0.3 [15]	Hitachi [15]
Corporation Hyper Cycle Systems Co., Ltd.	1.05 [11]	Mitsubishi [16]
Tokyo Eco Recycle Co., Ltd.	0.6 [11]	Hitachi [17]
Fuji Eco Cycle Co., Ltd.	0.56 [18]	Fujitsu [18]
Green Cycle Co., Ltd	0.85 [11]	Sony [19]

Table 5 Producer-built recycling plants in the "Group B" collective system under the Japanese HARL implementation

We assume that the capacity at the recycling plants in Table 5 will be accessible by its major shareholder if it operates independently. In cases where two major shareholders exist, we assume that the capacity will be equally divided if both shareholders operate independently. Accordingly, we can calculate the amount of recycling capacity each of these major producers has access to in an individual system. We summarize the resulting capacity levels as well as the publicly available return volume information of these major producers in Table 6.

[9] A representative example is the East Japan Recycling Systems Co., Ltd, where Mitsubishi holds 82.18% and each of the other manufacturers holds 3.58% of the total shares.

[10] A representative example is the Kansai Recycling Systems Co., Ltd., where Sharp and Mitsubishi each hold 43.3%, and each of the other manufacturers holds 3.3% of the total shares.

[11] Waste electrical and electronic equipment (WEEE): innovating novel recovery and recycling technologies in Japan. DTI Global Watch Mission Report. 2005

[12] `http://www.krsc.co.jp/corporate/index.html`

[13] `http://www.go-hers.co.jp/company/infomation.html`

[14] `http://www.ejrs.co.jp/info.html#info02`

[15] `http://www.hitachi.co.jp/New/cnews/9905/0506a.html`

[16] `http://www2u.biglobe.ne.jp/GOMIKAN/rep/katudo-3.htm`

[17] `http://www.tokyo-eco.co.jp/company/outline.html`

[18] `http://www.fujitsu-general.com/jp/news/2000/12/00-V07-41/\#top`

[19] `https://www.sony.co.jp/SonyInfo/csr/eco/recycle/concept/index.html`

[20] `http://www.mitsubishielectric.com/company/environment/report/recycling/recycle/index.html`

[21] `http://www.hitachi.com/csr/download/pdf/csr2015e.pdf` (page 75).

[22] `https://www.sony.net/SonyInfo/csr_report/environment/recycle/japan.html`

[23] `http://www.fujitsu.com/id/Images/2005report47-48-e.pdf`

[24] `http://www.sharp-world.com/corporate/eco/ssr/environment/recycling/used_products/`

Producer	Annual capacity in an individual system (million units)	Annual return volume (million units)
Mitsubishi	2.74	0.985 [20]
Hitachi	1.595	1.454 [21]
Sony	0.85	0.474 [22]
Fujitsu	0.56	0.341 [23]
Sharp	0.395	1.42 [24]

Table 6 Summary of the recycling capacity and the return volume of the major producers in the "Group B" collective system under the Japanese HARL implementation.

Return volumes: Note that in Table 6, the return volumes of Hitachi and Fujitsu are calculated based on the reported figures of 0.06 and 0.01407 million tons, respectively (from the references cited in the corresponding footnotes). The conversion rate is assumed to be 1 ton for 24.2329 units, calculated based on the total recycling volume data in Japan: A total of 11.341 million units and 0.468 million tons of appliances were recycled in 2012 according to the AEHA (Association for Electric Home Appliances in Japan) annual report[25]. Moreover, to ensure data accuracy, we also estimated the producers' return volumes based on (i) the annual recycling volume of each type of appliance covered under HARL (i.e., air conditioners, CRT TVs, LCD TVs, refrigerators, and washing machines) reported by AEHA[25], (ii) the market share of the producers in 2006[26] and 2016[27]. The resulting return volume numbers are close to those reported in Table 6.

It can be observed from Table 6 that except Sharp, all other producers are capable of operating independently using the capacity at the recycling plants where they are the major shareholder. Moreover, it can be observed that if Sharp were to operate independently, it needs to work with Mitsubishi since no other producer or producer group has sufficient capacity to process its own volume plus Sharp's volume. To capture these two observations, we construct an RN instance consisting of four producer nodes that represent the Mitsubishi-Sharp subgroup, and Hitachi, Sony and Fujitsu by themselves, respectively.

Processing technology efficiency: In terms of processing technology efficiency levels between the producers' capacity, note that the Mitsubishi group has a member company (Mitsubishi Materials) that has the expertise in electronic materials/components recycling and metal processing. Hence, it is reasonable to assume that the capacity associated with the Mitsubishi-Sharp subgroup is the most efficient. In addition, there exists evidence that Hitachi and Sony have had initiatives to improve the recycling technology at their facilities[28]. Since Hitachi has more volume and therefore potentially benefits more from scale economies, we assume Hitachi's capacity is the second most efficient, and Sony's the third most efficient.

A summary of the RN instance: Motivated by all the above, we construct an RN instance consisting of four representative producers $\{MS, H, S, F\}$ with return volume equal to $d^{\pi(MS)} = 0.985 + 1.42 = 2.405$, $d^{\pi(H)} = 1.454$, $d^{\pi(S)} = 0.474$ and $d^{\pi(F)} = 0.341$ (all numbers in this example are in million units). We assume

[25] http://www.aeha.or.jp/recycling_report/pdf/kadennenji27.pdf

[26] Tomoaki Shimada and Luk N. Van Wassenhove, Closed- Loop Supply Chain Activities in Japanese Home Appliance/Personal Computer Manufacturers: A Case Study, Intern. Journal of Production Economics, http://dx.doi.org/10.1016/j.ijpe.2016.11.010

[27] http://nbakki.hatenablog.com/entry/2015_Tablet_PC_Market_Share_in_Japan

[28] Waste electrical and electronic equipment (WEEE): innovating novel recovery and recycling technologies in Japan. DTI Global Watch Mission Report. 2005

these producers have access to sufficient capacity for their own volumes at four different processors, respectively, and their capacity levels equal $k_{r(MS)} = 2.74 + 0.395 = 3.135$, $k_{r(H)} = 1.595$, $k_{r(S)} = 0.85$ and $k_{r(F)} = 0.56$. The efficiency ranking of these four processors is $\tau_{r(MS)} < \tau_{r(H)} < \tau_{r(S)} < \tau_{r(F)}$. It can be calculated that this RN satisfies the design reinforcing condition identified in Proposition 5 of the paper when product recycability and processing technology efficiency are complementary, i.e.,

$$d^{\pi(H)} + d^{\pi(S)} + d^{\pi(F)} < k_{r(MS)}; \quad d^{\pi(S)} + d^{\pi(F)} < k_{r(H)}; \quad d^{\pi(F)} < k_{r(S)} \tag{2}$$

Appendix B: Two Additional Extensions

B.1. Extension to Product-differentiated Processor Efficiency

In practice, the recycling efficiency at a given processor may be differentiated by product. This can happen when the processing technology required for different products varies, and processors specialize in processing certain product types. For example, Apple's Liam robot currently specializes in recycling iPhone 6 devices (Reuters 2016)[29], and processors equipped with toxic material separation and recycling capabilities specialize in processing electronics that contain these materials. To capture this heterogeneity, we consider a more general version of the multiplicative model of the unit recycling cost considered in the main analysis. That is, in this subsection, we allow the efficiency measure of a processor r to depend on the product π and denote it as τ_r^π. Thus unit recycling cost c_r^π equals

$$c_r^\pi = \lambda^\pi \cdot \tau_r^\pi + \bar{c}^\pi \quad \forall \pi \in \Pi \quad \forall r \in R. \tag{3}$$

PROPOSITION 7. *Given any RN instance where the unit recycling cost is modeled as in (3), if there exists a linear design-based allocation x_l that is group incentive compatible and ensures a superior equilibrium design profile compared to an individual system, i.e., $\Lambda_l^{ne} \le \Lambda_{ind}$, then the RN instance satisfies $d^{\pi(i)} \ge \sum_{j:\tau_{r(j)}^{\pi(i)} < \tau_{r(i)}^{\pi(i)}} k_{r(j)}$ $\forall i$ for which $|\{j : \tau_{r(j)}^{\pi(i)} < \tau_{r(i)}^{\pi(i)}\}| \ge 2$.*

Proof of Proposition 7. Please refer to Appendix C.4.1 for the proof.

Note that the set $\{j : \tau_{r(j)}^{\pi(i)} < \tau_{r(i)}^{\pi(i)}\}$, which is product dependent, reduces to $\{j : \tau_{r(j)} < \tau_{r(i)}\}$ when the cost efficiency of a given processor is not differentiated by product, as in the main analysis. Hence, the condition identified above generalizes the low-synergy condition in Theorem 1 to the case with product-dependent heterogeneity in processing efficiency. We conclude that the design-reinforcing characterization derived in the main analysis can be extended to this more general setting.

B.2. Extension to Return Volume Uncertainty

In this extension, we analyze the impact of return volume uncertainty. To do so, we model each volume parameter $d^{\pi(i)}$ as an independent random variable whose probability distribution is known to the producers. We extend the definition of an RN instance to include these probabilistic parameters, and call it an *RN instance under return volume uncertainty*. Accordingly, we generalize the definition of design outcomes under EPR as follows: The optimal design profile induced by an individual system is the design profile that minimizes producers' expected recycling costs plus their design investments. The equilibrium design profile

[29] Reuters. 2016. Apple's robot rips apart iphones for recycling. `http://www.reuters.com/article/us-apple-products-recycling-idUSKCN0WN1Y0`.

in a collective implementation is a design profile such that no producer can reduce the sum of its expected recycling cost and its design investment by unilateral deviation.

We show that given an RN instance under return volume uncertainty, whether a group incentive compatible allocation leads to superior design incentives for a producer critically hinges on the probability of the RN realization being design-reinforcing. To see this in the context of the two-producer setting, recall that superior design incentives (under no return volume uncertainty) require $d^{\pi(2)} \geq k_{r(1)}$.

This condition can be relaxed and still ensure superior design incentives for producer 1 under return volume uncertainty. That is, $d^{\pi(2)}$ can take a value smaller than $k_{r(1)}$ as long as the associated probability is sufficiently low. To see this, note that in the deterministic case, the rate of change of producer 1's dual-based cost allocation can be strictly higher than the rate of change of its stand-alone cost under a design-reinforcing RN (e.g., in the first RN instance considered in Example 1; see Table 2). Hence, under return volume uncertainty, whether producer 1's expected cost allocation changes at a faster or slower rate in design depends on the relative probability of a design-reinforcing realization of the RN versus a non-design-reinforcing realization. In fact, for a two-producer RN instance, we can show that a sufficient condition to ensure superior design incentives for producer 1 under the dual-based cost allocation is $\frac{Pr(d^{\pi(2)}<k_{r(1)})}{Pr(d^{\pi(2)}\geq k_{r(1)})} \leq \frac{E(d^{\pi(1)})}{k_{r(1)}}$, where $E(d^{\pi(1)})$ is the expectation of $\pi(1)$'s return volume (see Appendix C.4.2 for the derivation).

The above discussion indicates that uncertainty in RN parameters may increase the likelihood of some producers choosing a superior design in a collective implementation compared to under an individual system. Nevertheless, in general, if superior design incentives are required for all producers in conjunction with group incentive compatibility, then we need the condition that for all i such that $|\{j : \tau_{r(j)} < \tau_{r(i)}\}| \geq 2$, the sample space of the random variable $d^{\pi(i)}$ is contained in the interval $[\sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}, \infty)$ (see Proposition 10 in Appendix C.4.2). This generalizes Theorem 1, implying that the fundamental insights regarding the design-stability tradeoff continue to apply under RN uncertainty.

Appendix C: Proofs and Technical Analysis

For convenience, in this appendix (except for in the proof of Proposition 7 where the processor efficiency is product-differentiated), we assume that the indices of the producers are arranged according to the efficiency levels of the capacities that they have access to in an individual system. Specifically, $i < j$ implies that $\tau_{r(i)} < \tau_{r(j)}$, i.e., $r(i)$ is more efficient.

C.1. Technical Details in §4

In this section, we present the equilibrium analysis of the design outcome under the dual-based cost allocation. We first study RNs with two producers, and present Proposition 8 that explains the equilibrium analysis in Example 1. We then extend the analysis to RNs with n producers and provide the detailed proof of Proposition 1.

C.1.1. RNs with Two Producers

Proposition 8. *Consider an RN instance with two producers. Under the dual-based cost allocation mechanism x_d,*

1. *when $d^{\pi(2)} \geq k_{r(1)}$, i.e., the capacity at $r(1)$ is no larger than producer 2's return volume, there exists a unique equilibrium design profile Λ_d^{ne} that is superior to the design profile induced by an individual system, i.e., $\Lambda_d^{ne} \leq \Lambda_{ind}$.*

2. *when $d^{\pi(2)} < k_{r(1)}$, if an equilibrium design profile exists, it is inferior to the design profile induced by an individual system, i.e., $\Lambda_d^{ne} \geq \Lambda_{ind}$; otherwise, there exists a mixed-strategy equilibrium under which any design profile that can occur with a positive probability is inferior to Λ_{ind}.*

Proof of Proposition 8. The proof of Proposition 8 is based on the following three-step procedure.

• Step 1 (compute the best response functions): For each producer $i = 1, 2$, we calculate the recyclability level of $\pi(i)$ that minimizes the sum of i's dual-based cost allocation and its design investment, i.e., $x_d^i + Q^i$. We calculate this design choice for i as a function of the other producer's design strategy. This function, denoted as $\lambda_d^{*\pi(i)}$, is called producer i's *best response function* to j's design choice.

• Step 2 (find the equilibrium Λ_d^{ne}): We calculate the equilibrium design profile by solving the two best response functions simultaneously. This can be done by finding the intersection point of the two functions.

• Step 3 (compare Λ_d^{ne} with Λ_{ind}^*): We compare the equilibrium design profile with the design profile induced by an individual system (i.e., Λ_{ind}^*).

Below we explain the calculation details in each step under the two RN cases described in Proposition 8.

Case 1: When the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$

• Step 1: To compute the best response functions, we first analyze the dual-based cost allocation to each producer. This cost allocation is calculated based on the optimal dual solution of the centralized allotment problem (C), and thus depends on the socially optimal allotment f^*. It can be observed that when $d^{\pi(2)} \geq k_{r(1)}$, f^* varies depending on the relative recyclability of the products (as depicted in Figure 4(b) and (c)). Accordingly, we can calculate producer 1's dual-based cost allocation x_d^1 as follows.

$$x_d^1 = \begin{cases} d^{\pi(1)} \cdot c_{r(2)}^{\pi(1)} - k_{r(1)} \cdot (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}) & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(1)} \cdot [c_{r(1)}^{\pi(1)} + (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)})] - k_{r(1)} \cdot (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}) & \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{4}$$

Recall that in our main model, $c_r^\pi = \lambda^\pi \cdot \tau_r + \bar{c}^\pi$ for each pair of product π and processor r. Hence, it can be observed that given producer 2's product design $\lambda^{\pi(2)}$, formula (4) is a piecewise linear function of producer 1's own design variable $\lambda^{\pi(1)}$, and the break point occurs when $\lambda^{\pi(1)} = \lambda^{\pi(2)}$. Since the investment function Q^1 is convex with respect to $\lambda^{\pi(1)}$, we can show that producer 1's total cost $x_d^1 + Q^1$ is a piecewise convex function of $\lambda^{\pi(1)}$, whose shape depends on the given $\lambda^{\pi(2)}$ and is as depicted in Figure 6. It can be observed based on the figure that the minimizer of $x_d^1 + Q^1$ (which is by definition producer 1's best response $\lambda_d^{*\pi(1)}$) can occur at two different values defined as below.

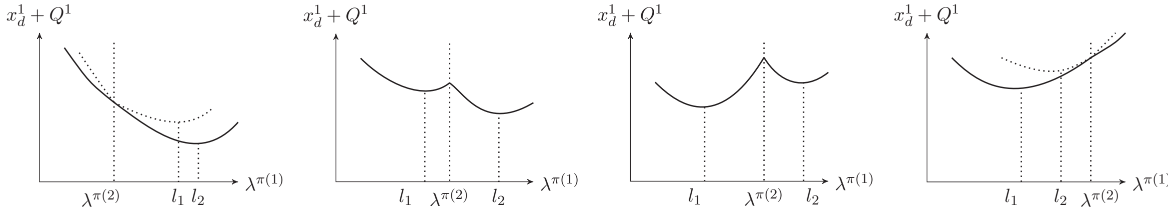
$$l_1 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(2)}\right); \quad l_2 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(1)}\right) \tag{5}$$

It can be observed that $l_1 < l_2$ since $\tau_{r(1)} < \tau_{r(2)}$ (i.e., $r(1)$ is the more cost efficient processor) and the investment function Q^1 is convex decreasing. Further calculation indicates that producer 1's best response $\lambda_d^{*\pi(1)}$ is a two-piece step function based on l_1 and l_2.

$$\lambda_d^{*\pi(1)} = \begin{cases} l_2 & \text{if } \lambda^{\pi(2)} \leq \lambda_0^{\pi(2)} \\ l_1 & \text{if } \lambda^{\pi(2)} > \lambda_0^{\pi(2)} \end{cases} \tag{6}$$

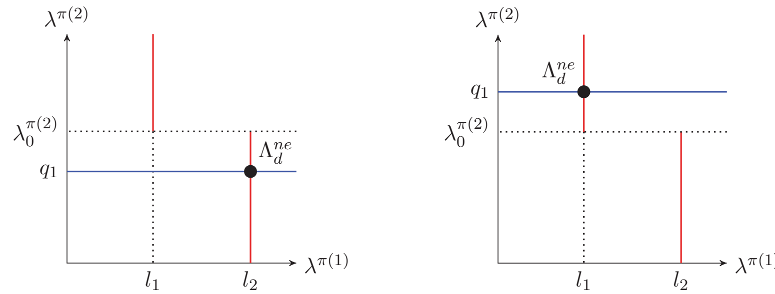
where $\lambda_0^{\pi(2)}$ is a constant defined by the formula $\frac{Q^1(l_2) + d^{\pi(1)} \cdot \tau_{r(2)} \cdot l_2 - Q^1(l_1) - d^{\pi(1)} \cdot \tau_{r(1)} \cdot l_1}{d^{\pi(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})}$. It can be verified that $\lambda_0^{\pi(2)}$ is a constant between l_1 and l_2.

Figure 6 **This figure shows producer 1's total cost function $x_d^1 + Q^1$ when $d^{\pi(2)} \geq k_{r(1)}$. In the figure, l_1 and l_2 are as defined in (5), and $\lambda_0^{\pi(2)}$ is a constant in (l_1, l_2), calculated based on the given RN instance.**



(a) When $\lambda^{\pi(2)} < l_2$ (b) When $l_1 \leq \lambda^{\pi(2)} \leq \lambda_0^{\pi(2)}$ (c) When $\lambda_0^{\pi(2)} < \lambda^{\pi(2)} \leq l_2$ (d) When $\lambda^{\pi(2)} > l_2$

Figure 7 **This figure shows the equilibrium design profile Λ_d^{ne} under the dual-based allocation when $d^{\pi(2)} \geq k_{r(1)}$. In this figure, l_1, l_2 and q_1 are as defined in (5) and (8), and $\lambda_0^{\pi(2)}$ is the same constant as shown in Figure 6. The vertical and horizontal lines represent the best response functions of producer 1 and 2 respectively.**



(a) When $q_1 \leq \lambda_0^{\pi(2)}$ (b) When $q_1 > \lambda_0^{\pi(2)}$

We continue to compute the best response function for producer 2, which is more straightforward. This is because we can calculate that producer 2's dual-based cost allocation equals

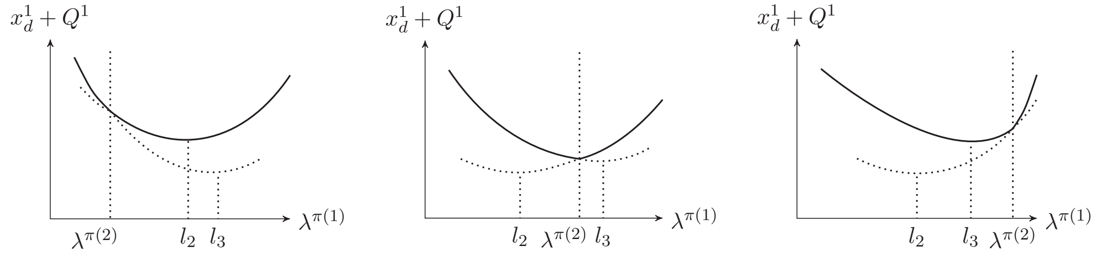
$$x_d^2 = d^{\pi(2)} \cdot c_{r(2)}^{\pi(2)} \tag{7}$$

under both patterns of the socially optimal allotment f^*. Hence, we conclude that when the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$, x_d^2 is independent of producer 1's design choice. Accordingly, it can be calculated that producer 2's best response equals a constant q_1 defined as

$$q_1 \doteq (Q^{2\prime})^{-1} \left(-d^{\pi(2)} \cdot \tau_{r(2)}\right) . \tag{8}$$

- Step 2: We locate the equilibrium design profile Λ_d^{ne} at the intersection the two best response functions calculated above (Figure 7). Based on this figure, we conclude that when the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$, there exists a unique equilibrium design profile under the dual-based cost allocation, and it equals either $\{l_1, q_1\}$ or $\{l_2, q_1\}$ depending on the specification of the RN considered.
- Step 3: We have shown in §3.2 that for each producer i, the product design induced by an individual system is $\lambda_{ind}^{\pi(i)} = (Q^{i\prime})^{-1} \left(-d^{\pi(i)} \cdot \tau_{r(i)}\right)$. Comparing this formula with (5) and (8), we immediately obtain that $l_2 = \lambda_{ind}^{\pi(1)}$ and $q_1 = \lambda_{ind}^{\pi(2)}$. Moreover, as we have shown $l_1 < l_2$, we know that $l_1 < \lambda_{ind}^{\pi(1)}$, indicating the possibility of a strictly superior design of the product $\pi(1)$ under the dual-based cost allocation compared to that in an individual system. This completes the proof of the first result in Proposition 8.

Figure 8 This figure shows producer 1's total cost function $x_d^1 + Q^1$ when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$. In the figure, l_2 and l_3 are as defined in (10).



(a) When $\lambda^{\pi(2)} \le l_2$ (b) When $l_2 < \lambda^{\pi(2)} < l_3$ (c) When $\lambda^{\pi(2)} \ge l_3$

Case 2: When the RN satisfies $d^{\pi(2)} < k_{r(1)}$ We continue to prove the second result in Proposition 8, i.e., the dual-based cost allocation leads to inferior product designs than those induced in an individual system when $d^{\pi(2)} < k_{r(1)}$. We analyze two sub-cases as follows.

Case 2(a): We consider the sub-case where the capacity at $r(1)$ is sufficient to process the return volume of $\pi(2)$ but not all products returned, i.e., $\underline{d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}}$. We follow the three-step procedure introduced at the beginning of the proof to analyze this sub-case.

- Step 1: We first observe that when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, similar to Case 1, there are two possible patterns of the socially optimal allotment depending on the relative recyclability of the products (Figure 4(d) and (e)). Accordingly, we can calculate producer 1's dual-based cost allocation x_d^1 as follows.

$$x_d^1 = \begin{cases} d^{\pi(1)} \cdot c_{r(2)}^{\pi(1)} - k_{r(1)} \cdot (c_{r(2)}^{\pi(1)} - c_{r(1)}^{\pi(1)}) & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(1)} \cdot [c_{r(1)}^{\pi(1)} + c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}] - k_{r(1)} \cdot (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}) & \text{if } \lambda^{\pi(1)} \ge \lambda^{\pi(2)} \end{cases} \tag{9}$$

It is easy to see that x_d^1 is a piecewise linear function of producer 1's design variable $\lambda^{\pi(1)}$ given any value of $\lambda^{\pi(2)}$, just as in Case 1. However, the difference is that x_d^1 defined in (9) is convex in $\lambda^{\pi(1)}$ as opposed to being concave in Case 1. This results in a different functional structure of producer 1's total cost $x_d^1 + Q^1$ (Figure 8), whose minimum may be attained at the following two values of $\lambda^{\pi(1)}$.

$$l_2 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(1)}\right); \quad l_3 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(2)} + k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})\right). \tag{10}$$

Since we can calculate that $d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) \le d^{\pi(1)} \cdot \tau_{r(1)}$, we conclude that $l_3 > l_2$ as Q^1 is convex decreasing. Hence, it can be shown that the best response of producer 1, $\lambda_d^{*\pi(1)}$, is no longer a step function as in Case 1. Instead, it equals

$$\lambda_d^{*\pi(1)} = \begin{cases} l_2 & \text{if } \lambda^{\pi(2)} \le l_2 \\ \lambda^{\pi(2)} & \text{if } \lambda^{\pi(2)} \in (l_2, l_3) \\ l_3 & \text{if } \lambda^{\pi(2)} \ge l_3 \end{cases} \tag{11}$$

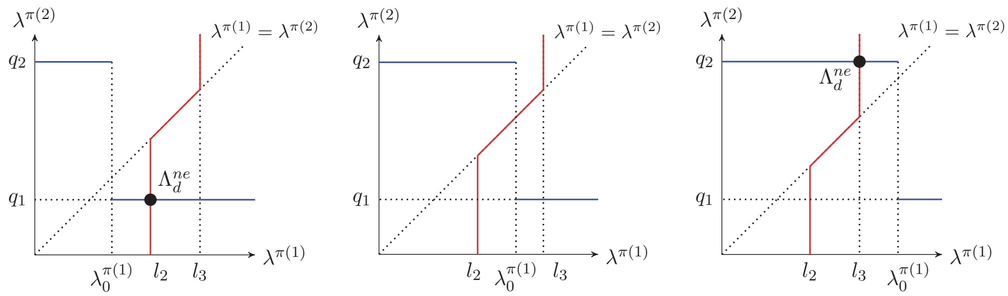
We continue to compute the best response of producer 2 in this sub-case. We can calculate that in this case, producer 2's dual-based cost allocation equals

$$x_d^2 = \begin{cases} d^{\pi(2)} \cdot [c_{r(1)}^{\pi(2)} + c_{r(2)}^{\pi(1)} - c_{r(1)}^{\pi(1)}] & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(2)} \cdot c_{r(2)}^{\pi(2)} & \text{if } \lambda^{\pi(1)} \ge \lambda^{\pi(2)} \end{cases} \tag{12}$$

Following the same argument that we use to derive producer 1's best response in Case 1, we can show that $\lambda_d^{*\pi(2)}$ is the following two-piece step function:

$$\lambda_d^{*\pi(2)} = \begin{cases} q_2 & \text{if } \lambda^{\pi(1)} \le \lambda_0^{\pi(1)} \\ q_1 & \text{if } \lambda^{\pi(1)} > \lambda_0^{\pi(1)} \end{cases} \tag{13}$$

Figure 9 This figure shows the equilibrium design profile Λ_d^{ne} under the dual-based allocation when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$. In this figure, l_2, l_3, q_1 and q_2 are as defined in (10) and (14) respectively, and $\lambda_0^{\pi(1)}$ is the constant used in formula (13). The best response function of producer 1 is represented by the horizontal lines, and that of producer 2's is represented by the vertical and the diagonal lines. Note that in panel (b), no pure-strategy equilibrium design profile exists.



