Online Appendix: Two-Part Tariffs with Refined Forecasting Systems

In this section, the manufacturer designs two-part tariff contracts[1] in the form of a fixed transfer, T, and a per-unit wholesale price, w. The analysis below is general and holds for any number of signals: $s \in \Omega \equiv \{1, \cdots, N\}$. (Table 1 and Figure 1 illustrate the results by presenting specific outcomes based on the simpler set-up of §4.1.) Upon observing signal, s, the retailer's optimal stocking strategy is $q_s^*(w)$, which yields optimal retail revenues of $\bar{R}_s\left(q_s^*(w)\right)$ (Equations 6 and 7). Hence, if she observes signal s, the retailer accepts a contract only if the following IR constraint holds: $T \leq \bar{R}_s\left(q_s^*(w)\right) - w \cdot q_s^*(w)$.

Common Signal Game SPNE. The manufacturer offers a signal-specific contract, (T_s, w_s), which he designs to solve $\max_{T_s, w_s} T_s + (w_s - c) \cdot q_s^*\left(w_s\right)$, subject to $T_s \leq \bar{R}_s\left(q_s^*\left(w_s\right)\right) - w_s \cdot q_s^*\left(w_s\right)$. The IR constraint is binding in equilibrium, and the manufacturer's problem reduces to

$$\max_{w_s} \bar{R}_s\left(q_s^*\left(w_s\right)\right) - c \cdot q_s^*\left(w_s\right). \tag{1}$$

In equilibrium,

$$w_s^{com*} = c, \qquad q_s^{com*} = q_s^*\left(c\right), \qquad T_s^{com*} = \Pi_s^{chl*}, \qquad \text{and} \qquad \mathbb{E}\left[\Pi^{com*}\right] = \sum_{s \in \Omega} \mathbb{P}(s) \cdot T_s^{com*}, \tag{2}$$

where $\Pi_s^{chl*} \equiv \bar{R}_s\left(q_s^*(c)\right) - c \cdot q_s^*(c)$ denotes the maximum combined channel profits conditional on signal, s. By setting $w_s^{com*} = c$, the manufacturer makes the retailer order $q_s^{com*} = q_s^*\left(c\right)$, which is the most efficient quantity that maximizes the combined channel profits. He uses the transfer to fully extract the combined (maximized) profits.

Private Signal Game SPNE. Any contract, (T, w), the manufacturer offers, will result in a minimum signal, denoted by $\hat{s} \in \Omega$, that the retailer needs to observe to accept the contract.[2] For an arbitrary $\hat{s} \in \Omega$, the manufacturer designs the contract to solve $\max_{T, w} T + \sum_{s \geq \hat{s}} \mathbb{P}(s) \cdot (w - c) \cdot q_s^*\left(w\right)$, subject to $T \leq \bar{R}_{\hat{s}}\left(q_{\hat{s}}^*\left(w\right)\right) - w \cdot q_{\hat{s}}^*\left(w\right)$. The IR constraint is binding in equilibrium, and the SPNE

[1] Identical results can be derived under contracts involving (incentive compatible) quantity-transfer menus.

[2] It could be optimal to offer a transfer that is too high to be accepted by the retailer if she observes a low signal. In general, the higher the variance of posterior beliefs, $\text{Var}\left(\bar{V}_s\right)$, the higher the cutoff signal, $\hat{s}$. The manufacturer solves for the optimal contract for every possible $\hat{s} \in \Omega$ and then chooses the one that yields the highest profits. We keep the analysis general and derive results that hold for any $\hat{s} \in \Omega$.

wholesale price, stocking level, transfer, and profits are

$$w^{prv*} = \arg\max_{w} \bar{R}_{\hat{s}}\left(q_{\hat{s}}^{*}(w)\right) - w \cdot q_{\hat{s}}^{*}(w) + \sum_{s \geq \hat{s}} \mathbb{P}(s) \cdot (w-c) \cdot q_{s}^{*}(w), \qquad q_{s}^{prv*} = q_{s}^{*}\left(w^{prv*}\right),$$

$$T^{prv*} = \bar{R}_{\hat{s}}\left(q_{\hat{s}}^{prv*}\right) - w^{prv*} \cdot q_{\hat{s}}^{prv*}, \text{ and } \qquad \mathbb{E}\left[\Pi^{prv*}\right] = T^{prv*} + \sum_{s \geq \hat{s}} \mathbb{P}(s) \cdot \left(w^{prv*} - c\right) \cdot q_{s}^{prv*}. \quad (3)$$

