

Online Appendix

“Macroeconomic Risks and Asset Pricing: Evidence from a Dynamic Stochastic General Equilibrium Model”

Erica X.N. Li ^{*} Haitao Li [†] Shujing Wang, [‡] and Cindy Yu [§]

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1 Data

The raw data used in constructing the observed macroeconomic variables are:

Nominal GDP (GDP): nominal gross domestic product, billions of dollars, seasonally adjusted at annual rates, NIPA.

GDP Deflator (P): price index of nominal gross domestic product, index numbers, 2005=100, seasonally adjusted, NIPA.

Nominal nondurable consumption ($C_{nondurables}^{nom}$): nominal personal consumption expenditures: nondurable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal durable consumption ($C_{durables}^{nom}$): nominal personal consumption expenditures: durable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal consumption services ($C_{services}^{nom}$): nominal personal consumption expenditures: services, billions of dollars, seasonally adjusted at annual rates, NIPA.

^{*}Department of Finance, Cheung Kong Graduate School of Business, Beijing, China, 100738; Tel: (86)10 85188888 ext. 3075; E-mail: xnli@ckgsb.edu.cn.

[†]Department of Finance, Cheung Kong Graduate School of Business, Beijing, China, 100738; Tel: (86)10 85188888 ext. 3083; E-mail: htli@ckgsb.edu.cn.

[‡]Department of Economics and Finance, School of Economics and Management, Tongji University, Shanghai, China 200092; Tel: (86) 21 65982274; E-mail: shujingwang@connect.ust.hk.

[§]Department of Statistics, Iowa State University, Ames, IA 50011; Tel: 1 515 2946885; E-mail: cindywu@iastate.edu.

Nominal investment (I^{nom}): nominal gross private domestic investment, billions of dollars, seasonally adjusted at annual rates, NIPA.

Price index (PC^{nom}): price index of nondurable goods, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.

Price index (PI^{nom}): nominal investment: price index of nominal gross private domestic investment, Nonresidential, Equipment & Software index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.

Employment: (E) civilian employment, CE16OV, seasonally adjusted, monthly, thousands, persons 16 years of age and older, FRED2.

Population (POP): civilian noninstitutional population, not seasonally adjusted, thousands, FRED2. **Federal Funds Rate: (FF)** effective federal funds rate, H.15 selected interest rates, monthly, percent, averages of daily figures, FRED2.

Average hours: (H^{avg}) average weekly hours, PRS85006023, sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.

Credit ($Credit^{nom}$): Nonfinancial Business; Credit Market Instruments; Liability, Level, Billions of Dollars, Quarterly, Seasonally Adjusted, FRED2

Loan Rate (R_{BAA}): Interest rate on BAA-rated corporate bond, FRED2

Government bond rate (R_{10yr}): interest rate of ten-year US government bond, FRED2

Here NIPA, BLS and FRED2 stand for

FRED2: Database of the Federal Reserve Bank of St. Louis available at:

<http://research.stlouisfed.org/fred2/>.

BLS: Database of the Bureau of Labor Statistics available at: <http://www.bls.gov/>.

NIPA: Database of the National Income And Product Accounts available at:

<http://www.bea.gov/national/nipaweb/index.asp>.

BGOV: Database of the Board of Governors of the Federal Reserve System available at:

<http://www.federalreserve.gov/econresdata/default.htm>.

The variables used in the estimation is constructed as follows:

- output = $\frac{GDP}{P \times POP}$
- inflation = growth rate of P
- hours = $\frac{H^{avg} \times E}{POP}$
- consumption = $C_{nondurables}^{nom} + C_{services}^{nom}$
- nominal investment = $I^{nom} + C_{durables}^{nom}$
- relative price of investment = $\frac{PI^{nom}}{PC^{nom}}$
- credit supply = $R_{BAA} - R_{10yr}$
- credit supply = $\frac{Credit^{nom}}{P \times POP}$

All variables are detrended by their sample means.

2 Full-Information Bayesian MCMC Estimation

In this section, we discuss the full-information Bayesian MCMC method for estimating DSGE models based on observed macroeconomic variables. The Bayesian MCMC method makes it possible to estimate both model parameters and the latent economic shocks by fully exploiting the conditional information contained in the likelihood function of the macroeconomic data.

2.1 Model Solution

We first discuss numerical solutions of our model. While DSGE models are generally solved to the first order for macroeconomic applications, for asset pricing applications, we have to solve our model to the second order in Dynare so that all the endogenous variables of the model are expressed as second-order polynomial functions of the state variables.¹ The coefficients in those polynomials depend on the model parameters.

We use quarterly observations on nine variables covering the period of 1957q2 - 2015q4 to estimate model parameters and exogenous shocks:² per capita GDP growth $dy_t = \log(Y_t/Y_{t-1})$, per capita consumption growth $dc = \log(C_t/C_{t-1})$, per capita investment growth $di = \log(I_t/I_{t-1})$, growth of relative price of investment goods μ_t^ψ , inflation π_t , 3-month T-bill rate $r_t = \log(R_t)$, weekly hours per capita normalized by sample average $h_t = \log(H_t/\bar{H})$, per capita real growth of credit $db_t = \log(B_t/(\pi_t * B_{t-1}))$, and credit spread $cs_t \equiv Z_t - R_t$.³

Define the vector of the state variables at time t as \mathcal{S}_t , the vector of endogenous variables that we would like to match with actual data as $\mathcal{O}_t = [dy_t, dc_t, di_t, \mu_t^\psi, \pi_t, r_t, h_t, db_t, cs_t]'$, and the vector of exogenous shocks as $U_t = [e_t^z, e_t^\psi, e_t^V, e_t^\sigma]'$. Among these state variables, $\mathcal{S}^{ex} \equiv [\mu_t^z, \mu_t^\psi, V_t, \sigma_t]$ is the vector of exogenous/latent state variables, whose evolution follows

$$\mathcal{S}_t^{ex} = \Gamma^{ex}(\mathcal{S}_{t-1}^{ex}, U_t; \Theta) \equiv \bar{\mathcal{S}}^{ex} + \rho \mathcal{S}_{t-1}^{ex} + \Sigma U_t \quad (1)$$

¹Detailed information on Dynare is at www.dynare.org and the model solution is in the online appendix.

²The sample of 1956q2 - 2015q4 is the longest sample during which data on the nine macroeconomic variables are available. We use the data during 1956q2 - 1957q1 as the burnin period in our estimation.

³The construction and data resources of these nine variables are provided in the Online Appendix.

where Θ is the vector of model parameters, $\bar{S}^{ex} = [(1 - \rho_z)\mu_z, (1 - \rho_\psi)\mu_\psi, 0, (1 - \rho_\sigma)\ln(\bar{\sigma})]'$ and ϱ and Σ are two 4×4 diagonal matrices whose diagonal elements are $[\rho_z, \rho_\psi, \rho_V, \rho_\sigma]$ and $[\sigma_z, \sigma_\psi, \sigma_V, \sigma_\sigma]$, respectively. Thus, equation (1) is a combination of equations (3), (4), (11), and (8). The rest of the state variables are endogenous, defined as S^{en} .

Given the values of the state variables at $t - 1$ and the latent shocks at t , the value of any endogenous variable at t can be approximated as a second-order polynomial of the state variables and the latent shocks. Specifically, the endogenous state variables and the variables to be matched with data evolve according the following rules, respectively,

$$S_t^{en} = \Gamma^{en}(S_{t-1}, U_t; \Theta) \quad \text{and} \quad \mathcal{O}_t = \Gamma(S_{t-1}, U_t; \Theta), \quad (2)$$

where Γ^{en} and Γ are second-order polynomial functions of S_{t-1} and U_t . The coefficients in those polynomials depend on the vector of model parameters Θ . These parameters include:

$$\Theta = [\beta, \phi, b, \alpha, \delta, \eta_g, \xi_p, \xi_w, \lambda_f, \lambda_w, \sigma_a, \sigma_s, \pi, \rho_R, \rho_\pi, \rho_y, \mu_z, \mu_\psi, \sigma_z, \sigma_\psi, \sigma_v, \rho_z, \rho_\psi, \rho_v]'$$

For any given initial values of the state variables, S_0 , and the time-series of exogenous shocks $\{U_\tau\}_{\tau=1}^t$, the state variables can be obtained by iterating equations (1) and (2):

$$S_t = \Gamma^s(\Gamma^s(\dots \Gamma^s(\Gamma^s(S_0, U_1; \Theta), U_1; \Theta) \dots), U_t, \Theta) \equiv \Gamma^{s,(t)}(S_0, \{U_\tau\}_{\tau=1}^t; \Theta),$$

where $s \in \{en, ex\}$ and $\Gamma^{s,(t)}$ refers to t iterations of Γ^s . Given the values of the state variables at any point of time, the model-implied time-series of \mathcal{O}_t can be easily computed as $\mathcal{O}_t = \Gamma(S_{t-1}, U_t; \Theta)$.

2.2 MCMC Method

The numerical method in Dynare allows us to solve for the seven macroeconomic variables given the model parameters Θ and the latent shocks $\mathbf{U} = \{U_t\}_{t=1}^T$. The purpose of the MCMC method is to estimate the model parameters and latent shocks based on the observed seven macroeco-

conomic variables.