(a) When $\lambda_0^{\pi(1)} \le l_2$ (b) When $l_2 < \lambda_0^{\pi(1)} < l_3$ (c) When $\lambda_0^{\pi(1)} \ge l_3$

where q_1 and q_2 equal

$$q_1 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(2)}\right); \quad q_2 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(1)}\right). \tag{14}$$

and $\lambda_0^{\pi(1)}$ is a constant defined by the formula $\frac{Q^2(q^1)+d^{\pi(2)}\cdot\tau_{r(2)}\cdot q^1 - Q^2(q_2) - d^{\pi(2)}\cdot\tau_{r(1)}\cdot q_2}{d^{\pi(2)}\cdot(\tau_{r(2)}-\tau_{r(1)})}$. It can be verified that $\lambda_0^{\pi(1)} \in (q_1, q_2)$ (it is easy to show that $q_1 < q_2$ as $\tau_{r(2)} > \tau_{r(1)}$ and Q^2 is convex decreasing).

- Step 2: We find the equilibrium design profile at the intersection of the two best response functions calculated above (Figure 9). Note that in this sub-case, a pure-strategy equilibrium does not exist if the RN instance leads to the situation shown in Figure 9(b). In that situation, we show that a mixed-strategy equilibrium always exists (see Lemma 5 presented at the end of Step 3).
- Step 3: Based on the calculations already presented in this proof, we have $l_3 > l_2 = \lambda_{ind}^{\pi(1)}$ and $q_2 > q_1 = \lambda_{ind}^{\pi(2)}$. Hence, we conclude that when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, all possible values of the best response function of each producer i shown in formula (11) and (13) are no better than i's design choice induced by an individual system. Hence, x_d leads to an inferior equilibrium design profile if such an equilibrium exists (Figure 9(a) and (c)). We further show that this result can be extended to the mixed-strategy equilibrium in the situation shown in Figure 9(b).

Definition 4 (Mixed-strategy Equilibrium). Consider an RN instance and a cost allocation mechanism x. A mix-strategy deisgn profile $\{f^i(\lambda^{\pi(i)}), \forall i \in N\}$ is a *mixed-strategy equilibrium* if no producer can reduce the sum of its expected design investment and recycling cost allocation by unilateral deviation, i.e., for each producer i, $\int_0^\infty \int_0^\infty \dots \int_0^\infty [x^i(\{\lambda^{\pi(1)}, \dots, \lambda^{\pi(n)}\}) + Q^i(\lambda^{\pi(i)})] \cdot \prod_{j=1}^n f^j(\lambda^{\pi(j)}) d\lambda^{\pi(j)} \le \int_0^\infty \int_0^\infty \dots \int_0^\infty [x^i(\{\lambda^{\pi(1)}, \dots, \lambda^{\pi(n)}\}) + Q^i(\lambda^{\pi(i)})] \cdot \prod_{j\ne i}^n f^j(\lambda^{\pi(j)}) d\lambda^{\pi(j)} \cdot \tilde{f}^i(\lambda^{\pi(i)}) d\lambda^{\pi(i)}$ holds for any probability distribution $\tilde{f}^i$ of the $\lambda^{\pi(i)}$ variable.

Based on Definition 4, we prove the following result.

Lemma 5. *Given an RN instance such that $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$ and $l_1 < \lambda_0^{\pi(1)} < l_3$, there always exists a mixed-strategy equilibrium design profile $\{f^1(\lambda^{\pi(1)}), f^2(\lambda^{\pi(2)})\}$. Moreover, under this equilibrium, $\forall i = 1, 2$ and $\forall \lambda^{\pi(i)}$ such that $f^i(\lambda^{\pi(i)}) > 0$, the inequality $\lambda^{\pi(i)} \ge \lambda_{ind}^{\pi(i)}$ is satisfied.*

Proof of Lemma 5. We follow a two-step procedure to prove Lemma 5.

STEP 1: We propose the following mix-strategy design profile $\{f_1(\lambda^{\pi(1)}), f_2(\lambda^{\pi(2)})\}$.

— The probability distribution $f_1(\lambda^{\pi(1)})$ is defined such that $f^1(\lambda_0^{\pi(1)}) = 1$ where $\lambda_0^{\pi(1)}$ is the break point defined in formula (13). This is equivalent to producer 1 choosing a fixed recyclability level of $\lambda_0^{\pi(1)}$.

— The probability distribution $f_2(\lambda^{\pi(2)})$ is defined such that $f^2(q_1) \doteq \frac{-Q^{1'}(\lambda_0^{\pi(1)})-a_1}{a_2-a_1}$ and $f^2(q_2) \doteq 1 - f^2(q_1)$, where $q_1 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(2)}\right)$ and $q_2 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(1)}\right)$ are as defined in formula (13), and a_1 and a_2 are two constants such that $a_1 \doteq d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})$ and $a_2 \doteq d^{\pi(1)} \cdot \tau_{r(1)}$. That is, producer 2 randomizes between two recyclability levels q_1 and q_2.

We need to prove that in the RN instance where pure-strategy equilibrium does not exist, i.e., in the RN instance depicted in Figure 9(b), $f^2(q_1) \in (0,1)$ so that f_2 is indeed a probability distribution and $\{f_1, f_2\}$ is indeed a mix-strategy design profile. To see this, recall that the RN instance in Figure 9(b) satisfies $l_2 < \lambda_0^{\pi(1)} < l_3$. Since the design investment function Q^1 is convex decreasing, we know that $Q^{1'}(l_2) < Q^{1'}(\lambda_0^{\pi(1)}) < Q^{1'}(l_3)$. Recall that $l_2 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(1)}\right)$ and $l_3 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(2)} + k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})\right)$, it can be calculated that $Q^{1'}(l_2) = -d^{\pi(1)} \cdot \tau_{r(1)} = -a_2$ and $Q^{1'}(l_3) = -d^{\pi(1)} \cdot \tau_{r(2)} + k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) = -a_1$. Hence, $Q^{1'}(l_2) < Q^{1'}(\lambda_0^{\pi(1)}) < Q^{1'}(l_3)$ is equivalent to

$$-a_2 < Q^{1'}(\lambda_0^{\pi(1)}) < -a_1 \tag{15}$$

which implies that $0 < -Q^{1'}(\lambda_0^{\pi(1)}) - a_1 < a_2 - a_1$. Therefore, we conclude that $f^2(q_1) \doteq \frac{-Q^{1'}(\lambda_0^{\pi(1)})-a_1}{a_2-a_1} \in (0,1)$. Hence, f_2 is a probability distribution, and $\{f_1, f_2\}$ is a mix-strategy design profile.

STEP 2: Next, we show that (f_1, f_2) is an equilibrium design profile. To do this, we evaluate each producer's expected total cost (i.e., design investment plus its dual-based cost allocation) under this design profile.

— Given that producer 1 chooses its own design $\lambda^{\pi(1)}$ according to f^1 (i.e., selects the recyclability level $\lambda_0^{\pi(1)}$), then according to Figure 9(b), producer 2's total cost $x_d^2 + Q^2$ is minimized at either $\lambda^{\pi(2)} = q_1$ or $\lambda^{\pi(2)} = q_2$. Hence, any mixed strategy that randomizes between q_1 and q_2 also minimizes producer 2's total cost. Since f^2 is an example of such mixed strategies, producer 2's total cost is also minimized under f^2, that is, f^2 is a best response for producer 2 when producer 1 follows f_1.

— Given that producer 2 chooses its own design $\lambda^{\pi(2)}$ according to f^2, we calculate producer 1's expected dual-based cost allocation (denoted as $E_{f_2}[x_d^1]$) as follows. For notational brevity, in this formula, we denote the unit recycling cost of $\pi(2)$ at any processor r when its recyclability level equals $\lambda^{\pi(2)} = q$ as $c_r^{\pi(2)}(q) \doteq q \cdot \tau_r + \bar{c}^{\pi(2)}$

$$E_{f_2}[x_d^1] = \begin{cases} d^{\pi(1)} \cdot c_{r(2)}^{\pi(1)} - k_{r(1)} \cdot (c_{r(2)}^{\pi(1)} - c_{r(1)}^{\pi(1)}) \quad \text{if } \lambda^{\pi(1)} \leq q_1 \\ f^2(q_1) \cdot \left[d^{\pi(1)} \cdot [c_{r(1)}^{\pi(1)} + c_{r(2)}^{\pi(2)}(q_1) - c_{r(1)}^{\pi(2)}(q_1)] - k_{r(1)} \cdot [c_{r(2)}^{\pi(2)}(q_1) - c_{r(1)}^{\pi(2)}(q_1)]\right] \\ \qquad + f^2(q_2) \cdot \left[d^{\pi(1)} \cdot c_{r(2)}^{\pi(1)} - k_{r(1)} \cdot (c_{r(2)}^{\pi(1)} - c_{r(1)}^{\pi(1)})\right] \quad \text{if } \lambda^{\pi(1)} \in (q_1, q_2) \\ d^{\pi(1)} \cdot c_{r(1)}^{\pi(1)} + f^2(q_1) \cdot \left[(d^{\pi(1)} - k_{r(1)}) \cdot [c_{r(2)}^{\pi(2)}(q_1) - c_{r(1)}^{\pi(2)}(q_1)]\right] \\ \qquad + f^2(q_2) \cdot \left[(d^{\pi(1)} - k_{r(1)}) \cdot [c_{r(2)}^{\pi(2)}(q_2) - c_{r(1)}^{\pi(2)}(q_2)]\right] \quad \text{if } \lambda^{\pi(1)} \geq q_2 \end{cases} \tag{16}$$

Replacing c_r^{π} with $\lambda^{\pi} \cdot \tau_r + \bar{c}^{\pi}$ in the above formula, we can calculate that $E_{f_2}[x_d^1]$ equals the following piecewise linear function of producer 1's own design variable $\lambda^{\pi(1)}$.

$$E_{f_2}[x_d^1] = \begin{cases} a_1 \cdot \lambda^{\pi(1)} + b_1 \quad \text{if } \lambda^{\pi(1)} \leq q_1 \\ (f^2(q_1) \cdot a_2 + f^2(q_2) \cdot a_1) \cdot \lambda^{\pi(1)} + (f^2(q_1) \cdot b_2^1 + f^2(q_2) \cdot b_1) \quad \text{if } \lambda^{\pi(1)} \in (q_1, q_2) \\ a_2 \cdot \lambda^{\pi(1)} + (f^2(q_1) \cdot b_2^1 + f^2(q_2) \cdot b_2^2) \quad \text{if } \lambda^{\pi(1)} \geq q_2 \end{cases} \tag{17}$$

where a_1 and a_2 are the two constants introduced in the definition of the probability distribution f_2, and the other three constants equal $b_1 = d^{\pi(1)} \cdot \bar{c}^{\pi(1)}$, $b_2^1 = d^{\pi(1)} \cdot \bar{c}^{\pi(1)} + (d^{\pi(1)} - k_{r(1)}) \cdot q_1 \cdot (\tau_{r(2)} - \tau_{r(1)})]$, and $b_2^2 = d^{\pi(1)} \cdot \bar{c}^{\pi(1)} + (d^{\pi(1)} - k_{r(1)}) \cdot q_2 \cdot (\tau_{r(2)} - \tau_{r(1)})]$. Accordingly, we can calculate the derivative of $E_{f_2}[x_d^1] + Q^1$ with respect to $\lambda^{\pi(1)}$ as follows.

$$\frac{\partial \left[E_{f_2}(x_d^1)(\lambda^{\pi(1)}) + Q^1(\lambda^{\pi(1)})\right]}{\partial \lambda^{\pi(1)}} = \begin{cases} a_1 + Q^{1'}(\lambda^{\pi(1)}) & \text{if } \lambda^{\pi(1)} \le q_1 \\ (f^2(q_1) \cdot a_2 + f^2(q_2) \cdot a_1) + Q^{1'}(\lambda^{\pi(1)}) & \text{if } \lambda^{\pi(1)} \in (q_1, q_2) \\ a_2 + Q^{1'}(\lambda^{\pi(1)}) & \text{if } \lambda^{\pi(1)} \ge q_2 \end{cases} \tag{18}$$

According to formula (15), $a_1 + Q^{1'}(\lambda^{\pi(1)}) < 0$ holds. This indicates that when $\lambda^{\pi(1)} \le q_1$, the derivative of $E_{f_2}[x_d^1] + Q^1$ with respect to $\lambda^{\pi(1)}$ is negative. Hence, producer 1's total expected cost decreases in $\lambda^{\pi(1)}$ when $\lambda^{\pi(1)} \in (0, q_1]$, and is minimized at q_1 in this interval. Similarly, formula (15) also indicates that $Q^{1'}(\lambda^{\pi(1)}) + a_2 > 0$. Hence, in the interval where $\lambda^{\pi(1)} \in [q_2, \infty)$, producer 1's expected total cost $E_{f_2}[x_d^1] + Q^1$ increases in $\lambda^{\pi(1)}$ and thus is minimized at q_2. Therefore, the global minimizer of the cost function $E_{f_2}[x_d^1] + Q^1$ must reside in the interval $[q_1, q_2]$, which can be obtained by solving the equation $\frac{\partial \left[E_{f_2}(x_d^1)(\lambda^{\pi(1)}) + Q^1(\lambda^{\pi(1)})\right]}{\partial \lambda^{\pi(1)}} = (f^2(q_1) \cdot a_2 + f^2(q_2) \cdot a_1) + Q^{1'}(\lambda^{\pi(1)}) = 0$. The solution to this equation is $\lambda^{\pi(1)} = (Q^{1\prime})^{-1}(-f^2(q_1) \cdot a_2 - f^2(q_2) \cdot a_1)$, which equals

$$\begin{aligned} (Q^{1\prime})^{-1}\left(-f^2(q_1) \cdot a_2 - (1 - f^2(q_1)) \cdot a_1\right) &= (Q^{1\prime})^{-1}\left(\frac{Q^{1'}(\lambda_0^{\pi(1)}) + a_1}{a_2 - a_1}(a_2 - a_1) - a_1\right) \\ &= (Q^{1\prime})^{-1}(Q^{1'}(\lambda_0^{\pi(1)})) = \lambda_0^{\pi(1)} \end{aligned} \tag{19}$$

Hence, we conclude that if producer 2 adopts the mixed strategy f^2, $\lambda^{\pi(1)} = \lambda_0^{\pi(1)}$ minimizes producer 1's expected total cost and is the best response for producer 1. We then conclude that (f^1, f^2) is a mixed-strategy equilibrium design profile based on Definition 4. This completes the proof of Lemma 5. $\square$

The above lemma indicates that under the mixed-strategy equilibrium $\{f^1, f^2\}$, any product design outcome that can occur with positive probability is inferior to the one induced by an individual system.

Case 2(b): We finally consider the other sub-case where $r(1)$ has enough capacity to process the entire volume, i.e., $\underline{k_{r(1)} \ge d^{\pi(1)} + d^{\pi(2)}}$. In this case, the socially optimal allotment f^* sends all products to the cheaper processor $r(1)$ (Figure 4(f)) regardless of the design profile. Because of this, the optimal design choices of the two producers are not correlated in the collective system, and thus the equilibrium analysis becomes much simpler. Specifically, we can calculate that the dual-based cost allocation of each producer i equals

$$x_d^i = d^{\pi(i)} \cdot c_{r(1)}^{\pi(i)}. \tag{20}$$

Accordingly, the design choice for i that minimizes its total cost $x_d^i + Q^i$ equals $\lambda_d^{*\pi(i)} = (Q^{i\prime})^{-1}\left(-d^{\pi(i)} \cdot \tau_{r(1)}\right)$, which is no smaller than $\lambda_{ind}^{\pi(i)}$. Moreover, for producer 2, we can calculate that $\lambda_d^{*\pi(2)} > \lambda_{ind}^{\pi(2)}$ since $\tau_{r(1)} < \tau_{r(2)}$ and the investment function Q^2 is convex decreasing. That is, participating in a collective system induces producer 2 to adopt a strictly worse design in this case of the RN. Combining the above results, we conclude that Proposition 8 holds. $\square$

C.1.2. RNs with n Producers

Proof of Proposition 1 In this online appendix, we provide the detailed proof of Observation 1, which leads to Proposition 1 according to the sketch of the proof presented in the appendix of the paper. Recall that Observation 1 states the following.

Observation 1 *Under the low-synergy condition $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i \in N$, each producer i's dual-based cost allocation in the original RN instance G is equivalent to that in a sub-network G^i that consists of producer i and all other producers whose capacity is less efficient than $r(i)$.*

Proof of Observation 1 We show that Observation 1 holds in two steps. In Step 1, we provide an algorithm to compute the socially optimal allotment in any RN with a given design profile of the products; this enables us to come up with closed-form expressions of the optimal dual solution $[\beta^{*\pi}, \alpha_r^*]$ as functions of the design profile. In Step 2, we show that under the low-synergy condition, the optimal dual solution that corresponds to i's product and capacity is independent of any other producer with more efficient capacity (i.e., any producer $j < i$).

STEP 1: The algorithm we propose to compute the socially optimal allotment is a greedy one (referred to as the *greedy allotment algorithm*), described in Algorithm 2 shown on the next page.

Algorithm 3: A greedy product allotment algorithm in an n-producer RN

Input: An RN instance G
Output: A product allotment solution $\tilde{f}$
1. Rank the products in decreasing order of its recyclability level. Let $\sigma(\pi)$ denote the index of product π under the resulting ranking.
Let $\Pi_0 = \Pi$ and $R_0 = R$ equal the set of products and processors in G, respectively. Let $\tilde{f}_{[\pi,r]} = 0$ $\forall r$ and $\forall \pi$. Let $t = 0$.
while $\Pi_t \neq \emptyset$ **do**
  1. Let $\bar{\pi} \doteq \arg\max_{\pi\in\Pi_t} \lambda^\pi$ be the product with the worst design in Π_t.
     Let $\bar{r} \doteq \arg\min_{r\in R_t} \tau_r$ be the most efficient processor in R_t.
  2. Let $\tilde{f}_{[\bar{\pi},\bar{r}]} = \min\{d^{\bar{\pi}}, k_{\bar{r}}\}$. Let $d^{\bar{\pi}} = d^{\bar{\pi}} - \tilde{f}_{[\bar{\pi},\bar{r}]}$ and $k_{\bar{r}} = k_{\bar{r}} - \tilde{f}_{[\bar{\pi},\bar{r}]}$.
  3. Let $\Pi_{t+1} \doteq \Pi_t \setminus \{\pi \in \Pi_t : d^\pi = 0\}$, and $R_{t+1} \doteq R_t \setminus \{r \in R_t : k_r = 0\}$. Let $t = t + 1$.
end
Output the product allotment $\tilde{f}$.

The optimality of the $\tilde{f}$ solution generated by the greedy allotment algorithm (Algorithm 3) can be proven based on duality theory.

Claim 1. *The routing $\tilde{f}$ computed by the greedy allotment algorithm (Algorithm 3) is optimal to the centralized transportation problem (C).*

Proof of Claim 1 For convenience, we assume in this proof that $\tilde{f}$ is nondegenerate; the proof also applies to the degenerate case if we focus on one of the multiple potential dual solutions.

We prove the optimality of $\tilde{f}$ based on duality theory, i.e., we show the existence of a feasible dual solution of (C) that satisfies the complementary slackness constraints with respect to $\tilde{f}$. To this end, we first formulate the dual problem of (C) in the n-producer case.

$$(D) \quad \max \quad \sum_{\pi\in\Pi} d^\pi \beta^\pi + \sum_{r\in R} k_r \alpha_r \quad s.t. \quad \alpha_r + \beta^\pi \leq c_r^\pi \quad \forall \pi \in \Pi, \forall r \in R; \quad \alpha_r \leq 0 \quad \forall r \in R \tag{21}$$

Below, we show that the dual solution defined by formula (A.1)-(A.2) in the appendix of the paper is feasible to (D) and satisfies complementary slackness with respect to the allotment solution $\tilde{f}$ computed by Algorithm 3. For discussion convenience, we duplicate the formulas below: Let $R_\pi \doteq \{r : \tilde{f}_{[\pi,r]} > 0\}$ be the set of processors where product π is processed under $\tilde{f}$. We also denote the least and the most cost efficient processor in R_π as $\underline{r}_\pi \doteq r_{\max\{j:r_j\in R_\pi\}}$ and $\bar{r}_\pi \doteq r_{\min\{j:r_j\in R_\pi\}}$ respectively. For each producer $i \in N$, we first define $\beta^{\pi(i)}$ as

$$\beta^{\pi(i)} = c^{\pi(i)}_{\underline{r}_{\pi(i)}} - \sum_{j:\lambda^{\pi(j)}<\lambda^{\pi(i)}} (c^{\pi(j)}_{\bar{r}_{\pi(j)}} - c^{\pi(j)}_{\underline{r}_{\pi(j)}}). \tag{22}$$

Then consider the processor $r(i)$. Assume there exists $t \in N$ such that $r(i) \in R_{\pi(t)} \setminus \underline{r}_{\pi(t)}$, i.e., $r(i)$ processes product $\pi(t)$ under $\tilde{f}$ yet $r(i)$ is not the least cost efficient processor to do so. In other words, $\pi(t)$ is the most recyclable product processed at r under $\tilde{f}$ (which we denote as $\bar{\pi}_r$ in the appendix of the paper). We define $\alpha_{r(i)}$ as follows.

$$\alpha_{r(i)} = (c^{\pi(t)}_{r(i)} - c^{\pi(t)}_{\underline{r}_{\pi(t)}}) + \sum_{j:\lambda^{\pi(j)}<\lambda^{\pi(t)}} (c^{\pi(j)}_{\bar{r}_{\pi(j)}} - c^{\pi(j)}_{\underline{r}_{\pi(j)}}) \tag{23}$$

Note that $\alpha_{r(i)}$ is well-defined, since there cannot exist $t_1 \neq t_2$ such that $r(i)$ belongs to both sets $R_{\pi(t_1)} \setminus \underline{r}_{\pi(t_1)}$ and $R_{\pi(t_1)} \setminus \underline{r}_{\pi(t_2)}$ under the greedy routing algorithm. In case that $r(i) \notin \bigcup_{i=1}^n R_{\pi(i)} \setminus \underline{r}_{\pi(i)}$, we define $\alpha_r = 0$. We show that $[\alpha_r, \beta^\pi]$ is a feasible solution to (D) that satisfies complementary slackness with respect to $\tilde{f}$.

- Dual feasibility:

—For each $i \in N$, by definition, $\underline{r}_{\pi(i)}$ is the least cost efficient processor in $R_{\pi(i)}$. Thus $c^{\pi(i)}_r \le c^{\pi(i)}_{\underline{r}_{\pi(i)}}$ holds for all $r \in R_\pi$. This indicates that $\alpha_r \le 0 \ \forall r \in R$.

—Consider any product $\pi(i)$ and any processor r. Assume that $r \in R_{\pi(t)} \setminus \underline{r}_{\pi(t)}$ for some $t \in N$. We can calculate that

$$\alpha_r + \beta^{\pi(i)} - c^{\pi(i)}_r = \begin{cases} \sum_{j:\lambda^{\pi(t)}<\lambda^{\pi(j)}\le\lambda^{\pi(i)}} (\lambda^{\pi(j)} - \lambda^{\pi(j')}) \cdot (\tau_{\underline{r}_{\pi(j)}} - \tau_r) & \text{if } \lambda^{\pi(t)} < \lambda^{\pi(i)} \\ 0 \quad \text{if } t = i \\ \sum_{j:\lambda^{\pi(i)}<\lambda^{\pi(j)}\le\lambda^{\pi(t)}} (\lambda^{\pi(j)} - \lambda^{\pi(j')}) \cdot (\tau_r - \tau_{\underline{r}_{\pi(j)}}) & \text{if } \lambda^{\pi(t)} > \lambda^{\pi(i)} \end{cases} \tag{24}$$

In the above formula, $\pi(j')$ denotes the product that is the least recyclable among those with a better design compared to $\pi(j)$. Hence, we know that $\lambda^{\pi(j)} > \lambda^{\pi(j')}$. Furthermore, since t is such that $r \in R_{\pi(t)} \setminus \underline{r}_{\pi(t)}$, we know that $\pi(t)$ is processed at r. Since under the greedy routing algorithm, less recyclable products are processed using more efficient capacity, we conclude that (i) when $\lambda^{\pi(t)} < \lambda^{\pi(j)}$, $\tau_{\underline{r}_{\pi(j)}} \le \tau_r$ holds, and (ii) when $\lambda^{\pi(t)} > \lambda^{\pi(j)}$, $\tau_{\underline{r}_{\pi(j)}} \ge \tau_r$ holds. According to formula (24), this indicates that the dual constraint $\alpha_r + \beta^{\pi(i)} - c^{\pi(i)}_r \le 0$ is satisfied for every pair of product $\pi(i)$ and processor r such that $r \in R_{\pi(t)} \setminus \underline{r}_{\pi(t)}$ for some t.