Proposition 1. *Under two-part tariff contracts, the manufacturer incurs a hidden information cost in the private signal game compared to the common signal game, i.e.,* $\mathbb{E}\left[\Pi^{com*}\right] - \mathbb{E}\left[\Pi^{prv*}\right] \geq 0$.

Because of the information asymmetry of the private signal game, the manufacturer faces a two-fold dilemma. First, the higher the transfer he sets, the greater the number of low signals that will cause the retailer to not order at all; the lower the transfer, the more surplus he leaves for the retailer when she observe a high signal. Second, because any transfer he sets leaves surplus to the retailer when she observes a high signal, it becomes suboptimal for him to set $w = c$; thus, he chooses $w^{prv*} > c$. As a result, the final stocking levels are distorted as $q_s^{prv*} < q_s^*(c), \forall s \in \Omega$. In other words, he can neither maximize the combined channel profits nor extract the entire combined profits.

TCA Game SPNE. The manufacturer designs pre-signal terms, (T_y, w_y), and (optional) post-signal terms, (T_z, w_z).[3] At Stage 2, the retailer places a pre-signal order, y. At Stage 4, she will place a post-signal order, z, only if $s \in H$, where $H \equiv \Omega \backslash L$ and $L \equiv \{1, \cdots, \tilde{s}\}$ (analogous to §4). Specifically, a post-signal order of $z_s^* = q_s^*(w_z) - y$ (see Lemma 3) will be placed if $T_z \leq \Delta_s \equiv \bar{R}_s\left(q_s^*(w_z)\right) - \bar{R}_s(y) - w_z\left(q_s^*(w_z) - y\right)$, and $\tilde{s}$ will be defined by $\Delta_{\tilde{s}} < T_z \leq \Delta_{\tilde{s}+1}$. The retailer's optimal pre-signal order quantity is $y^*(w_y, w_z) = \left(\bar{R}_L'\right)^{-1}\left(w_z - \frac{w_z - w_y}{\mathbb{P}(L)}\right)$, where $\left(\bar{R}_L\right)^{-1} \equiv \mathbb{E}\left[\left(\bar{R}_s\right)^{-1} \mid S \in L\right]$ (see the derivation of Equation 27). The manufacturer designs the contract at Stage 1 to solve

$$\max_{w_y, T_y, w_z, T_z} T_y + (w_y - c) \cdot y^* + \sum_{s \in H} \mathbb{P}(s) \cdot \left(T_z + (w_z - c) \cdot \left(q_s^*(w_z) - y^*\right)\right), \quad (4)$$

$$\text{s.t.} \quad T_y \leq -w_y \cdot y^* + \sum_{s \in L} \mathbb{P}(s) \cdot \bar{R}_s(y^*) + \sum_{s \in H} \mathbb{P}(s) \cdot \left(\bar{R}_s\left(q_s^*(w_z)\right) - w_z \cdot \left(q_s^*(w_z) - y^*\right) - T_z\right). \quad (\text{IR}_y)$$

Theorem 1. *Under two-part tariffs, the SPNE pre-signal terms of the TCA game are characterized by* $T_y^{tca*} = \sum_{s \in \Omega} \mathbb{P}(s) \cdot \left(\bar{R}_s\left(q_s^{com*}\right) - c \cdot q_s^{com*}\right) - \sum_{s > 1} \mathbb{P}(s) \cdot T_z^{tca*}$ *, and* $w_y = c$. *Depending on the post-signal terms, the following two types of SPNE can arise:*

(i) The first type of SPNE has post-signal terms of $T_z^{tca*} = \bar{R}_2\left(q_2^{com*}\right) - \bar{R}_2\left(q_1^{com*}\right) - c\left(q_2^{com*} - q_1^{com*}\right)$, *and* $w_z = c$. *The retailer orders* $y^{tca*} = q_1^{com*}$ *pre-signal. Later, she places a post-signal order of* $z_s^{tca*} = q_s^{com*} - y^{tca*}$ *only if observes* $s > 1$. *Her final stocking level is* $q_s^{tca*} = q_s^{com*}$, $\forall s \in \Omega$.