Specifically, we denote the time series of the observed macroeconomic variables as $\mathcal{O}^{obs} = \{\mathcal{O}_t^{obs}\}_{t=1}^T$ and assume that \mathcal{O}_t^{obs} are equal to model-implied values plus some independent measurement errors:

$$\mathcal{O}_t^{obs} = \mathcal{O}_t + \varepsilon_t = \Gamma(S_{t-1}, U_t, \Theta) + \varepsilon_t,$$

where $\mathcal{O}_t^{obs} = \{dy_t, dc_t, di_t, \mu_t^\psi, \pi_t, r_t, h_t, db_t, cs_t\}^{obs}$, and $\varepsilon_t = \{\varepsilon_{1t}, \dots, \varepsilon_{9t}\}$ with $\varepsilon_{it} \sim N(0, \sigma_i^2)$ for $i = 1, \dots, 9$. \mathcal{O}_t are the model-implied values from the $\Gamma(\cdot)$ function that is solved numerically using Dynare. The main objective of the MCMC method is to obtain the posterior distributions of both the model parameters and the latent shocks based on the observed macroeconomic variables, i.e., $p(\Theta, \mathbf{U} | \mathcal{O}^{obs})$.⁴

We choose the MCMC method because the traditional likelihood approach would be computationally challenging to implement due to the need to integrate out high dimensional latent shocks (of the order of $4 \times T$) to obtain the marginal likelihood function $L(\Theta) \equiv p(\mathcal{O}^{obs} | \Theta)$. In contrast, our MCMC method simulates posterior samples of (Θ, \mathbf{U}) from a complicated posterior distribution $p(\Theta, \mathbf{U} | \mathcal{O}^{obs})$. We estimate the means and standard deviations of (Θ, \mathbf{U}) using the means and standard deviations of the simulated posterior samples. By Bayes rule,

$$p(\Theta, \mathbf{U} | \mathcal{O}^{obs}) \propto p(\mathcal{O}^{obs}, \mathbf{U}, \Theta) = p(\mathcal{O}^{obs} | \mathbf{U}, \Theta) p(\mathbf{U} | \Theta) p(\Theta),$$

where the likelihood functions equal:

$$p(\mathcal{O}^{obs} | \mathbf{U}, \Theta) = \prod_{t=1}^T \prod_{i=1}^9 \frac{1}{\sqrt{2\pi}\sigma_i} \exp \left\{ -\frac{[\mathcal{O}_t^{obs}(i) - \mathcal{O}_t(i)]^2}{2\sigma_i^2} \right\},$$

⁴In our empirical analysis, we need to estimate σ_i ($i = 1, \dots, 9$) as well. For brevity, we treat them as part of the model parameters in our discussion of the MCMC method.

$$\begin{aligned}
p(\mathbf{U}|\Theta) &= \prod_{t=1}^T \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left\{-\frac{1}{2\sigma_z^2}[\mu_{z,t} - \mu_z(1 - \rho_z) - \rho_z\mu_{z,t-1}]^2\right\} \\
&\times \prod_{t=1}^T \frac{1}{\sqrt{2\pi}\sigma_\psi} \exp\left\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t} - \mu_\psi(1 - \rho_\psi) - \rho_\psi\mu_{\psi,t-1}]^2\right\} \\
&\times \prod_{t=1}^T \frac{1}{\sqrt{2\pi}\sigma_V} \exp\left\{-\frac{1}{2\sigma_V^2}[V_t - \rho_V V_{t-1}]^2\right\} \\
&\times \prod_{t=1}^T \frac{1}{\sqrt{2\pi}\sigma_\sigma} \exp\left\{-\frac{1}{2\sigma_\sigma^2}[\ln(\sigma_t/\bar{\sigma}) - \rho_\sigma \ln(\sigma_{t-1}/\bar{\sigma})]^2\right\},
\end{aligned}$$

where $\mathcal{O}_t^{obs}(i)$ and $\mathcal{O}_t(i)$ are the observed and model-implied values for the i^{th} macroeconomic variable, respectively, and $p(\Theta)$ is the prior distribution of the model parameters.

Direct draws from the joint posterior distribution $p(\Theta, \mathbf{U}|\mathcal{O}^{obs})$ is very hard. However, the Clifford-Hammersley theorem indicates that the draws from the joint posterior are equivalent to the draws from their complete conditionals, i.e.,

$$p(\Theta, \mathbf{U}|\mathcal{O}^{obs}) \iff p(\Theta|\mathbf{U}, \mathcal{O}^{obs}), p(\mathbf{U}|\Theta, \mathcal{O}^{obs}).$$

Therefore, we simulate the posterior samples of each parameter and the latent shocks (U_t for each t) from the complete conditionals as follows. Given initial values $(\Theta^{(0)}, \mathbf{U}^{(0)})$,

- draw $\Theta^{(g+1)} \sim p(\Theta|\mathbf{U}^{(g)}, \mathcal{O}^{obs})$,
- draw $\mathbf{U}^{(g+1)} \sim p(\mathbf{U}|\Theta^{(g+1)}, \mathcal{O}^{obs})$.

One challenge we face in the implementation of the MCMC method is that the posterior distributions of model parameters and the latent shocks are generally not available in closed form. Since our model has to be solved numerically through Dynare, (Θ, \mathbf{U}) are involved in the likelihood function inexplicitly, resulting in intractable posterior distributions. We therefore turn to the Metropolis-Hastings (MH) algorithm for updating (Θ, \mathbf{U}) . The MH algorithm is an adaptive rejection sampling method where candidate draw is proposed and then accepted with probability proportional to the ratio of the likelihood of the proposed draw to the current draw. If the new position has a higher likelihood (defined using the posterior distribution), then

the parameter values are updated with probability 1. Alternatively, if they are less likely, the parameter values are updated with a probability according to the likelihood ratio. Thus the parameter values will tend to stay near the highest probability regions when being sampled and adequately cover the probability space.

Given a vector of starting values for the parameters and latent variables, the steps of the MH algorithm are as follows.

For the j^{th} parameter θ_j in Θ ($j = 1, \dots, 23$) in the g^{th} iteration:

- Step 1: Specify a candidate distribution, $h(\theta_j|\theta_j^{(g-1)})$.
- Step 2: Generate a proposed value for the parameter, $\theta_j^* \sim \mathbf{h}(\theta_j|\theta_j^{(g-1)})$.
- Step 3: Compute the acceptance ratio:

$$r_g = \frac{p(\theta_j^*|\theta_{[-j]}, \mathbf{U}, \mathcal{O}^{obs}) \times \mathbf{h}(\theta_j^*|\theta_j^{(g-1)})}{p(\theta_j^{(g-1)}|\theta_{[-j]}, \mathbf{U}, \mathcal{O}^{obs}) \times \mathbf{h}(\theta_j^{(g-1)}|\theta_j^*)}$$

where $p(\cdot|\theta_{[-j]}, \mathbf{U}, \mathcal{O}^{obs})$ represents a complete conditional distribution for parameter θ_j and the notation $\theta_{[-j]}$ contains the most updated parameters except for θ_j in Θ (see details in Appendix 2.3).

- Step 4: Generate u from an uniform distribution $Uniform[0, 1]$, then set

$$\theta_j^{(g)} = \begin{cases} \theta_j^* & \text{if } r_g \geq u \\ \theta_j^{(g-1)} & \text{if } r_g < u \end{cases}.$$

- Step 5: Do this for all θ_j for $j = 1, \dots, 23$. Then set $g = g + 1$ and return to Step 1.

If the candidate distribution is symmetric, the MH algorithm has an acceptance ratio equivalent to $p(\theta_j^*|\theta_{[-j]}, \mathbf{U}, \mathcal{O}^{obs}) / p(\theta_j^{(g-1)}|\theta_{[-j]}, \mathbf{U}, \mathcal{O}^{obs})$. In implementation, we chose $N(\theta_j^{(g-1)}, c^2)$ with a constant variance c^2 as a candidate distribution for $\mathbf{h}(\theta_j|\theta_j^{(g-1)})$. The MH algorithm is conducted iteratively on each parameter in Θ and on each latent variable at each time point $t = 1, \dots, T$.

In estimation, in order to have priors that are as uninformative as possible, we choose normal distribution as priors for all the parameters. The means and standard deviations of the prior normal distributions are chosen to be close to the estimated values in Christiano, Motto, and Rostagno (2014). We draw posterior samples based on the above described MCMC method, and use the means of the posterior draws as parameter estimates and the standard deviations of the posterior draws as standard errors of the parameter estimates. We discard the first 20,000 simulations as burn in and use the mean of the last 30,000 simulations as the parameter and state estimates.

2.3 Bayesian MCMC Estimation

In this subsection, we provide a brief description about the priors, posterior distributions, and the updating procedures for parameters and latent variables in our DSGE model. The hyperparameters of the priors are provided in Table 2.

- *Posterior of σ_i ($i = 1, \dots, 7$):* Set the prior of σ_i as $\sigma_i^2 \sim IG(a, b)$, where a, b are hyperparameters. The posterior of σ_i^2 is:

$$\sigma_i^2 \sim IG\left(\frac{T}{2} + a, A\right),$$

where

$$A = \sum_{t=1}^T \frac{1}{2} (\mathcal{O}_t^{obs}(i) - \mathcal{O}_t(i))^2 + b.$$

- *Posterior of θ_j ($j = 1, \dots, 25$):* Set the prior of θ_j as $\theta_j \sim N(m, M^2)$, where m, M are hyperparameters. The posterior of θ_j is:

$$\begin{aligned} p\left(\theta_j | \theta_{[-j]}, \mathbf{U}, \mathcal{O}^{obs}\right) &\propto \prod_{t=1}^T \prod_{i=1}^7 \frac{1}{\sigma_i} \exp\left\{-\frac{1}{2\sigma_i^2} [\mathcal{O}_t^{obs}(i) - \mathcal{O}_t(i)]^2\right\} \\ &\times \prod_{t=1}^T \frac{1}{\sigma_z} \exp\left\{-\frac{1}{2\sigma_z^2} [\mu_{z,t} - \mu_z(1 - \rho_z) - \rho_z \mu_{z,t-1}]^2\right\} \\ &\times \prod_{t=1}^T \frac{1}{\sigma_\psi} \exp\left\{-\frac{1}{2\sigma_\psi^2} [\mu_{\psi,t} - \mu_\psi(1 - \rho_\psi) - \rho_\psi \mu_{\psi,t-1}]^2\right\} \\ &\times \prod_{t=1}^T \frac{1}{\sigma_V} \exp\left\{-\frac{1}{2\sigma_V^2} [V_t - \rho_V V_{t-1}]^2\right\} \times \exp\left\{-\frac{(\theta_j - m)^2}{2M^2}\right\}, \end{aligned}$$

where $\theta_{[-j]}$ contains the most updated parameters except for θ_j in Θ . In implementation, we simplify the above posterior by abandoning terms that do not depend on θ_j , and use the MH algorithm to update θ_j .

