Electronic Companion to
The Implications of Visibility on the Use of Strategic Inventory in a Supply Chain

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Proofs of Lemmas and Propositions

EC.1 Proof of Lemma 1

In this section, all of the decision variables refer to the dynamic contract with no inventory observability. We eliminate the use of the superscript N to simplify reading. In the dynamic contract with no inventory observability, the manufacturer chooses $w_2^*(q_1)$ without being able to observe the retailer's decision $(I^*(q_1), s_1^*(q_1))$. The manufacturer offers $w_2^*(q_1)$ solely based on the retailer's first-period purchase and in anticipation of the retailer's inventory decision. The retailer knows this, and decides to sell $s_1^*(q_1)$ and hold $I^*(q_1)$ in period 1, in anticipation of the manufacturer's second-period offer $w_2^*(q_1)$. Thus, $(I^*(q_1), s_1^*(q_1))$ and $w_2^*(q_1)$ are the best responses of the retailer and the manufacturer to each other's decisions in a simultaneous game.

The manufacturer's best response maximizes his second-period profit given by (4):

$$\Pi_2^N(w_2, I) = w_2 \cdot q_2^{N^*}(w_2, I) = \begin{cases} \frac{1}{2}w_2(a - w_2 - 2I) & \text{if } w_2 < a - 2I, \\ 0 & \text{if } w_2 \geq a - 2I. \end{cases}$$

By applying first-order conditions on (4), we obtain the manufacturer's best response:

$$w_2^{BR}(I) = \left[\frac{1}{2}(a - 2I)\right]^+ . \tag{EC.1}$$

Similarly, the retailer's best response maximizes her profit-to-go in (5), rewritten below:

$$\pi_1^N(q_1, s_1, I, w_2) = (a - s_1)(s_1) + \pi_2^{N^*}(w_2, I) - h \cdot I, \text{ s.t. } s_1 + I \leq q_1,$$

where

$$\pi_2^{N^*}(w_2, I) = \begin{cases} \frac{1}{4}(a - w_2)^2 + w_2 I & \text{if } w_2 < a - 2I, \\ (a - I) \cdot I & \text{if } w_2 \geq a - 2I. \end{cases}$$

When the constraint $s_1 + I \leq q_1$ is binding, we can replace s_1 by $I - q_1$. Applying first-order conditions on (5), we obtain the retailer's best response:

$$I^{BR}(w_2) = \left[\frac{1}{2}(2q_1 - a - h + w_2)\right]^+ \text{ when } s_1 + I = q_1. \tag{EC.2}$$

When the constraint $s_1 + I \leq q_1$ is non-binding, the retailer maximizes $\pi_1^N(q_1, s_1, I, w_2)$ by choosing s_1 and I independently. Observe that this happens when q_1 is big enough, so that the retailer can choose to salvage some quantity $q_1 - s_1 - I$ in the first period. Applying first-order conditions on (5) again, $s_1^{BR}(w_1) = \frac{a}{2}$ when $s_1 + I \geq q_1$. The retailer choice of I maximizes $\pi_2^{N^*}(w_2, I) - h \cdot I$, given the manufacturer's choice of w_2 maximizes his second-period profit in (4). Applying first-order conditions on $\pi_2^{N^*}(w_2, I) - h \cdot I$, it can be confirmed that the retailer is indifferent across any inventory position I as long as the manufacturer sets $w_2 = h$. However, from (EC.1), the manufacturer sets $w_2 = h$ only if $I = \frac{a}{2} - h$. Observe that the lowest value for q_1 at which $s_1 = \frac{a}{2}$ is $q_1 = a - h \equiv \overline{q_1}$. Below this threshold, $w_2^{BR}(I)$ and $I^{BR}(w_2)$ from (EC.1) and (EC.2) intersect at $I^{N^*}(q_1) = \frac{1}{6}(4q_1 - a - 2h)$ and $w_2^{N^*}(q_1) = \frac{1}{3}(2a + h - 2q_1)$. It is easy to confirm that $I^{N^*}(q_1) = 0$ for $q_1 \leq \frac{1}{4}(a + 2h) \equiv \underline{q_1}$. Thus, we obtain