—We then show that $\alpha_r + \beta^{\pi(i)} - c^{\pi(i)}_r \le 0$ remains valid in the case where $r \notin \bigcup_{i=1}^n R_{\pi(i)} \setminus \underline{r}_{\pi(i)}$. In the non-degenerate case, we know that if $r \notin \bigcup_{i=1}^n R_{\pi(i)} \setminus \underline{r}_{\pi(i)}$, then r must be only partially used or not used at all under $\tilde{f}$. Let r' be the processor that is partially used (there is only one such processor under the greedy routing algorithm). Note that if $r \notin \bigcup_{i=1}^n R_{\pi(i)} \setminus \underline{r}_{\pi(i)}$, then r must be no more cost efficient than r', i.e., $c^{\pi(i)}_r \ge c^{\pi(i)}_{r'}$. Hence, we can calculate that

$$\begin{aligned} \alpha_r + \beta^{\pi(i)} - c^{\pi(i)}_r = \beta^{\pi(i)} - c^{\pi(i)}_r &\le c^{\pi(i)}_{\underline{r}_{\pi(i)}} - c^{\pi(i)}_{r'} - \sum_{j:\lambda^{\pi(j)}<\lambda^{\pi(i)}} (c^{\pi(j)}_{\bar{r}_{\pi(j)}} - c^{\pi(j)}_{\underline{r}_{\pi(j)}}) \\ &= \sum_{j:\lambda^{\pi(j)}<\lambda^{\pi(i)}} (\lambda^{\pi(i)} - \lambda^{\pi(j)}) \cdot (\tau_{\bar{r}_{\pi(j)}} - \tau_{\underline{r}_{\pi(j)}}) \le 0 \end{aligned} \tag{25}$$

Combining the above observations, we conclude that $[\alpha_r, \beta^\pi]$ is a feasible solution to the dual problem (D).

- Complementary slackness:

—Any processor r that is not fully used under $\tilde{f}$ is not in the set $\bigcup_{i=1}^n R_{\pi(i)} \setminus \underline{r}_{\pi(i)}$, and therefore is assigned a dual value $\alpha_r = 0$.

—Consider any product $\pi(i)$ and any processor r. Assume that $f_{[\pi(i),r]} > 0$. Then by definition $r \in R_{\pi(i)}$.

∗ If $r \in R_{\pi(i)} \setminus \underline{r}_{\pi(i)}$, then by formula (24) we know that $\alpha_r + \beta^{\pi(i)} - c_r^{\pi(i)} = 0$.

∗ If r is the processor $\underline{r}_{\pi(i)}$, we consider two situations.

· In the first situation, we assume that $\pi(i)$ is the most recyclable product in Π. Then we know that $r = \underline{r}_{\pi(i)}$ is actually the partially used processor r'. Hence, by formula (25), $\alpha_r + \beta^{\pi(i)} - c_r^{\pi(i)} = c_{r'}^{\pi(i)} - c_{r'}^{\pi(i)} = 0$.

· In the second situation, $\pi(i)$ is not the most recyclable product. Consider the set S_r of products processed at $r = \underline{r}_{\pi(i)}$. According to the greedy routing algorithm, we know that $\pi(i)$ is the least recyclable product in S_r. In addition, if we let $\pi(t)$ denote the most recyclable product in S_r, then all product $\pi(j)$ such that $\lambda^{\pi(i)} < \lambda^{\pi(j)} < \lambda^{\pi(t)}$ are in S_r and are entirely processed at r, indicating that $\underline{r}_{\pi(j)} = \underline{r}_{\pi(i)}$, which is exactly r. Hence, we know that $\tau_{\underline{r}_{\pi(j)}} = \tau_r \ \forall j : \lambda^{\pi(i)} < \lambda^{\pi(j)} < \lambda^{\pi(t)}$. Thus by formula (24), $\alpha_r + \beta^{\pi(i)} - c_r^{\pi(i)} = 0$ as well.

Combining the above results, we prove that $[\alpha_r, \beta^\pi]$ is a feasible dual solution to (C) that satisfies complementary slackness with respect to $\tilde{f}$. By duality theory, this indicates that $\tilde{f}$ is a socially optimal routing in the RN. This completes the proof of Claim 1. □

STEP 2: In this step, we prove that, under the low-synergy condition $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)} \ \forall i \in N$, for each producer i, the optimal dual solution $[\beta^{*\pi(i)}, \alpha^{*\pi(i)}]$ is independent of any producer $j < i$.

We first show this for $\beta^{*\pi(i)}$. For convenience, we rewrite formula (A.1) that calculates $\beta^{*\pi(i)}$ as follows.

$$\beta^{*\pi(i)} = \underbrace{c^{\pi(i)}_{\bar{r}_{\pi(i)}}}_{①} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)}<\lambda^{\pi(i)} \text{ and } j>i} (c^{\pi(j)}_{\bar{r}_{\pi(j)}} - c^{\pi(j)}_{\underline{r}_{\pi(j)}})}_{②} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)}<\lambda^{\pi(i)} \text{ and } j<i} (c^{\pi(j)}_{\bar{r}_{\pi(j)}} - c^{\pi(j)}_{\underline{r}_{\pi(j)}})}_{③} \tag{26}$$

In the following points $(i)-(iii)$, we show that each term in the above formula is independent of both the product and the capacity of any producer $j > i$, respectively.

(i) *Part ① in (26) is independent of producers $j < i$: This is due to the fact that for any i, $\bar{r}_{\pi(i)}$ corresponds to a processor $r(l)$ where $l \geq i$.* To illustrate this, consider an example RN instance with four producers that satisfies $d^{\pi(i)} > \sum_{j<i} k_{r(j)}$[30]. Assuming the design profile is such that $\lambda^{\pi(3)} > \lambda^{\pi(1)} > \lambda^{\pi(2)} > \lambda^{\pi(4)}$, the socially optimal allotment is as depicted in Figure 10. We can make the following observations.

In general RNs, we can show that for any product $\pi(i)$, either its last unit is allotted to $r(i)$ (as in the case of $\pi(3)$ and $\pi(4)$ in this example), or it is entirely allotted to some $r(j)$ where $j > i$ (as in the case of $\pi(1)$ and $\pi(2)$ in this example). Hence, $\bar{r}_{\pi(i)}$ always corresponds to a processor $r(l)$ where $l \geq i$.

(ii) *Part ② in (26) is independent of producers $j < i$: This is the case as we can show that for any $\pi(j)$ such that $\lambda^{\pi(j)} < \lambda^{\pi(i)}$, both $\bar{r}_{\pi(j)}$ and $\underline{r}_{\pi(j)}$ correspond to $r(l)$'s where $l \geq i$.* In other words, $\pi(j)$ is recycled at processors less efficient than $r(i)$ under the socially optimal allotment. To see this, note

[30] For convenience, we consider a non-degenerate version of the low-synergy condition. The argument can be generalized to degenerate cases as well.

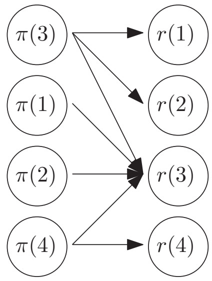


Figure 10 An RN example

- For product $\pi(3)$, due to the low-synergy condition $d^{\pi(3)} > k_{r(1)} + k_{r(2)}$, its last unit will be allotted to $r(3)$. This is also the case for product $\pi(4)$: Since $d^{\pi(4)} > k_{r(1)} + k_{r(2)} + k_{r(3)}$, the last unit of $\pi(4)$ will be allotted to $r(4)$.
- For product $\pi(1)$ and $\pi(2)$, we can see that $r(1)$ and $r(2)$ are already saturated by $\pi(3)$ when these two products are allotted in the greedy algorithm. Moreover, due to the modeling assumption, we know that $r(1), r(2), r(3)$ combined have sufficient capacity to process $\pi(1), \pi(2), \pi(3)$. Hence, $\pi(1)$ and $\pi(2)$ are both alloted to $r(3)$ only.

that such a product $\pi(j)$ will be alloted after $\pi(i)$ in the greedy algorithm and thus will only be alloted to processors that are less efficient than $\bar{r}_{\pi(i)}$, which we know from point (i) is less or equally efficient compared to $r(i)$.

(iii) *Part ③ in (26) equals zero: This is because for any $\pi(j)$ such that $\lambda^{\pi(j)} < \lambda^{\pi(i)}$ and whose index $j < i$, it must be recycled at only one processor under the socially optimal allotment, i.e., $\bar{r}_{\pi'} = \underline{r}_{\pi'}$.* We have shown this in point (i), illustrated by the allotment of product $\pi(1)$ and $\pi(2)$ in the RN in Figure 10.

Combining the above observations, we conclude that $\beta^{*\pi(i)}$ is independent of any producer $j < i$.

We continue to show that $\alpha^*_{r(i)}$ is also independent of any producer $j < i$. Similar to the above analysis to $\beta^{*\pi(i)}$, we also rewrite formula (A.2) that calculates $\alpha^*_{r(i)}$ as the summation of three parts.

$$\alpha^*_{r(i)} = \underbrace{(c^{\bar{\pi}_{r(i)}}_{r(i)} - c^{\bar{\pi}_{r(i)}}_{\bar{r}_{\bar{\pi}_{r(i)}}})}_{①} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)}<\lambda^{\bar{\pi}_{r(i)}} \text{ and } j>i} (c^{\pi(j)}_{\underline{r}_{\pi(j)}} - c^{\pi(j)}_{\bar{r}_{\pi(j)}})}_{②} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)}<\lambda^{\bar{\pi}_{r(i)}} \text{ and } j<i} (c^{\pi(j)}_{\underline{r}_{\pi(j)}} - c^{\pi(j)}_{\bar{r}_{\pi(j)}})}_{③}. \quad (27)$$

We also show that each part in this formula is independent of either the product or the capacity of any producer $j < i$. The proof is also based on the argument in (i)-(iii) above, plus some additional observations.

(iv) *Part ① in (27) is independent of producers $j < i$: This is because for any i, $\bar{\pi}_{r(i)}$ corresponds to a product $\pi(l)$ where $l \geq i$.* To see this, note that according to point (i) above, if there exists a product $\pi(j)$ where $j > i$ and that is less recyclable than $r(i)$ (i.e., allotted before $\pi(i)$ in the greedy algorithm), then due to the low-synergy condition, $r(i)$ will be saturated by $\pi(j)$, i.e., $\bar{\pi}_{r(i)} = \pi(j)$; this is the case for $i = 1, 2$ in the example shown in Figure 10. Otherwise, we show that the last unit of $\pi(i)$ is allotted to $r(i)$, and the remaining capacity at $r(i)$ is sufficient to process the products that follow $r(i)$ in the greedy algorithm until a product with a larger index appears; this is the case for $i = 3$ in the example, and thus $\bar{\pi}_{r(3)} = 4$.

(v) *Part ② in (27) is independent of producers $j < i$ and part ③ there equals zero.* This can be shown by combining points (iv) with the arguments in points (ii) and (iii), respectively.

Combining the above observations, we conclude that $\alpha^*_{r(i)}$ is independent of any producer $j < i$. This completes the proof of Observation 1. Hence Proposition 1 follows according to the proof presented in the appendix of the paper. $\square$

C.2. Technical Details in §5

In this section, we provide the technical details in the analysis of the marginal contribution-based cost allocation x_m.

Proof of Proposition 2 Recall in the proof presented in the appendix of the paper, we have shown that within an interval of $\lambda^{\pi(i)}$ where $f^{*\{j:j\geq i\}}$ is unchanged, we can calculate that the local minimum of the function $x_m^i + Q^i$ must satisfy the equation below:

$$\frac{\partial(x_m^i + Q^i)}{\partial\lambda^{\pi(i)}} = \sum_{t=i}^{n} f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \frac{\partial c^{\pi(i)}_{r(t)}}{\partial\lambda^{\pi(i)}} + Q^{i\prime} = \sum_{t=i}^{n} f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)} + Q^{i\prime} = 0 \tag{28}$$

and thus equals $(Q^{i\prime})^{-1}(-\sum_{t=i}^n f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$.

A key technical step that follows is that the global minimizer of the function $x_m^i + Q^i$ is achieved at such a local minimum under some $f^{*\{j:j\geq i\}}$ solution. We provide the details of this step in this online appendix. Assume this is not the case, and the global minimizer of $x_m^i + Q^i$ equals some other value $\lambda_0^{\pi(i)}$. Then we know that there exist intervals $[c_1, \lambda_0^{\pi(i)}]$ and $[\lambda_0^{\pi(i)}, c_2]$, where the function $x_m^i + Q^i$ strictly decreases and increases, respectively. Assume c_1 and c_2 are sufficiently close to $\lambda_0^{\pi(i)}$ such that when producer i's design choice varies within each of the two intervals, the optimal product allotment in the coalition $\{j : j \geq i\}$ is not changed. Denote the corresponding optimal product allotment solutions as $f_1^{*\{j:j\geq i\}}$ and $f_2^{*\{j:j\geq i\}}$. Since $x_m^i + Q^i$ strictly decreases on $[c_1, \lambda_0^{\pi(i)}]$, we know that $\lambda_0^{\pi(i)} < (Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. Similarly, we can conclude that $\lambda_0^{\pi(i)} > (Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. This leads to the inequality $(Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)}) > (Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. However, this cannot be the case: The optimal allotment changes from $f_2^{*\{j:j\geq i\}}$ to $f_1^{*\{j:j\geq i\}}$ after producer i improves its own design (i.e., the value of $\lambda^{\pi(i)}$ is smaller in the interval $[\lambda_0^{\pi(i)}, c_2]$ compared to in the interval $[c_1, \lambda_0^{\pi(i)}]$). According to the greedy allotment algorithm, product $\pi(i)$ will be allotted to less efficient processors under $f_1^{*\{j:j\geq i\}}$ than $f_2^{*\{j:j\geq i\}}$. This means that $\sum_{t=i}^n f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)} \geq \sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)}$, and thus $(Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$ should be *no larger than* $(Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. This leads to a contradiction. Thus we conclude that the global minimum of $x_m^i + Q^i$ equals $(Q^{i\prime})^{-1}(-\sum_{t=i}^n f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$ under some $f^{*\{j:j\geq i\}}$ solution. □

Proof of Proposition 3 In order to show that x_m is individually rational (i.e., $x_m^i = v(\{j : j \geq i\}) - v(\{j : j > i\}) \geq v(i)$ holds for each producer i), we show that the PA game is sub-additive. Recall that we use f^{*S} to denote the socially optimal allotment within the sub-coalition S. Let S_1 and S_2 denote any two mutually exclusive sub-coalitions in N. Consider their union $S_1 \cup S_2$ and the following product allotment within this combined coalition: For all products manufactured by producers in S_1 and S_2, it is allotted according to f^{*S_1} and f^{*S_2}, respectively. This is obviously a feasible allotment solution and the total cost incurred equals $v(S_1) + v(S_2)$. Since $v(S_1 \cup S_2)$ is the minimum total cost that can be achieved in $S_1 \cup S_2$, we know that $v(S_1 \cup S_2) \leq v(S_1) + v(S_2)$. Since this inequality holds for any mutually exclusive sub-coalitions S_1 and S_2, by definition, the PA game is sub-additive. This means that for any producer i, $v(i \cup \{j : \tau_{r(j)} > \tau_{r(i)}\}) \leq v(\{j : \tau_{r(j)} > \tau_{r(i)}\}) + v(i)$. Since x_m^i is calculated as the difference, $v(i \cup \{j : \tau_{r(j)} > \tau_{r(i)}\}) - v(\{j : \tau_{r(j)} > \tau_{r(i)}\})$, we conclude that $x_m^i \leq v(i)$ holds for each producer i, i.e., x_m is by definition individually rational.

Proof of Proposition 4. We provide the complete proof of Proposition 4 in this online appendix. Consider an RN instance that satisfies the condition $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i > 2$. In order to show weak convexity of the PA game, we prove that given any design profile Λ, for each producer i, $v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S)$ holds $\forall S \subset T \subset \{i+1, ..., n\}$. We do so by studying two cases of the sub-coalition T. In each case, we first compute the optimal routing within a coalition in closed-form, and characterize the change of this routing when i joins the coalition. We then calculate the marginal contribution of i to this coalition and prove $v(T\cup\{i\})-v(T)\le v(S\cup\{i\})-v(S)$.

Case 1: $2\notin T$: We first consider the case where the sub-coalition T does not contain producer 2 (which is the case considered in the sketch of the proof in Appendix C.2). Recall that in the Appendix of the paper, we introduced some additional notation for convenience of the proof. We duplicate these notations below: Given a design profile Λ, define a set $J(T)\doteq\{\pi(j),j\in T:\lambda^{\pi(l)}<\lambda^{\pi(j)},\forall l>j\}$. We rank the indices of the elements in $J(T)$ in increasing order and relabel them as $j_1^T,j_2^T,...,j_{|J(T)|}^T$. (In the rest of this proof, for notational simplicity, we sometimes refer to $\pi(j)\in J(T)$ as $j\in T$.) By the definition of $J(T)$, we know that $\lambda^{\pi(j_1^T)}>\lambda^{\pi(j_2^T)}>...>\lambda^{j_{|J(T)|}^T}$. Moreover, it can also be observed that $\pi(j_1^T)$ is the least recyclable product in T (to see this, note that such a product must be contained in $J(T)$, hence $\pi(j_1^T)$ is the least recyclable product in $J(T)$.) For convenience, we also relabel i as j_0^T.

Since T does not contain producer 2, we know that the RN that corresponds to T satisfies the low-synergy condition $d^{\pi(j)}\ge\sum_{l\in S:l<j}k_{r(l)}$ $\forall j\in T$. We implement the greedy routing algorithm on this RN to compute the optimal routing within T, denoted as f^{*T}. Using the $J(T)$ notation defined above, we can write f^{*T} in closed-form.

- Since product $\pi(j_1^T)$ is the least recyclable product in T, it is routed first under the greedy routing algorithm. Due to the low-synergy condition, we know that $\pi(j_1^T)$ will saturate all processors that are more efficient than $r(j_1^T)$ (i.e., all processors $r(l):l<j_1^T$). The rest will be routed to $r(j_1^T)$, which has sufficient capacity to process $\pi(j_1^T)$. Hence, we obtain that

$$f^{*T}_{[\pi(j_1^T),r(l)]}=\begin{cases}k_{r(l)} & \forall l<j_1^T\\ d^{\pi(j_1^T)}-\sum_{s<j_1^T}k_{r(s)} & l=j_1^T\\ 0 & \forall l>j_1^T\end{cases}\tag{29}$$

- Consider the product $\pi(j_t^T)$, $t=2,...,|J(T)|$.

—First, note that this product will not be routed to any processor that are more efficient than $r(j_{t-1}^T)$. This is because under the greedy routing algorithm, product $\pi(j_{t-1}^T)$ is routed before $\pi(j_t^T)$. Due to the low-synergy condition, the total capacity at processors that are more efficient than $r(j_{t-1}^T)$ is no larger than the return volume of $\pi(j_{t-1}^T)$, and therefore will be saturated after $\pi(j_{t-1}^T)$ is routed by the greedy algorithm.

—Second, at the processor $r(j_{t-1}^T)$, the amount of capacity left when the greedy algorithm starts to route $\pi(j_{t-1}^T)$ is $\sum_{s\in T:s\le j_{t-1}^T}k_{r(s)}-\sum_{s\in T:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}}d^{\pi(s)}$. We show that this value is nonnegative.

Claim 2. *For each $t\in\{2,...,|J(T)|\}$, $\sum_{s\in T:s\le j_{t-1}^T}k_{r(s)}-\sum_{s\in T:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}}d^{\pi(s)}\ge0$ holds.*

Proof of Claim 2 We first prove that the set $\{s\in T:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}\}\subset\{s\in T:s\le j_{t-1}^T\}$. Assume this is not the case. Let $s_0\in T$ be the largest index such that $\lambda^{\pi(s_0)}>\lambda^{\pi(j_t^T)}$ yet $s_0>j_{t-1}^T$. First, since $j_t^T\in J(T)$ and $\lambda^{\pi(s_0)}>\lambda^{\pi(j_t^T)}$, we know $s_0<j_t^T$. Then we can show the following:

$*$ $\forall l:l>j_t^T$, $l>s_0$ holds, and since $j_t^T\in J(T)$, we know that $\lambda^{\pi(l)}<\lambda^{\pi(j_t^T)}<\lambda^{\pi(s_0)}$ also holds.

$*$ $\forall l:j_t^T>l>s_0$, we show that $\lambda^{\pi(l)}<\lambda^{\pi(s_0)}$ should also hold, as otherwise, l satisfies $\lambda^{\pi(l)}>\lambda^{\pi(s_0)}>\lambda^{\pi(j_t^T)}$ and $l>s_0>j_{t-1}^T$, which contradicts the definition of s_0 as the largest index such that $\lambda^{\pi(s_0)}>\lambda^{\pi(j_t^T)}$ and $s_0>j_{t-1}^T$.

Therefore, we conclude that $\forall l$ such that $l > s_0$, $\lambda^{\pi(l)} < \lambda^{\pi(s_0)}$ is satisfied. Hence $s_0 \in J(T)$. However, by the definition of s_0, $j_{t-1}^T < s_0 < j_t^T$ holds, and thus $s_0 \notin J(T)$. Accordingly, we conclude that $\{s \in T : \lambda^{\pi(s)} > \lambda^{\pi(j_t^T)}\} \subset \{s \in T : s \leq j_{t-1}^T\}$. Based on this, we obtain $\sum_{s\in T: s\leq j_{t-1}^T} k_{r(s)} \geq \sum_{s\in T:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}} k_{r(s)}$. Since by assumption, $d^{\pi(s)} \leq k_{r(s)}$ is satisfied $\forall s \in N$, we can further show that $\sum_{s\in T:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}} k_{r(s)} \geq \sum_{s\in T:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}} d^{\pi(s)}$. Combining these two inequalities leads to Claim 2. $\square$

Based on the two observations above, we can characterize how $\pi(j_t^T)$ is routed under f^{*T}. Due to the low-synergy condition, the product $\pi(j_t^T)$ will saturate the capacity left at $r(j_{t-1}^T)$, as well as the full capacity at each processor where the efficiency level is between $r(j_{t-1}^T)$ and $r(j_t^T)$ (i.e., $r(l) : j_{t-1}^T \leq l \leq j_t^T$). The rest of the return volume will be routed to $r(j_t^T)$, which has sufficient capacity to process $\pi(j_t^T)$. To summarize, we have

$$f^{*T}_{[\pi(j_t^T),r(l)]} = \begin{cases} 0 \quad \forall l < j_{t-1}^T \\ \sum_{s=\min T}^{j_{t-1}^T} k_{r(s)} - \sum_{s:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}} d^{\pi(s)} \quad l = j_{t-1}^T \\ k_{r(l)} \quad \forall j_{t-1}^T < l < j_t^T \\ d^{\pi(j(T)_1)} - f^*_{[\pi(j_t^T),r(j_{t-1}^T)]} - \sum_{j_{t-1}^T<s<j_t^T} k_{r(s)} \quad l = j_t^T \\ 0 \quad \forall l > j_t^T \end{cases} \qquad \forall t = 2, ..., |J(T)|. \tag{30}$$

- Consider the product $\pi(h)$ such that $h \in T \setminus J(T)$. In this case, either there exists $t \in \{2, 3, ..., |J(T)|\}$ such that $\lambda^{\pi(j_{t-1}^T)} < \lambda^{\pi(h)} < \lambda^{\pi(j_t^T)}$, or $\lambda^{\pi(j_{|J(T)|}^T)} < \lambda^{\pi(h)}$. In the first situation, under the greedy routing algorithm, we know that $\pi(h)$ will be routed after $\pi(j_{t-1}^T)$. Hence, it will not be routed to any processor that is more efficient than $r(j_{t-1}^T)$. Moreover, Claim 2 indicates that $r(j_{t-1}^T)$ has sufficient capacity to process the entire return volume of $\pi(h)$. This is because we can calculate that the capacity left at $r(j_{t-1}^T)$ after $\pi(h)$ is routed there equals $\sum_{s\in T:s\leq j_{t-1}^T} k_{r(s)} - \sum_{s\in T:\lambda^{\pi(s)}\geq\lambda^{\pi(h)}} d^{\pi(s)} \geq \sum_{s\in T:s\leq j_{t-1}^T} k_{r(s)} - \sum_{s\in T:\lambda^{\pi(s)}>\lambda^{\pi(j_t^T)}} d^{\pi(s)} \geq 0$. Therefore, we conclude that under f^{*T}, the entire return volume of this product $\pi(h)$ is routed to $r(j_{t-1}^T)$. Mathematically, $\forall t = 2, 3, ..., |J(T)|$,

$$f^{*T}_{[\pi(h),r(l)]} = \begin{cases} d^{\pi(h)} \quad l = j_{t-1}^T \\ 0 \quad \forall l \neq j_{t-1}^T \end{cases} \qquad \forall \pi : \lambda^{\pi(j_{t-1}^T)} < \lambda^{\pi(h)} < \lambda^{\pi(j_t^T)} \tag{31}$$

This argument also applies to any product $\pi(h)$ such that $\lambda^{\pi(j_{|J(T)|}^T)} < \lambda^{\pi(h)}$. In this case, $f^{*T}_{[\pi(h),r(l)]}$ equals $d^{\pi(h)}$ if $l = j_{|J(T)|}^T$, and 0 otherwise.

Now consider the case after producer i joins T. Since $T \subset \{i+1, ..., n\}$, we know that $\pi(i)$ is the most cost efficient capacity in the coalition $T \cup \{i\}$ and is the first processor used under the greedy routing algorithm. Comparing the optimal routing $f^{*T\cup\{i\}}$ with f^{*T}, we make the following observations.

- According to formula (31), we know that every product $\pi(h)$ such that $h \notin J(T)$ is routed in the same way under $f^{*T\cup\{i\}}$ as under f^{*T}.
- Consider the product $\pi(j_t^T)$ such that $j_t^T \in J(T)$. According to formula (29) - (30),

—if $\lambda^{\pi(j_t^T)} > \lambda^{\pi(i)}$, $f^{*T\cup\{i\}}$ is different from f^{*T} in that $k_{r(i)}$ amount of the return volume of product $\pi(j_t^T)$ is rerouted from $r(j_t^T)$ to $r(j_{t-1}^T)$ (this is well-defined even if $t = 1$ as we have relabeled i as j_0^T.)

—if $\lambda^{\pi(j_t^T)} < \lambda^{\pi(i)}$, we observe the same situation except that only $k_{r(i)} - d^{\pi(i)}$ amount of the return volume of product $\pi(j_t^T)$ is rerouted.