- *Posterior of $\{\mu_{z,t}, \mu_{\psi,t}, V_t\}$ ($t = 1, \dots, T$):* The posterior distribution of $\mu_{z,t}$ (for $1 \leq t < T$)

is:

$$\begin{aligned}
& p\left(\mu_{z,t} \mid \Theta, \{\mu_{z,1}, \dots, \mu_{z,t-1}, \mu_{z,t+1}, \dots, \mu_{z,T}\}, \{\mu_{\psi,t}\}_{t=1}^T, \{V_t\}_{t=1}^T, \mathcal{O}^{obs}\right) \\
& \propto \prod_{s=t}^T \prod_{i=1}^N \exp\left\{-\frac{1}{2\sigma_i^2} [\mathcal{O}_t^{obs}(i) - \mathcal{O}_t(i)]^2\right\} \\
& \quad \times \exp\left\{-\frac{1}{2\sigma_z^2} [\mu_{z,t} - \mu_z(1 - \rho_z) - \rho_z \mu_{z,t-1}]^2\right\} \\
& \quad \times \exp\left\{-\frac{1}{2\sigma_z^2} [\mu_{z,t+1} - \mu_z(1 - \rho_z) - \rho_z \mu_{z,t}]^2\right\}.
\end{aligned}$$

For $t = T$, the posterior distribution only involves the first two terms in the above equation. Again, the MH algorithm is used to update $\mu_{z,t}$. Updating of $\mu_{\psi,t}$ and V_t ($t = 1, \dots, T$) are done in the same way. The analogous posterior distribution for $\mu_{\psi,t}$ is:

$$\begin{aligned}
& p\left(\mu_{\psi,t} \mid \Theta, \{\mu_{\psi,1}, \dots, \mu_{\psi,t-1}, \mu_{\psi,t+1}, \dots, \mu_{\psi,T}\}, \{\mu_{z,t}\}_{t=1}^T, \{V_t\}_{t=1}^T, \mathcal{O}^{obs}\right) \\
& \propto \prod_{s=t}^T \prod_{i=1}^N \exp\left\{-\frac{1}{2\sigma_i^2} [\mathcal{O}_t^{obs}(i) - \mathcal{O}_t(i)]^2\right\} \\
& \quad \times \exp\left\{-\frac{1}{2\sigma_\psi^2} [\mu_{\psi,t} - \mu_\psi(1 - \rho_\psi) - \rho_\psi \mu_{\psi,t-1}]^2\right\} \\
& \quad \times \exp\left\{-\frac{1}{2\sigma_\psi^2} [\mu_{\psi,t+1} - \mu_\psi(1 - \rho_\psi) - \rho_\psi \mu_{\psi,t}]^2\right\}.
\end{aligned}$$

The analogous posterior distribution for V_t is:

$$\begin{aligned}
& p\left(V_t \mid \Theta, \{V_1, \dots, V_{t-1}, V_{t+1}, \dots, V_T\}, \{\mu_{z,t}\}_{t=1}^T, \{\mu_{\psi,t}\}_{t=1}^T, \mathcal{O}^{obs}\right) \\
& \propto \prod_{s=t}^T \prod_{i=1}^N \exp\left\{-\frac{1}{2\sigma_i^2} [\mathcal{O}_t^{obs}(i) - \mathcal{O}_t(i)]^2\right\} \\
& \quad \times \exp\left\{-\frac{1}{2\sigma_V^2} [V_t - \rho_V V_{t-1}]^2\right\} \\
& \quad \times \exp\left\{-\frac{1}{2\sigma_V^2} [V_{t+1} - \rho_V V_t]^2\right\}.
\end{aligned}$$

The analogous posterior distribution for V_t is:

$$\begin{aligned}
& p \left(V_t | \Theta, \{\sigma_1, \dots, \sigma_{t-1}, \sigma_{t+1}, \dots, \sigma_T\}, \{\mu_{z,t}\}_{t=1}^T, \{\mu_{\psi,t}\}_{t=1}^T, \{V_t\}_{t=1}^T, \mathcal{O}^{obs} \right) \\
& \propto \prod_{s=t}^T \prod_{i=1}^N \exp \left\{ -\frac{1}{2\sigma_i^2} [\mathcal{O}_i^{obs}(i) - \mathcal{O}_t(i)]^2 \right\} \\
& \quad \times \exp \left\{ -\frac{1}{2\sigma_\sigma^2} [\ln(\sigma_t/\bar{\sigma}) - \rho_\sigma \ln(\sigma_{t-1}/\bar{\sigma})]^2 \right\} \\
& \quad \times \exp \left\{ -\frac{1}{2\sigma_\sigma^2} [\ln(\sigma_{t+1}/\bar{\sigma}) - \rho_\sigma \ln(\sigma_t/\bar{\sigma})]^2 \right\}.
\end{aligned}$$

3 Model Solution

3.1 Household Maximization Problem

The household's life-time utility is given by:

$$\max_{\{C_{t+\tau}, L_{t+\tau}, I_{t+\tau}, \bar{K}_{t+\tau}\}_{\tau=0}^{\infty}} V_t$$

Subject to

$$\begin{aligned} & B_{t+\tau-1}^{\infty} (Q_{t+\tau}^{\infty} \rho + 1) + R_{t+\tau-1} B_{t+\tau-1} + Q_{t+\tau}^k \left[1 - S \left(\frac{I_{t+\tau}}{I_{t+\tau-1}} \right) \right] I_{t+\tau} \\ & + P_{t+\tau} [L I_{t+\tau} + D_t + T_t^e] - \left[P_{t+\tau} C_{t+\tau} + Q_{t+\tau}^{\infty} B_{t+\tau}^{\infty} + B_{t+\tau} + \frac{P_{t+\tau} I_{t+\tau}}{\Psi_{t+\tau}} \right] \geq 0. \end{aligned}$$

Solving this maximization problem leads to :

- The pricing real and nominal kernels $M_{t,t+1}$ and $M_{t,t+1}^{\$}$, and the risk-free rate are given by:

$$M_{t,t+1} = \beta_t \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^{-\psi} \left(\frac{V_{t+1}^{1/(1-\psi)}}{\mathbb{E}_t [V_{t+1}^{(1-\gamma)/(1-\psi)}]^{1/(1-\gamma)}} \right)^{\psi-\gamma}, \quad (3)$$

$$M_{t,t+1}^{\$} = M_{t,t+1} \left(\frac{P_{t+1}}{P_t} \right)^{-1}. \quad (4)$$

$$\log(R_t) \equiv i_t = -\log \mathbb{E}_t [M_{t,t+1}^{\$}]$$

respectively. The nominal interest rate i_t is the instrument of monetary policy.

- The FOC wrt I_t leads to

$$\frac{Q_t^k}{P_t} \left[1 - S_t - S'_t \frac{I_t}{I_{t-1}} \right] - 1/\Psi_t + \mathbb{E}_t \left\{ M_{t,t+1} \frac{Q_{t+1}^k}{P_{t+1}} S'_{t+1} \frac{I_{t+1}^2}{I_t^2} \right\} = 0. \quad (5)$$

where

$$S_t = S \left(\frac{I_t}{I_{t-1}} \right) \text{ and } S'_t = \left. \frac{dS(\cdot)}{d\cdot} \right|_{\cdot = I_t/I_{t-1}}.$$

3.2 Financial Intermediation

Assume that some members of household are entrepreneurs who have the ability to turn raw capital into productive capital, which is used in production. How much productive capital can be produced by entrepreneur e depends on his net worth $N_{e,t}$ and a random productive shock $\omega_{e,t}$:

$$K_{t+1} = \int_0^\infty dF(\omega) \int_0^1 de [u_{e,t+1}\omega_{e,t}\bar{K}_{e,t}] = \int_0^\infty f(\omega)d\omega \int_0^1 de \left[u_{e,t+1}\omega_{e,t}\frac{N_{e,t}\chi_{e,t}}{Q_t^k} \right]$$

where $\omega_{e,t}$ is entrepreneur's productivity, and $u_{e,t+1}$ is the optimal utilization rate of capital chosen by entrepreneur e at time $t + 1$. Entrepreneurs' productivity $\omega_{e,t}$ follows a lognormal distribution with time-varying standard deviation of σ_t .

$\chi_{e,t}$ is leverage ratio that the entrepreneur can take:

$$\chi_{e,t} = \frac{N_{e,t} + B_{e,t}}{N_{e,t}},$$

where $B_{e,t}$ is the one-period loan to e that matures at $t + 1$. In aggregate, we have

$$N_t = \int_0^1 N_{e,t}de \quad \text{and} \quad B_t = \int_0^1 B_{e,t}de.$$

It can be shown that the leverage ratio χ is the same to all entrepreneurs

$$\chi_{e,t} = \chi_t = \frac{N_t + B_t}{N_t}, \tag{6}$$

$$Q_t^k \bar{K}_t = N_t + B_t. \tag{7}$$

Assume that the banking industry is competitive and banks earn risk-free interest rate on loans in every state of $t + 1$, i.e.,

$$[1 - F(\bar{\omega}_{t+1})]Z_t B_{e,t} + (1 - \mu_b) \int_0^{\bar{\omega}_{t+1}} \omega_t dF(\omega_t) R_{t+1}^k Q_{k,t} \bar{K}_{e,t} = R_t B_{e,t},$$

where μ_b is the bankruptcy cost and $\bar{\omega}_{t+1}$ is the threshold above which entrepreneur is productive

enough to pay back the loan, i.e.,

$$R_{t+1}^k \bar{\omega}_{t+1} Q_t^k \bar{K}_{e,t} = B_{e,t} Z_{t+1}, \Rightarrow$$

$$\bar{\omega}_{t+1} = \frac{Z_{t+1}(\chi_t - 1)}{\chi_t} \frac{1}{R_{t+1}^k},$$

where Z_{t+1} is the $t + 1$ state-contingent nominal return on bank loan. It can be shown that

$$R_{t+1}^k [\Gamma(\bar{\omega}_{t+1}) - \mu_b G(\bar{\omega}_{t+1})] = \frac{\chi_t - 1}{\chi_t} R_t. \quad (8)$$

which needs to be held at every $t + 1$ state.