$$\left(I^{N^*}(q_1), s_1^{N^*}(q_1), w_2^{N^*}(q_1)\right) = \begin{cases} \left(0, q_1, \frac{a}{2}\right) & \text{if } q_1 \in [0, \underline{q_1}), \\ \left(\frac{1}{6}(4q_1 - a - 2h), \frac{1}{6}(2q_1 + a - 2h), \frac{1}{3}(2a + h - 2q_1)\right) & \text{if } q_1 \in [\underline{q_1}, \overline{q_1}], \\ \left(\frac{a}{2} - h, \frac{a}{2}, h\right) & \text{if } q_1 \in (\overline{q_1}, \infty), \end{cases} \tag{EC.3}$$

where $\underline{q_1} = \frac{1}{4}(a + 2h)$, and $\overline{q_1} = a - h$, are two thresholds on the retailer's purchase quantity q_1. It can be seen that these are sub-game perfect Nash equilibrium, and that neither the manufacturer nor the retailer gains from unilaterally deviating.

EC.2 Proof of Lemma 2

First, we show that $\pi^N(w_1, q_1)$ lacks concavity at $q_1 = \underline{q_1} = \frac{1}{4}(a + 2h)$, and $q_1 = \overline{q_1} = a - h$. From Table A.2, and (7), we get $\pi^N(w_1, q_1)$ for three possible q_1: when $q_1 \in [0, \underline{q_1}]$, $q_1 \in (\underline{q_1}, \overline{q_1})$, and $q_1 \in [\overline{q_1}, \infty)$. The slope of $\pi^N(w_1, q_1)$ is given by the partial derivative $\frac{\partial \pi^N(w_1, q_1)}{\partial q_1}$ in each of these four intervals.

$$\frac{\partial \pi^N(w_1, q_1)}{\partial q_1} = \begin{cases} a - 2q_1 - w_1 & \text{if } q_1 \in [0, \underline{q_1}), \\ \frac{1}{9}(8a - 5h - 8q_1 - 9w_1) & \text{if } q_1 \in [\underline{q_1}, \overline{q_1}], \\ -w_1 & \text{if } q_1 \in (\overline{q_1}, \infty). \end{cases} \tag{EC.4}$$

Differentiating a second time with respect to q_1 yields

$$\frac{\partial^2 \pi^N(w_1, q_1)}{\partial q_1^2} = \begin{cases} -2 & \text{if } q_1 \in [0, \underline{q_1}), \\ -\frac{8}{9} & \text{if } q_1 \in [\underline{q_1}, \overline{q_1}], \\ 0 & \text{if } q_1 \in (\overline{q_1}, \infty). \end{cases} \tag{EC.5}$$

Therefore, $\pi^N(w_1, q_1)$ is piece-wise concave in q_1 in each of the three intervals $[0, \underline{q_1}]$, $(\underline{q_1}, \overline{q_1})$, and $[\overline{q_1}, \infty)$. However, the increase in the second derivative $\frac{\partial^2 \pi^N(w_1, q_1)}{\partial q_1^2}$ at $\underline{q_1}$ and $\overline{q_1}$ implies that $\pi^N(w_1, q_1)$ is not concave at $q_1 = \underline{q_1} = \frac{1}{4}(a + 2h)$, and $q_1 = \overline{q_1} = a - h$. A graphical representation of this phenomenon is made in Figure 1, with $a = 1$ and $h = 0.1$.

Maximizing (7) with respect to q_1, the retailer is better off purchasing $q_1 > \underline{q_1}$ and holding strategic inventory when $w_1 < W = \frac{3a}{5} - h$; and purchasing $q_1 < \underline{q_1}$ and not holding strategic inventory when $w_1 > W = \frac{3a}{5} - h$. She is indifferent between purchasing a higher quantity (holding inventory) and purchasing much lower quantity (not holding inventory) at $w_1 \equiv W = \frac{3a}{5} - h$ (two local maxima). Mathematically,

$$q_1^{N^*}(w_1) = \begin{cases} a - \frac{5h}{8} - \frac{9w_1}{8} \in (\underline{q_1}, \overline{q_1}) & \text{if } 0 < w_1 < W. \\ \frac{a-w_1}{2} \in [0, \underline{q_1}] & \text{if } W < w_1 \leq a. \end{cases} \tag{EC.6}$$

Note that the retailer's first-period purchase quantity $q_1^{N^*}(w_1)$ is discontinuous at $W = \frac{3a}{5} - h$.