Note that product $\pi(i)$ will also be routed differently under $f^{*T\cup\{i\}}$ compared to when i operates independently, if $\pi(i)$ is not the least recyclable product in $T\cup\{i\}$. In that case, we know that $i\notin J(T\cup\{i\})$ (and thus $J(T\cup\{i\})=J(T)$). Hence, according to formula (31), if $\lambda^{\pi(j^T_{t-1})}<\lambda^{\pi(i)}<\lambda^{\pi(j^T_t)}$ for some $t\in\{2,3,...,|J(T)|\}$, then the entire return volume of $\pi(i)$ is routed to $r(j^T_{t-1})$ under $f^{*T\cup\{i\}}$. If $\lambda^{\pi(j^T_{|J(T)|})}<\lambda^{\pi(i)}$, then $\pi(i)$ is routed to $r(j^T_{|J(T)|})$ under $f^{*T\cup\{i\}}$

In order to capture the above observations mathematically, we define $\bar{t}(T)\doteq\min\{t:\lambda^{\pi(j^T_t)}<\lambda^{\pi(i)}\}$, i.e., $\bar{t}(T)$ is the smallest index such that $\pi(j^T_{\bar{t}(T)})$ is more recyclable than $\pi(i)$. In case $\pi(i)$ is more recyclable than all products $\pi(j^T_t)$ such that $j^T_t\in J(T)$, we set $\bar{t}(T)=|J(T)|+1$. We can then calculate that

$$
\begin{aligned}
& v(T\cup\{i\})-v(T)-v(i)\\
=& \sum_{t=1}^{\bar{t}(T)-1} k_{r(i)}\cdot\left(c^{\pi(j^T_t)}_{r(j^T_{t-1})}-c^{\pi(j^T_t)}_{r(j^T_t)}\right)+\sum_{t=\bar{t}(T)}^{|J(T)|}(k_{r(i)}-d^{\pi(i)})\cdot\left(c^{\pi(j^T_t)}_{r(j^T_{t-1})}-c^{\pi(j^T_t)}_{r(j^T_t)}\right)+d^{\pi(i)}\cdot\left(c^{\pi(i)}_{r(j^T_{\bar{t}(T)-1})}-c^{\pi(i)}_{r(i)}\right)\\
=& \sum_{t=1}^{\bar{t}(T)-1} k_{r(i)}\cdot\lambda^{\pi(j^T_t)}\cdot\left(\tau_{r(j^T_{t-1})}-\tau_{r(j^T_t)}\right)+\sum_{t=\bar{t}(T)}^{|J(T)|}(k_{r(i)}-d^{\pi(i)})\cdot\lambda^{\pi(j^T_t)}\cdot\left(\tau_{r(j^T_{t-1})}-\tau_{r(j^T_t)}\right)\\
&+d^{\pi(i)}\cdot\lambda^{\pi(i)}\cdot\left(\tau_{r(j^T_{\bar{t}(T)-1})}-\tau_{r(i)}\right) &(32)\\
=& \underbrace{\sum_{t=1}^{\bar{t}(T)-1} d^{\pi(i)}\cdot\left(\lambda^{\pi(j^T_t)}-\lambda^{\pi(i)}\right)\cdot\left(\tau_{r(j^T_{t-1})}-\tau_{r(j^T_t)}\right)}_{\Delta^1(T,i)}+\underbrace{\sum_{t=1}^{|J(T)|}(k_{r(i)}-d^{\pi(i)})\cdot\lambda^{\pi(j^T_t)}\cdot\left(\tau_{r(j^T_{t-1})}-\tau_{r(j^T_t)}\right)}_{\Delta^2(T,i)} &(33)
\end{aligned}
$$

Note that in this proof, we treat a summation from $t=t_1$ to $t=t_2$ where $t_1>t_2$ as 0. Similarly, we can also calculate $v(S\cup\{i\})-v(S)-v(i)$. In the following, we show that $\Delta^1(T,i)\le\Delta^1(S,i)$ and $\Delta^2(T,i)\le\Delta^2(S,i)$. To this end, we prove the following three claims regarding the relationship between $J(T)$ and $J(S)$.

Claim 3. *Consider any element $j^T_{t'}\in J(T)$ and $j^S_{t''}\in J(S)$. If $(j^T_{t'-1},j^T_{t'})\cap(j^S_{t''-1},j^S_{t''})\neq\emptyset$, then $\lambda^{\pi(j^T_{t'})}\ge\lambda^{\pi(j^S_{t''})}$.*

Proof of Claim 3. Assume $\lambda^{\pi(j^T_{t'})}<\lambda^{\pi(j^S_{t''})}$. In that case, we know that $j^T_{t'}>j^S_{t''}$, since otherwise it contradicts the fact that $j^T_{t'}\in J(T)$ by the definition of $J(T)$. On the other hand, since $(j^T_{t'-1},j^T_{t'})\cap(j^S_{t''-1},j^S_{t''})\neq\emptyset$, we know that $j^T_{t'-1}<j^S_{t''}$. This implies $\lambda^{\pi(j^S_{t''})}<\lambda^{\pi(j^T_{t'-1})}$ as $j^T_{t'-1}\in J(T)$. Combining the above observations, we conclude that $j^T_{t'-1}<j^S_{t''}<j^T_{t'}$ and $\lambda^{\pi(j^T_{t'})}<\lambda^{\pi(j^S_{t''})}<\lambda^{\pi(j^T_{t'-1})}$ both hold. However, according to Claim 2, we know that $\nexists s$ such that $s>j^T_{t-1}$ and $\lambda^{\pi(s)}>\lambda^{\pi(j^T_t)}$. By contradiction, we conclude that $\lambda^{\pi(j^T_{t'})}\ge\lambda^{\pi(j^S_{t''})}$ must hold. □

Claim 4. *$j^T_{\bar{t}(T)-1}$ in coalition T is no smaller than $j^S_{\bar{t}(S)-1}$ in coalition S.*

Proof of Claim 4. Assume that $j^T_{\bar{t}(T)-1}<j^S_{\bar{t}(S)-1}$. According to the definition of $\bar{t}(S)$ in coalition S, $\lambda^{\pi(j^S_{\bar{t}(S)-1})}>\lambda^{\pi(i)}$ holds. Hence, we know that $j^S_{\bar{t}(S)-1}\notin J(T)$, since the definition of $\bar{t}(T)$ implies that $\forall j^T_t\in J(T): j^T_t>j^T_{\bar{t}(T)-1}$, we have $\lambda^{\pi(j^T_t)}<\lambda^{\pi(i)}$. However, since $S\subset T$, $j^S_{\bar{t}(S)-1}\in T\setminus J(T)$. Hence, by the definition of $J(T)$, there must exist $l>j^S_{\bar{t}(S)-1}$ such that $\lambda^{\pi(l)}>\lambda^{\pi(j^S_{\bar{t}(S)-1})}$. Let l_0 be the largest such index. We show that $l_0\in J(T)$ as follows: Assume $l_0\notin J(T)$, then by the definition of $J(T)$, $\exists l'>l_0$ and $\lambda^{\pi(l')}>\lambda^{\pi(l_0)}$. Hence, we know that l' is a larger index than l_0 that satisfies $l'>l_0>j^S_{\bar{t}(S)-1}$ and $\lambda^{\pi(l')}>\lambda^{\pi(l_0)}>\lambda^{\pi(j^S_{\bar{t}(S)-1})}$.

Consider l_0. Since it is in $J(T)$, there must exist $t=1,2,...,|J(T)|$ such that $l_0=j^T_t$. According to the above analysis, we know that $j^T_t>j^S_{\bar{t}(S)-1}>j^T_{\bar{t}(T)-1}$, thus $t\ge\bar{t}(T)$. In addition, we can also derive $\lambda^{\pi(j^T_t)}>\lambda^{\pi(j^S_{\bar{t}(S)-1})}>\lambda^{\pi(i)}$. Hence, by the definition of $\bar{t}(T)$, we know that $\bar{t}(T)>t\ge\bar{t}(T)$, which cannot happen. This indicates that $j^T_{\bar{t}(T)-1}\ge j^S_{\bar{t}(S)-1}$ must hold. □

CLAIM 5. *$j^T_{|J(T)|}$ is no smaller than $j^S_{|J(S)|}$ in coalition S.*

Proof of Claim 5. By definition of $J(T)$, we know that $j^T_{|J(T)|}$ is always the largest index in T. Since $S \subset T$, we know that $j^T_{|J(T)|} = \max T \geq \max S = j^S_{|J(S)|}$. $\square$

Continuing the proof of Proposition 4, we first show that $\Delta^1(T,i) \leq \Delta^1(S,i)$ based on Claim 3 and Claim 4. First, let $t_0 \in \{1,2,...,|J(T)|\}$ such that $j^S_{\bar{t}(S)-1} \in (j^T_{t_0-1}, j^T_{t_0}]$. Since $j^S_{\bar{t}(S)-1} \leq j^T_{\bar{t}(T)-1}$, we know that $t_0 \leq \bar{t}(T)-1$. Divide $\Delta^1(T,i)$ into two parts as follows.

$$\begin{aligned}\Delta^1(T,i) = & \underbrace{\sum_{t=1}^{t_0-1} d^{\pi(i)} \cdot \left[(\lambda^{\pi(j^T_t)} - \lambda^{\pi(i)}) \cdot (\tau_{r(j^T_{t-1})} - \tau_{r(j^T_t)}) + (\lambda^{\pi(j^T_{t_0})} - \lambda^{\pi(i)}) \cdot (\tau_{r(j^T_{t_0-1})} - \tau_{r(j^S_{\bar{t}(S)-1})})\right]}_{①} \\ & \underbrace{+ d^{\pi(i)} \cdot \left[(\lambda^{\pi(j^T_{t_0})} - \lambda^{\pi(i)}) \cdot (\tau_{r(j^S_{\bar{t}(S)-1})} - \tau_{r(j^T_{t_0})}) + \sum_{t=t_0+1}^{\bar{t}(T)-1} (\lambda^{\pi(j^T_t)} - \lambda^{\pi(i)}) \cdot (\tau_{r(j^T_{t-1})} - \tau_{r(j^T_t)})\right]}_{②} \end{aligned} \tag{34}$$

Because (i) $\tau_{r(j^S_{\bar{t}(S)-1})} \leq \tau_{r(j^T_{t_2})}$, (ii) $\tau_{r(j^T_{t-1})} \leq \tau_{r(j^T_t)}$ $\forall t = 1,2,...,|J(T)|$, and (iii) $\lambda^{\pi(j^T_t)} \geq \lambda^{\pi(i)}$ $\forall t < \bar{t}(T)$, we know that ② ≤ 0 holds.

We then prove that ① $\leq \Delta^1(S,i)$. We can interpret ① as follows: The set of numbers $\{\tau_{r(j^T_t)} : 1 \leq t \leq t_0 - 1\}$ divide the interval $[\tau_{r(i)}, \tau_{r(j^S_{\bar{t}(S)-1})})]$ into several sub-intervals. Each sub-interval $[\tau_{r(j^T_{t-1})}, \tau_{r(j^T_t)}] : t \in \{1,...,t_0-1\}$ is associated with a weight equal to $d^{\pi(i)} \cdot (\lambda^{\pi(j^T_t)} - \lambda^{\pi(i)})$; the last interval $[\tau_{r(j^T_{t_0-1})}, \tau_{r(j^S_{\bar{t}(S)-1})}]$ is associated with a weight of $d^{\pi(i)} \cdot (\lambda^{\pi(j^T_{t_0})} - \lambda^{\pi(i)})$. Then the absolute value of ①, |①|, is the sum of the weighted length of these sub-intervals. Note that $\Delta^1(S,i)$ can also be interpreted in this way. In that case, $[\tau_{r(i)}, \tau_{r(j^S_{\bar{t}(S)-1})}]$ is divided into sub-intervals $[\tau_{r(j^S_{t-1})}, \tau_{r(j^S_t)}]$, and each of these intervals is associated with a weight $d^{\pi(i)} \cdot (\lambda^{\pi(j^S_t)} - \lambda^{\pi(i)})$.

Assume that ① $> \Delta^1(S,i)$, i.e., |①| $< |\Delta^1(S,i)|$. Then we know that there exist $j^T_{t'} \in J(T) : t' \leq t_0 < \bar{t}(T)$ and $j^S_{t''} \in J(S) : t'' < \bar{t}(S)$ such that $(j^T_{t'-1}, j^T_{t'}) \cap (j^S_{t''-1}, j^S_{t''}) \neq \emptyset$ and the weight $d^{\pi(i)} \cdot (\lambda^{\pi(j^T_{t'})} - \lambda^{\pi(i)}) < d^{\pi(i)} \cdot (\lambda^{\pi(j^S_{t''})} - \lambda^{\pi(i)})$, which implies $\lambda^{\pi(j^T_{t'})} < \lambda^{\pi(j^S_{t''})}$. This contradicts Claim 3, thus ① $\leq \Delta^1(S,i)$ must hold.

To summarize, we have shown that $\Delta^1(T,i) =$ ① + ② $\leq \Delta^1(S,i)$. Similarly, we can prove $\Delta^2(T,i) \leq \Delta^2(S,i)$ based on Claim 3 and Claim 5 (basically we treat $j^T_{|J(T)|}$ and $j^S_{|J(S)|}$ in the same way as $j^T_{\bar{t}(T)-1}$ and $j^S_{\bar{t}(S)-1}$ when proving $\Delta^1(T,i) \leq \Delta^1(S,i)$). Therefore $v(T \cup \{i\}) - v(T) - v(i) = \Delta^1(T,i) + \Delta^2(T,i) \leq \Delta^1(S,i) + \Delta^2(S,i) = v(S \cup \{i\}) - v(S) - v(i)$ holds. This completes our proof that under the low-synergy condition specified in Proposition 4, for each producer i, $v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S)$ holds $\forall S \subset T \subset \{i+1,...,n\}$ if T does not contain producer 2.

Case 2: $2 \in T$: Now we prove $v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S)$ when T contains producer 2. Before presenting the proof, we make the following preliminary observations.

- We know that since $T \subset \{i+1,...,n\}$, i must equal 1. Hence, $1 \notin T$ and the inequality we need to prove in this case is $v(T \cup \{1\}) - v(T) \leq v(S \cup \{1\}) - v(S)$.
- It can be observed that the proof in Case 1 is based on the closed-form characterization of the optimal routing f^{*T} in (29) - (31). Hence, we can apply this proof as long as the optimal routings in coalitions T, S, $T \cup \{i\}$ and $S \cup \{i\}$ all follows (29) - (31). We observe this will be this case if

—(*Condition 1*) the low-synergy condition also applies to producer 2, $d^{\pi(2)} \geq k_{r(1)}$;

—(*Condition 2*) $\pi(2)$ is not the least recyclable product in T (i.e., $\exists j \in T \setminus \{2\}$ such that $\lambda^{\pi(j)} > \lambda^{\pi(2)}$).

Hence, in the rest of the discussion of Case 2, we focus on RN instances where the above two conditions do not hold. In particular, we assume $d^{\pi(2)} < k_{r(1)}$, and depending on the relative recyclability of the products involved in T and S, we have the following sub-cases.

1. (Case 2a) $2 \notin S$, and $\pi(2)$ is the least recyclable product in T, i.e., $\lambda^{\pi(2)} > \lambda^{\pi(j)}\ \forall j \in T$.
2. (Case 2b) $2 \in S$, and $\pi(2)$ is the least recyclable product in T (as well as in S).

Case 2a: $2 \notin S$ **and** $\lambda^{\pi(2)} > \lambda^{\pi(j)}\ \forall j \in T$: In that case, we only need to prove that $v(T \cup \{1\}) - v(T) \leq v(T \setminus \{2\} \cup \{1\}) - v(T \setminus \{2\})$, since we know that $v(T \setminus \{2\} \cup \{1\}) - v(T \setminus \{2\}) \leq v(S \cup \{1\}) - v(S)$ according to the result obtained in Case 1.

Recall that we assume $d^{\pi(2)} < k_{r(1)}$ and $\lambda^{\pi(2)} > \lambda^{\pi(j)}\ \forall j \in T$ (i.e., $j_1^T = 2$ by the definition of $J(T)$). First, since producer 1 is not T, the RN corresponds to coalition T satisfies the low-synergy condition, and thus the optimal routing in T, f^{*T}, follows (29) - (31). However, $f^{*T\cup\{i\}}$ will be different since the low-synergy condition is violated between producer 1 and 2 who are both in the coalition $T \cup \{i\}$. As a consequence, $v(T \cup \{1\}) - v(T) - v(1)$ also needs to be calculated in a different way. We discuss this in three situations as follows.

- When $\bar{t}(T) > 2$, by the definition of $\bar{t}(T)$, we know that producer 2 (which is assumed to be the least recyclable product in T) will be first routed to $r(1)$ under the greedy algorithm. Since $d^{\pi(2)} < k_{r(1)}$, the entire return volume of 2 is sent to $r(1)$ under $f^{*T\cup\{1\}}$. The remaining capacity at $r(1)$ will be used to process the second least recyclable product. We can show that this product is exactly $\pi(j_2^T)$, since otherwise, the index associated with this product must be smaller than 2, contradicting with the fact that $2 = \min T$. Hence, comparing $f^{*T\cup\{1\}}$ with f^{*T}, it can be shown that $k_{r(1)} - d^{\pi(2)}$ amount of product $\pi(j_2^T)$ is rerouted from $r(j_2^T)$ to $r(1)$. Moreover, $d^{\pi(2)}$ amount of $\pi(j_2^T)$ is rerouted from $r(j_2^T)$ to $r(2)$ (as the entire volume of $\pi(2)$ goes to $r(1)$, freeing up the capacity at $r(2)$ to process $\pi(j_2^T)$.) As for the other products $\pi(j_t^T)$, their routing is the same as in Case 1, i.e., either $k_{r(1)}$ or $k_{r(1)} - d^{\pi(1)}$ amount of the return volume is rerouted from $r(j_t^T)$ to $r(j_{t-1}^T)$, depending on whether $t < \bar{t}(T)$ or $t \geq \bar{t}(T)$. Finally, the routing of all products $\pi(l) : l \notin J(T)$ remain unchanged after producer 1 joins the coalition. Combining these observations, we can calculate that

$$
\begin{aligned}
& v(T \cup \{1\}) - v(T) - v(1) \\
= {} & \lambda^{\pi(2)} \cdot d^{\pi(2)} \cdot (\tau_{r(1)} - \tau_{r(2)}) + \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) + \sum_{t=2}^{\bar{t}(T)-1} \lambda^{\pi(j_t^T)} \cdot k_{r(1)} \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) \\
& + \sum_{t=\bar{t}(T)}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) + \lambda^{\pi(1)} \cdot d^{\pi(1)} \cdot (\tau_{r(j_{\bar{t}(T)-1}^T)} - \tau_{r(1)}) \qquad (35)
\end{aligned}
$$

Since $\lambda^{\pi(2)} < \lambda^{\pi(j_2^T)}$ and $\tau_{r(1)} < \tau_{r(2)}$, we have $\lambda^{\pi(2)} \cdot d^{\pi(2)} \cdot (\tau_{r(1)} - \tau_{r(2)}) < \lambda^{\pi(j_2^T)} \cdot d^{\pi(2)} \cdot (\tau_{r(1)} - \tau_{r(2)})$. Accordingly, we can further derive that

$$
\begin{aligned}
(35) < {} & \lambda^{\pi(j_2^T)} \cdot k_{r(1)} \cdot (\tau_{r(1)} - \tau_{r(j_2^T)}) + \sum_{t=3}^{\bar{t}(T)-1} \lambda^{\pi(j_t^T)} \cdot k_{r(1)} \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) \\
& + \sum_{t=\bar{t}(T)}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) + \lambda^{\pi(1)} \cdot d^{\pi(1)} \cdot (\tau_{r(j_{\bar{t}(T)-1}^T)} - \tau_{r(1)}) \\
= {} & v(T \setminus \{2\} \cup \{1\}) - v(T \setminus \{2\}) - v(1) \qquad (36)
\end{aligned}
$$

Hence, we conclude that $v(T \cup \{1\}) - v(T) - v(1) < v(T \setminus \{2\} \cup \{1\}) - v(T \setminus \{2\}) - v(1)$.

- When $\bar{t}(T)=2$ in coalition T, we know that $\lambda^{\pi(2)} > \lambda^{\pi(1)} > \lambda^{\pi(j_2^T)}$. Under the assumption that $d^{\pi(2)} < k_{r(1)}$, we observe that all products except $\pi(1)$ and $\pi(j_2^T)$ are rerouted in the same way as when $\bar{t}(T)>2$ after producer 1 joins the coalition T. As for product $\pi(1)$ and $\pi(j_2^T)$, how the optimal routing $f^{*T\cup\{1\}}$ differs from f^{*T} depends on whether $r(1)$ has sufficient capacity to process both $\pi(1)$ and $\pi(2)$.

—If $k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, then $r(1)$ will be saturated by $\pi(1)$ and $\pi(2)$. Moreover, $d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}$ amount of the return volume of $\pi(1)$ is rerouted to $r(2)$, and $k_{r(1)} - d^{\pi(1)}$ amount of $\pi(j_2^T)$ is rerouted from $r(j_2^T)$ to $r(2)$.

—If $k_{r(1)} \geq d^{\pi(1)} + d^{\pi(2)}$, then $\pi(1)$ remains being fully processed at $r(1)$, and any capacity left at $r(1)$ is used to process $\pi(j_2^T)$. That is, $k_{r(1)} - d^{\pi(2)} - d^{\pi(1)}$ amount of $\pi(j_2^T)$ is rerouted from $r(j_2^T)$ to $\pi(1)$. In addition, $d^{\pi(2)}$ amount of $\pi(j_2^T)$ is rerouted from $r(j_2^T)$ to $\pi(2)$.

Combining these two situations, we can calculate $v(T\cup\{1\}) - v(T) - v(1)$ as follows.

$$
\begin{aligned}
& v(T\cup\{1\}) - v(T) - v(1) \\
= & \underbrace{\lambda^{\pi(2)} \cdot d^{\pi(2)} \cdot (\tau_{r(1)} - \tau_{r(2)}) + \lambda^{\pi(j_2^T)} \cdot \max\{k_{r(1)} - d^{\pi(1)} - d^{\pi(2)}, 0\} \cdot (\tau_{r(1)} - \tau_{r(2)})}_{③} \\
& \underbrace{+\lambda^{\pi(1)} \cdot \max\{d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}, 0\} \cdot (\tau_{r(2)} - \tau_{r(1)})}_{④} + \sum_{t=\bar{t}(T)}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) \quad (37)
\end{aligned}
$$

We can show that ③+④ $\leq \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)})$. Specifically,

—If $k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, since $\lambda^{\pi(2)} > \lambda^{\pi(1)} > \lambda^{\pi(j_2^T)}$ and $\tau_{r(1)} < \tau_{r(2)}$, we can calculate that ③+④ $= \lambda^{\pi(2)} \cdot d^{\pi(2)} \cdot (\tau_{r(1)} - \tau_{r(2)}) + \lambda^{\pi(1)} \cdot (d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(2)} - \tau_{r(1)}) < \lambda^{\pi(2)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) < \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)})$.

—Similarly, if $k_{r(1)} \geq d^{\pi(1)} + d^{\pi(2)}$, we can calculate that ③+④ $= \lambda^{\pi(2)} \cdot d^{\pi(2)} \cdot (\tau_{r(1)} - \tau_{r(2)}) + \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) \leq \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)})$.

Hence, we can show $v(T\cup\{1\}) - v(T) - v(1) < v(T\setminus\{2\}\cup\{1\}) - v(T\setminus\{2\}) - v(1)$ since

$$
\begin{aligned}
(37) \leq\ & \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) + \sum_{t=\bar{t}(T)}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) \\
=\ & \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(j_2^T)}) + \sum_{t=\bar{t}(T)+1}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) \\
=\ & v(T\setminus\{2\}\cup\{1\}) - v(T\setminus\{2\}) - v(1). \qquad (38)
\end{aligned}
$$

- When $\bar{t}(T)=1$ (i.e., $\pi(1)$ is the least recyclable product in $T\cup\{1\}$), $\pi(1)$ is fully processed at $r(1)$ in both coalitions $T\cup\{1\}$ and T. The remaining capacity at $r(1)$ is then utilized by the products in T. If $k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, then this remaining capacity is used to process part of $\pi(2)$'s volume; otherwise, it is used to process the entire volume of $\pi(2)$ and $k_{r(1)} - d^{\pi(1)} - d^{\pi(2)}$ amount of $\pi(j_2^T)$. Accordingly, we can calculate that

$$
\begin{aligned}
& v(T\cup\{1\}) - v(T) - v(1) \\
= & \begin{cases}
\sum_{t=\bar{t}(T)}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) & \text{if } k_{r(1)} < d^{\pi(1)} + d^{\pi(1)} \\
\lambda^{\pi(2)} \cdot d^{\pi(2)} \cdot (\tau_{r(1)} - \tau_{r(2)}) + \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(2)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) & \\
\qquad + \sum_{t=\bar{t}(T)+1}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) & \text{if } k_{r(1)} \geq d^{\pi(2)} - d^{\pi(1)}
\end{cases} . \qquad (39)
\end{aligned}
$$

Since $\lambda^{\pi(2)} < \lambda^{\pi(j_2^T)}$, we can show that

$$\begin{aligned}(39) &\le \lambda^{\pi(j_2^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(j_2^T)}) + \sum_{t=\bar{t}(T)+2}^{|J(T)|} \lambda^{\pi(j_t^T)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^T)} - \tau_{r(j_t^T)}) \\ &= v(T \setminus \{2\} \cup \{1\}) - v(T \setminus \{2\}) - v(1).\end{aligned}$$

To summarize, we have shown that when $2 \in T$ but $2 \notin S$, $v(T \cup \{1\}) - v(T) - v(1) \le v(T \setminus \{2\} \cup \{1\}) - v(T \setminus \{2\}) - v(1)$ holds. Since $S \subset T \setminus \{2\}$, the above inequality leads to $v(T \cup \{1\}) - v(T) - v(1) \le v(S \cup \{1\}) - v(S) - v(1)$ since $v(T \setminus \{2\} \cup \{1\}) - v(T \setminus \{2\}) - v(1) \le v(S \cup \{1\}) - v(S) - v(1)$ according to our analysis in Case 1.