The definitions of $G(\bar{\omega}_{t+1})$ and $\Gamma(\bar{\omega}_{t+1})$ are given as follows:

$$G(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega_t dF(\omega_t), \quad (9)$$

$$\Gamma(\bar{\omega}_{t+1}) = [1 - F(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G(\bar{\omega}_{t+1}), \quad (10)$$

and

$$R_t^k = \frac{(1 - \tau^k) [u_t r_t^k - a(u_t)/\Psi_t] P_t + (1 - \delta) Q_t^k + \tau^k \delta Q_{t-1}^k}{Q_{t-1}^k}, \quad (11)$$

where R_t^k is the nominal return on raw capital, r_t^k is the real rental rate of productive capital paid by producers, and τ^k is the tax rate on capital income. The nominal cost of utilization per unit of raw capital is $\frac{P_t}{\Psi_t} a(u_t)$, where

$$a(u) = r^k [\exp(\sigma_a(u - 1)) - 1] / \sigma_a,$$

where $\sigma_a > 0$. Maximizing the above equation leads to the optimal utilization rate u_t :

$$r_t^k = a'(u_t) / \Psi_t. \quad (12)$$

For entrepreneur e , his total worth at the end of t is given by

$$\begin{aligned} N_t &= \gamma_e R_t^k Q_{k,t-1} \bar{K}_{t-1} \left[\int_{\bar{\omega}}^{-\infty} (\omega - \bar{\omega}) dF(\omega) \right] + W_t^e \\ &= \gamma_e [1 - \Gamma(\bar{\omega}_t)] R_t^k Q_{k,t-1} \bar{K}_{t-1} + W_t^e, \end{aligned} \quad (13)$$

where $1 - \gamma_e$ is fraction of wealth transferred from entrepreneur to households and W_t^e is the transfer from household to entrepreneur. The latter serves as an insurance to entrepreneurs so that they have consumptions even if they bankrupt. Therefore, the net transfer from entrepreneurs to household is

$$T_t^e = (1 - \gamma_e) [1 - \Gamma(\bar{\omega}_t)] R_t^k Q_{k,t-1} \bar{K}_{t-1} - W_t^e.$$

Entrepreneurs maximize his expected wealth by choosing the optimal leverage, which leads to

$$\mathbb{E}_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma'(\bar{\omega}_{t+1})}{\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})} \left[\frac{R_{t+1}^k}{R_t} (\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})) - 1 \right] \right\} = 0 \quad (14)$$

Since all entrepreneurs choose the same utilization rate and leverage ratio, we have the following aggregation:

$$K_t = u_t \bar{K}_{t-1} \quad (15)$$

3.3 Intermediate-Good Production Sector

The production of intermediate goods i uses both capital and labor via the following homogenous production technology:

$$Y_{i,t} = (z_t L_{i,t})^{1-\alpha} K_{i,t-1}^\alpha - z_t^+ \varphi$$

Cost minimization problem gives the relationship between capital rental rate and wage:

$$\frac{K_t}{L_t} = \frac{\alpha}{1 - \alpha} \frac{W_t}{r_t^k}. \quad (16)$$

Intermediate goods producer i rents capital service K_{it} from households and its net profit at period t is given by $P_{it}Y_{it} - r_t^k K_{it} - W_t L_{it}$, where L is the labor service demanded by firms. L is a combination of all labor types and will be defined later. The producer takes the nominal rent of capital service r_t^k and nominal wage rate W_t as given but has market power to set the price of its product in a Calvo (1983) staggered price setting to maximize profits. With probability ξ_p , producer i cannot reoptimize its price and has to set it according to the following rule,

$$P_{i,t} = \tilde{\pi}_{p,t} P_{i,t-1}$$

where

$$\tilde{\pi}_{p,t} = (\pi_t^*)^\ell (\pi_{t-1})^{1-\ell} \quad (17)$$

is the inflation indexation, π_t^* is the target inflation rate or steady state inflation rate, and $\pi_t \equiv P_t/P_{t-1}$ is the inflation rate. Producer i sets price $P_{i,t}$ with probability $1 - \xi_p$ to maximize its profits, i.e.,

$$\max_{\{P_{i,t}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \xi_p^\tau M_{t,t+\tau}^\$ [\tilde{\theta}_{p,t \oplus \tau} P_{i,t} Y_{i,t+\tau|t} - s_{t+\tau} P_{t+\tau} Y_{i,t+\tau|t}]$$

subject to the demand function

$$Y_{i,t+\tau} = Y_{t+\tau} \left(\frac{\tilde{\theta}_{p,t \oplus \tau} P_{i,t}}{P_{t+\tau}} \right)^{-\frac{\lambda_p}{\lambda_p - 1}}$$

where $\tilde{\theta}_{p,t \oplus \tau} = (\prod_{s=1}^{\tau} \tilde{\pi}_{p,t+s})$ for $\tau \geq 1$ and equals 1 for $\tau = 0$. Here, $Y_{i,t+\tau|t}$ is the output by producer i at time $t + \tau$ if the last time P_i is reoptimized is period t , and $s_{t+\tau}$ is the real marginal cost, given by

$$s_{t+\tau} \equiv MC_{t+\tau} = \frac{1}{z_{t+\tau}^{1-\alpha} P_{t+\tau}} \left(\frac{W_{t+\tau}}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_{t+\tau}^k}{\alpha} \right)^\alpha. \quad (18)$$

The first order condition of this problem w.r.t. $P_{i,t}$ is

$$\sum_{\tau=0}^{\infty} \xi_p^\tau M_{t,t+\tau}^\$ \left[\tilde{\theta}_{p,t \oplus \tau}^{1+\sigma_p} (1 + \sigma_p) P_{i,t}^{\sigma_p} P_{t+\tau}^{-\sigma_p} Y_{t+\tau} - \sigma_t s_{t+\tau} \tilde{\theta}_{p,t \oplus \tau}^{\sigma_p} P_{i,t}^{\sigma_p - 1} P_{t+\tau}^{1-\sigma_p} Y_{t+\tau} \right] = 0$$

where $\sigma_p = \lambda_p / (1 - \lambda_p)$. Define the following auxiliary variables

$$H_t = \sum_{\tau=0}^{\infty} \zeta_p^\tau M_{t,t+\tau}^\$ \tilde{\theta}_{p,t \oplus \tau}^{1+\sigma_p} \left(\frac{Y_{t+\tau}}{Y_t} \right) \left(\frac{P_{t+\tau}}{P_t} \right)^{-\sigma_p},$$

$$J_t = \sum_{\tau=0}^{\infty} \zeta_p^\tau M_{t,t+\tau}^\$ \tilde{\theta}_{p,t \oplus \tau}^{\sigma_p} \left(\frac{S_{t+\tau}}{S_t} \right) \left(\frac{Y_{t+\tau}}{Y_t} \right) \left(\frac{P_{t+\tau}}{P_t} \right)^{1-\sigma_p}.$$

We can show that H_t and J_t can be rewritten in recursive form:

$$H_t = 1 + \zeta_p M_{t,t+1}^\$ \tilde{\pi}_{p,t+1}^{1+\sigma_p} \pi_{t+1}^{-\sigma_p} \left(\frac{Y_{t+1}}{Y_t} \right) H_{t+1}, \quad (19)$$

$$J_t = 1 + \zeta_p M_{t,t+1}^\$ \tilde{\pi}_{p,t+1}^{\sigma_p} \pi_{t+1}^{1-\sigma_p} \left(\frac{S_{t+1}}{S_t} \right) \left(\frac{Y_{t+1}}{Y_t} \right) J_{t+1}. \quad (20)$$

Therefore, the optimal price is then written as

$$\frac{P_{i,t}^*}{P_t} = \frac{P_t^*}{P_t} = \frac{\sigma_p}{1 + \sigma_p} \frac{J_t}{H_t} S_t.$$

where the first equality holds because ex-post every firm is identical and firms that are able to change price would pick the same optimal price P_t^* . The rest ζ_p fraction of the firms that cannot optimize over price will adopt a price according to equation (3.3). Therefore

$$P_t^{\frac{1}{1-\lambda_p}} = (1 - \zeta_p) (P_t^*)^{\frac{1}{1-\lambda_p}} + \zeta_p (\tilde{\pi}_{p,t} P_{t-1})^{\frac{1}{1-\lambda_p}},$$

which leads to the law of motion for inflation:

$$1 = (1 - \zeta_p) \left[\frac{\sigma_p}{1 + \sigma_p} \frac{J_t}{H_t} S_t \right]^{\frac{1}{1-\lambda_p}} + \zeta_p \left[\frac{\tilde{\pi}_{p,t}}{\pi_t} \right]^{\frac{1}{1-\lambda_p}}. \quad (21)$$

3.4 Labor Unions

Labor contractors hire workers of different labor types through labor unions and produce homogenous labor service L_t , according to the following production function:

$$L_t = \left[\int_0^1 L_{jt}^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \quad \lambda_w > 1,$$

where λ_w measures the elasticity of substitution among different labor types. The intermediate goods producers employ the homogenous labor service for production. Labor contractors are perfectly competitive, and their profit maximization leads to the demand function for labor type j :

$$L_{jt} = L_t \left(\frac{W_{jt}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}}.$$