The retailer strictly prefers to order the larger quantity for any $w_1 < W$, and for simplicity, we assume that the retailer orders the larger quantity at the limiting value of $w_1 = W$. Recall from (6),

$$I^{N^*}(q_1) = \begin{cases} 0 & \text{if } q_1 \in [0, \underline{q_1}), \\ \frac{1}{6}(4q_1 - a - 2h) & \text{if } q_1 \in [\underline{q_1}, \overline{q_1}], \\ \frac{a}{2} - h & \text{if } q_1 \in (\overline{q_1}, \infty). \end{cases}$$

Substituting (EC.6) in (6), we derive the retailer's equilibrium strategic inventory as a function of the manufacturer's first-period wholesale price:

$$I^{N^*}(w_1) = \begin{cases} \frac{1}{4}(2a - 3h - 3w_1) & \text{if } 0 < w_1 < W, \\ 0 & \text{if } W < w_1 \leq a. \end{cases} \tag{EC.7}$$

Together, (EC.6) and (EC.7) give us (8).

EC.3 Proof of Lemma 3

Substituting $q_1^{N^*}(w_1)$ from (8) in $\Pi_2^{N^*}(q_1)$ from Table A.2, we obtain the manufacturer's total profit over the two periods as the following function of w_1:

$$\begin{aligned} \Pi^N(w_1) &= w_1 \cdot q_1^{N^*}(w_1) + \Pi_2^{N^*}(w_1) \\ &= \begin{cases} \frac{1}{32}(9h^2 - 2hw_1 + (32a - 27w_1)w_1) & \text{if } 0 < w_1 < W, \\ \frac{1}{8}(a^2 + 4aw_1 - 4(w_1)^2) & \text{if } W < w_1 \leq a. \end{cases} \end{aligned} \tag{EC.8}$$

We maximize the manufacturer's total profit $\Pi^N(w_1)$ in (EC.8) with respect to w_1. It can be easily shown that $\Pi^N(w_1)$ is piece-wise concave in w_1 in the two intervals $(0, W)$ and $(W, a]$. However, $\Pi^N(w_1)$ is discontinuous at $w_1^N = W \equiv \frac{3a}{5} - h$. Further, for low inventory holding cost, offering w_1 marginally higher than W causes the manufacturer's profit to drop significantly. By comparing the upper ($w_1 < W$) and lower ($w_1 > W$) branches of (EC.8) at W, it can be seen that at the limit, as $w_1 \to W$,

$$\lim_{w_1 \to W^-} \Pi^N(w_1) \quad > \quad \lim_{w_1 \to W^+} \Pi^N(w_1) \quad \forall h < \frac{41a}{100} \tag{EC.9}$$

The discontinuity at $w_1 = W \equiv \frac{3a}{5} - h$ implies that: (a) for low holding cost ($h < \frac{41a}{100}$), the manufacturer is better off offering w_1 just under W (inducing strategic inventory) than a marginally higher price (does not induce strategic inventory); and that (b) in order to induce the retailer to order large enough q_1 to hold strategic inventory ($q_1 > \underline{q_1}$), the manufacturer must offer $w_1^N < W$.

From the lower branch of (EC.8), it is easy to confirm that, among all $w_1 > W$, the best the manufacturer can do is to set $w_1 = \frac{a}{2}$. For $h < \underline{h} = \frac{a}{130}$, the manufacturer maximizes his profit by setting $w_1^{N^*} = \frac{1}{27}(16a - h)$, i.e., the value that satisfies the first-order condition for the upper branch of (EC.8). For$h \in [\underline{h}, \overline{h})$, he maximizes his profit by setting $w_1^{N^*} = W$, i.e., the largest possible value that induces the retailer to order $q_1 > \underline{q_1}$. Finally, for $h \geq \overline{h}$, he maximizes his profit by setting $w_1^{N^*} = \frac{a}{2}$.

EC.4 Proof of Lemma 4

(a) In the interval $[\underline{h}, \overline{h})$, $w_1^{N^*}$ slightly under $W \equiv \frac{3a}{5} - h$ decreases at a faster rate in h than either $w_1^{F^*} = \frac{1}{16}(9a - 4h)$, or $w^{C^*} = \frac{a}{2}$, which is constant in h. For all $h < \overline{h}$, it is easily shown that $w_1^{C^*} \leq w_1^{F^*}$. By comparing $w_1^{N^*}$ just under W, and $w^{C^*} = \frac{a}{2}$, we find that they intersect at $h = h_{w_1} = \frac{a}{10} < \widehat{h}$. Since $w_1^{N^*}$ is decreasing in h when $h \in (\underline{h}, \overline{h})$, we obtain that when $h \in (h_{w_1}, \overline{h})$, the manufacturer offers $w_1^{N^*} < w^{C^*} \leq w_1^{F^*}$.