[3] The pre-signal terms are mandatory in the sense that accepting them is required to exercise the post-signal terms. Of course, the retailer is free to refuse the pre-signal terms; in which case, she cannot order again post-signal.

(ii) The second type of SPNE has post-signal terms of $T_z^{tca*} = 0$, and $w_z = c$. The retailer orders $y^{tca*} \in [0, q_1^{com*}]$ pre-signal. Later, she places a post-signal order of $z_s^{tca*} = q_s^{com*} - y^{tca*}$, for any signal. Her final stocking level is $q_s^{tca*} = q_s^{com*}$, $\forall s \in \Omega$.

In type (i) SPNE, before observing the signal, the retailer pays the pre-signal transfer and orders the efficient quantity for the lowest signal, $y^{tca*} = q_1^{com*}$. Then later, if she observes any signal $s > 1$, she pays the post-signal transfer and adjusts her stock to reach the efficient quantity for that signal, $z_s^{tca*} = q_s^{com*} - y^{tca*}$. In type (ii) SPNE, before observing the signal, she pays the pre-signal transfer and is indifferent to ordering any quantity between 0 and q_1^{com*}. Later, she tops off to the efficient quantity, $z_s^{tca*} = q_s^{com*} - y^{tca*}$, and pays zero (post-signal) transfer. In both SPNE types, the retailer stocks the efficient quantities, $q_s^{tca*} = q_s^{com*}$ and the combined channel profits are maximized. Through the expected value of both transfers, the manufacturer fully extracts the combined expected profits.

Corollary 1. *Under two-part tariff contracts and any number of forecast signals, the manufacturer achieves the same expected SPNE profits in the TCA and the common signal games.*

Applying the mechanism to two-part tariff contracts is analogous to applying it to wholesale price contracts, where each contract is offered with two sets of terms. With two-part tariffs, the pre-signal terms are designed to make the retailer pay a transfer and order some stock before she observes the signal. (With the wholesale price contract, the discounted wholesale price represented pre-signal terms.) The post-signal terms are designed to make the retailer adjust her stock after she observes the signal. (With the wholesale price contract, the undiscounted wholesale price represented post-signal terms.) The fixed transfer component of the two-part tariff contract makes the TCA mechanism more powerful and restores full information profits for any refined forecasting system.

Note that the two-part tariff contract alone is *not* sufficient to eliminate the hidden information cost. On the one hand, if the manufacturer designs a two-part tariff contract after the retailer observes the signal, then he achieves the private signal game profits. In such case, he bears the full burden of the hidden information cost. On the other hand, if he designs a two-part tariff contract before the retailer observes the signal, then the value of the signal information will be lost and he will not be able to maximize the combined channel profits as the equilibrium quantities will be distorted (see §A.3). Consequently, neither strategy can maximize the combined channel profits. The TCA mechanism eliminates hidden information cost and avoids distortion by engaging the retailer before and after she observes the signal.

The TCA dynamics with two-part tariffs are similar to those with wholesale prices, except that the fixed transfer components of the former makes the mechanism even more powerful. Paying a pre-signal transfer makes the retailer play the odds, and she ends up over-invested if the signal turns out low. The over-investment occurs because the mechanism allows her to adjust her quantity post-signal. On the one hand, if the signal turns out high, her investment pays off because she she can adjust her stock at a low price and make profits that exceed both pre-signal and post-signal transfers. On the other hand, if the signal turns out low, she ends up over-invested because she paid for a benefit that did not materialize; that is, she cannot make enough profits to recoup the pre-signal transfer that she had already paid. This becomes clear if we write the the retailer's profits conditional on signal s as $\pi_s^{tca*} = \Pi_s^{chl*} - \mathbb{E}\left[\Pi_s^{chl*}\right]$, where Π_s^{ch*} denotes maximum combined channel profits conditional on signal s. In the common signal game, the manufacturer uses the fixed transfer to extract all (maximized) channel profits for each signal. In the TCA game, however, he extracts the expected (maximized) channel profits through the two transfers. As such, when the signal is low (high), he makes more (less) in the TCA game compared to the common signal game. These effects cancel out in expectation.