It is easy to show that wages satisfy the following relation:

$$W_t = \left(\int_0^1 W_{jt}^{\frac{1}{1-\lambda_w}} dj \right)^{1-\lambda_w},$$

where W_{jt} is the wage of labor type j and W_t is the wage of the homogenous labor service. The aggregate labor income at $t + \tau$ as

$$LI_{t+\tau} = \int_0^1 \frac{W_{j,t+\tau}}{P_{t+\tau}} L_{j,t+\tau} dj = \frac{L_{t+\tau} W_{t+\tau}}{P_{t+\tau}} \int_0^1 \left(\frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} dj.$$

Assume that labor unions face the same Calvo (1983) type of wage rigidities. Each period, with probability ξ_w , labor union j cannot reoptimize the wage rate of labor type j and has to set the wage rate according to the following rule:

$$W_{jt} = \tilde{\pi}_{w,t} e^{\tilde{\mu}_{w,t}} W_{jt-1},$$

where

$$\tilde{\pi}_{w,t} = (\pi_t^*)^{\ell_w} (\pi_{t-1})^{1-\ell_w} \quad (22)$$

is the inflation indexation and $\tilde{\mu}_{w,t} = \ell_w \mu_{z^+,t} + (1 - \ell_w) \mu_{z^+}$ is the growth indexation. With probability $1 - \xi_w$, labor union j chooses W_{jt}^* to maximize households' utility. Conditional on being able to choose optimal wage at t , there are $\tau + 1$ possible values for the wage of type j labor at $t + \tau$:

$$W_{j,t+\tau} = \begin{cases} \tilde{\theta}_{w,t+\tau-s \oplus s} W_{t+\tau-s}^* & \text{with prob} = (1 - \xi_w) \xi_w^s \quad \text{for } s = 0, 1, \dots, \tau \\ \tilde{\theta}_{w,t \oplus \tau} W_{j,t}^* & \text{with prob} = \xi_w^\tau \end{cases}$$

where $\tilde{\theta}_{w,t\oplus\tau} = \prod_{s=1}^{\tau} (\tilde{\pi}_{w,t+s} e^{\tilde{\mu}_{w,t+s}})$ for $\tau \geq 1$ and equals 1 for $\tau = 0$. Assume that the union chooses wages to maximize household's utility. The maximization leads to

$$\begin{aligned} & \left(\frac{1 + \sigma_w}{\sigma_w} \right) C_{h,t}^{-\varphi} W_t^{\sigma_w \phi} \left(W_{j,t}^* \right)^{1 - \sigma_w \phi} P_t^{-1} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \zeta_w^{\tau} \frac{L_{t+\tau}}{L_t} \left(\frac{\tilde{\theta}_{w,t\oplus\tau} W_t}{W_{t+\tau}} \right)^{\sigma_w} \right] = \\ & A_{L,t} L_t^{\phi} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \zeta_w^{\tau} \frac{P_{t+\tau}}{P_t} \left(\frac{C_{h,t+\tau}}{C_{h,t}} \right)^{\varphi} \left(\frac{A_{L,t+\tau}}{A_{L,t}} \right) \left(\frac{L_{t+\tau}}{L_t} \right)^{1+\phi} \left(\frac{\tilde{\theta}_{w,t\oplus\tau} W_t}{W_{t+\tau}} \right)^{\sigma_w(1+\phi)} \right]. \end{aligned}$$

Define

$$J_{w,t} = 1 + \zeta_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \frac{L_{t+1}}{L_t} \left(\frac{W_{t+1}}{W_t} \right)^{-\sigma_w} \left(\tilde{\pi}_{w,t+1} e^{\tilde{\mu}_{w,t+1}} \right)^{\sigma_w} J_{w,t+1} \right] \quad (23)$$

$$\begin{aligned} H_{w,t} &= 1 + \zeta_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \frac{P_{t+1}}{P_t} \frac{A_{L,t+1}}{A_{L,t}} \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^{\psi} \left(\frac{L_{t+1}}{L_t} \right)^{1+\phi} \left(\frac{W_{t+1}}{W_t} \right)^{-\sigma_w(1+\phi)} \right. \\ & \quad \left. \times \left(\tilde{\pi}_{w,t+1} e^{\tilde{\mu}_{w,t+1}} \right)^{\sigma_w(1+\phi)} H_{w,t+1} \right]. \end{aligned} \quad (24)$$

The FOC can be written as

$$P_t^{-1} C_{h,t}^{-\varphi} W_{j,t}^{\sigma_w} L_t W_t^{-\sigma_w} G_{w,t} = \frac{\sigma_w}{1 + \sigma_w} A_{L,t} L_t^{1+\phi} W_t^{-\sigma_w(1+\phi)} \left(W_{j,t}^* \right)^{\sigma_w(1+\phi)-1}.$$

Since all labor types face the same demand curve, we have $W_{j,t}^* = W_t^*$ for all j . The optimal real wage and the optimal wage markup $\mu_{w,t}$ are then given by

$$\left(W_t^* \right)^{1 - \phi \sigma_w} = \mu_w P_t C_{h,t}^{\varphi} A_{L,t} L_t^{\phi} W_t^{-\sigma_w \phi} \left(\frac{G_{w,t}}{H_{w,t}} \right), \quad \text{and} \quad \mu_w = \frac{\sigma_w}{1 + \sigma_w}. \quad (25)$$

Hence, the law of motion of aggregate wage level is:

$$W_t^{1/(1-\lambda_w)} = (1 - \zeta_w) (W_t^*)^{1/(1-\lambda_w)} + \zeta_w \left(\tilde{\pi}_{w,t} W_{t-1} \right)^{1/(1-\lambda_w)}. \quad (26)$$

3.5 Monetary Policy

The monetary policy follows the Taylor rule:

$$\log(R_t) = \phi_R \log(R_{t-1}) + (1 - \phi_R) \left[\phi_{\pi} \log(\pi_t / \pi^*) + \phi_y \log(Y_t / Y_t^N) \right] + \sigma_R e_{R,t}. \quad (27)$$

The policy rule has an interest-rate smoothing component captured by the sensitivity ϕ_R to the lagged term, R_{t-1} , and responds to the difference between aggregate inflation $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and inflation target, the output gap, and a policy shock $e_{R,t} \sim \text{IID}\mathcal{N}(0,1)$. The coefficients ϕ_π and ϕ_y capture the response of the monetary authority to the deviations of inflation and the output gap from their targets, respectively.

3.6 Equilibrium

— All intermediate good producers take the same actions and all markets clear:

$$P_{j,t} = P_t, \quad Y_{j,t} = Y_t, \quad L_{j,t} = L_t.$$

— Interest rate under Taylor rule

$$\log(R_t) = \phi_R \log(R_{t-1}) + (1 - \phi_R) \left[\phi_\pi \log(\pi_t / \pi^*) + \phi_y \log(Y_t / Y_t^N) \right] + \sigma_R e_{R,t}.$$

satisfies the Euler equation:

$$\mathbb{E}_t[M_{t,t+1}R_{f,t}] = 1.$$

— Resource constraint:

$$Y_t = C_t + I_t / \Psi_t + G_t + a(u_t)\bar{K}_t + \mathcal{D}_t \tag{28}$$

where G_t is government spending, equal to a constant fraction g_y of output Y_t and \mathcal{D}_t is the bankruptcy cost in real terms, equal to $\mu G(\bar{\omega}_t)R_t^k Q_{k,t-1} \bar{K}_{t-1} / P_t$.

3.7 Decomposition of the Pricing Kernel

Define $\tilde{V}_t = \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right]$ and

$$\begin{aligned} \beta \tilde{V}_t^{\frac{1-\psi}{1-\gamma}} &= \beta \tilde{V}_t \tilde{V}_t^{-\frac{\psi-\gamma}{1-\gamma}} = \mathbb{E}_t \left[V_{t+1} V_{t+1}^{\frac{\psi-\gamma}{1-\gamma}} \tilde{V}_{t+1}^{-\frac{\psi-\gamma}{1-\gamma}} \right] \\ &= C_{h,t}^{-\psi} \mathbb{E}_t \left[M_{t,t+1} C_{h,t+1}^\psi V_{t+1} \right] \end{aligned}$$

which leads to

$$C_{h,t}^\psi V_t = (1 - \beta)C_{h,t}^\psi U_t + \mathbb{E}_t \left[M_{t,t+1} C_{h,t+1}^\psi V_{t+1} \right]. \quad (29)$$

Define dividend payout $D_{a,t} = (1 - \psi)C_{h,t}^\psi U_t$, we can rewrite equation (29) as

$$\begin{aligned} P_{a,t} &= \frac{1 - \psi}{1 - \beta} C_{h,t}^\psi V_t = D_{a,t} + \mathbb{E}_t [P_{a,t+1}] \\ R_{a,t+1} &= \frac{P_{a,t+1}}{P_{a,t} - C_{h,t}^\psi U_t}. \end{aligned}$$

It can be shown that the pricing kernel can be written as

$$M_{t,t+1} = \left[\beta \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^{-\psi} \right]^{\frac{1-\gamma}{1-\psi}} R_{a,t+1}^{\frac{\psi-\gamma}{1-\psi}}.$$

Dividend $D_{a,t}$ can be rewritten as

$$D_{a,t} = C_{h,t} - LI_t \quad \text{and} \quad LI_t \equiv \kappa \frac{W_t L_t}{P_t},$$

where LI_t can be interpreted as labor income and κ is a coefficient adjusting for the relative weight between utility from consumption and disutility from labor and for the difference in wages due to stickiness, given by

$$\kappa = \frac{1 - \psi}{1 + \phi} \left[\frac{(\hat{W}_t / W_t)^{\frac{\lambda_w(1+\phi)}{1-\lambda_w}}}{\mu_w (W_t / W_t^*)^{-\sigma_w \phi} J_{w,t} / H_{w,t}} \right].$$

We can thus interpret $R_{a,t}$ as the return on the wealth portfolio, which is a claim on all future habit-adjusted consumption subtract labor income. Without habit and agent's disutility from labor, the wealth portfolio is simply a claim on all future consumption.