(b) From Table A.3, it can be seen that when $h \in [\underline{h}, \overline{h})$, the retailer's first-period purchase quantity under no inventory observability, $q_1^{N^*} = \frac{1}{40}(13a + 20h)$, increases in the holding cost h. In contrast, under full observability, $q_1^{F^*} = \frac{1}{12}(4a - 7h)$ decreases in h in the interval $h < \widehat{h}$. These two quantity functions intersect at $h_{q_1} = \frac{39a}{700} \in (\underline{h}, h_{w_1})$. Further, by comparing them with the retailer's purchase quantity under the commitment contract $q^{C^*} = \frac{a}{4}$, it is easy to confirm that when $h \in (h_{q_1}, \overline{h})$, the retailer purchases $q_1^{N^*} > q_1^{F^*} \geq q^{C^*}$.

EC.5 Proof of Proposition 1

Recognizing that the upper (lower) branches of (8) correspond to the case where strategic inventory is (is not) held by the retailer, we can substitute back into (7) to obtain the following expression of her total profit as a function of w_1 only:

$$\pi^N(w_1) = \pi^N(w_1, q_1^{N^*}(w_1)) = \begin{cases} \frac{1}{16}(8a^2 + 5h^2 + 10hw_1 + 9w_1^2 - 8a(h + 2w_1)) & \text{if } 0 < w_1 < W, \\ \frac{1}{16}(5a^2 - 8aw_1 + 4w_1^2) & \text{if } W < w_1 \leq a. \end{cases} \tag{EC.10}$$

The manufacturer's total profit $\Pi^N(w_1)$ is given by (EC.8), rewritten below:

$$\begin{aligned} \Pi^N(w_1) &= w_1 \cdot q_1^{N^*}(w_1) + \Pi_2^{N^*}(w_1) \\ &= \begin{cases} \frac{1}{32}(9h^2 - 2hw_1 + (32a - 27w_1)w_1) & \text{if } 0 < w_1 < W, \\ \frac{1}{8}(a^2 + 4aw_1 - 4(w_1)^2) & \text{if } W < w_1 \leq a. \end{cases} \end{aligned}$$

From Lemma 3, we know that in equilibrium, the manufacturer's first-period wholesale price offer is

$$w_1^{N^*} = \begin{cases} \frac{1}{27}(16a - h) < W & \text{if } h \in [0, \underline{h}), \\ \frac{3a}{5} - h = W & \text{if } h \in [\underline{h}, \overline{h}), \\ \frac{a}{2} = w^{C^*} & \text{if } h \in [\overline{h}, \infty). \end{cases}$$

Substituting $w_1^{N^*}$ back in (EC.10) and (EC.8), we obtain the equilibrium profits of the retailer and the manufacturer, respectively, under no inventory observability:

$$\pi^{N^*} = \begin{cases} \frac{1}{162}(17a^2 - 19ah + 47h^2) & \text{if } h \in [0, \underline{h}), \\ \frac{1}{400}(41a^2 + 80ah + 100h^2) & \text{if } h \in [\underline{h}, \overline{h}), \\ \frac{a^2}{8} & \text{if } h \in [\overline{h}, \infty), \end{cases} \tag{EC.11}$$

and

$$\Pi^{N^*} = \begin{cases} \frac{1}{216}(64a^2 - 8ah + 61h^2) & \text{if } h \in [0, \underline{h}), \\ \frac{1}{800}(237a^2 - 20ah - 400h^2) & \text{if } h \in [\underline{h}, \overline{h}), \\ \frac{a^2}{4} & \text{if } h \in [\overline{h}, \infty). \end{cases} \tag{EC.12}$$