Table 1: SPNE Outcomes with Two-Part Tariff Contract[(i)]

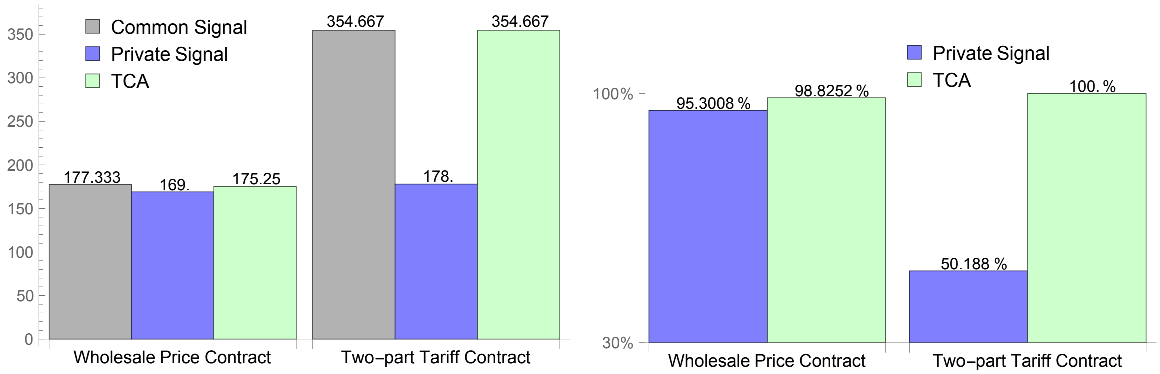
		Private-Signal SPNE	Common-Signal SPNE	TCA SPNE
Realized outcomes at $s = v \in [a, b]$	Pre-signal T	N/A	N/A	$T_y^{tca*} = \mathbb{E}\left[\Pi_s^{chl*}\right]$ [(ii)]
	Pre-signal w	N/A	N/A	$w_y^{tca*} = c$
	Pre-signal Qty	N/A	N/A	$y^{tca*} \in [0, a-c]$
	Post-signal T	$T^{prv*} = \frac{\left(a-c-\sqrt{3}\sigma\right)^2}{2}$	$T^{com*} = \Pi_s^{chl*} = \frac{(v-c)^2}{2}$	$T_z^{tca*} = 0$
	Post-signal w	$w^{prv*} = c + \sqrt{3}\sigma$	$w_s^{com*} = c$	$w_z^{tca*} = c$
	Post-signal Qty	$q_s^{prv*} = q_s^*(c) - \sqrt{3}\sigma$	$q_s^{com*} = q_s^*(c) = v - c$	$z_s^{ca*} = q_s^*(c) - y^{tca*}$
	Total Qty	$q_s^{prv*} = q_s^*(c) - \sqrt{3}\sigma$	$q_s^{com*} = q_s^*(c) = v - c$	$q_s^{tca*} = q_s^{com*} = q_s^*(c)$
	Rtr. profit	$\pi_s^{prv*} = \frac{\left(v-c-\sqrt{3}\sigma\right)^2 - \left(a-c-\sqrt{3}\sigma\right)^2}{2}$	$\pi_s^{com*} = 0$	$\pi_s^{tca*} = \Pi_s^{chl*} - \mathbb{E}\left[\Pi_s^{chl*}\right]$
	Mfr. profit	$\Pi_s^{prv*} = \mathbb{E}\left[\Pi_s^{chl*}\right] - \frac{2\sqrt{3}\sigma\left(2\bar{V}-v-c-\frac{5\sigma}{2\sqrt{3}}\right)}{2}$	$\Pi_s^{com*} = \Pi_s^{chl*}$	$\Pi_s^{tca*} = \mathbb{E}\left[\Pi_s^{chl*}\right]$
	Exp. Rtr. Profit	$\mathbb{E}[\pi^{prv*}] = \sqrt{3}\sigma\left(\bar{V} - c - \frac{4\sigma}{\sqrt{3}}\right)$	$\mathbb{E}[\pi^{com*}] = 0$	$\mathbb{E}[\pi_{tpt}^{tca*}] = 0$
	Exp. Mfr. Profit	$\mathbb{E}[\Pi^{prv*}] = \mathbb{E}[\Pi^{com*}] - \Psi$ [(iii)]	$\mathbb{E}[\Pi^{com*}] = \frac{(\bar{V}-c)^2+\sigma^2}{2}$	$\mathbb{E}[\Pi^{tca*}] = \frac{(\bar{V}-c)^2+\sigma^2}{2}$
	Efficiency[(iv)]	$\eta^{prv} = 1 - \frac{2\Psi}{(\bar{V}-c)^2+\sigma^2}$	$\eta^{com} = 1$	$\eta^{tca} = 1$