To understand the dynamics of the stochastic pricing kernel, we rewrite the pricing kernel in natural log,

$$m_{t,t+1} = -\psi \frac{1 - \gamma}{1 - \psi} \ln \beta - \gamma \Delta c_{h,t+1} - \frac{\gamma - \psi}{1 - \psi} (r_{a,t+1} - \Delta c_{h,t+1}),$$

where the first two terms appear in the pricing kernel under power utility preference, and the third term only appears under the recursive preference. Define wealth-consumption ratio $wc_t =$

$\ln(P_{a,t}/C_{h,t})$, shock to the pricing kernel can be written as

$$m_{t,t+1} = \mathbb{E}_t[m_{t,t+1}] - \gamma \varepsilon_{t+1}^{ch} - \frac{\gamma - \psi}{1 - \psi} \varepsilon_{t+1}^{wc}$$

where ε_{t+1}^c is the shock to habit-adjusted consumption growth, $\varepsilon_{t+1}^{ch} \equiv \Delta c_{h,t+1} - \mathbb{E}_t[\Delta c_{h,t+1}]$, and ε_{t+1}^{wc} is the shock to wealth-consumption ratio, $\varepsilon_{t+1}^{wc} \equiv wc_{t+1} - \mathbb{E}_t[wc_{t+1}]$.

Using a long-linear approximation in Campbell (1999), we can write the shock to wealth-consumption ratio as

$$\begin{aligned} wc_{t+1} - \mathbb{E}_t[wc_{t+1}] &\approx (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \kappa_1^j (\Delta d_{a,t+1} - r_{a,t+1}) \\ &= (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \kappa_1^j (\Delta d_{a,t+1} - \psi \Delta c_{h,t+1}) \end{aligned} \quad (30)$$

where $\kappa_1 \equiv \frac{PD_a - 1}{PD_a} < 1$ and PD_a is the nonstochastic steady-state price-to-dividend ratio of the wealth portfolio. The equality in equation (30) holds due to the fact that

$$\mathbb{E}_t[\exp(m_{t,t+1} + r_{a,t+1})] = 1$$

and thus

$$\mathbb{E}_t \left[\exp \left(-\psi \frac{1 - \gamma}{1 - \psi} \ln \beta + \frac{1 - \gamma}{1 - \psi} [r_{a,t+1} - \psi \Delta c_{h,t+1}] \right) \right] = 1.$$

For the above equality to hold under any condition, we must have

$$(\mathbb{E}_{t+1} - \mathbb{E}_t)[r_{a,t+1}] = (\mathbb{E}_{t+1} - \mathbb{E}_t)[\psi \Delta c_{h,t+1}].$$

which leads to Equation (30). Finally, define $\tilde{D}_{a,t} = D_{a,t}/C_{h,t} = 1 - \kappa LI_t/C_{h,t}$, equation (30) can be written as

$$wc_{t+1} - \mathbb{E}_t[wc_{t+1}] \approx (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \kappa_1^j [\Delta \tilde{d}_{a,t+1} + (1 - \psi) \Delta c_{h,t+1}].$$

3.8 Detrending

The economy grow at the rate of z_t^+ . Investment and capital grow at the rate of $z_t^+ \Psi_t$. We detrend the model to get stationary solution:

$$\begin{aligned}
C_{h,t} &= c_{h,t} z_t^+, Y_t = y_t z_t^+, G_t = g_t z_t^+, T_t = \tau_t z_t^+, V_t = v_t (z_t^+)^{1-\varphi} \\
I_t &= i_t z_t^+ \Psi_t, \bar{K}_t = \bar{k}_t z_t^+ \Psi_t, K_t = k_t z_t^+ \Psi_t, \\
W_t &= w_t z_t^+ P_t, W_t^* = w_t^* z_t^+ P_t, \hat{W}_t = \hat{w}_t z_t^+ P_t, \tilde{W}_t = \tilde{w}_t z_t^+ P_t, A_{L,t} = a_{L,t} (z_t^+)^{1-\varphi} \\
N_t &= n_t P_t z_t^+, B_t = b_t P_t z_t^+, B_t^\infty = b_t^\infty z_t^+, P_t z_t^+, \pi_{w,t} = \frac{W_t}{W_{t-1}} = \frac{w_t z_t^+ \pi_t}{w_{t-1}} \\
q_t^\infty &= \frac{Q_t^\infty}{P_t}, q_t^k = \frac{Q_t^k}{P_t} \Psi_t, r_t^k = \tilde{r}_t^k / \Psi_t, W_t^e = w_t^e P_t z_t^+ \\
D_{u,t} &= d_{u,t} z_t^+, P_{u,t} = p_{u,t} z_t^+
\end{aligned}$$

where we use lowercase letters to represent the corresponding detrended variables.

4 Additional Empirical Tests

4.1 Principle Component Analysis

We perform the principal component analysis on the nine macroeconomic variables we use to estimate our model. Our purpose is to investigate how far it can go if we use shocks based on simple principal component analysis in explaining the cross-sectional return spreads.

We report the eigenvalues of the correlation matrix of the nine macroeconomic variables and percentage of the standardized variance explained by each principal component (Panel A in Table A3). The larger eigenvalues are extracted first. Based on the commonly used eigenvalues-greater-than-one rule, four principal components are retained, the eigenvalues of which are 2.952, 1.869, 1.279, and 1.015, respectively. The four principal components can explain 32.8%, 20.8%, 14.2%, and 11.3% of the standardized variance, respectively. Taken together, the four principal components account for 79.1% of the standardized variance, which provides an adequate summary of the data.

Panel B and C of Table A3 present the correlation between the principal components and the nine macroeconomic variables and the correlation between the principle components and our

model-implied shocks, respectively.

The risk premia of the principal components are estimated via the two-step Fama and MacBeth (1973) regression and presented in Table A4. The expected cross-sectional return spreads of size, book-to-market, investment, earnings, momentum, and long-term reversal predicted by the four principal components are reported in Table A5.

4.2 Portfolio Returns and Factor Loadings on Model-Implied Shocks: Additional Assets

In addition to size and book-to-market portfolios, we estimate factor loadings of other test asset returns, including the investment, earnings, momentum, long-term reversal, and industry decile portfolios, with respect to our four model-implied shocks. The results are reported in Table A6.

4.3 Correlation and Persistence of Model-Implied Shocks

We report the correlation between our four model-implied shocks with four business cycle variables in Panel A of Table A7, including GDP growth, consumption growth, real investment growth, and credit spread. The NT shock is the embedded technology shock. A positive NT shock increases the productivity of both capital and labor. Thus we observe a significant positive correlation between the NT shock and GDP growth, consumption growth, and real investment growth. The correlation is 0.273, 0.200, and 0.185, respectively, which are all significant at the 1% level.

A positive IST shock lowers the price of investment goods, and the price of capital because of lower replacement costs. This in turn makes growth options more valuable due to lower installment costs but assets-in-place less valuable due to lower price of capital. We find that the correlation between IST shock and GDP growth, consumption growth, and real investment growth are positive but insignificant. Our results are similar to the findings in Garlappi and Song (2016), who show that the IST shock measured by change in investment price has insignificant correlation with consumption and the growth rate of total factor productivity during post-1964

sample period. The correlation with GDP is positive but its significance depends on data frequency and sample period.

MP shock is the unexpected shock in the Taylor rule. A positive MP shock leads to unexpected increase in nominal interest rate, which generally leads to higher real interest rate under a stable Taylor rule. Thus, a positive MP shock leads to the contraction of the economy. We show that the MP shock has a significant and negative correlation with GDP growth, consumption growth, and real investment growth. The correlation coefficient is -0.488, -0.276, and -0.434, respectively, which are all significant at the 1% level. The MP shock has a significant and positive correlation with the credit spread.

Entrepreneurs face idiosyncratic uncertainty when they combine their own wealth with bank loans to acquire raw capital and transform it into effective capital. The magnitude of this uncertainty is referred to as the Risk shock. A positive Risk shock leads to a larger dispersion in the efficacy of transforming raw capital into effective capital, which in turn leads to more defaults. In equilibrium, banks require higher credit spread on loans and total credit extended to entrepreneurs drops. With fewer financial resources, entrepreneurs acquire less raw capital, as a result, investment, output, and consumption all fall. We show that the Risk shock has negative correlation with GDP growth, consumption growth, and real investment growth, although the correlation is not significant. The Risk shock is significantly correlated with the credit spread with a correlation coefficient of 0.163 (p-value = 0.013).

All the four shocks in our model are simulated as i.i.d. shocks. Theoretically, their persistence should be zero. Nonetheless, we estimate the persistence of each shock by running an AR(1) and report the estimates in Panel B of Table A7. We show that none of the AR(1) persistence coefficient is significant at the 5% level.

References

- Calvo, Guillermo. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12:383–398.
- Campbell, John Y. 1999. "Asset Prices, Consumption, and the Business Cycle." *Handbook of Macroeconomics* 1:1231–1303.
- Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review* 104 (1):27–65.
- Fama, Eugene F. and James D. MacBeth. 1973. "Risk, Return, and Equilibrium: Empirical Tests." *Journal of Political Economy* 81 (3):607–636.
- Garlappi, Lorenzo and Zhongzhi Song. 2016. "Can Investment Shocks Explain Value Premium and Momentum Profits." *Management Science* Forthcoming.
- Newey, Whitney K. and Kenneth. D. West. 1987. "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix." *Econometrica* 55:703–708.

Table A1: Risk premium of model-implied shocks: alternative test assets

This table reports the estimated risk premia (in percentage) of four model-implied shocks via the two-step Fama-Macbeth cross-sectional regressions. In Panel A, the test assets are the value-weighted ten size, ten book-to-market, and ten industry portfolios. In Panel B, the test assets are the value-weighted ten size, ten book-to-market, ten investment, ten operating profitability, and ten industry portfolios. We consider both univariate model for each shock and multivariate model with all four shocks included. The t-statistics adjusted for autocorrelation and heteroscedasticity following Newey and West (1987) are reported in parenthesis.