Adding (EC.11) and (EC.12), we obtain the total supply chain profit under no inventory observability:

$$\Pi^{N^*} + \pi^{N^*} = \begin{cases} \frac{1}{648}(260a^2 - 100ah + 371h^2) & \text{if } h \in [0, \underline{h}), \\ \frac{1}{800}(319a^2 + 140ah - 200h^2) & \text{if } h \in [\underline{h}, \overline{h}), \\ \frac{3a^2}{8} & \text{if } h \in [\overline{h}, \infty). \end{cases} \tag{EC.13}$$

These profits are shown in Table A.3, under N in the last two columns. We do not separately tabulate the case when $h \in [\overline{h}, \infty)$, as the dynamic contract then converges to the commitment contract, which is already tabulated under C. Note that Table A.3 also provides the benchmark profits under the vertically integrated channel (V), and the full inventory observability scenario (F).

By comparing the profits of the manufacturer, the retailer, and the supply chain under the dynamic contract with no inventory observability (N) with the corresponding profits under the commitment contract (C) and the dynamic contract with full inventory observability (F) as given by Table A.3, we obtain the results in Proposition 1.

EC.6 Details of the Analysis of the Cost of Unobservability: Sales and Operations Planning Errors

We will first describe the model of how sales and operations planning errors can affect strategic inventory under full inventory observability (FE), and then do so under no inventory observability (NE).

Full-Observability Error (FE)

When the manufacturer can observe the inventory, stage $2a$ follows stage $1c$, and he can adjust w_2 accordingly. From (2), we know that having carried I in inventory, the retailer's response to the manufacturer's second period offer w_2 is independent of the manufacturer's observability, and is given by $q_2^{FE^*}(w_2, I) = \left[\frac{a-w_2}{2} - I\right]^+$. With probability α, the retailer has carried $I = 0$, and with probability $1-\alpha$, the retailer has carried $I = I^{FE^*}$. The manufacturer chooses the appropriate $w_2(I)$ in each case to maximize his second period profit given by (EC.14). When $I = 0$, the second-period decisions converge to the commitment contract: the manufacturer sets $w_2 = \frac{a}{2}$, and the retailer responds by purchasing $q_2 = \frac{a}{4}$.

$$\mathbb{E}\left[\Pi_2^{FE}(I)\right] \quad = \quad \alpha \cdot \frac{a}{2} \cdot \frac{a}{4} + (1-\alpha) \cdot w_2(I) \cdot q_2^{FE^*}(w_2(I), I). \tag{EC.14}$$

After observing the retailer's inventory, the manufacturer offers $w_2^{FE^*}(I) = \left[\frac{1}{2}(a-2I)\right]^+$, and the retailer responds by purchasing $q_2^{FE^*}(I) = \left[\frac{1}{4}(a-2I)\right]^+$, when the retailer has held $I \geq 0$. Thus, the expected second-period profits of the manufacturer and the retailer are given by (EC.15) and (EC.16), respectively.

$$\mathbb{E}\left[\Pi_2^{FE^*}(I)\right] = \alpha\left(\frac{a^2}{8}\right) + (1-\alpha)\left(w_2^{FE^*}(I) \cdot q_2^{FE^*}(I)\right) \tag{EC.15}$$

$$\begin{aligned}\mathbb{E}\left[\pi_2^{FE^*}(I)\right] &= \alpha\left[\left(a - q_2^{FE^*}(0)\right) \cdot q_2^{FE^*}(0) - w_2^{FE^*}(0) \cdot q_2^{FE^*}(0)\right] \\ &\quad + (1-\alpha)\left[\left(a - (q_2^{FE^*}(I) + I\right) \cdot \left(q_2^{FE^*}(I) + I\right) - w_2^{FE^*}(I) \cdot q_2^{FE^*}(I)\right]\end{aligned} \tag{EC.16}$$

In stage $1c$, the retailer decides how much to hold in inventory when she does not make a mistake, knowing that with probability α, she is going to make a mistake, and the manufacturer will respond accordingly. The retailer's expected profit-to-go after purchasing q_1 in period 1 is given by (EC.17):

$$\begin{aligned}\mathbb{E}\left[\pi_1^{FE}(q_1, s_1, I, \alpha)\right] &= \alpha\left[(a-q_1)(q_1)\right] + (1-\alpha)\left[(a-s_1)\cdot s_1 - h\cdot I\right] \\ &\quad + \mathbb{E}\left[\pi_2^{FE^*}(I)\right], \text{ s.t. } s_1 + I \leq q_1. \end{aligned} \tag{EC.17}$$