[(i)] Based on the set-up in §4.1: the prior is $V \sim U[a, b]$, with a mean of $\bar{V} = \frac{b+a}{2}$, and standard deviation of $\sigma = \frac{b-a}{2\sqrt{3}}$; the signal space is continuous ($N \to \infty$) and fully accurate, such that $s = v \Rightarrow V = v$.

[(ii)] $\Pi_s^{ch*} = \frac{(v-c)^2}{2}$ denotes the maximum combined channel profits conditional on observing signal s; and $\mathbb{E}[\Pi_s^{ch*}] = \frac{(\bar{V}-c)^2+\sigma^2}{2}$. In the common signal game, the manufacturer collects $T_s^{com*} = \Pi_s^{ch*}$ for every signal, s. In the TCA game, he collects $T_y^{tca*} = \mathbb{E}[\Pi_s^{ch*}]$ pre-signal. The retailer accepts to pay that pre-signal transfer *only* because the TCA mechanism allows her to adjust quantity post-signal.

[(iii)] $\Psi \equiv \mathbb{E}[\Pi^{com*}] - \mathbb{E}[\Pi^{prv*}] = \sqrt{3}\sigma\left(\bar{V} - c - \frac{5\sqrt{3}\sigma}{6}\right)$; it denotes the hidden information cost in the private game.

[(iv)] Ratio of the game's SPNE expected profits to the full information (common signal game) SPNE expected profits.



(a) Manufacturer's Expected Profits (b) Informational Efficiency

Figure 1: Effect of Implementing TCA with Different Contracts

Note. Plotted at $V \sim U[20, 40]$, and $c = 4$, under the assumptions of §4.1 and Table 1

Table 1 lists two-part tariff SPNE outcomes in closed form under the setup of of §4.1. Figure 1 shows a snapshot of the SPNE profits/efficiency of the different games under both wholesale price and two-part tariff contracts. It illustrates how the hidden information cost results in a greater loss of relative efficiency under two-part tariff contracts versus simple wholesale price contracts. A manufacturer implementing a two-part tariff contract incurs a relatively larger hidden information cost in the private signal game compared to one implementing a simple wholesale price contract. It also illustrates that the TCA mechanism has a greater impact in restoring full information profits under a strong contract (two-part tariff) versus a weak contract (simple wholesale price).

Online Appendix Proofs

Proof of Proposition 1. From (1), the manufacturer's F.O.C. in the common signal game is $q_s^{*\prime}\left(w_s^{com*}\right) \cdot \bar{R}_s'\left(q_s^*\left(w_s^{com*}\right)\right) - c \cdot q_s^{*\prime}\left(w_s^{com*}\right) = 0$. From (7), $q_s^*(\cdot) = (\bar{R}_s')^{-1}(\cdot) \Rightarrow \bar{R}_s'\left(q_s^*\left(w_s^{com*}\right)\right) = w_s^{com*}$. The F.O.C. reduces to $q_s^{*\prime}\left(w_s^{com*}\right) \cdot \left(w_s^{com*} - c\right) = 0 \Rightarrow w_s^{com*} = c$, and (2) follows. Since