Case 2b: $2 \in S$ **and** $\lambda^{\pi(2)} > \lambda^{\pi(j)}\ \forall j \in S$: To analyze this sub-case, we first let $T' = \{j \in T : \lambda^{\pi(2)} > \lambda^{\pi(j)}\}$ be the set of products in T that are more recyclable than $\pi(2)$. That is, we have $j_1^{T'} = 2$. Our first step in analyzing Case 2b is to show that $v(T' \cup \{1\}) - v(T') - v(1) \le v(T \cup \{1\}) - v(T) - v(1)$. Note that $v(T' \cup \{1\}) - v(T') - v(1)$ can be calculated by the formula (35), (37) and (39) depending on the value of $\bar{t}(T')$. We can show that either formula is no smaller than $\sum_{t=1}^{\bar{t}(T')-1} \lambda^{\pi(j_t^{T'})} \cdot k_{r(1)} \cdot (\tau_{r(j_{t-1}^{T'})} - \tau_{r(j_t^{T'})}) + \sum_{t=\bar{t}(T')}^{|J(T')|} \lambda^{\pi(j_t^{T'})} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^{T'})} - \tau_{r(j_t^{T'})}) + \lambda^{\pi(1)} \cdot d^{\pi(1)} \cdot (\tau_{r(j_{\bar{t}(T')-1}^{T'})} - \tau_{r(1)})$. To see this, we label the latter formula as ⑤ and calculate that

$$\begin{aligned}&v(T' \cup \{1\}) - v(T') - v(1) - ⑤ \\ &= \begin{cases} (\lambda^{\pi(2)} - \lambda^{\pi(j_2^{T'})}) \cdot (d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) \ge 0 \quad \text{if } \bar{t}(T') > 2 \\ (\lambda^{\pi(2)} - \lambda^{\pi(1)}) \cdot (d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) \ge 0 \quad \text{if } \bar{t}(T') = 2 \text{ and } k_{r(1)} < d^{\pi(1)} + d^{\pi(2)} \\ \lambda^{\pi(2)} \cdot (d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) + \lambda^{\pi(j_2^{T'})} \cdot (k_{r(1)} - d^{\pi(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) - \lambda^{\pi(1)} \cdot d^{\pi(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) \\ \qquad\qquad \ge (\lambda^{\pi(2)} - \lambda^{\pi(1)}) \cdot (d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) \ge 0 \quad \text{if } \bar{t}(T') = 2 \text{ and } k_{r(1)} \ge d^{\pi(1)} + d^{\pi(2)} \\ 0 \quad \text{if } \bar{t}(T') = 1 \text{ and } k_{r(1)} < d^{\pi(1)} + d^{\pi(2)} \\ (\lambda^{\pi(2)} - \lambda^{\pi(j_2^{T'})}) \cdot (d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) \ge 0 \quad \text{if } \bar{t}(T') = 1 \text{ and } k_{r(1)} \ge d^{\pi(1)} + d^{\pi(2)} \end{cases}\end{aligned} \tag{40}$$

To see how (40) is derived, recall that in analyzing Case 2, we assume $d^{\pi(2)} < k_{r(1)}$. Moreover, since T' is defined such that $\pi(2)$ is the least recyclable product in the set, we have $\lambda^{\pi(2)} > \lambda^{\pi(j_2^{T'})}$. When $\bar{t}(T') = 2$, we know that $\lambda^{\pi(2)} > \lambda^{\pi(1)}$ as well. Finally, we also have $\tau_{r(1)} < \tau_{r(2)}$. It is based on these inequalities that we derive $v(T' \cup \{1\}) - v(T') - v(1) - ⑤ \ge 0$ in (40).

Assuming $T \ne T'$, we have $j_1^T \ne 2$, and thus $v(T \cup \{1\}) - v(T) - v(1)$ can be calculated as in (32). Hence, we can apply the same proof in Case 1 to show that ⑤ $\ge v(T \cup \{1\}) - v(T) - v(1)$. Combining this inequality with (40), we obtain that $v(T \cup \{1\}) - v(T) \le v(T' \cup \{1\}) - v(T')$.

The second step in our analysis in this sub-case is to prove that $v(T' \cup \{1\}) - v(T') \le v(S \cup \{1\}) - v(S)$. Note that since we assume that $\pi(2)$ is the least recyclable product in S, we know that $S \subset T'$, and $j_1^{T'} = j_1^S = 2$. Hence, according to Claim 3, $\lambda^{\pi(j_2^{T'})} \ge \lambda^{\pi(j_2^S)}$ holds. Hence only the following three situations can happen.

- When $\bar{t}(T') \le 2$, we can show that $\bar{t}(T') = \bar{t}(S)$. This is because if $\bar{t}(T') = 1$, we have $\lambda^{\pi(1)} > \lambda^{\pi(2)}$, thus $\bar{t}(S) = 1$ as well; if $\bar{t}(T') = 2$, we have $\lambda^{\pi(2)} > \lambda^{\pi(1)} > \lambda^{\pi(j_2^{T'})} \ge \lambda^{\pi(j_2^S)}$, thus $\bar{t}(S) = 2$ as well. According to formula (37) and (39), we can calculate that

$$\begin{aligned}&[v(T' \cup \{1\}) - v(T') - v(1)] - [v(S \cup \{1\}) - v(S) - v(1)] \\ &= (\lambda^{\pi(j_2^{T'})} - \lambda^{\pi(j_2^S)}) \cdot \max\{k_{r(1)} - d^{\pi(2)} - d^{\pi(1)}, 0\} \cdot (\tau_{r(1)} - \tau_{r(2)}) \\ &\quad + \underbrace{\left[\sum_{t=\bar{t}(T')}^{|J(T')|} \lambda^{\pi(j_t^{T'})} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^{T'})} - \tau_{r(j_t^{T'})}) - \sum_{t=\bar{t}(S)}^{|J(S)|} \lambda^{\pi(j_t^S)} \cdot (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(j_{t-1}^S)} - \tau_{r(j_t^S)})\right]}_{⑥}\end{aligned} \tag{41}$$

To further analyze (41), we observe that ⑥ $= [\Delta^1(T',1)+\Delta^2(T',1)] - [\Delta^1(S,1)+\Delta^2(S,1)]$ where Δ^1 and Δ^2 functions are as defined in (33). To see this, note that since $\bar{t}(T') = \bar{t}(S)$ in this situation, ⑥ can be calculated as the difference between the values of formula (32) with respect to T' and S. According to the analysis in Case 1, we know that ⑥ $= [\Delta^1(T',1)+\Delta^2(T',1)] - [\Delta^1(S,1)+\Delta^2(S,1)] \le 0$. Moreover, $(\lambda^{\pi(j_2^{T'})} - \lambda^{\pi(j_2^S)}) \cdot \max\{k_{r(1)} - d^{\pi(2)} - d^{\pi(1)}, 0\} \cdot (\tau_{r(1)} - \tau_{r(2)}) \le 0$ also holds since $\lambda^{\pi(j_2^{T'})} \ge \lambda^{\pi(j_2^S)}$ and $\tau_{r(1)} < \tau_{r(2)}$. These observations indicate that (41)≤ 0, i.e., $[v(T'\cup\{1\}) - v(T') - v(1)] - [v(S\cup\{1\}) - v(S) - v(1)] \le 0$.

- When $\bar{t}(T') > 2$, $v(T'\cup\{1\}) - v(T') - v(1)$ can be calculated based on (35). Comparing (35) with (32) - (33), we obtain

$$v(T'\cup\{1\}) - v(T') - v(1) = \Delta^1(T',1)+\Delta^2(T',1) + (\lambda^{\pi(j_2^{T'})} - \lambda^{\pi(2)}) \cdot (k_{r(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)}). \quad (42)$$

As for the sub-coalition S, $\bar{t}(T') > 2$ indicates that $\lambda^{\pi(2)} > \lambda^{\pi(1)}$. Hence, $\bar{t}(S) \ne 1$, i.e., either $\bar{t}(S) = 2$ or $\bar{t}(S) > 2$.

—If $\bar{t}(S) > 2$, then $v(S\cup\{1\}) - v(S) - v(1)$ can also be calculated as in (42). We can further show that

$$\begin{aligned}&v(T'\cup\{1\}) - v(T') - v(1) = \Delta^1(T',1)+\Delta^2(T',1) + (\lambda^{\pi(j_2^{T'})} - \lambda^{\pi(2)}) \cdot (k_{r(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)})\\ &\quad \le \Delta^1(S,1)+\Delta^2(S,1) + (\lambda^{\pi(j_2^{S})} - \lambda^{\pi(2)}) \cdot (k_{r(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) = v(S\cup\{1\}) - v(S) - v(1)\end{aligned}$$

The inequality is due to the fact that $\Delta^1(T',1)+\Delta^2(T',1) \le \Delta^1(S,1)+\Delta^2(S,1)$ according to our analysis in Case 1, as well as that $d^{\pi(2)} < k_{r(1)}$, $\lambda^{\pi(j_2^{T'})} \ge \lambda^{\pi(j_2^S)}$ and $\tau_{r(1)} < \tau_{r(2)}$.

—If $\bar{t}(S) = 2$, then comparing (37) with (32) - (33), we obtain

$$\begin{aligned} v(S\cup\{1\}) - v(S) - v(1) \;=\; & \Delta^1(S,1)+\Delta^2(S,1) + \lambda^{\pi(2)} \cdot (d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) \\ & + \lambda^{\pi(j_2^S)} \cdot \max\{k_{r(1)} - d^{\pi(2)} - d^{\pi(1)}, 0\} \cdot (\tau_{r(1)} - \tau_{r(2)}) \\ & + \lambda^{\pi(1)} \cdot \max\{d^{\pi(2)} + d^{\pi(1)} - k_{r(1)}, 0\} \cdot (\tau_{r(2)} - \tau_{r(1)}) - \lambda^{\pi(1)} \cdot d^{\pi(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) \quad (43)\end{aligned}$$

Based on (42) and (43), we can show that

$$\begin{aligned} & [v(T'\cup\{1\}) - v(T') - v(1)] - [v(S\cup\{1\}) - v(S) - v(1)] \\ =\; & [\Delta^1(T',1)+\Delta^2(T',1)] - [\Delta^1(S,1)+\Delta^2(S,1)] \\ +\; & \underbrace{\left[\lambda^{\pi(1)} \cdot d^{\pi(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) + \lambda^{\pi(j_2^{T'})} \cdot (k_{r(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)})\right]}_{⑦} \\ -\; & \underbrace{\left[\lambda^{\pi(j_2^S)} \cdot \max\{k_{r(1)} - d^{\pi(2)} - d^{\pi(1)}, 0\} \cdot (\tau_{r(1)} - \tau_{r(2)}) + \lambda^{\pi(1)} \cdot \max\{d^{\pi(2)} + d^{\pi(1)} - k_{r(1)}, 0\} \cdot (\tau_{r(2)} - \tau_{r(1)})\right]}_{⑧} \quad (44)\end{aligned}$$

We can calculate that

$$\begin{aligned} ⑧ \;=\; & \begin{cases} \lambda^{\pi(1)} \cdot (d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(2)} - \tau_{r(1)}) & \text{if } k_{r(1)} < d^{\pi(1)} + d^{\pi(2)} \\ \lambda^{\pi(j_2^S)} \cdot (k_{r(1)} - d^{\pi(2)} - d^{\pi(1)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) & \\ \qquad \ge^1 \lambda^{\pi(1)} \cdot (d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}) \cdot (\tau_{r(2)} - \tau_{r(1)}) & \text{if } k_{r(1)} \ge d^{\pi(1)} + d^{\pi(2)} \end{cases} \\ \ge^2\; & \lambda^{\pi(1)} \cdot d^{\pi(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) + \lambda^{\pi(j_2^{T'})} \cdot (k_{r(1)} - d^{\pi(2)}) \cdot (\tau_{r(1)} - \tau_{r(2)}) = ⑦ \quad (45)\end{aligned}$$

In (45), the inequalities $\ge^1$ and $\ge^2$ hold since $\lambda^{\pi(j_2^S)} \le \lambda^{\pi(1)} \le \lambda^{\pi(j_2^{T'})}$ (because $\bar{t}(S) = 2$ and $\bar{t}(T') > 2$), $d^{\pi(2)} < k_{r(1)}$ and $\tau_{r(1)} < \tau_{r(2)}$. Combining formula (44) and (45) we conclude that $[v(T'\cup\{1\}) - v(T') - v(1)] - [v(S\cup\{1\}) - v(S) - v(1)] \le [\Delta^1(T',1)+\Delta^2(T',1)] - [\Delta^1(S,1)+\Delta^2(S,1)]$, which is less or equal to zero according to our analysis in Case 1.

To summarize, we have shown that when $2 \in T$ and $2 \in S$, $v(T \cup \{1\}) - v(T) \leq v(T' \cup \{1\}) - v(T') \leq v(S \cup \{1\}) - v(S) - v(1)$ holds.

Combining the results in both Case 1 and Case 2 (including Case 2a and 2b), we have proven Proposition 4. □

C.3. Technical Details in §6

C.3.1. Examples of the Linear Design-based Allocation We first show that the dual-based allocation x_d, the marginal contribution-based allocation x_m, and the allocation by return share are all piecewise linear functions of the design variables.

Dual-based allocation x_d: Based on duality theory, the dual optimal solution is a linear function of the objective function coefficients of the primal problem given the optimal basis[31]. Hence, we know that both $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ are piecewise linear in the objective function coefficients of the centralized allotment problem (C), which are the unit recycling cost parameters $\{c_r^\pi\}$. Since each c_r^π parameter is linear in the design variables, we know that $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ are also piecewise linear the in design variables, and so is each producer i's dual-based allocation x_d^i according to its definition in (6).

Marginal contribution-based allocation x_m: Based on the way that the formula of x_m is written in (A.4), we know that each x_m^i is linear in the unit recycling cost parameters $\{c_r^\pi\}$ under a given product allotment solution within the coalition $\{j : j \geq i\}$. Thus x_m is a piecewise linear function in the design variables.

Allocation by return share: The allocation by return share to each producer i can be calculated as

$$x_r^i = \frac{d^{\pi(i)}}{\sum_{i \in N} d^{\pi(i)}} \cdot v(N) \tag{46}$$

Since $v(N) = \sum_{\pi \in \Pi} \sum_{r \in R} c_r^\pi \cdot f^*_{[\pi,r]}$, we know that $v(N)$ is linear in the design variables under a given optimal allotment f^*. Hence, x_r^i is a piecewise linear function in the design variables.

It can be shown that the allocation by return share undermines at least one producer's design incentives compared to that in an individual system. The result presented is based on RN instances with two producers. Note that according to the notation assumption in this appendix, producer 1 has the cheaper capacity, i.e., $\tau_{r(1)} < \tau_{r(2)}$.

PROPOSITION 9. *Consider an RN instance with two producers. Under the allocation by return share, there always exists an equilibrium design profile Λ_r^{ne}. Moreover, under this equilibrium,*

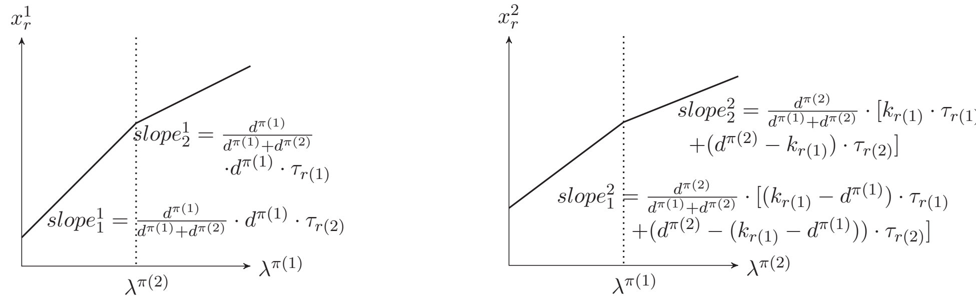
1. *producer 2 always adopts an inferior design than its design choice induced by an individual system, i.e., $(\lambda_r^{ne})^{\pi(2)} > \lambda_{ind}^{\pi(2)}$;*
2. *when $\frac{\tau_{r(1)}}{\tau_{r(2)}} \leq \min\{\frac{d^{\pi(1)}}{d^{\pi(1)}+d^{\pi(2)}}, \frac{d^{\pi(1)}+d^{\pi(2)}-k_{r(1)}}{d^{\pi(1)}+2d^{\pi(2)}-k_{r(1)}}\}$, producer 1 adopts an inferior design than its design choice induced by an individual system, i.e., $(\lambda_r^{ne})^{\pi(1)} > \lambda_{ind}^{\pi(1)}$.*

Proof of Proposition 9. We follow the three-step procedure introduced in the proof of Proposition 8.

Case 1: When the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$

[31] See page 148 in *Introduction to Linear Optimization* by Bertsimas D. and Tsitsiklis J. N.

Figure 10 **The cost allocation by return share for each producer i (x_r^i) when the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$.**



(a) Producer 1's allocation by return share (b) Producer 2's allocation by return share

• Step 1: In this situation, the socially optimal routing is as shown in panel (b) and (c) in Figure 4 of the paper. Accordingly, the minimum total cost equals

$$v(N) = \begin{cases} d^{\pi(1)} \cdot c_{r(2)}^{\pi(1)} + k_{r(1)} \cdot c_{r(1)}^{\pi(2)} + (d^{\pi(2)} - k_{r(1)}) \cdot c_{r(2)}^{\pi(2)} & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(1)} \cdot c_{r(1)}^{\pi(1)} + (k_{r(1)} - d^{\pi(1)}) \cdot c_{r(1)}^{\pi(2)} + [d^{\pi(2)} - (k_{r(1)} - d^{\pi(1)})] \cdot c_{r(2)}^{\pi(2)} & \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{47}$$

where each unit cost $c_{r(j)}^{\pi(i)} = \lambda^{\pi(i)} \cdot \tau_{r(j)} + \bar{c}^{\pi(i)}$ $\forall i, j = 1, 2$. It can be observed that $v(N)$ is a piecewise linear function of the design variables. The cost allocation by return share to each producer i is calculated as $x_r^i \doteq \frac{d^{\pi(i)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot v(N)$, which is proportional to $v(N)$. Hence, we know that given the other producer's design choice $\lambda^{\pi(j)}$, each x_r^i is a piecewise linear function with respect to i's own design variable $\lambda^{\pi(i)}$, where the break point occurs where $\lambda^{\pi(i)} = \lambda^{\pi(j)}$ (Figure 10). We denote the slopes of the two linear pieces in x_r^i as $slope_1^i$ and $slope_2^i$. Since $r(1)$ is assumed to be more cost efficient, i.e., $\tau_{r(1)} < \tau_{r(2)}$, it can be calculated that $slope_1^i > slope_2^i$ holds for each $i = 1, 2$ in this RN case (i.e., $d^{\pi(2)} \geq k_{r(1)}$ is satisfies). Based on the above observation, we can apply the same techniques described in the proof of Proposition 8 to find producer i's best response (denoted as $\lambda_r^{*\pi(i)}$) that minimizes its total cost $x_r^i + Q^i$ given the other producer's design choice.

$$\lambda_r^{*\pi(i)} = \begin{cases} (Q^{i\prime})^{-1}(slope_2^i) & \text{if } \lambda^{\pi(j)} \leq \lambda_0^{\pi(j)} \\ (Q^{i\prime})^{-1}(slope_1^i) & \text{if } \lambda^{\pi(j)} > \lambda_0^{\pi(j)} \end{cases} \tag{48}$$

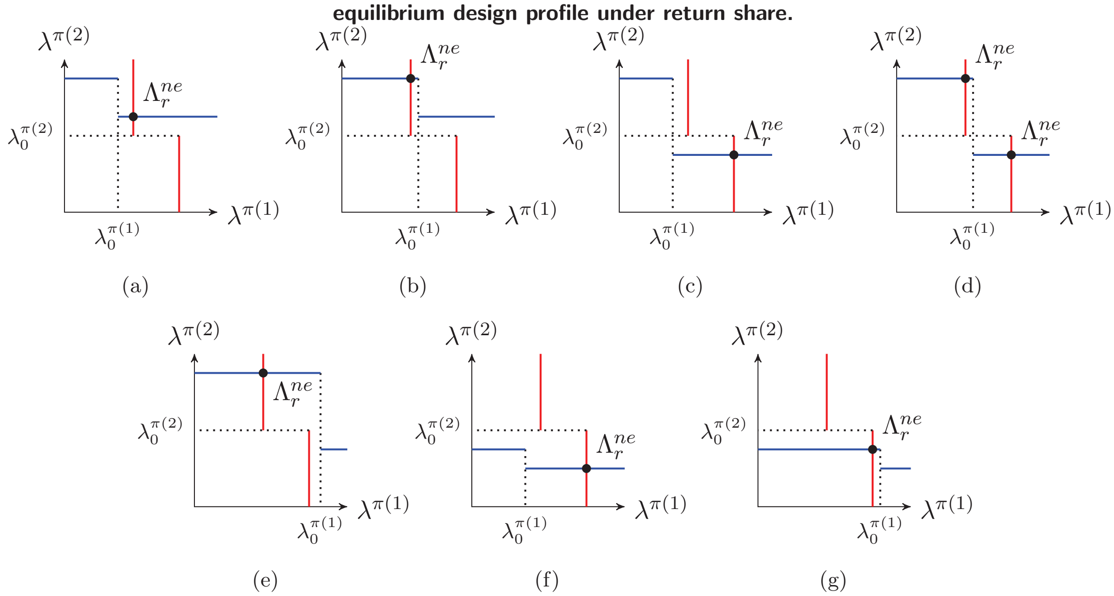
where $\lambda_0^{\pi(j)}$ corresponds to a constant between $(Q^{i\prime})^{-1}(slope_2^i)$ and $(Q^{i\prime})^{-1}(slope_2^i)$.

• Step 2: By definition, the equilibrium design profile is essentially the intersection point of the two producers' best response functions (Figure 11). As it is shown below in (49), when $d^{\pi(2)} \geq k_{r(1)}$ holds, the equilibrium design profile under return share can emerge in different ways depending on the specification of the RN instance.

$$\Lambda_r^{ne} = \begin{cases} \{(Q^{1\prime})^{-1}(slope_1^1), (Q^{2\prime})^{-1}(slope_1^2)\} & \text{if the RN leads to Figure 11(a)} \\ \{(Q^{1\prime})^{-1}(slope_1^1), (Q^{2\prime})^{-1}(slope_2^2)\} & \text{if the RN leads to Figure 11(b) (d) and (e)} \\ \{(Q^{1\prime})^{-1}(slope_2^1), (Q^{2\prime})^{-1}(slope_1^2)\} & \text{if the RN leads to Figure 11(c) (d) and (f)} \\ \{(Q^{1\prime})^{-1}(slope_2^1), (Q^{2\prime})^{-1}(slope_2^2)\} & \text{if the RN leads to Figure 11(g)} \end{cases} . \tag{49}$$

• Step 3: First consider producer 2. We can calculate that in this RN case, the slope of his return share allocation x_r^2 is always strictly smaller than that of his stand-alone end-of-life cost, i.e., $slope_1^2 < d^{\pi(2)} \cdot \tau_{r(2)}$ and $slope_2^2 < d^{\pi(2)} \cdot \tau_{r(2)}$. Due to the convexity of the investment function Q^2, we conclude that $(Q^{2\prime})^{-1}(-slope_1^2) > (Q^{2\prime})^{-1}(-d^{\pi(2)} \cdot \tau_{r(2)})$ and $(Q^{2\prime})^{-1}(-slope_2^2) > (Q^{2\prime})^{-1}(-d^{\pi(2)} \cdot \tau_{r(2)})$. Recall that we have

Figure 11 Equilibrium design profile under the cost allocation by return share. The vertical and the horizontal lines represent the best response function under return share for producer 1 and 2 respectively. Λ_r^{ne} denotes the equilibrium design profile under return share.



calculated in the paper that each producer i's optimal design choice in an individual system, $\lambda_{ind}^{*\pi(i)}$, equals $(Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(i)})$. Hence, we conclude that producer 2 always adopts an inferior design in equilibrium under return share compared to its optimal design induced by an individual system. Similarly, for producer 1, $slope_2^1 < d^{\pi(1)} \cdot \tau_{r(1)}$ always holds, and therefore $(Q^{1\prime})^{-1}(-slope_2^1) > (Q^{2\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(1)}) = \lambda_{ind}^{*\pi(1)}$. Moreover, when the condition $\frac{\tau_{r(1)}}{\tau_{r(2)}} \geq \frac{d^{\pi(1)}}{d^{\pi(1)}+d^{\pi(2)}}$ is satisfied, we show that

$$slope_1^1 = \frac{d^{\pi(1)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot d^{\pi(1)} \cdot \tau_{r(2)} \leq \frac{\tau_{r(1)}}{\tau_{r(2)}} \cdot d^{\pi(1)} \cdot \tau_{r(2)} = d^{\pi(1)} \cdot \tau_{r(1)} \tag{50}$$

and therefore $(Q^{1\prime})^{-1}(-slope_1^1) < (Q^{2\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(1)}) = \lambda_{ind}^{*\pi(1)}$ also holds. In this case, we conclude that producer 1 will always adopt an inferior design choice under return share compared to its optimal design induced by an individual system.

Case 2(a): When the RN satisfies $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$.

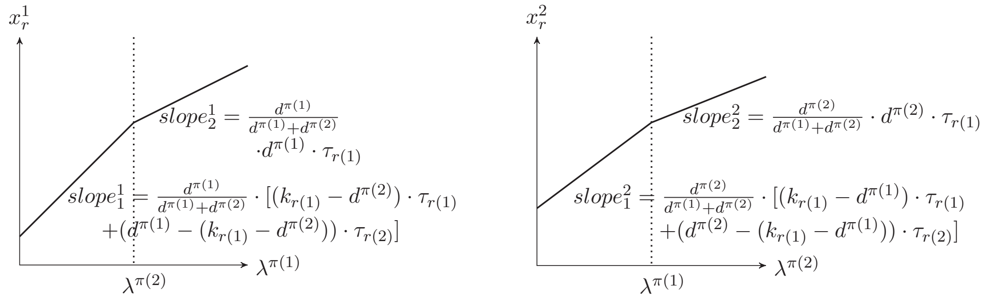
- Step 1: In this situation, the socially optimal routing is as shown in panel (d) and (e) in Figure 4 of the paper, and the minimum total cost can be calculated as

$$v(N) = \begin{cases} (k_{r(1)} - d^{\pi(2)}) \cdot c_{r(1)}^{\pi(1)} + [d^{\pi(1)} - (k_{r(1)} - d^{\pi(2)})] \cdot c_{r(2)}^{\pi(1)} + d^{\pi(2)} \cdot c_{r(1)}^{\pi(2)} & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(1)} \cdot c_{r(1)}^{\pi(1)} + (k_{r(1)} - d^{\pi(1)}) \cdot c_{r(1)}^{\pi(2)} + [d^{\pi(2)} - (k_{r(1)} - d^{\pi(1)})] \cdot c_{r(2)}^{\pi(2)} & \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{51}$$

where each unit cost $c_{r(j)}^{\pi(i)} = \lambda^{\pi(i)} \cdot \tau_{r(j)} + \bar{c}^{\pi(i)}$ $\forall i, j = 1, 2$. Figure 12 plots the cost allocation by return share in this case. Based on the same arguments in the analysis of Case 1, we can derive that the best response of each producer i in this case 2(a) can also be calculated by the same step function defined in (48).