The retailer maximizes (EC.17) with respect to s_1 and I. Similar to the thresholds $\underline{q_1}$ and $\overline{q_1}$ in Lemma 1, there exist two thresholds, $\underline{q_1}^{FE} = \frac{1}{8}(a+4h)$ and $\overline{q_1}^{FE} = a - \frac{2h}{3}$. In the full-observability error (FE) scenario, the retailer's equilibrium sales and inventory decision in stage $1c$ is given by

$$\left(I^{FE^*}(q_1), s_1^{FE^*}(q_1)\right) = \begin{cases} (0, q_1) & \text{if } q_1 \in [0, \underline{q_1}^{FE}], \\ \left(\frac{1}{14}(8q_1 - a - 4h), \frac{1}{14}(6q_1 + a + 4h)\right) & \text{if } q_1 \in (\underline{q_1}^{FE}, \overline{q_1}^{FE}), \\ \left(\frac{1}{6}(3a - 4h), \frac{a}{2}\right) & \text{if } q_1 \in [\overline{q_1}^{FE}, \infty). \end{cases} \tag{EC.18}$$

The retailer's equilibrium expected profit-to-go in the first period is

$$\mathbb{E}\left[\pi_1^{FE^*}(q_1)\right] = \mathbb{E}\left[\pi_1^{FE}(q_1, s_1^{FE^*}(q_1), I^{FE^*}(q_1))\right]. \tag{EC.19}$$

In stage $1b$, the retailer decides to purchase $q_1^{FE^*}(w_1, \alpha)$ that maximizes her expected total profit given by (EC.20):

$$\mathbb{E}\left[\pi^{FE^*}(w_1, q_1)\right] = \mathbb{E}\left[\pi_1^{FE}(q_1, \alpha)\right] - w_1 \cdot q_1. \tag{EC.20}$$

There exists a threshold $W^{FE} = \frac{3a}{4} - h$ that influences the retailer's response to the manufacturer's offer w_1, to purchase $q_1^{FE^*}(w_1, \alpha)$ and then hold corresponding inventory $I^{FE^*}(q_1^{FE^*}(w_1, \alpha), \alpha)$, as given by (EC.21).

$$\left(q_1^{FE^*}(w_1, \alpha), I^{FE^*}(w_1, \alpha)\right) = \begin{cases} \left(\frac{a(6+\alpha) - 4h(1-\alpha) - 7w_1}{6+8\alpha}, \frac{3a - 4(h+w_1)}{6+8\alpha}\right) & \text{if } w_1 \in [0, W^{FE}), \\ \left(\frac{a-w_1}{2}, 0\right) & \text{if } w_1 \in [W^{FE}, a], \\ (0, 0) & \text{if } w_1 \in (a, \infty). \end{cases} \tag{EC.21}$$

The manufacturer sets his first-period wholesale price to maximize his expected total profit given by (EC.22):

$$\mathbb{E}\left[\Pi^{FE^*}(w_1,\alpha)\right] = w_1 \cdot q_1^{FE^*}(w_1,\alpha) + \mathbb{E}\left[\Pi_2^{FE^*}(w_1,\alpha)\right], \tag{EC.22}$$

where

$$\mathbb{E}\left[\Pi_2^{FE^*}(w_1,\alpha)\right] = \mathbb{E}\left[\Pi_2^{FE^*}(I^{FE^*}(q_1^{FE^*}(w_1,\alpha),\alpha),\alpha)\right]. \tag{EC.23}$$

In equilibrium, as long as $h < \widehat{h}(\alpha) = \frac{a}{4}$, the manufacturer offers $w_1^{FE^*}(\alpha)$ low enough such that the retailer purchases $q_1^{FE^*}(w_1,\alpha) > \underline{q_1}^{FE}$ and holds positive expected inventory. In the FE scenario, the manufacturer offers the first-period wholesale price $w_1^{FE^*}(\alpha)$ and the retailer responds by purchasing the first-period quantity $q_1^{FE^*}(\alpha)$ as given by (EC.24):

$$\left(w_1^{FE^*}(\alpha), q_1^{FE^*}(\alpha)\right) = \begin{cases} \left(\frac{a(18+\alpha(35-4\alpha))-4h(1-\alpha)(1+4\alpha)}{34+64\alpha}, \frac{a(26+23\alpha)-36h(1-\alpha)}{68+128\alpha}\right) & \text{if } h \in [0,\widehat{h}(\alpha)), \\ \left(\frac{a}{2}, \frac{a}{4}\right) & \text{if } h \in [\widehat{h}(\alpha),\infty). \end{cases} \tag{EC.24}$$