Panel A.					
	NT	IST	MP	Risk	AvgR2
1	1.89 (7.03)				0.56
2		-3.89 (-6.93)			0.51
3			-2.27 (-6.99)		0.51
4				-4.00 (-7.14)	0.52
5	0.71 (2.86)	-1.48 (-3.45)	-0.04 (-0.16)	-0.95 (-2.87)	0.66
Panel B.					
	NT	IST	MP	Risk	AvgR2
1	1.92 (6.78)				0.60
2		-3.84 (-6.74)			0.56
3			-2.46 (-6.76)		0.53
4				-3.73 (-6.87)	0.56
5	0.68 (2.85)	-1.66 (-3.91)	-0.17 (-0.65)	-0.55 (-2.56)	0.67

Table A2: Risk premium of model-implied shocks: subperiod analysis

This table reports the estimated risk premia (in percentage) of four model-implied shocks via the two-step Fama-Macbeth cross-sectional regressions. The test assets are the value-weighted ten size, ten book-to-market, ten momentum, and ten industry portfolios. Panel A reports the risk premia estimated during 1957q2-1989q4. Panel B reports the risk premia estimated during 1990q1-2015q4. We consider both univariate model for each shock and multivariate model with all four shocks included. The t-statistics adjusted for autocorrelation and heteroscedasticity following Newey and West (1987) are reported in parenthesis.

Panel A. 1957q2-1989q4					
	NT	IST	MP	Risk	AvgR2
1	1.57 (5.51)				0.61
2		-2.85 (-5.71)			0.59
3			-2.08 (-4.32)		0.14
4				-4.47 (-5.47)	0.49
5	0.77 (3.19)	-2.44 (-7.95)	0.06 (0.24)	-1.15 (-3.00)	0.71
Panel B. 1990q1-2015q4					
	NT	IST	MP	Risk	AvgR2
1	1.36 (3.53)				0.38
2		-1.85 (-3.20)			0.10
3			-0.92 (-3.77)		0.54
4				-2.80 (-3.98)	0.44
5	-0.82 (-3.86)	0.15 (0.84)	-0.81 (-3.27)	-0.91 (-2.43)	0.62

Table A3: **Principal component analysis of nine macroeconomic variables**

This table reports principal component analysis of nine macroeconomic variables, including per capita GDP growth (dy), per capita consumption growth (dc), per capita investment growth (di), weekly hours per capita (h), (the negative of) the change in the relative price of investment goods (μ^ψ), per capita real growth of credit (db), credit spread (cs), 3-month T-bill rate (r), and inflation rate (π). Panel A reports the eigenvalues of the correlation matrix and percentage of the standardized variance explained by the principle components. Panel B reports the correlation matrix between the four principal components retained by the eigenvalues-greater-than-one rule and the nine macroeconomic variables. Panel C reports the correlation between the principle components and the model-implied shocks. P-values of correlation coefficients are reported in parenthesis.

Panel A.

Factor	Eigenvalue	VarExp (%)	CumVarExp (%)
1	2.952	32.8%	32.8%
2	1.869	20.8%	53.6%
3	1.279	14.2%	67.8%
4	1.015	11.3%	79.1%
5	0.622	6.9%	86.0%
6	0.562	6.2%	92.2%
7	0.464	5.2%	97.4%
8	0.170	1.9%	99.3%
9	0.067	0.7%	100.0%

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Panel B.

	dy	dc	di	h	μ^ψ	db	cs	r	π
PC1	0.834 (0.000)	0.742 (0.000)	0.690 (0.000)	0.428 (0.000)	0.397 (0.000)	0.649 (0.000)	-0.625 (0.000)	-0.013 (0.847)	-0.277 (0.000)
PC2	-0.218 (0.001)	0.021 (0.748)	-0.239 (0.000)	0.405 (0.000)	0.195 (0.003)	0.072 (0.271)	-0.412 (0.000)	0.893 (0.000)	0.767 (0.000)
PC3	0.430 (0.000)	-0.145 (0.026)	0.595 (0.000)	-0.574 (0.000)	-0.237 (0.000)	-0.197 (0.002)	0.088 (0.177)	0.216 (0.001)	0.490 (0.000)
PC4	0.010 (0.876)	0.235 (0.000)	0.023 (0.728)	-0.099 (0.132)	0.796 (0.000)	-0.516 (0.000)	0.178 (0.006)	-0.089 (0.173)	0.101 (0.123)

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Panel C.

	NT	IST	MP	Risk
PC1	0.174 (0.007)	-0.096 (0.141)	-0.362 (0.000)	-0.092 (0.161)
PC2	-0.452 (0.000)	0.391 (0.000)	-0.120 (0.067)	-0.181 (0.005)
PC3	-0.014 (0.833)	0.088 (0.181)	-0.393 (0.000)	-0.081 (0.218)
PC4	-0.019 (0.769)	0.272 (0.000)	-0.159 (0.015)	-0.012 (0.850)

Table A4: Risk premia of principal components

This table reports the estimated risk premia (in percentage) of the four principal components via the two-step Fama-Macbeth cross-sectional regressions. The principal components are extracted from nine macroeconomic variables. In Panel A, the test assets are the value-weighted ten size, ten book-to-market, and ten industry portfolios. In Panel B, the test assets are the value-weighted ten size, ten book-to-market, ten investment, ten operating profitability, and ten industry portfolios. We consider both univariate model for each principal component and multivariate model with all four principal components included. The t-statistics adjusted for autocorrelation and heteroscedasticity following Newey and West (1987) are reported in parenthesis.

Panel A.					
	PC1	PC2	PC3	PC4	AvgR2
1	4.01 (7.44)				0.27
2		-2.95 (-6.31)			0.50
3			5.20 (5.89)		0.26
4				3.56 (6.52)	0.43
5	2.16 (7.55)	-2.21 (-5.15)	-1.42 (-2.87)	0.68 (2.39)	0.62
Panel B.					
	PC1	PC2	PC3	PC4	AvgR2
1	4.22 (6.96)				0.34
2		-3.17 (-6.30)			0.53
3			4.85 (5.86)		0.22
4				3.86 (6.53)	0.44
5	1.98 (6.97)	-2.08 (-4.85)	-1.15 (-2.73)	0.91 (3.49)	0.63

Table A5: Expected cross-sectional return spreads predicted by principal components

This table reports the estimated expected cross-sectional return spreads (in percentage) predicted by principle components for six cross sections, including value-weighted size, book-to-market, investment, earnings, momentum, and long-term reversal decile portfolios. Panel A presents the average quarterly cross-sectional return spreads (R), t-statistics of R ($t(R)$), expected cross-sectional return spreads (ER), the ratio of ER/R, the difference between R and ER (effectively the alpha of the model), and t-statistics of the difference ($t(\text{diff})$). The expected risk premium due to the exposure to a risk factor is calculated as the risk exposure times the risk premium of the corresponding risk factor ($\beta_i \lambda_i$). The risk exposures are estimated from the full sample from 1957q2 and 2015q4. The risk premia of risk factors are estimated using the multivariate model as Panel A in Table 5. ER is calculated as the summation of the expected risk premia from all four model-implied shocks, $ER = \sum_i \beta_i \lambda_i$, $i = NT, IST, MP, \text{ and Risk}$. Panel B presents the expected risk premium for each principal component, respectively. The t-statistics adjusted for autocorrelation and heteroscedasticity following Newey and West (1987) are reported in parenthesis.

	R	$t(R)$	ER	ER/R	R-ER	$t(\text{diff})$		
Panel A.								
SIZE(s-b)	1.09	1.76	0.36	32.9%	0.73	1.18		
BM(h-l)	1.36	2.21	0.20	15.0%	1.16	1.88		
INV(l-h)	1.35	3.05	-1.01	-74.3%	2.36	5.32		
EP(h-l)	1.28	2.40	-0.98	-77.0%	2.26	4.25		
MOM(h-l)	3.94	4.97	0.54	13.7%	3.40	4.29		
LT(l-h)	1.13	1.75	-2.41	-214.3%	3.54	5.49		
Panel B.								
	$\beta_{NT} \lambda_{NT}$	$\beta_{IST} \lambda_{IST}$	$\beta_{MP} \lambda_{MP}$	$\beta_{Risk} \lambda_{Risk}$	$\beta_{NT} \lambda_{NT} / R$	$\beta_{IST} \lambda_{IST} / R$	$\beta_{MP} \lambda_{MP} / R$	$\beta_{Risk} \lambda_{Risk} / R$
SIZE(s-b)	-0.31	1.24	-1.02	0.45	-28.5%	114.4%	-94.3%	41.3%
BM(h-l)	1.59	-0.83	-1.05	0.49	116.5%	-60.9%	-76.9%	36.2%
INV(l-h)	-0.56	-0.93	0.05	0.44	-41.7%	-68.7%	3.4%	32.6%
EP(h-l)	-0.03	-0.65	-0.14	-0.17	-2.2%	-50.7%	-10.8%	-13.3%
MOM(h-l)	3.94	-3.54	0.81	-0.66	100.0%	-89.9%	20.5%	-16.8%
LT(l-h)	-2.29	-0.13	-0.39	0.39	-203.3%	-11.6%	-34.5%	35.0%

Table A6: **Portfolio returns and factor loadings on model-implied shocks: additional assets**

This table reports the quarterly portfolio returns (in percentage) and their factor loadings on the four model-implied shocks. Panel A-E present results for portfolios sorted on investment, earnings, momentum, long-term reversal, and industry, respectively. Due to data availability, betas for investment and earnings portfolios are estimated using data from 1963q3 to 2015q4. Betas for momentum, long-term reversal, and industry portfolios are estimated using the full sample from 1957q2 to 2015q4. The t-statistics adjusted for autocorrelation and heteroscedasticity following Newey and West (1987) are reported in parenthesis.