- Step 2: Similar to Case 1, we find the equilibrium design profile by locating the intersection of the two best response functions as illustrated by Figure 11. Hence, we conclude that depending on the specification of the RN instance, each producer i may choose either $(Q^{i\prime})^{-1}(-slope_1^i)$ or $(Q^{i\prime})^{-1}(-slope_2^i)$ under the equilibrium design profile.

Figure 12 The cost allocation by return share for each producer i, x_r^i, in the high-synergy case where the CRN instance satisfies $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$.



(a) Producer 1's allocation by return share (b) Producer 2's allocation by return share

- Step 3: It can be observed that in this case, the slopes of the linear pieces in producer 2' allocation by return share satisfy $slope_1^2 < d^{\pi(2)} \cdot \tau_{r(2)}$ and $slope_2^2 < d^{\pi(2)} \cdot \tau_{r(2)}$. Due to the convexity of the function Q^2, we know that $(Q^{2\prime})^{-1}(-slope_1^2) > (Q^{2\prime})^{-1}(-d^{\pi(2)} \cdot \tau_{r(2)}) = \lambda_{ind}^{*\pi(2)}$ and $(Q^{2\prime})^{-1}(-slope_2^2) > (Q^{2\prime})^{-1}(-d^{\pi(2)} \cdot \tau_{r(2)}) = \lambda_{ind}^{*\pi(2)}$ hold. In other words, producer 2's equilibrium design choice is always strictly inferior to his design choice in an individual system. This is also the case for producer 1 if the condition $\frac{\tau_{r(1)}}{\tau_{r(2)}} \geq \frac{d^{\pi(1)}+d^{\pi(2)}-k_{r(1)}}{d^{\pi(1)}+2d^{\pi(2)}-k_{r(1)}}$ is satisfied: It is easy to see that $slope_2^1 < d^{\pi(1)} \cdot \tau_{r(1)}$. Moreover, we can calculate that

$$\begin{aligned} slope_1^1 &= \frac{d^{\pi(1)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot [(k_{r(1)} - d^{\pi(2)}) \cdot \tau_{r(1)} + (d^{\pi(1)} - (k_{r(1)} - d^{\pi(2)})) \cdot \tau_{r(2)}] \\ &\leq \frac{d^{\pi(1)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot [(k_{r(1)} - d^{\pi(2)}) \cdot \tau_{r(1)} + (d^{\pi(1)} - (k_{r(1)} - d^{\pi(2)})) \cdot \tau_{r(1)} \cdot \frac{d^{\pi(1)} + 2d^{\pi(2)} - k_{r(1)}}{d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}}] = d^{\pi(1)} \cdot \tau_{r(1)}. \end{aligned} \tag{52}$$

Hence, we conclude that $(Q^{1\prime})^{-1}(-slope_1^1) \geq (Q^{1\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(1)}) = \lambda_{ind}^{*\pi(1)}$ and $(Q^{1\prime})^{-1}(-slope_2^1) > (Q^{1\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(1)}) = \lambda_{ind}^{*\pi(1)}$ hold. That is, producer 1 will adopt an inferior design choice under return share compared to its optimal design induced by an individual system under the condition identified.

Case 2(b): When the RN satisfies $k_{r(1)} \geq d^{\pi(1)} + d^{\pi(2)}$

- Step 1: In this case, the socially optimal routing routes all products to $r(1)$. Hence, the total cost $v(M) = d^{\pi(1)} \cdot c_{r(1)}^{\pi(1)} + d^{\pi(2)} \cdot c_{r(1)}^{\pi(2)}$. For each producer i, his allocation by return share x_r^i is a linear function that equals $\frac{d^{\pi(i)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot [d^{\pi(1)} \cdot (\lambda^{\pi(1)} \cdot \tau_{r(1)} + \bar{c}^{\pi(1)}) + d^{\pi(2)} \cdot (\lambda^{\pi(2)} \cdot \tau_{r(1)} + \bar{c}^{\pi(2)})]$. Hence, i's total cost $x_r^i + Q^i$ is minimized at $\lambda_r^{*\pi(i)} = (Q^{i\prime})^{-1}(-\frac{d^{\pi(i)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot d^{\pi(i)} \cdot \tau_{r(1)})$ regardless of the other producer's design choice.
- Step 2: It is easy to see that in this case, the equilibrium design profile is such that each producer i adopts the recyclability level $(Q^{i\prime})^{-1}(-\frac{d^{\pi(i)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot d^{\pi(i)} \cdot \tau_{r(1)})$ for its own product.
- Step 3: Since for each producer i, $\frac{d^{\pi(i)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot d^{\pi(i)} \cdot \tau_{r(1)} < d^{\pi(i)} \cdot \tau_{r(i)}$, we obtain that $(Q^{i\prime})^{-1}(-\frac{d^{\pi(i)}}{d^{\pi(1)}+d^{\pi(2)}} \cdot d^{\pi(i)} \cdot \tau_{r(1)}) > (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(i)})$, i.e., the equilibrium design profile under return share is strictly worse than $\lambda_{ind}^{*\pi(i)}$ in this case.

Combining the results in all cases above, we obtain Proposition 9. □

C.3.2. Technical Details of Theorem 1

Proof of Theorem 1. In this online appendix, we provide two pieces of technical details. First, we prove Observation 2, which is proposed as a key claim that the proof presented in the appendix of the paper is based on. Second, we present the calculation details in deriving that comparing the formula (A.12)-(A.13) in the proof presented in the appendix of the paper.

- *Proof of Observation 2* We begin by rewriting the inequality $-b_u^i \leq v(S)+v(T)-v(N)-v(i)$ as follows: For every coalition T, let $cost_T^i$ denote the part of $v(T)$ that depends on $\lambda^{\pi(i)}$. Hence, $-b_u^i \leq v(S)+v(T)-v(N)-v(i)$ can be written as

$$cost_S^i + cost_T^i - cost_N^i - v(i) \geq -b_u^i - (v(S)-cost_S^i) - (v(T)-cost_T^i) - (v(N)-cost_N^i) \tag{53}$$

Note that while the left-hand-side of (53) is a piecewise linear function of $\lambda^{\pi(i)}$ (since for each coalition T, $v(T)$ is a piecewise linear function of $\lambda^{\pi(i)}$), the right-hand-side of the formula is independent of $\lambda^{\pi(i)}$. We next show that if (53) is guaranteed, then the inequality $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ must hold under any design profile. To see this, assume that there exists a design profile Λ_0 under which $cost_S^i + cost_T^i - cost_N^i - v(i) < 0$. Consider another design profile Λ_h where i's design choice equals $\lambda_h^{\pi(j)} = h \cdot \lambda_0^{\pi(j)}$, and other producers' design choices equal $\lambda_h^{\pi(j)} = \lambda_0^{\pi(j)}\ \forall j \neq i$. It can be observed that under Λ^h, the left-hand-side of (53) is h times the value calculated under Λ_0, i.e., it linearly decreases in h. Meanwhile, the right-hand-side of (53) equals the value calculated under Λ_0. In this case, we can always find a sufficiently large h value under which (53) is violated. Thus we conclude that $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ must hold in order to ensure the inequality in (53). This completes the proof of Observation 2. □

- In Step 2 of the proof presented in the appendix of the paper, we evaluate the inequality $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ based any producer $i \geq 3$, and two sub-coalitions $S = \{1, 2, ..., i-2, i\}$, and $T = \{i-1, i, i+1, ..., n\}$. We also consider a product design profile where $\pi(i)$ is the least recyclable, i.e., $\lambda^{\pi(i)}$ is the largest among all products. Here we present the associated calculation details. According to the greedy allotment algorithm (Algorithm 3 in the proof of Proposition 1), we know that $\pi(i)$ has the priority to use the most cost efficient capacity. Hence, we know that

—Comparing to producer i's operating alone, the cost to recycle $\pi(i)$ is reduced in the sub-coalition T if $\pi(i)$ is allotted to processor $r(i-1)$, which is more efficient than $r(i)$.

—The cost savings in recycling $\pi(i)$ in the grand coalition N versus in the sub-coalition S is also derived from re-allotting the portion of the $\pi(i)$ volume that is recycled at $r(i)$ when $i \in S$ to $r(i-1)$ when $i \in N$. Hence,

∗ when $d^{\pi(i)} \leq \sum_{j=1}^{i-2} k_{r(j)}$, the entire volume of $\pi(i)$ is already recycled at more efficient processors in S. Thus zero cost saving is generated when i participates in the grand coalition N.

∗ when $\sum_{j=1}^{i-2} k_{r(j)} < d^{\pi(i)} < \sum_{j=1}^{i-1} k_{r(j)}$, $d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}$ amount of $\pi(i)$ is recycled at $r(i)$ in S and will be re-allotted to $r(i-1)$ when i participates in N.

∗ when $d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)}$, $d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}$ amount of $\pi(i)$ is recycled at $r(i)$ in S. However, this volume exceeds the capacity at $r(i-1)$, out of which $k_{r(i-1)}$ amount of $\pi(i)$ will be re-allotted to $r(i-1)$ when i participates in N.

Accordingly, we can derive formula (A.12)-(A.13), which is reproduced below for convenience.

$$cost_T^i - v(i) = \min\{d^{\pi(i)}, k_{r(i-1)}\} \cdot (\tau_{r(i-1)} - \tau_{r(i)}) \cdot \lambda^{\pi(i)} \tag{54}$$

$$cost_S^i - cost_N^i = \begin{cases} 0 \quad \text{if } d^{\pi(i)} \leq \sum_{j=1}^{i-2} k_{r(j)} \\ (d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}) \cdot (\tau_{r(i)} - \tau_{r(i-1)}) \cdot \lambda^{\pi(i)} \quad \text{if } \sum_{j=1}^{i-2} k_{r(j)} < d^{\pi(i)} < \sum_{j=1}^{i-1} k_{r(j)} \\ k_{r(i-1)} \cdot (\tau_{r(i)} - \tau_{r(i-1)}) \cdot \lambda^{\pi(i)} \quad \text{if } d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)} \end{cases} \tag{55}$$

Since we assume that $\tau_{r(i-1)} < \tau_{r(i)}$, it is obvious that when $d^{\pi(i)} \leq \sum_{j=1}^{i-2} k_{r(j)}$, $cost_S^i + cost_T^i - cost_N^i - v(i) = cost_T^i - v(i) < 0$. In the second situation where $\sum_{j=1}^{i-2} k_{r(j)} < d^{\pi(i)} < \sum_{j=1}^{i-1} k_{r(j)}$, since $d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}$ is strictly smaller than both $d^{\pi(i)}$ and $k_{r(i-1)}$, we can calculate that $cost_S^i + cost_T^i - cost_N^i - v(i) < 0$ as well. In the third situation, since $d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)}$, we have $\min\{d^{\pi(i)}, k_{r(i-1)}\} = k_{r(i-1)}$, and thus we can calculate that $cost_S^i + cost_T^i - cost_N^i - v(i) = 0$. Hence, we conclude that $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ only holds in the third situation, i.e., where $d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)}$. Since this inequality must hold $\forall i \geq 3$, we conclude that the low synergy condition should be satisfied. This completes the proof that statement 3 implies statement 1 in Theorem 1. Combining this result with Proposition 4, Lemma 1 and Proposition 2, we derive Theorem 1. $\square$

The proof of Theorem 1 provides insights into the tradeoff between design incentives and group incentive compatibility. Specifically, we can derive the following corollary from the proof.

COROLLARY 1. *Given an RN instance, if the PA game is not guaranteed to be weakly convex, then for any linear design-based allocation x_l that is group incentive compatible, there must exist a producer i and an interval I_u in which the linear piece of the allocation satisfies $a_u^i < \tau_{r(i)} \cdot d^{\pi(i)}$.*

The corollary indicates that when a PA game is not weakly convex, group incentive compatibility of a linear design-based allocation requires an additional condition on at least one parameter a_u^i, i.e., it should be smaller than a threshold. This observation echoes the existing studies on convex cooperative games, which show that group incentive compatibility is harder to achieve without convexity of the cooperative game. In the problem context considered in this paper, this additional constraint means that there must be at least one producer i who is charged a strictly smaller "design cost" than when i operates individually. This undermines producer i's design incentives. This provides a technical explanation of the root cause of the tradeoff between design incentives and coalitional stability in a collective implementation.

C.4. Technical Details in §7

C.4.1. Technical Details in §7.1 and §B.1

Proof of Proposition 5. Consider an RN instance where design improvement leads to higher cost savings at more efficient processors (as it is modeled in Assumption 2). We first consider the case where the dual-based cost allocation is adopted. We prove that under the condition $\sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} \leq k_{r(i)}$ $\forall i \in N$, any pure-strategy equilibrium design profile Λ_d^{ne} is superior to the optimal design profile induced by an individual system.

STEP 1: We analyze the socially optimal allotment f^* given a design profile under the network condition $\sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} \leq k_{r(i)}$. We first show that in such an RN, the total volume of less recyclable products than $\pi(i)$ (i.e., $\sum_{j:\lambda^{\pi(j)}>\lambda^{\pi(i)}} d^{\pi(j)}$) does not exceed the total capacity at processors that are at least as efficient as $r(i)$ (i.e., $\sum_{l:\tau_{r(l)}\leq\tau_{r(i)}} k_{r(l)}$). To show this, let J_1 and J_2 be two sets where $J_1 = \{j : \lambda^{\pi(j)} > \lambda^{\pi(i)}, \tau_{r(j)} > \tau_{r(i)}\}$ and $J_2 = \{j : \lambda^{\pi(j)} > \lambda^{\pi(i)}, \tau_{r(j)} < \tau_{r(i)}\}$. We can calculate that

$$\sum_{j:\lambda^{\pi(j)}>\lambda^{\pi(i)}} d^{\pi(j)} = \sum_{j\in J_1} d^{\pi(j)} + \sum_{j\in J_2} d^{\pi(j)} \leq \sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} + \sum_{j\in J_2} d^{\pi(j)} \leq^1 k_{r(i)} + \sum_{j\in J_2} k_{r(j)} \leq \sum_{l:\tau_{r(l)}\leq\tau_{r(i)}} k_{r(l)}\,. \tag{56}$$

The inequality $\leq^1$ in the above formula is due to the condition that $\sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} \leq k_{r(i)}$, and the assumption that $d^{\pi(j)} \leq k_{r(j)}$ $\forall j$. According to the greedy allotment algorithm (Algorithm 2 introduced in the proof of Proposition 1), we know that under the socially optimal allotment f^*, all products less recyclable than $\pi(i)$ will be recycled at processors that are at least as efficient as $r(i)$. Accordingly, we know that only one of the following two situations can happen under f^*.

Situation (a) $\pi(i)$ is recycled at either processors that are more efficient than $r(i)$ or at $r(i)$ itself;

Situation (b) $\pi(i)$ is recycled at a processor less efficient than $r(i)$, and if the capacity at $r(i)$ is increased by 1 unit, this additional unit of capacity will be used to process $\pi(i)$.

STEP 2: We continue to analyze the equilibrium design profile when the dual-based cost allocation is used. Let f^*_{ne} denote the socially optimal allotment under the equilibrium design profile. Based on a similar argument as in the proof of Proposition 1, we can show that the design choice for producer i under a pure strategy equilibrium design profile equals $(Q^{i\prime})^{-1}(-\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}}) = (Q^{i\prime})^{-1}(-\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} - \frac{\partial \alpha^*_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot k_{r(i)})$, where $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ are optimal dual solutions corresponding to f^*_{ne}. Furthermore, based on equations (A.1)-(A.2), we can calculate the derivatives of $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ with respect to $\lambda^{\pi(i)}$ as follows.

- If situation (a) occurs under f^*_{ne}, then

1. there exists $r(j_0)$ more or equally efficient as $r(i)$ such that $\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} = \frac{\partial c^{\pi(i)}_{r(j_0)}}{\partial \lambda^{\pi(i)}}$. Hence, according to Assumption 2, we know that $\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \geq \frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}}$.
2. $\frac{\partial \alpha^*_{r(i)}}{\partial \lambda^{\pi(i)}} = 0$.

Hence, the equilibrium design choice for producer i satisfies

$$(\lambda^{ne}_d)^{\pi(i)} = (Q^{i\prime})^{-1}(-\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}}) = (Q^{i\prime})^{-1}(-\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}) \leq^2 (Q^{i\prime})^{-1}(-\frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}) = \lambda^{\pi(i)}_{ind} \tag{57}$$

The inequality $\leq^2$ in the above formula is derived due to the fact that Q^i is a convex decreasing function.

- If situation (b) occurs under f^*_{ne}, then there exists $r(j_0)$ less efficient than $r(i)$ such that

1. $\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} = \frac{\partial c^{\pi(i)}_{r(j)}}{\partial \lambda^{\pi(i)}}$;
2. $\frac{\partial \alpha^*_{r(i)}}{\partial \lambda^{\pi(i)}} = \frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}}$

Hence, we can calculate that

$$\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}} = \frac{\partial c^{\pi(i)}_{r(j)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} + \frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}} \cdot k_{r(i)} \geq^3 \frac{\partial c^{\pi(i)}_{r(j)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} + \frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} = \frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} \tag{58}$$

The inequality $\geq^3$ is due to the fact that (i) $r(j)$ is less efficient than $r(i)$, thus according to Assumption 2, $\frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}} \geq 0$, and (ii) $k_{r(i)} \geq d^{\pi(i)}$. Hence, we can conclude that in this situation, producer i's equilibrium design choice satisfies

$$(\lambda^{ne}_d)^{\pi(i)} = (Q^{i\prime})^{-1}(-\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}}) \leq (Q^{i\prime})^{-1}(-\frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}) = \lambda^{\pi(i)}_{ind} \tag{59}$$

Since (57) - (59) hold for each producer i, we conclude that $\Lambda^{ne}_d \leq \Lambda_{ind}$. This completes the proof of the first result in Proposition 5.

Next we consider the marginal contribution-based allocation $\bar{x}_m$ proposed in Proposition 5. Recall that in this appendix, we assume that $i < j$ indicates $\tau_{r(i)} < \tau_{r(j)}$. Hence, the allocation can be rewritten as $\bar{x}^i_m = v(i \cup \{j : j < i\}) - v(\{j : j < i\}) \; \forall i \in N$.

The individual rationality of the allocation is due to the sub-additivity of the PA game, which is shown in the proof of Proposition 3. To show that $\bar{x}_m$ induces superior design incentives, we adopt the same arguments used in the proof of Proposition 2. That is, we define $f^{*\{j:j\leq i\}}$ as the optimal product allotment in the sub-coalition $\{j : j \leq i\}$, and rewrite producer i's cost allocation x^i_m as

$$\bar{x}^i_m = \underbrace{\sum_{t=1}^{i} f^{*\{j:j\leq i\}}_{[\pi(i),r(t)]} \cdot c^{\pi(i)}_{r(t)}}_{\substack{\text{recycling cost of } \pi(i) \\ \text{in the coalition } \{j:j\leq i\}}} + \underbrace{\sum_{l=1}^{i-1} \sum_{t=1}^{i} f^{*\{j:j\leq i\}}_{[\pi(l),r(t)]} \cdot c^{\pi(l)}_{r(t)} - v(\{j : j < i\})}_{\substack{\text{recycling cost reduction for products } \pi(l) \; \forall l < i \\ \text{after producer } i \text{ joins the coalition}}} \tag{60}$$

Under Assumption 2, we know that $\forall t < i$, processor $r(t)$ is more efficient than $r(i)$ and $\frac{\partial c_{r(t)}^{\pi(i)}}{\partial \lambda^{\pi(i)}} > \frac{\partial c_{r(i)}^{\pi(i)}}{\partial \lambda^{\pi(i)}}$ holds. Hence, for each i, we have

$$\frac{\partial \bar{x}_m^i}{\partial \lambda^{\pi(i)}} = \sum_{t=1}^{i} f_{[\pi(i),r(t)]}^{*\{j:j\leq i\}} \cdot \frac{\partial c_{r(t)}^{\pi(i)}}{\partial \lambda^{\pi(i)}} \geq \frac{\partial c_{r(i)}^{\pi(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}$$

$$\Rightarrow \quad (\lambda_m^{ne})^{\pi(i)} = (Q^{i\prime})^{-1}(-\frac{\partial \bar{x}_m^i}{\partial \lambda^{\pi(i)}}) \leq (Q^{i\prime})^{-1}(-\frac{\partial c_{r(i)}^{\pi(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}) = \lambda_{ind}^{\pi(i)} \tag{61}$$

That is, the equilibrium design profile under the allocation $\bar{x}_m^i$ is superior than that induced by an individual system. This completes the proof of the second result in Proposition 5. □

Proof of Proposition 7. Assume there exists a piecewise linear allocation mechanism that is group incentive compatible and also induces superior design incentives. According to the proof of Theorem 1 in Appendix C.3.2, this implies that for each $i \in N$, and any pair of subsets S, T such that $S \cup T = N$ and $S \cap T = \{i\}$, $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ holds. In the following, we show that when processor efficiency is differentiated by products, this inequality leads to the low synergy condition identified in Proposition 7. For notational convenience, in the rest of this proof, we denote the set $\{j : \tau_{r(j)}^{\pi(i)} < \tau_{r(i)}^{\pi(i)}\}$ in the proposition as J^i.

Consider any producer i such that $|J^i| \geq 2$. Let $j(i)$ denote the producer whose capacity is the most expensive among J^i when processing $\pi(i)$. We define two sets of producers, $S = \{i, J^i \setminus j(i)\}$ and $T = \{j(i), i, N \setminus J^i\}$. Consider a product design profile where $\lambda^{\pi(i)}$ is sufficiently large so that under the optimal allotment solution within S, T and N, $\pi(i)$ always has the priority to use the most efficient capacity. We can then calculate the following:

$$cost_T^i - v(i) = \min\{d^{\pi(i)}, k_{r(j(i))}\} \cdot (\tau_{r(j(i))} - \tau_{r(i)}) \cdot \lambda^{\pi(i)} \tag{62}$$

$$cost_S^i - cost_N^i = \begin{cases} 0 \quad \text{if } d^{\pi(i)} \leq \sum_{j \in J^i \setminus j(i)} k_{r(j)} \\ (d^{\pi(i)} - \sum_{j \in J^i \setminus j(i)} k_{r(j)}) \cdot (\tau_{r(i)} - \tau_{r(j(i))}) \cdot \lambda^{\pi(i)} \quad \text{if } \sum_{j \in J^i \setminus j(i)} k_{r(j)} < d^{\pi(i)} < \sum_{j \in J^i} k_{r(j)} \\ k_{r(j(i))} \cdot (\tau_{r(i)} - \tau_{r(j(i))}) \cdot \lambda^{\pi(i)} \quad \text{if } d^{\pi(i)} \geq \sum_{j \in J^i} k_{r(j)} \end{cases} \tag{63}$$

Note that the above formulas are similar to equation (A.12) - (A.13) in the proof Theorem 1. Applying the same arguments used to analyze (A.12) - (A.13), we can derive that in order for $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ to hold the RN instance must satisfy $d^{\pi(i)} \geq \sum_{j \in J^i} k_{r(j)}$. Since this inequality is required for any i such that $|J^i| \geq 2$, we derive the low-synergy condition proposed in Proposition 7. □

C.4.2. Technical Details in §B.2 In this section, we present the detailed analysis of the extension to return volume uncertainty. First, we introduce the additional notation used in this study. Let P_i and S_i denote the probability density function and the sample space of each random variable $d^{\pi(i)}$, respectively. The expected volume of $\pi(i)$ is denoted by $E(d^{\pi(i)})$. Each producer is assumed to make the design choice based on the *expected* recycling cost under EPR implementation.

- For each producer i, its optimal design choice induced by an individual system is defined as the value of $\lambda^{\pi(i)}$ that minimizes its expected total cost $c_{r(i)}^{\pi(i)} \cdot E(d^{\pi(i)}) + Q^i(\lambda^{\pi(i)})$. This optimal choice can be calculated as $\lambda_{ind}^{\pi(i)} = (Q^{i\prime})^{-1}\left(-E(d^{\pi(i)}) \cdot \tau_{r(i)}\right)$.
- In a collective system with a cost allocation mechanism x, due to return volume uncertainty, the cost allocation to each producer i is a random variable as well, whose expectation is denoted as $E(x^i)$. The equilibrium design profile under the mechanism x is defined as the set of design choices $\{(\lambda_x^{ne})^{\pi(i)}\}$ such that $E_d\left(x^i\left(\Lambda_x^{ne}\right)\right) + Q^i\left((\lambda_x^{ne})^{\pi(i)}\right) \leq E_d\left(x^i\left(\lambda^{\pi(i)} \cup (\Lambda_x^{ne})^{-i}\right)\right) + Q^i\left(\lambda^{\pi(i)}\right)$ holds $\forall \lambda^{\pi(i)}$.