$$\mathbb{E}\left[\Pi^{com*}\right] = \sum_{s \in \Omega} \mathbb{P}(s) \cdot \left(\bar{R}_s\left(q_s^*(c)\right) - c \cdot q_s^*(c)\right) = \sum_{s \in \Omega} \mathbb{P}(s) \cdot \max_{w_s} \bar{R}_s\left(q_s^*\left(w_s\right)\right) - c \cdot q_s^*\left(w_s\right),$$

then we can establish that for any $\hat{s} \in \Omega$,

$$\mathbb{E}\left[\Pi^{com*}\right] \geq \sum_{s \geq \hat{s}} \mathbb{P}(s) \cdot \max_{w_s} \bar{R}_s\left(q_s^*\left(w_s\right)\right) - c \cdot q_s^*\left(w_s\right) \equiv B. \tag{5}$$

Taken together, (3), (5), and $\bar{R}_s\left(q_s^*(\cdot)\right) \leq \bar{R}_{s+1}\left(q_{s+1}^*(\cdot)\right)$, $\forall s \in \Omega$ impliy that, for any $\hat{s} \in \Omega$, $\mathbb{E}\left[\Pi^{prv*}\right] = \max_w \bar{R}_{\hat{s}}\left(q_{\hat{s}}^*(w)\right) - w \cdot q_{\hat{s}}^*(w) + \sum_{s \geq \hat{s}} \mathbb{P}(s) \cdot (w - c) \cdot q_s^*(w) \leq B \leq \mathbb{E}\left[\Pi^{com*}\right]$. □

Proof of Theorem 1. The IR constraint in (4) is binding. For an arbitrary $\tilde{s}$, the manufacturer's optimization problem becomes

$$\max_{w_y, w_z,} - w_y \cdot y^* + \sum_{s \in L} \mathbb{P}(s) \cdot \bar{R}_s(y^*) + \sum_{s \in H} \mathbb{P}(s) \cdot \left(\bar{R}_s\left(q_s^*(w_z)\right) - w_z \cdot \left(q_s^*(w_z) - y^*\right) - T_z\right)$$
$$+ (w_y - c) \cdot y^* + \sum_{s \in H} \mathbb{P}(s) \cdot \left(T_z + (w_z - c) \cdot \left(q_s^*(w_z) - y^*\right)\right).$$

Cancel out terms containing T_z, and collect on y^* and $q_s^*(w_z)$ to get

$$\max_{w_y, w_z,} \sum_{s \in L} \mathbb{P}(s) \cdot \left(\bar{R}_s(y^*) - c \cdot y^*\right) + \sum_{s \in H} \mathbb{P}(s) \cdot \left(\bar{R}_s\left(q_s^*(w_z)\right) - c \cdot q_s^*(w_z)\right).$$

Since $y^*(w_y, w_z) = \left(\bar{R}'_L\right)^{-1}\left(w_z - \frac{w_z - w_y}{\mathbb{P}(L)}\right)$, and $q_s^*(w_z) = \left(\bar{R}'_s\right)^{-1}(w_z)$, then, for any $\tilde{s} \in \Omega$, the SPNE wholesale prices must be $w_y^{tca*} = w_z^{tca*} = c$. These will result in efficient quantities within set H: $q_s^*(c)$, for all $s \in H$. However, they will result in the average efficient quantity within set L: $y^* = \left(\bar{R}'_L\right)^{-1}(c)$ for all $s \in L$. As such, it is optimal for the manufacturer to set $\tilde{s} \leq 1$ such that set L contains at most one signal (i.e., $L = \{1\}$ or $L = \{\}$). □

Proof of Corollary 1. From Equation (2) and Proposition 1, $\mathbb{E}\left[\Pi^{tca*}\right] = T_y^{tca*} + \sum_{s>1} \mathbb{P}(s) \cdot T_z^{tca*} - \sum_{s \in \Omega} \mathbb{P}(s) \cdot c \cdot q_s^{com*} = \mathbb{E}\left[\Pi^{com*}\right]$. □