The inventory held by the retailer in the event she does not commit a mistake, is given by (EC.25):

$$I^{FE^*}(\alpha) = \begin{cases} \frac{(a-4h)(5+2\alpha)}{34+64\alpha} & \text{if } h \in [0,\widehat{h}(\alpha)), \\ 0 & \text{if } h \in [\widehat{h}(\alpha),\infty). \end{cases} \tag{EC.25}$$

Observe that the retailer's first-period purchase quantity $q_1^{FE^*}(\alpha)$, her equilibrium inventory $I^{FE^*}(\alpha)$, as well as her expected inventory $\mathbb{E}\left[I^{FE^*}(\alpha)\right]$ are decreasing in α when $h \in [0,\widehat{h}(\alpha))$. Substituting (EC.24) in (EC.20) and (EC.22) gives us $\mathbb{E}\left[\pi^{FE^*}(\alpha)\right]$ and $\mathbb{E}\left[\Pi^{FE^*}(\alpha)\right]$, the expected total profits for the retailer and the manufacturer, respectively. The retailer's expected total profit is

$$\mathbb{E}\left[\pi^{FE^*}(\alpha)\right] = \begin{cases} \frac{X^{FE}}{16(32\alpha+17)^2} & \text{if } h \in [0,\widehat{h}(\alpha)), \\ \frac{a^2}{8} & \text{if } h \in [\widehat{h}(\alpha),\infty), \end{cases} \tag{EC.26}$$

where

$$
\begin{aligned}
X^{FE} &= (\alpha(\alpha(156\alpha+1925)+2101)+620)a^2 \\
&\quad -8(\alpha(\alpha(28\alpha-95)+8)+59)ah \\
&\quad -16(\alpha-1)(\alpha(100\alpha+167)+76)h^2.
\end{aligned}
$$

The manufacturer's expected total profit is

$$
\mathbb{E}\left[\Pi^{FE^*}(\alpha)\right] = \begin{cases} \frac{Y^{FE}}{8(32\alpha+17)} & \text{if } h \in [0, \widehat{h}(\alpha)), \\ \frac{a^2}{4} & \text{if } h \in [\widehat{h}(\alpha), \infty), \end{cases} \tag{EC.27}
$$

where

$$
Y^{FE} = (\alpha(\alpha+61)+36)a^2 - 8(\alpha-2)(\alpha-1)ah + 16(\alpha-2)(\alpha-1)h^2.
$$

No-Observability Error (NE)

When inventory is not observable to the manufacturer, he does not have the ability to tailor w_2 based on the actual event of the retailer committing the mistake. However, he knows that with probability α, the retailer carries zero inventory, and with probability $1-\alpha$, she carries her best response inventory I^{NE^*}. In either case, her response to any w_2 offered by the manufacturer is $q_2^{NE^*}(w_2, I) = \left[\frac{a-w_2}{2} - I\right]^+$. The manufacturer's expected second-period profit given by (EC.28):

$$
\mathbb{E}\left[\Pi_2^{NE}(w_2, I, \alpha)\right] = w_2\left(\alpha \cdot q_2^{NE^*}(w_2, 0) + (1-\alpha)\cdot q_2^{NE^*}(w_2, I)\right) \tag{EC.28}
$$

The retailer's expected second-period profit is

$$
\begin{aligned}
\mathbb{E}\left[\pi_2^{NE}(w_2, I, \alpha)\right] &= \alpha\left[\left(a - q_2^{NE^*}(w_2,0)\right)\cdot q_2^{NE^*}(w_2,0) - w_2 \cdot q_2^{NE^*}(w_2,0)\right] \\
&\quad +(1-\alpha)[\left(a - (q_2^{NE^*}(w_2,I)+I\right)\cdot\left(q_2^{NE^*}(w_2,I)+I\right) \\
&\quad -w_2\cdot q_2^{NE^*}(w_2,I)]
\end{aligned} \tag{EC.29}
$$