Based on the above notation, we first show that given an RN instance under volume uncertainty, producer 1 is guaranteed to have superior design incentives in a collective system with the dual-based allocation if the condition $\frac{Pr(d^{\pi(2)}<k_{r(1)})}{Pr(d^{\pi(2)}\geq k_{r(1)})} \leq \frac{E(d^{\pi(1)})}{k_{r(1)}}$ is satisfied. Note that based on the proof of Proposition 8, we know that whether superior design incentives can be achieved is determined by the rate of change of the cost allocation in design. Based on a similar argument, we can show that under return volume uncertainty, this holds true if we consider the expected cost allocation. Hence, in the following, we first calculate the rate at which the expected dual-based cost allocation to producer 1 $E(x_d^1)$ changes with respect to the design variable $d^{\pi(1)}$. According to the proof of Proposition 8, we know that the formula that calculates x_d^1 depends on whether the RN realization satisfies $d^{\pi(2)} \geq k_{r(1)}$ (which defines a low-synergy RN), $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$ or $k_{r(1)} \geq d^{\pi(1)} + d^{\pi(2)}$ (which define the two situations of a high-synergy RN). Hence, for convenience, we denote the dual-based allocation to producer 1 under these three situations as $(x_d^1)_l$, $(x_d^1)_h^1$ and $(x_d^1)_h^2$. We also divide the sample space $S_1 \times S_2$ into three parts that correspond to these three situations, and denote them as $\boldsymbol{s}_l$, $\boldsymbol{s}_h^1$ and $\boldsymbol{s}_h^2$. Thus we can calculate that

$$E\left(x_d^1\right) = \int_{\boldsymbol{s}_l} (x_d^1)_l \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}_h^1} (x_d^1)_h^1 \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}_h^2} (x_d^1)_h^2 \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) \tag{64}$$

Since $(x_d^1)_l$, $(x_d^1)_h^1$ and $(x_d^1)_h^2$ are all piecewise linear functions of $\lambda^{\pi(1)}$ whose formulas are as shown in equations (4), (9) and (20), we can calculate that

$$\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}} = \int_{\boldsymbol{s}_l} \frac{\partial (x_d^1)_l}{\partial \lambda^{\pi(1)}} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}_h^1} \frac{\partial (x_d^1)_h^1}{\partial \lambda^{\pi(1)}} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}_h^2} \frac{\partial (x_d^1)_h^2}{\partial \lambda^{\pi(1)}} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) \tag{65}$$

We next replace $(x_d^1)_l$, $(x_d^1)_h^1$ and $(x_d^1)_h^2$ in (65) with equations (4), (9) and (20). Specifically, according to equations (4), (9) and (20), we can calculate that if $\lambda^{\pi(1)} < \lambda^{\pi(2)}$,

$$\frac{\partial (x_d^1)_l}{\partial \lambda^{\pi(1)}} = d^{\pi(1)} \cdot \tau_{r(2)}, \quad \frac{\partial (x_d^1)_h^1}{\partial \lambda^{\pi(1)}} = d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}), \quad \frac{\partial (x_d^1)_h^2}{\partial \lambda^{\pi(1)}} = d^{\pi(1)} \cdot \tau_{r(1)} \tag{66}$$

Plugging the above values into formula (65), we can calculate that $\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}} = (\bar{d}_l + \bar{d}_h^1) \cdot \tau_{r(2)} - p_h^1 \cdot k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) + \bar{d}_h^2 \cdot \tau_{r(1)}$ if $\lambda^{\pi(1)} < \lambda^{\pi(2)}$. Similarly, if $\lambda^{\pi(1)} \geq \lambda^{\pi(2)}$, we can calculate based on equations (4), (9) and (20) that

$$\frac{\partial (x_d^1)_l}{\partial \lambda^{\pi(1)}} = \frac{\partial (x_d^1)_h^1}{\partial \lambda^{\pi(1)}} = \frac{\partial (x_d^1)_h^2}{\partial \lambda^{\pi(1)}} - = d^{\pi(1)} \cdot \tau_{r(1)} \tag{67}$$

Hence, it is easy to calculate that in that case, $\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}} = E(d^{\pi(1)}) \cdot \tau_{r(1)}$. Combining (66)-(67), we can derive formula (68).

$$\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}} = \begin{cases} (\bar{d}_l + \bar{d}_h^1) \cdot \tau_{r(2)} - p_h^1 \cdot k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) + \bar{d}_h^2 \cdot \tau_{r(1)} & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ E(d^{\pi(1)}) \cdot \tau_{r(1)} \quad \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{68}$$

where the notation $\bar{d}_l$ is defined as $\int_{\boldsymbol{s}_l} d^{\pi(1)} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)})$, i.e., the expected volume of product $\pi(1)$ if the realization of the RN is a low-synergy one. Similarly, we denote $\bar{d}_h^t \doteq \int_{\boldsymbol{s}_h^t} d^{\pi(1)} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)})$ $t = 1, 2$ as the expected volume of $\pi(1)$ in each of the two scenarios of a high-synergy RN. The notation p_h^1 denotes the probability that the RN realization belongs to the first scenario of the high-synergy case, i.e., $\int_{\boldsymbol{s}_h^1} P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)})$.

Based on the equilibrium analysis in §C.1.1, we know that in order for producer 1's equilibrium design outcome to be superior than its optimal design in an individual system, i.e., $(Q^{i\prime})^{-1}\left(-E(d^{\pi(1)}) \cdot \tau_{r(1)}\right)$, the rate at which its expected cost allocation changes in $\lambda^{\pi(1)}$, i.e., $\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}}$, should be at least as large as $E(d^{\pi(1)}) \cdot \tau_{r(1)}$. Accordingly, since it can be calculated that $\bar{d}_l + \bar{d}_h^1 + \bar{d}_h^2 = E(d^{\pi(1)})$, we derive the following condition based on formula (68):

$$(\bar{d}_l + \bar{d}_h^1) \cdot \tau_{r(2)} - p_h^1 \cdot k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) + \bar{d}_h^2 \cdot \tau_{r(1)} \geq E(d^{\pi(1)}) \cdot \tau_{r(1)} \quad \Rightarrow \quad p_h^1 \leq \frac{\bar{d}_l + \bar{d}_h^1}{k_{r(1)}} \tag{69}$$

Hence, we conclude that producer 1 is guaranteed to have superior design incentives in a collective implementation under the dual-based cost allocation than its operating alone if and only if the probability associated with the first scenario of low-synergy RNs is smaller than a threshold, i.e., $p_h^1 \leq \frac{\bar{d}_l + \bar{d}_h^1}{k_{r(1)}}$. To better demonstrate this insight, we next consider a sufficient condition that ensures the above inequality. That is, since p_h^1 is at most as large as the total probability of high-synergy RNs, i.e., $Pr\left(d^{\pi(2)} < k_{r(1)}\right)$, $p_h^1 \leq \frac{\bar{d}_l + \bar{d}_h^1}{k_{r(1)}}$ holds if $Pr\left(d^{\pi(2)} < k_{r(1)}\right) \leq \frac{\bar{d}_l}{k_{r(1)}}$. Moreover, note that the sample space of the volume parameters that correspond to the low-synergy RNs equals $s_l = S_1 \times [k_{r(1)}, \infty)$. Thus we can calculate that $\bar{d}_l = \int_{S_1} d^{\pi(1)} \cdot P_1 d(d^{\pi(1)}) \int_{[k_{r(1)},\infty)} P_2 d(d^{\pi(2)}) = E(d^{\pi(1)}) \cdot Pr\left(d^{\pi(2)} \geq k_{r(1)}\right)$. Hence, the inequality $Pr\left(d^{\pi(2)} < k_{r(1)}\right) \leq \frac{\bar{d}_l}{k_{r(1)}}$ is equivalent to $Pr\left(d^{\pi(2)} < k_{r(1)}\right) \leq \frac{E(d^{\pi(1)}) \cdot Pr\left(d^{\pi(2)} \geq k_{r(1)}\right)}{k_{r(1)}}$, that is,

$$\frac{Pr\left(d^{\pi(2)} < k_{r(1)}\right)}{Pr\left(d^{\pi(2)} \geq k_{r(1)}\right)} \leq \frac{E(d^{\pi(1)})}{k_{r(1)}} \tag{70}$$

However, note that if the probability of high-synergy RNs is strictly larger than zero, a superior design profile cannot be ensured in a two-producer RN. This is because producer 2 will make a strictly inferior design choice based on its expected dual-based cost allocation. To see this, note that with fixed return volumes, the rate at which producer 2's cost allocation x_d^2 changes in design is smaller or equal to the rate of change of its stand-alone cost (see Table 2 and the proof of Proposition 8). Hence, producer 2 will at best choose the same design outcome as in an individual system, and this is only ensured if the RN is design-reinforcing (e.g., in the first RN instance considered in Example 1). Hence, in the probabilistic case, even a small probability associated with non-design-reinforcing RNs undermines the rate at which producer 2's expected cost allocation changes in design, and thus leads to strictly inferior design incentives. We generalize this finding to RNs with any number of producers in the next proposition.

PROPOSITION 10. *Given an RN instance under return volume uncertainty and the probability distribution P_i of each volume parameter $d^{\pi(i)}$, the following statements are equivalent.*

1. *The probability that the RN realization is design reinforcing is one, i.e., the sample space of $d^{\pi(i)}$ is contained in the interval $[\sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}, \infty)$ $\forall i > 2$.*
2. *There exists a linear design-based allocation mechanism x_l such that (i) x_l is group incentive compatible, and (ii) any equilibrium design profile under x_l is superior to the design profile induced by an individual system, i.e., $\Lambda_l^{ne} \leq \Lambda_{ind}$.*

Proof of Proposition 10. "$1 \Rightarrow 2$": According to Proposition 4, the marginal contribution based allocation x_m is group incentive compatible and also guarantees superior design incentives in any RN instance that satisfies $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i > 2$. Hence, given the condition in Proposition 10 which states that the probability that the RN realization is design reinforcing is one, it can be concluded that the allocation x_m will continue to be group incentive compatible and provide superior design incentives.

"$2 \Rightarrow 1$": The proof is similar to the one of Theorem 1. Assume there exists a piecewise linear allocation mechanism x_l that is group incentive compatible and also induces superior design incentives given the probability distribution Pr over the RN instances. This indicates that under this distribution, design improvement leads to a higher reduction in each producer i's expected cost allocation compared to that in i's expected stand-alone cost. Hence, we conclude that for each linear piece $a_u^i \cdot \lambda^{\pi(i)} - b_u^i$ of the allocation x_l, the linear coefficient a_u^i satisfies $E_{Pr}[a_u^i] \geq E_{Pr}[d^{\pi(i)} \cdot \tau_{r(i)}]$. Moreover, since x_l is group incentive compatible, $\sum_{j\in S} x_l^j + \sum_{j\in T} x_l^j - \sum_{j\in N} x_l^j \leq v(S) + v(T) - v(N)$ holds in any RN instance given two sets of producers S and T such that $S \cup T = N$ and $S \cap T = \{i\}$. Taking the expectation of the equation, we obtain $\sum_{j\in S} E_{Pr}[x_l^j] + \sum_{j\in T} E_{Pr}[x_l^j] - \sum_{j\in N} E_{Pr}[x_l^j] \leq E_{Pr}[v(S)] + E_{Pr}[v(T)] - E_{Pr}[v(N)]$. Combining these two inequalities, we can derive the following.

$$\begin{aligned} E_{Pr}[v(i) - b_u^i] \leq E_{Pr}[a_u^i \cdot \lambda^{\pi(i)} - b_u^i] &= \sum_{j\in S} E_{Pr}[x_l^j] + \sum_{j\in T} E_{Pr}[x_l^j] - \sum_{j\in N} E_{Pr}[x_l^j] \\ &\leq E_{Pr}[v(S)] + E_{Pr}[v(T)] - E_{Pr}[v(N)] \end{aligned} \tag{71}$$

Applying the same argument in the proof of Theorem 1, we conclude that in order to guarantee (71), $E_{Pr}[cost_S^i + cost_T^i - cost_N^i - v(i)] \geq 0$ must hold. According to formulas (A.12)-(A.13), we know that for any $i \geq 3$, if the design profile is such that $\pi(i)$ is the least recyclable, then

$$cost_S^i + cost_T^i - cost_N^i - v(i) \begin{cases} < 0 & \text{if } d^{\pi(i)} < \sum_{j\in J^i} k_{r(j)} \\ = 0 & \text{if } d^{\pi(i)} \geq \sum_{j\in J^i} k_{r(j)} \end{cases} \tag{72}$$

where $S = \{1, 2, ..., i-2, i\}$, and $T = \{i-1, i, i+1, ..., m\}$. Hence, in order for $E_{Pr}[cost_S^i + cost_T^i - cost_N^i - v(i)] \geq 0$ to hold, the probability of the first situation (i.e., $d^{\pi(i)} < \sum_{j\in J^i} k_{r(j)}$) happening should be zero. In other words, the probability that the RN realization is design reinforcing must be one. $\square$

C.4.3. Technical Details in §7.2

Proof of Proposition 6. We provide the complete proof of Proposition 6 in this online appendix. To do so, we present detailed proofs of Lemma 3 and 4.

- *Proof of Lemma 3.* The Lemma is proven based on two observations.

Observation 3 *The rate at which producer i's per unit recycling and design cost (i.e., $\frac{C^i(\tau)}{d^{\pi(i)}}$) is reduced by processor efficiency improvement equals the optimal recyclability level for i's product in an individual system (i.e., $\Lambda_{ind}^{\pi(i)}$) given the current efficiency level τ.*

To show Observation 3, we first present the formula that calculates the total cost $C^i(\tau)$. Since it is shown in the paper that producer i's optimal design choice when working with a processor with efficiency level τ equals $\lambda_{ind}^{\pi(i)} = (Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)})$, we can calculate that

$$C^i(\tau) = \underbrace{(\lambda_{ind}^{\pi(i)} \cdot \tau + \bar{c}^{\pi(i)}) \cdot d^{\pi(i)}}_{\text{recycling cost}} + \underbrace{Q(\lambda_{ind}^{\pi(i)})}_{\text{design investment}} = ((Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)}) \cdot \tau + \bar{c}^{\pi(i)}) \cdot d^{\pi(i)} + Q((Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)})) \, . \tag{73}$$

Hence, the derivative of $\frac{C^i(\tau)}{d^{\pi(i)}}$ with respect to τ can be calculated as

$$\begin{aligned} \frac{1}{d^{\pi(i)}} \cdot \frac{\partial C^i(\tau)}{\partial \tau} &= \frac{\partial (Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)})}{\partial \tau} \cdot \tau + (Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)}) \\ &+ \frac{1}{d^{\pi(i)}} \cdot \underbrace{Q^{i\prime}((Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)}))}_{= -\tau \cdot d^{\pi(i)}} \cdot \frac{\partial (Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)})}{\partial \tau} = (Q^{i\prime})^{-1}(-\tau \cdot d^{\pi(i)}) = \lambda_{ind}^{\pi(i)} \, . \end{aligned} \tag{74}$$

Observation 4 *Given any τ, the optimal design choice of the smaller producer (producer A) is worse than that of the larger producer (producer B), i.e., $\lambda_{ind}^{\pi(A)} > \lambda_{ind}^{\pi(B)}$.*

To prove Observation 4, we introduce some additional notation: Recall that in §7.4, for the sake of comparison, we assume that the design investment functions for both producers consist of the same fixed cost and unit variable cost components. Hence, we model that for each producer $i = A, B$, its total design investment cost equals $Q^i(\lambda) = Q_f(\lambda) + d^{\pi(i)} \cdot Q_v(\lambda)$. We further assume that both $Q_f(\lambda)$ and $Q_v(\lambda)$ decrease in λ, i.e., a higher fixed and variable investment cost are required for a more recyclable product. Accordingly, it can be calculated that for the smaller producer (producer A), its optimal design choice, which equals $\lambda_{ind}^{\pi(A)} \doteq (Q^{A\prime})^{-1}(-\tau \cdot d^{\pi(A)})$, satisfies $Q^{A\prime}(\lambda_{ind}^{\pi(A)}) = Q_f'(\lambda_{ind}^{\pi(A)}) + d^{\pi(A)} \cdot Q_v'(\lambda_{ind}^{\pi(A)}) = -\tau \cdot d^{\pi(A)}$. This indicates that $Q_f'(\lambda_{ind}^{\pi(A)}) = -d^{\pi(A)} \cdot \left(Q_v'(\lambda_{ind}^{\pi(A)}) + \tau\right) > -d^{\pi(B)} \cdot \left(Q_v'(\lambda_{ind}^{\pi(A)}) + \tau\right)$, where the inequality is due to the fact that Q_f is decreasing in λ, and therefore $Q_f'(\lambda_{ind}^{\pi(A)}) < 0$, thus $Q_v'(\lambda_{ind}^{\pi(A)}) + \tau$ must be positive. This inequality can be rewritten as $Q_f'(\lambda_{ind}^{\pi(A)}) + d^{\pi(B)} \cdot Q_v'(\lambda_{ind}^{\pi(A)}) > -d^{\pi(B)} \cdot \tau$, indicating that $Q^{B\prime}(\lambda_{ind}^{\pi(A)}) > -\tau \cdot d^{\pi(B)}$. Since Q^B is convex decreasing in τ, it can be concluded that $(Q^{B\prime})^{-1}(-\tau \cdot d^{\pi(B)}) < \lambda_{ind}^{\pi(A)}$, i.e., $\lambda_{ind}^{\pi(B)} < \lambda_{ind}^{\pi(A)}$ is satisfied.

Finally, by definition, $\Delta^i(\tau_{r_a}, \tau_{r_b})$ can be calculated as the integral of $\frac{1}{d^{\pi(i)}} \cdot \frac{\partial C^i(\tau)}{\partial \tau}$ over the interval $[\tau_{r_a}, \tau_{r_b}]$. Combining the above two observations, we show that

$$\Delta^A(\tau_{r_a}, \tau_{r_b}) = \int_{\tau_{r_a}}^{\tau_{r_b}} \frac{1}{d^{\pi(A)}} \cdot \frac{\partial C^A(\tau)}{\partial \tau} d\tau = \int_{\tau_{r_a}}^{\tau_{r_b}} \lambda_{ind}^{\pi(A)} d\tau > \int_{\tau_{r_a}}^{\tau_{r_b}} \lambda_{ind}^{\pi(B)} d\tau = \int_{\tau_{r_a}}^{\tau_{r_b}} \frac{1}{d^{\pi(B)}} \cdot \frac{\partial C^B(\tau)}{\partial \tau} d\tau = \Delta^B(\tau_{r_a}, \tau_{r_b}). \tag{75}$$

This completes the proof of Lemma 3. □

- *Proof of Lemma 4.* The proof of Lemma 4 on whether the producers invest in in-house processing capacity or contract with competing contractors, which are both discussed below.

In-house Investment Scenario: In the in-house investment scenario, each producer's goal is to minimize the sum of its design and recycling costs plus the investment cost to build capacity. Hence, producer i ($i = A, B$) will choose the more efficient processing technology r_a over the less efficient one r_b if and only if its per-unit design and recycling cost benefit $\Delta^i(\tau_{r_a}, \tau_{r_b})$ exceeds the per-unit investment cost required to enhance processing efficiency from τ_{r_b} to τ_{r_a}. Based on the model of the investment cost to build capacity introduced in §7.2 and the assumption that each producer builds just enough capacity for its volume, this condition can be presented as $\Delta^i(\tau_{r_a}, \tau_{r_b}) > \frac{1}{d^{\pi(i)}} \cdot \left((v(\tau_{r_a}) + d^{\pi(i)} \cdot u(\tau_{r_a})) - (v(\tau_{r_b}) + d^{\pi(i)} \cdot u(\tau_{r_b}))\right) = \frac{1}{d^{\pi(i)}} \cdot (v(\tau_{r_a}) - v(\tau_{r_b})) + (u(\tau_{r_a}) - u(\tau_{r_b}))$. Hence, it is easy to see that the smaller (larger) producer, i.e., producer A (B) will choose the more efficient technology r_a (the less efficient technology r_b) if and only if $\frac{1}{d^{\pi(A)}} \cdot (v(\tau_{r_a}) - v(\tau_{r_b})) + (u(\tau_{r_a}) - u(\tau_{r_b})) < \Delta^A(\tau_{r_a}, \tau_{r_b})$ and $\frac{1}{d^{\pi(B)}} \cdot (v(\tau_{r_a}) - v(\tau_{r_b})) + (u(\tau_{r_a}) - u(\tau_{r_b})) > \Delta^B(\tau_{r_a}, \tau_{r_b})$. Combining these two inequalities, we can derive the following conditions on the fixed cost $v(\tau)$ and the variable cost $u(\tau)$. For notational convenience, we suppress the arguments in $\Delta^i(\tau_{r_a}, \tau_{r_b})$ and use Δ^i for $i = A, B$.

Observation 5 *A pairing between the smaller (larger) producer and the more (less) efficient processing technology emerges if and only if the inequalities below hold.*

$$v(\tau_{r_a}) - v(\tau_{r_b}) < \bar{v} \doteq \max\{(\Delta^A - \Delta^B) \cdot \left(\frac{1}{d^{\pi(A)}} - \frac{1}{d^{\pi(B)}}\right)^{-1}, \Delta^A \cdot d^{\pi(A)}, \Delta^B \cdot d^{\pi(B)}\} \tag{76}$$

$$\Delta^B - \frac{1}{d^{\pi(B)}} \cdot (v(\tau_{r_a}) - v(\tau_{r_b})) \doteq \underline{u} < u(\tau_{r_a}) - u(\tau_{r_b}) < \bar{u} \doteq \Delta^A - \frac{1}{d^{\pi(A)}} \cdot (v(\tau_{r_a}) - v(\tau_{r_b})) \tag{77}$$

Hence, the first statement in Lemma 4 has been proven. It can be observed that the condition in (77) depends on the fixed cost $v(\tau)$. It can be verified that when the fixed cost satisfies condition (76), the thresholds $\underline{u}$ and $\bar{u}$ in (77) are both positive, and satisfy $\underline{u} < \bar{u}$, thus the range identified in (77) is non-empty. It can be observed that if inequality (77) is violated, both producers will choose the same technology. Hence, inequality (77) essentially identifies conditions for the variable cost that permits processing technologies that differ in cost efficiency to coexist in the market. As for inequality (76), note that it implies $\left(\frac{1}{d^{\pi(A)}} - \frac{1}{d^{\pi(B)}}\right) \cdot v(\tau_{r_a}) - v(\tau_{r_b}) \leq \Delta^A(\tau_{r_a}, \tau_{r_b}) - \Delta^B(\tau_{r_a}, \tau_{r_b})$, where the left-hand-side calculates the difference between the per-unit fixed-cost investment of the two producers. Inequality (76) implies that this investment cost difference must be smaller than the benefit difference $\Delta^A(\tau_1, \tau_2) - \Delta^B(\tau_1, \tau_2)$. This happens when $v(\tau_1) - v(\tau_2)$ is small, i.e., the technology-based fixed cost in improving capacity efficiency is low.

We then show that Observation 5 (i.e., the first statement in Lemma 4) leads to the first statement in Proposition 7. To do so, we note that a pairing between the smaller (larger) producer and the more (less) efficient processing technology is equivalent to a design-reinforcing network in the two-producer setting considered in this analysis. Hence, under the conditions identified in Lemma 4, a design-reinforcing network is guaranteed to emerge. This completes the proof of the first statement in Proposition 7.

We further show that Observation 5 can be extended to cases to settings where (i) producers build processing capacities based on their cost allocation in a collective system, and (ii) the capacity levels are such that the smaller producer builds less capacity than the larger producer (i.e., the capacity each producer builds is not necessarily the same as its return volume). We briefly discuss these two extensions below.

1. *Extension 1:* First, we consider the extension where producers build/contract for processing capacities based on their cost allocation in a collective system. For illustration purposes, we assume the dual-based cost allocation x_d will be used in the collective system. The main change to our analysis is that in this case, each producer i evaluates $C^i(\tau)$ (i.e., the total recycling cost and design investment cost when working with a processor with efficiency level τ) based on its dual-based cost allocation x_d^i instead of its stand-alone recycling cost. We show that Observation 5 is robust to the above change, i.e., the producer with the smaller volume continues to benefits more from a more efficient processor on a per unit basis.

Observation 5(a) *Assume in the in-house investment scenario, each producer aims to minimize the sum of (i) its dual-based cost allocation, (ii) design investment, (iii) investment cost to build capacity. A pairing between the smaller (larger) producer and the more (less) efficient processing technology is guaranteed to be an equilibrium if and only if inequalities (76)-(77)* holds.

Proof of Observation 5(a). We show that inequalities (76)-(77) constitute an if and only if condition under which it is the best response for the smaller producer to choose the more efficient processing technology if the larger one chooses the less efficient technology, and vise versa. For example, assume the larger producer (producer B) chooses the less efficient technology (τ_{r_b}). In this case,

- if producer A chooses τ_{r_b}, then the RN is a homogeneous one (i.e., the efficiency levels of all capacities are the same). Thus the dual-based allocation to A is the same as its stand-alone recycling cost, and producer A's total design and recycling equals $C^A(\tau_{r_b})$ as defined in (73).

- if producer A chooses τ_{r_a}, then the RN satisfies the design-reinforcing condition identified in Proposition 8. Hence, we can adopt the analysis of Case 1 in the proof of Proposition 8 to calculate the equilibrium design profile. In particular, note that for the sake of comparison, we assume that the design investment function is the same for both producers (i.e., $Q^1 = Q^2$). Hence, it can be calculated that under the equilibrium design profile, producer A will choose $(Q')^{-1}\left(-d^{\pi(A)} \cdot \tau_{r_a}\right)$ and producer B will choose $(Q')^{-1}\left(-d^{\pi(B)} \cdot \tau_{r_b}\right)$. Notice that in this case, the dual-based allocation to A continues to be the same as its stand-alone recycling cost, and thus its total design and recycling equals $C^A(\tau_{r_a})$ as defined in (73).

  To summarize, given that the larger producer chooses the less efficient technology r_b, the smaller producer can reduce its total design and recycling cost (under the dual-based cost allocation) from $C^A(\tau_{r_b})$ to $C^A(\tau_{r_a})$ if it switches from the less efficient technology r_b to the more efficient technology r_a. This cost reduction is the same as when the smaller producer's stand-alone recycling cost is considered. Hence, we can apply the same arguments in the main analysis in the in-house investment scenario and show that inequalities (76)-(77) constitute an if and only if condition for the smaller producer to choose the more efficient technology. Based on similar arguments, we can prove that under the same condition, the larger producer's best response is to choose the less efficient technology given that the smaller producer chooses the more efficient one. □

2. *Extension 2:* In the main analysis, we assume each producer builds just enough processing capacity for its own return volume. In this extension, we relax this assumption and consider the case where the capacity levels are such that the smaller producer builds less capacity than the larger producer. To illustrate the main idea of this extension, the discussion below focuses on the case where the processing capacity level that each producer builds is proportional to its return volume, i.e., $k_{r(i)} = \zeta \cdot d^{\pi(i)}\ \forall i = A, B$. In this case, we can generalize our main analysis and derive that a pairing between the smaller (larger) producer and the more (less) efficient processing technology emerges if and only if $\frac{1}{d^{\pi(A)}} \cdot (v(\tau_{r_a}) - v(\tau_{r_b})) + \zeta \cdot (u(\tau_{r_a}) - u(\tau_{r_b})) < \Delta^A(\tau_{r_a}, \tau_{r_b})$ and $\frac{1}{d^{\pi(B)}} \cdot (v(\tau_{r_a}) - v(\tau_{r_b})) + \zeta \cdot (u(\tau_{r_a}) - u(\tau_{r_b})) > \Delta^B(\tau_{r_a}, \tau_{r_b})$. Hence, it is easy to see that Observation 5 continues to hold as long as we replace the thresholds $\underline{u}$ and $\bar{u}$ in inequality (77) by $\frac{1}{\zeta} \cdot \underline{u}$ and $\frac{1}{\zeta} \cdot \bar{u}$.