Panel A. Investment portfolios					
	R	$Beta_{NT}$	$Beta_{IST}$	$Beta_{MP}$	$Beta_{Risk}$
Low	3.65 (6.15)	1.39 (1.80)	-0.83 (-1.02)	-1.70 (-2.14)	-1.28 (-1.77)
2	3.55 (6.80)	1.41 (2.22)	-0.68 (-1.03)	-1.43 (-2.20)	-0.83 (-1.39)
3	3.11 (6.47)	0.49 (0.83)	-0.83 (-1.36)	-0.87 (-1.45)	-0.92 (-1.68)
4	3.01 (6.46)	1.12 (2.03)	-0.56 (-0.98)	-0.65 (-1.14)	-0.82 (-1.58)
5	2.93 (6.22)	0.97 (1.73)	-0.90 (-1.55)	-0.88 (-1.53)	-0.86 (-1.65)
6	2.85 (5.60)	1.12 (1.89)	-0.95 (-1.53)	-1.22 (-2.02)	-0.87 (-1.56)
7	3.02 (5.33)	1.44 (2.38)	-0.80 (-1.27)	-0.99 (-1.60)	-1.07 (-1.90)
8	2.80 (5.40)	1.73 (2.70)	-0.94 (-1.41)	-1.42 (-2.16)	-1.22 (-2.04)
9	2.98 (4.52)	1.61 (2.22)	-0.98 (-1.30)	-1.48 (-2.00)	-1.55 (-2.28)
High	2.29 (3.28)	2.12 (2.48)	-0.53 (-0.60)	-1.94 (-2.22)	-1.21 (-1.51)
Low-High	1.35 (3.05)	-0.73 (-1.57)	-0.29 (-0.60)	0.25 (0.52)	-0.07 (-0.17)

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Panel B. Earnings portfolios					
	R	$Beta_{NT}$	$Beta_{IST}$	$Beta_{MP}$	$Beta_{Risk}$
Low	2.67 (4.52)	2.37 (3.27)	-0.53 (-0.70)	-1.70 (-2.41)	-0.74 (-1.08)
2	2.52 (5.67)	1.66 (2.75)	-0.52 (-0.82)	-1.03 (-1.75)	-0.69 (-1.19)
3	2.79 (6.18)	1.45 (2.48)	-0.41 (-0.67)	-1.06 (-1.85)	-0.78 (-1.41)
4	2.82 (6.03)	1.05 (1.89)	-0.77 (-1.34)	-0.94 (-1.74)	-0.47 (-0.89)
5	2.91 (6.61)	0.77 (1.34)	-0.87 (-1.45)	-0.97 (-1.72)	-0.68 (-1.23)
6	3.18 (7.43)	0.85 (1.56)	-0.60 (-1.04)	-0.86 (-1.61)	-0.24 (-0.45)
7	3.47 (7.45)	0.80 (1.45)	-0.53 (-0.91)	-1.07 (-1.98)	-0.68 (-1.28)
8	3.61 (6.80)	0.87 (1.45)	-0.94 (-1.49)	-1.28 (-2.18)	-1.28 (-2.22)
9	3.78 (7.60)	1.02 (1.62)	-0.72 (-1.09)	-1.58 (-2.57)	-0.90 (-1.49)
High	3.95 (7.75)	1.54 (2.21)	-0.73 (-1.00)	-1.91 (-2.80)	-1.28 (-1.92)
High-Low	1.28 (2.40)	-0.83 (-1.54)	-0.20 (-0.36)	-0.20 (-0.38)	-0.53 (-1.03)

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Panel C. Momentum portfolios					
	R	$Beta_{NT}$	$Beta_{IST}$	$Beta_{MP}$	$Beta_{Risk}$
Low	0.80 (1.02)	3.24 (3.14)	-0.59 (-0.55)	-2.81 (-2.79)	-1.55 (-1.58)
2	2.07 (3.66)	2.78 (3.45)	-0.23 (-0.27)	-2.02 (-2.57)	-1.13 (-1.48)
3	2.55 (4.71)	2.19 (3.24)	-0.27 (-0.38)	-1.19 (-1.81)	-0.96 (-1.50)
4	2.66 (5.80)	1.87 (3.06)	-0.50 (-0.79)	-1.51 (-2.53)	-0.77 (-1.32)
5	2.63 (6.52)	1.58 (2.71)	-0.53 (-0.87)	-0.88 (-1.55)	-0.51 (-0.93)
6	2.79 (6.07)	1.42 (2.42)	-0.31 (-0.50)	-1.05 (-1.83)	-0.70 (-1.25)
7	2.84 (6.50)	0.77 (1.42)	-0.71 (-1.24)	-0.80 (-1.51)	-0.51 (-0.98)
8	3.30 (6.96)	0.72 (1.32)	-0.85 (-1.48)	-1.14 (-2.12)	-0.81 (-1.55)
9	3.53 (6.84)	1.13 (1.90)	-0.57 (-0.92)	-1.10 (-1.89)	-0.95 (-1.67)
High	4.74 (7.55)	1.08 (1.38)	-1.04 (-1.27)	-1.90 (-2.49)	-1.34 (-1.80)
High-Low	3.94 (4.97)	-2.16 (-2.62)	-0.44 (-0.51)	0.91 (1.12)	0.21 (0.27)

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Panel D. Long-term reversal portfolios					
	R	$Beta_{NT}$	$Beta_{IST}$	$Beta_{MP}$	$Beta_{Risk}$
Low	3.95 (6.45)	2.27 (2.50)	-0.48 (-0.50)	-2.17 (-2.45)	-1.31 (-1.51)
2	3.42 (6.41)	1.33 (1.94)	-0.63 (-0.88)	-1.17 (-1.76)	-1.24 (-1.91)
3	3.39 (7.69)	1.18 (1.94)	-0.70 (-1.11)	-0.85 (-1.43)	-1.11 (-1.92)
4	3.19 (6.63)	1.21 (2.06)	-0.60 (-0.98)	-0.91 (-1.60)	-0.71 (-1.27)
5	3.19 (7.72)	1.26 (2.22)	-0.58 (-0.97)	-1.11 (-2.00)	-0.86 (-1.59)
6	3.15 (7.49)	1.09 (2.00)	-0.55 (-0.98)	-0.70 (-1.32)	-0.45 (-0.87)
7	2.98 (6.91)	0.96 (1.77)	-1.00 (-1.76)	-1.15 (-2.17)	-0.63 (-1.22)
8	2.91 (6.96)	1.10 (1.97)	-0.49 (-0.84)	-1.32 (-2.42)	-0.50 (-0.93)
9	2.62 (5.02)	1.57 (2.58)	-0.59 (-0.93)	-1.59 (-2.67)	-0.71 (-1.22)
High	2.83 (4.31)	2.19 (2.82)	-0.36 (-0.44)	-1.76 (-2.33)	-0.66 (-0.89)
Low-High	1.13 (1.75)	0.08 (0.11)	-0.12 (-0.16)	-0.40 (-0.58)	-0.65 (-0.95)

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Panel E. Industry portfolios					
	R	$Beta_{NT}$	$Beta_{IST}$	$Beta_{MP}$	$Beta_{Risk}$
NoDur	3.95 (6.45)	2.27 (2.50)	-0.48 (-0.50)	-2.17 (-2.45)	-1.31 (-1.51)
Durbl	3.42 (6.41)	1.33 (1.94)	-0.63 (-0.88)	-1.17 (-1.76)	-1.24 (-1.91)
Manuf	3.39 (7.69)	1.18 (1.94)	-0.70 (-1.11)	-0.85 (-1.43)	-1.11 (-1.92)
Enrgy	3.19 (6.63)	1.21 (2.06)	-0.60 (-0.98)	-0.91 (-1.60)	-0.71 (-1.27)
HiTec	3.19 (7.72)	1.26 (2.22)	-0.58 (-0.97)	-1.11 (-2.00)	-0.86 (-1.59)
Telcm	3.15 (7.49)	1.09 (2.00)	-0.55 (-0.98)	-0.70 (-1.32)	-0.45 (-0.87)
Shops	2.98 (6.91)	0.96 (1.77)	-1.00 (-1.76)	-1.15 (-2.17)	-0.63 (-1.22)
Hlth	2.91 (6.96)	1.10 (1.97)	-0.49 (-0.84)	-1.32 (-2.42)	-0.50 (-0.93)
Utils	2.62 (5.02)	1.57 (2.58)	-0.59 (-0.93)	-1.59 (-2.67)	-0.71 (-1.22)
Other	2.83 (4.31)	2.19 (2.82)	-0.36 (-0.44)	-1.76 (-2.33)	-0.66 (-0.89)

Table A7: Correlation and persistence of model-implied shocks

This table reports the correlation between the four model-implied shocks and four business cycle variables (Panel A) and persistence (ρ) of four model-implied shocks estimated from a AR(1) process (Panel B). The four business cycle variables are per capita GDP growth (dy), per capita consumption growth (dc), per capita investment growth (di), and credit spread (cs). P-values for correlation coefficients and t-statistics for regression estimates are reported in parenthesis.

Panel A.				
	gGDP	gC	gI	Default
NT	0.273 (0.000)	0.200 (0.002)	0.185 (0.004)	-0.103 (0.114)
IST	0.129 (0.148)	0.056 (0.395)	0.162 (0.113)	-0.093 (0.157)
MP	-0.488 (0.000)	-0.276 (0.000)	-0.434 (0.000)	0.150 (0.021)
Risk	-0.097 (0.140)	-0.060 (0.361)	-0.065 (0.324)	0.163 (0.013)
Panel B.				
	Intercept	ρ	$AdjR^2$	
NT	0.003 (0.03)	0.139 (1.35)	0.015	
IST	0.003 (0.04)	-0.173 (-1.70)	0.026	
MP	0.003 (0.03)	0.087 (1.12)	0.003	
Risk	0.004 (0.04)	0.090 (1.52)	0.004	