Contracting Scenario: We continue to prove the second statement in Lemma 4 when producers contract processing capacity with competing processors. Before presenting the proof of the result for the contracting scenario, we first formally introduce the model of the strategic efficiency level and pricing decisions of competing processors. We consider two symmetric processors (denoted as m and n) that incur the same investment cost to build the same capacity (similar assumptions have been widely used in product and price competition literature such as Moorthy (1988)[32]). Each processor determines the efficiency level for its own capacity (denoted as τ^m, τ^n) between the two choices τ_{r_a} and τ_{r_b}. They also decide a unit mark-up (denoted as ϕ^m and ϕ^n) on top of the unit recycling cost incurred. Given the $(\tau^m, \phi^m, \tau^n, \phi^n)$ decisions of the two processors, a producer contracts with the processor that minimizes the sum of the price paid for recycling (i.e., the recycling cost incurred plus the mark-up) and its design investment cost. Recall that

[32] Moorthy, K. S. 1988. Product and price competition in a duopoly. *Marketing Science* 7(2) 141–168.

$\frac{C^i(\tau)}{d^{\pi(i)}}$ represents the per-unit design and recycling cost of producer i working with a processor of efficiency level τ. Accordingly, producer i contracts with processor m if $\frac{C^i(\tau^m)}{d^{\pi(i)}} + \phi^m \le \frac{C^i(\tau^n)}{d^{\pi(i)}} + \phi^n$ (we assume the producer will choose the more efficient capacity if the costs are the same), and contracts with processor n if $\frac{C^i(\tau^m)}{d^{\pi(i)}} + \phi^m > \frac{C^i(\tau^n)}{d^{\pi(i)}} + \phi^n$. Each processor's total profit (which we denote as Π^m and Π^n respectively) is then calculated as the unit mark-up multiplied by the volume contracted, minus the investment in building the capacity to satisfy that volume.

Given the above game dynamics, it can be observed that Π^m and Π^n are functions of both processors' decisions. A set of $(\tau^m, \phi^m, \tau^n, \phi^n)$ decisions is called an equilibrium if (τ^m, ϕ^m) maximizes Π^m given that processor n adopts (τ^n, ϕ^n), and vice versa. The question is whether a design-reinforcing recycling network can be achieved under such an equilibrium. The technical difficulty in addressing this question is that the setting considered is known to be generally intractable and to possess no Nash equilibrium solution (Tirole 1988)[33]. This is due to the fact that we consider a discrete set of consumers (i.e., the producers), thus the profit functions of the processors are discontinuous. To overcome this issue and still provide a meaningful analysis, we adopt a generalization of the Nash equilibrium notion, called the farsighted equilibrium. This notion assumes the processors take into account their competitors' responses to their own moves when evaluating their benefit from such moves. Such a farsighted assumption has been extensively utilized in the game theory literature (e.g., Sošic 2006)[34]. We formalize the concept in Definition 5. In this definition, we denote processor j's $(j = m, n)$ best response to the other processor's (τ, ϕ) decision as $b^j(\tau, \phi)$.

Definition 5. We call a set of decisions $(\tau^m, \phi^m, \tau^n, \phi^n)$ a farsighted equilibrium if

1. $\forall(\tau, \phi) \neq (\tau^m, \phi^m)$, $\Pi^m(\tau^m, \phi^m, \tau^n, \phi^n) \ge \Pi^m(\tau, \phi, b^n(\tau, \phi))$ holds;
2. $\forall(\tau, \phi) \neq (\tau^n, \phi^n)$, $\Pi^n(\tau^m, \phi^m, \tau^n, \phi^n) \ge \Pi^n(b^m(\tau, \phi), \tau, \phi)$ holds.

We next present a sufficient condition for the smaller (larger) producer to contract with the more (less) efficient processor under a farsighted equilibrium. Similarly to Lemma 4, we also use Δ^i in place of $\Delta^i(\tau_{r_a}, \tau_{r_b})$ for notational convenience.

Observation 6 *A farsighted equilibrium $(\tau^m, \phi^m, \tau^n, \phi^n)$ is guaranteed to exist under which the smaller (larger) producer contracts with the more (less) efficient processor if the inequalities below hold.*

$$v(\tau_{r_b}) \le v(\tau_{r_a}) < \bar{v}' \doteq \frac{\left(d^{\pi(A)}\right)^2}{d^{\pi(B)}}(\Delta^A - \Delta^B) \quad (78)$$

$$\Delta^B + \left(\frac{d^{\pi(B)}}{d^{\pi(A)}} - \frac{d^{\pi(A)}}{d^{\pi(B)}}\right) \cdot (\Delta^A - \Delta^B) \doteq \underline{u}' < u(\tau_{r_a}) - u(\tau_{r_b}) < \bar{u}' \doteq \Delta^A + \left(\frac{d^{\pi(B)}}{d^{\pi(A)}} - \frac{d^{\pi(A)}}{d^{\pi(B)}}\right) \cdot (\Delta^A - \Delta^B) \quad (79)$$

Proof of Observation 6 To prove this observation, we propose a set of efficiency and mark-up decisions of the processors and show it is a farsighted equilibrium. To define this set of decisions, we first introduce two threshold values ϕ_0^m and ϕ_0^n. To present the definition of these thresholds, let us consider the case where the two processors select different efficiency levels (without loss of generality, we assume processor m has the more efficient capacity, i.e., $\tau^m = \tau_{r_a}$ and $\tau^n = \tau_{r_b}$). Recall that in our model, producer i prefers processor m over processor n if $\frac{C^i(\tau^m)}{d^{\pi(i)}} + \phi^m \le \frac{C^i(\tau^n)}{d^{\pi(i)}} + \phi^n$, that is, $\phi^m - \phi^n \le \Delta^i$. Since $\Delta^A > \Delta^B$ by Lemma 3, we

[33] Tirole, J. 1988. *The theory of industrial organization.* MIT press.

[34] Sošic, G. 2006. Transshipment of inventories among retailers: Myopic vs. farsighted stability. *Management Science* 52(10) 1493–1508.

conclude that if $\phi^m - \phi^n > \Delta^A$, both producers prefer processor n; if $\Delta^A \geq \phi^m - \phi^n > \Delta^B$, producer A and B prefer processor m and n respectively; if $\phi^m - \phi^n \leq \Delta^B$, both producers prefer processor m. Hence, given processor n's mark-up ϕ^n, processor m's profit function is the following. For discussion convenience, we introduce some additional notation: Let $U^i(\tau) = v(\tau) + u(\tau) \cdot d^{\pi(i)}$ be the processor's investment cost to build capacity at efficiency level τ to process producer i's volume ($i = A, B$). If both producers contract with the processor, its investment cost is denoted by $U^{A\cup B}(\tau) = v(\tau) + u(\tau) \cdot \left(d^{\pi(A)} + d^{\pi(B)}\right)$.

$$\Pi^m = \begin{cases} 0 \quad \text{if } \phi^m > \phi^n + \Delta^A \text{ (processor } m \text{ obtains no volume)} \\ \phi^m \cdot d^{\pi(A)} - U^A(\tau_{r_a}) \quad \text{if } \phi^n + \Delta^B < \phi^m \leq \phi^n + \Delta^A \text{ (processor } m \text{ works with producer } A) \\ \phi^m \cdot \left(d^{\pi(A)} + d^{\pi(B)}\right) - U^{A\cup B}(\tau_{r_a}) \quad \text{if } \phi^m \leq \phi^n + \Delta^B \\ \qquad\qquad \text{(processor } m \text{ works with both producer } A \text{ and } B) \end{cases} \tag{80}$$

Next, we analyze the mark-up decision for processor m to maximize Π^m in formula (80). Note that this decision depends on processor n's mark-up. In particular, we observe the following.

- On one hand, when ϕ^n is sufficiently high, processor m obtains a higher profit when working with both producer A and B compared to working with only producer A. That is, the maximum value achievable in the third scenario in formula (80), $(\phi^n + \Delta^B) \cdot \left(d^{\pi(A)} + d^{\pi(B)}\right) - U^{A\cup B}(\tau_{r_a})$, is higher than that in the second scenario, $(\phi^n + \Delta^A) \cdot d^{\pi(A)} - U^A(\tau_{r_a})$. We can calculate that this happens when ϕ^n is higher than a threshold defined as $\phi_0^n \doteq \min\{\phi^n : (\phi^n + \Delta^A) \cdot d^{\pi(A)} - U^A(\tau_{r_a}) \leq (\phi^n + \Delta^B) \cdot \left(d^{\pi(A)} + d^{\pi(B)}\right) - U^{A\cup B}(\tau_{r_a})\}$. According to this definition, we can calculate that $\phi_0^n = \frac{d^{\pi(A)}}{d^{\pi(B)}} (\Delta^A - \Delta^B) - \Delta^B + u(\tau_{r_a})$.
- On the other hand, when ϕ^n is sufficiently low, there does not exist a mark-up for processor m that justifies its technology investment cost and at the same time guarantees positive volume from the producers. That is, processor m cannot make a positive profit. We can calculate that this happens when $\phi^n \leq \frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A$. To see this, note that the maximum value in the second scenario in formula (80), $(\phi^n + \Delta^A) \cdot d^{\pi(A)} - U^A(\tau_{r_a})$, is non-positive when ϕ^n is below the threshold identified above. In other words, processor m cannot obtain a positive profit if it works with producer A. This is also the case in the third scenario in formula (80), i.e., when processor m works with both producer A and B. To show this, we first calculate that $\phi^n + \Delta^B \leq \frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A + \Delta^B = \frac{v(\tau_{r_a})}{d^{\pi(A)}} + u(\tau_{r_a}) - \Delta^A + \Delta^B$, which is smaller than $\left(\frac{d^{\pi(A)}}{d^{\pi(B)}} - 1\right)(\Delta^A - \Delta^B) + u(\tau_{r_a}) < u(\tau_{r_a})$ due to condition 1 in the observation and the fact that $d^{\pi(A)} < d^{\pi(B)}$. Therefore, the maximum value in the third-scenario in formula (80) satisfies $(\phi^n + \Delta^B) \cdot \left(d^{\pi(A)} + d^{\pi(B)}\right) - U^{A\cup B}(\tau_{r_a}) < u(\tau_{r_a}) \cdot \left(d^{\pi(A)} + d^{\pi(B)}\right) - U^{A\cup B}(\tau_{r_a}) = -v(\tau_{r_a}) \leq 0$.

We can further calculate that the threshold ϕ_0^n is larger than $\frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A$; this is because, according to condition 1 in the observation and the fact that $\Delta^A > \Delta^B$ shown in Lemma 3, we show that $\phi_0^n = \frac{d^{\pi(A)}}{d^{\pi(B)}} (\Delta^A - \Delta^B) - \Delta^B + u(\tau_{r_a}) > \frac{v(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A + u(\tau_{r_a}) = \frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A$. Therefore, we conclude that in the case where processor m and n choose τ_{r_a} and τ_{r_b} respectively, processor m's best mark-up strategy is the following.

$$\begin{cases} \text{If } \phi^n > \phi_0^n, \text{ processor } m \text{ adopts a mark-up of } \phi^n + \Delta^B. \\ \text{If } \frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A < \phi^n \leq \phi_0^n, \text{ processor } m \text{ adopts a mark-up of } \phi^n + \Delta^A. \\ \text{If } \phi^n \leq \frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A, \text{ processor } m \text{ cannot make a positive profit.} \end{cases} \tag{81}$$

Similarly, we can analyze processor n's best mark-up strategy as a function of processor m's mark-up, and define a parallel threshold $\phi_0^m \doteq \lim_{\epsilon\to 0^+} \min\{\phi^m : (\phi^m - \Delta^B - \epsilon) \cdot d^{\pi(B)} - U^B(\tau_{r_b}) \leq (\phi^m - \Delta^A - \epsilon) \cdot$

$\big(d^{\pi(A)}+d^{\pi(B)}\big)-U^{A\cup B}(\tau_{r_b})\}$. The meaning of this threshold is similar to that of ϕ_0^n: When processor m'mark-up ϕ^m is higher than ϕ_0^m, there exists $\bar{\epsilon}>0$ such that $\forall\epsilon\in(0,\bar{\epsilon})$, it is more profitable for processor n to adopt $\phi^m-\Delta^A-\epsilon$ and work with both producer A and B compared to adopting $\phi^m-\Delta^B-\epsilon$ and working with only producer B. We can calculate that $\phi_0^m=\frac{d^{\pi(B)}}{d^{\pi(A)}}(\Delta^A-\Delta^B)+\Delta^A+u(\tau_{r_b})$. A similar analysis leads to the following mark-up strategy for processor n.

$$\begin{cases}\text{If } \phi^m>\phi_0^m, \text{ processor } n \text{ adopts a mark-up of } \phi^m-\Delta^A-\epsilon \text{ for some small } \epsilon>0 \\ \qquad\qquad\qquad\qquad \text{that guarantees } \phi-\Delta^A-\epsilon>\frac{U^B(\tau_{r_b})}{d^{\pi(B)}}. \\ \text{If } \frac{U^B(\tau_{r_b})}{d^{\pi(B)}}-\Delta^B<\phi^m\le\phi_0^m, \text{ processor } n \text{ adopts a mark-up of } \phi^m-\Delta^B-\epsilon \text{ for} \\ \qquad\qquad\qquad \text{some small } \epsilon>0 \text{ that guarantees } \phi-\Delta^B-\epsilon>\frac{U^B(\tau_{r_b})}{d^{\pi(B)}}. \\ \text{If } \phi^m\le\frac{U^B(\tau_{r_b})}{d^{\pi(B)}}-\Delta^B, \text{ processor } n \text{ cannot make a positive profit.}\end{cases} \tag{82}$$

Based on the two thresholds ϕ_0^m and ϕ_0^n introduced above, we propose the following efficiency and mark-up decisions of the processors.

$$\tau^m=\tau_{r_a};\quad \phi^m=\phi_0^m;\quad \tau^n=\tau_{r_b};\quad \phi^n=\phi_0^n \tag{83}$$

In the rest of this proof, we show that $(\tau_{r_a},\phi_0^m,\tau_{r_b},\phi_0^n)$ defined in (83) is a farsighted equilibrium. To do this, we first show that under this set of decisions, both processors obtain a positive profit. We calculate that $\phi_0^m-\phi_0^n=(\frac{d^{\pi(B)}}{d^{\pi(A)}}-\frac{d^{\pi(A)}}{d^{\pi(B)}})\cdot(\Delta^A-\Delta^B)+\Delta^A+\Delta^B-(u(\tau_{r_a})-u(\tau_{r_b}))$, which is in between Δ^B and Δ^A due to condition 2 in the observation. Accordingly, producer A and B will prefer processor m and n respectively, and the profit of processor m and n can be calculated as $\Pi_0^m=\phi_0^m\cdot d^{\pi(A)}-U^A(\tau_{r_a})=\Big(\phi_0^m-\frac{v(\tau_{r_a})}{d^{\pi(A)}}-u(\tau_{r_a})\Big)\cdot d^{\pi(A)}$ and $\Pi_0^n=\phi_0^n\cdot d^{\pi(B)}-U^B(\tau_{r_b})=\Big(\phi_0^n-\frac{v(\tau_{r_b})}{d^{\pi(B)}}-u(\tau_{r_b})\Big)\cdot d^{\pi(B)}$ respectively. We next show that both profits are strictly positive. For processor m, $\Pi_0^m>0$ because it can be calculated that the threshold ϕ_0^m is larger than $\frac{v(\tau_{r_a})}{d^{\pi(A)}}+u(\tau_{r_a})$, since $\phi_0^m-\frac{v(\tau_{r_a})}{d^{\pi(A)}}-u(\tau_{r_a})=\Delta^A-(u(\tau_{r_a})-u(\tau_{r_b}))+\frac{d^{\pi(B)}}{d^{\pi(A)}}(\Delta^A-\Delta^B)-\frac{v(\tau_{r_a})}{d^{\pi(A)}}\ge^1\Delta^A-(u(\tau_{r_a})-u(\tau_{r_b}))+(\frac{d^{\pi(B)}}{d^{\pi(A)}}-\frac{d^{\pi(A)}}{d^{\pi(B)}})\cdot(\Delta^A-\Delta^B)>^2 0$, where inequality $\ge^1$ and $>^2$ are due to condition 1 and 2 in the observation, respectively. For processor n, it can be calculated that the threshold ϕ_0^n is larger than $\frac{v(\tau_{r_b})}{d^{\pi(B)}}+u(\tau_{r_b})$ in a similar way, and $\Pi_0^n>0$ follows.

Next we show that $(\tau_{r_a},\phi_0^m,\tau_{r_b},\phi_0^n)$ defined in (83) is a farsighted equilibrium, i.e., neither producers can strictly improve its profit by deviating from this set of decisions if it takes into account its competitor's best response to its own move. Let us start with processor m and assume it deviates to a decision $(\tau,\phi)\neq(\tau_{r_a},\phi_0^m)$. We consider the following cases.

- (**Case 1**) Consider the case where processor m switches to the less efficient capacity (i.e., it switches to $\tau=\tau_{r_b}$), and chooses any mark-up ϕ that leads to a positive profit for itself. This means that ϕ has to be at least strictly larger than $\frac{U^{A\cup B}(\tau_{r_b})}{d^{\pi(A)}+d^{\pi(B)}}$, since $\frac{U^{A\cup B}(\tau_{r_b})}{d^{\pi(A)}+d^{\pi(B)}}$ is smaller than both $\frac{U^A(\tau_{r_b})}{d^{\pi(A)}}$ and $\frac{U^B(\tau_{r_b})}{d^{\pi(B)}}$, i.e., $\frac{U^{A\cup B}(\tau_{r_b})}{d^{\pi(A)}+d^{\pi(B)}}$ is the smallest per-unit investment cost to build processing capacity at efficiency level τ_{r_b}. In that case, processor n's best response will be one of the following two strategies:

— (Strategy S_1^n) Processor n keeps its efficiency level at τ_{r_b} and selects a lower mark-up than ϕ (the new mark-up adopted by processor m) that leads to a positive profit for itself. That is, strategy $S_1^n=(\tau_{r_b},\phi-\epsilon)$ for some $\epsilon>0$ such that $\phi-\epsilon>\frac{U^{A\cup B}(\tau_{r_b})}{d^{\pi(A)}+d^{\pi(B)}}$ (such an ϵ exists since ϕ is chosen to be strictly larger than $\frac{U^{A\cup B}(\tau_{r_b})}{d^{\pi(A)}+d^{\pi(B)}}$). This strategy leaves processor m with zero volume and a zero profit.

— (Strategy S_2^n) Processor n improves its efficiency level to τ_{r_a}. In this case, the calculation of processor n's profit is similar to formula (80). Hence, processor n's best mark-up strategy is similar to the one illustrated in (81). Specifically,

$*$ if the mark-up adopted by processor m satisfies $\phi > \phi_0^n$, processor n will choose a mark-up of $\phi + \Delta^B$. That is, strategy $S_2^n = (\tau_{r_a}, \phi + \Delta^B)$, under which processor n obtains all volumes. This leaves processor m with zero volume and a zero profit.

$*$ if $\frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A < \phi \le \phi_0^n$, processor n will choose a mark-up of $\phi + \Delta^A$. That is, strategy $S_2^n = (\tau_{r_a}, \phi + \Delta^A)$, under which producer A and B contract with processor n and m, respectively. Hence, processor m's new profit is $\phi \cdot d^{\pi(B)} - U^B(\tau_{r_b})$. We can calculate that this new profit is smaller than Π_0^m if strategy S_2^n is indeed the best response for processor n, i.e., processor n's profit is higher under S_2^n than that under S_1^n. To see this, note that processor n obtains a higher profit under S_2^n if $\forall \epsilon > 0$, $(\phi - \epsilon) \cdot (d^{\pi(A)} + d^{\pi(B)}) - U^{A\cup B}(\tau_{r_b}) < (\phi + \Delta^A) \cdot d^{\pi(A)} - U^A(\tau_{r_a})$, which is equivalent to $\phi \cdot d^{\pi(B)} \le U^{A\cup B}(\tau_{r_b}) - U^A(\tau_{r_a}) + \Delta^A \cdot d^{\pi(A)}$. In that case, we can calculate after a few algebraic manipulations that processor m's new profit satisfies $\phi \cdot d^{\pi(B)} - U^B(\tau_{r_b}) \le (\Delta^A + u(\tau_{r_b})) \cdot d^{\pi(A)} - U^A(\tau_{r_a})$. Furthermore, based on the definition of ϕ_0^m, we can calculate that $\phi_0^m > \Delta^A + u(\tau_{r_b})$. Combining these inequalities, we can conclude that $\phi \cdot d^{\pi(B)} - U^B(\tau_{r_b}) < \phi_0^m \cdot d^{\pi(A)} - U^A(\tau_{r_a}) = \Pi_0^m$, i.e., processor m's new profit is smaller than Π_0^m.

$*$ if $\phi \le \frac{U^A(\tau_{r_a})}{d^{\pi(A)}} - \Delta^A$, processor n cannot make a positive profit under any mark-up when switching to the higher efficiency level τ_{r_a}. In this case, processor n's best response is to adopt strategy S_1^n.

Combining the above discussion, we conclude that when processor m deviates to a different strategy where its efficiency level is reduced to τ_{r_b}, it always obtains a lower profit than Π_0^m considering processor n's best response.

- (**Case 2**) Consider the case where processor m maintains the efficiency level at τ_{r_a} and chooses a different mark-up $\phi \neq \phi_0^m$ that leads to a positive profit for itself. Hence, this new mark-up is at least larger than $\frac{U^{A\cup B}(\tau_{r_a})}{d^{\pi(A)} + d^{\pi(B)}}$ as discussed in Case 1. Similarly, processor n's best response will be one of the following two strategies:

—(Strategy $S_1^{n\prime}$) Processor n improves its efficiency level to τ_{r_a} and selects a lower mark-up than ϕ (the new mark-up adopted by processor m) that leads to a positive profit for itself. That is, strategy $S_1^{n\prime} = (\tau_{r_a}, \phi - \epsilon)$ for some $\epsilon > 0$ such that $\phi - \epsilon > \frac{U^{A\cup B}(\tau_{r_a})}{d^{\pi(A)} + d^{\pi(B)}}$ (such an ϵ exists given that $\phi > \frac{U^{A\cup B}(\tau_{r_a})}{d^{\pi(A)} + d^{\pi(B)}}$). This leaves processor m with zero volume and a zero profit.

—(Strategy $S_2^{n\prime}$) Processor n keeps its efficiency level at τ_{r_b}. Then processor n's best mark-up strategy is as illustrated in (82). That is,

$*$ if processor m raises its mark-up, i.e., $\phi > \phi_0^m$, processor n chooses a mark-up of $\phi^m - \Delta^A - \epsilon$ for some small $\epsilon > 0$ such that $\phi - \Delta^A - \epsilon > \frac{U^B(\tau_{r_b})}{d^{\pi(B)}}$. That is, strategy $S_2^{n\prime} = (\tau_{r_b}, \phi - \Delta^A - \epsilon)$, under which processor n obtains all volumes. This leaves processor m with no volume and a zero profit.

$*$ if processor m lowers its mark-up and chooses ϕ such that $\frac{U^B(\tau_{r_b})}{d^{\pi(B)}} + \Delta^B < \phi < \phi_0^m$, processor n chooses a mark-up of $\phi - \Delta^B - \epsilon$ for some small $\epsilon > 0$ such that $\phi - \Delta^B - \epsilon > \frac{U^B(\tau_{r_b})}{d^{\pi(B)}}$. That is, strategy $S_2^{n\prime} = (\tau_{r_b}, \phi - \Delta^B - \epsilon)$, under which producer A and B continue to contract with processor m and n respectively. However, processor m's profit is reduced to $\phi \cdot d^{\pi(A)} - U^A(\tau_{r_a}) < \Pi_0^m$.

$*$ If $\phi \le \frac{U^B(\tau_{r_b})}{d^{\pi(B)}} + \Delta^B$, processor n cannot make a positive profit under any mark-up when keeping its efficiency level at τ_{r_b}. In that case, processor n's best response is to adopt strategy $S_1^{n\prime}$.

Combining the above discussion, we conclude that when processor m maintains its efficiency level at τ_{r_a} and adopts a different mark-up, it always obtains a lower profit than Π_0^m considering processor n's best response.

Hence, we conclude that processor m cannot obtain a higher profit deviating from $(\tau_{r_a}, \phi_0^m, \tau_{r_b}, \phi_0^n)$ when taking into account processor n's best response. We can derive the same conclusion for processor n by performing a parallel analysis with respect to processor n using the same arguments. Therefore, $(\tau_{r_a}, \phi_0^m, \tau_{r_b}, \phi_0^n)$ is a farsighted equilibrium by definition. Since we have already shown that under this solution, producer A and B prefer processor m (who has the more efficient capacity) and processor n (who has the less efficient capacity) respectively, Observation 6, which is the second statement in Lemma 4, is proven. $\square$

In Observation 6, condition (78) indicates that the fixed cost technology investment is smaller than a threshold. Condition (79) indicates that the variable cost needed to improve capacity efficiency from τ_{r_b} to τ_{r_a} is in between functions of the producers' unit benefits from such capacity improvement. The proof of Observation 6 indicates that this condition for the variable cost permits both technologies to be adopted by the processors. It can be verified that the ranges identified in the above conditions are non-empty because (i) $\Delta^A > \Delta^B$ by Lemma 3, and (ii) $d^{\pi(A)} < d^{\pi(B)}$ as we assume producer A has the smaller volume. To summarize, Observation 6 indicates that under certain conditions regarding the capacity investment costs, a pairing between the smaller (larger) producer and the more (less) efficient processing technology will emerge under a farsighted equilibrium when producers contract with strategic and competing processors. Next, we discuss whether such a pairing as discussed above will lead to a design reinforcing network. Note that in the contracting scenario, because of the inclusion of the price mark-up, the more efficient processor may not necessarily be the cheapest for the producers as it is assumed in the biform game in the paper. However, we can show that when capacity equals volume in the recycling network (which is the setting considered in §7.2), the inclusion of a price mark-up does not influence the optimal product allotment in the collective system, and therefore the condition of the design-reinforcing network remains equivalent to the smaller (larger) producer being paired with the more (less) efficient processor in the two-producer setting. Hence, we conclude that the conditions shown in Observation 6 ensure a design-reinforcing network under a farsighted equilibrium of producers' choices between strategic and competing processors. This completes the proof of the second statement in Proposition 6. $\square$