Having bought q_1 in stage $1b$, the retailer's expected profit-to-go is:

$$\begin{aligned}\mathbb{E}\left[\pi_1^{NE}(q_1, s_1, I, w_2, \alpha)\right] &= \alpha\left[(a-q_1)(q_1)\right] + (1-\alpha)\left[(a-s_1)\, s_1 - h\cdot I\right] \\ &\quad + \mathbb{E}\left[\pi_2^{NE}(w_2, I, \alpha)\right]. \end{aligned} \tag{EC.30}$$

Knowing that the retailer is prone to make a mistake with probability α, the retailer chooses $s_1^{NE^*}(q_1, \alpha)$ and $I^{NE^*}(q_1, \alpha)$, subject to $s_1 + I \leq q_1$, and the manufacturer simultaneously chooses $w_2^{NE^*}(q_1, \alpha)$, in anticipation that the retailer is choosing s_1 and I that maximize (EC.30). Note that given any α, when the manufacturer infers that I is high enough, he may prefer to set $w_2^{NE^*}(q_1, \alpha) = \frac{a}{2}$ and sell to the retailer only when she makes a mistake and carries zero inventory. Once again, we establish two thresholds $\underline{q_1}^{NE}$ and $\overline{q_1}^{NE}$ on the first-period purchase quantity. The first threshold, $\underline{q_1}^{NE} = \frac{1}{4}(a + 2h)$, is the minimum first-period purchase quantity above which the retailer holds inventory: $I^{NE^*}(q_1, \alpha) = 0$ when $q_1 \leq \underline{q_1}^{NE}$. The second threshold is defined as

$$\overline{q_1}^{NE} = \min\{\frac{a(2-\alpha) - 2h}{2(1-\alpha)}, \frac{2a(2-\alpha) + 2h(1-\alpha) - a(3-\alpha)\sqrt{\alpha}}{4(1-\alpha)}\}.$$

It can be seen that for any $w_1 \geq 0$, it is not optimal for the retailer to buy $q_1^{NE^*}(w_1, \alpha) > \overline{q_1}^{NE}$. Therefore, we focus on the region $q_1 \in [0, \overline{q_1}^{NE}]$, where $I^{NE^*}(q_1, \alpha) + s_1^{NE^*}(q_1, \alpha) = q_1$. In the NE scenario, the simultaneous inventory and wholesale price made respectively by the retailer and the manufacturer, are given by:

$$\left(I^{NE^*}(q_1, \alpha), w_2^{NE^*}(q_1, \alpha)\right) = \begin{cases} \left(0, \frac{a}{2}\right) & \text{if } q_1 \in [0, \underline{q_1}^{NE}], \\ \left(\frac{4q_1 - a - 2h}{2(3-\alpha)}, \frac{a(2-\alpha) + (h - 2q_1)(1-\alpha)}{3-\alpha}\right) & \text{if } q_1 \in (\underline{q_1}^{NE}, \overline{q_1}^{NE}). \end{cases} \tag{EC.31}$$

Observe that unlike the full-observability error scenario, now the manufacturer cannot observe the actual inventory being held. Thus, his wholesale price in period 2 when q_1 is high enough, depends on the expected inventory, and in turn, on α. The retailer anticipates an increase in w_2 across the board, since the manufacturer cannot determine whether the retailer has actually made a mistake or not, but knows the probability of the retailer making the mistake. Therefore, in the event she holds inventory $I^{NE^*}(q_1, \alpha)$ correctly, it is increasing in α when $q_1 \in (\underline{q_1}^{NE}, \overline{q_1}^{NE})$, so that she can moderate the effect of the increased w_2. However, in expectation, $\mathbb{E}[I^{NE^*}(q_1, \alpha)] = (1-\alpha)\cdot I^{NE^*}(q_1, \alpha)$ is decreasing in α in the same region, and the manufacturer correctly increases prices as his best response to reduced expected inventory. The retailer's equilibrium expected profit-to-go in the first period is

$$\mathbb{E}\left[\pi_1^{NE^*}(q_1,\alpha)\right] = \mathbb{E}\left[\pi_1^{NE}(q_1, s_1^{NE^*}(q_1,\alpha), I^{NE^*}(q_1,\alpha), w_2^{NE^*}(q_1,\alpha),\alpha)\right]. \tag{EC.32}$$

In stage 1b, the retailer decides to purchase $q_1^{NE^*}(w_1,\alpha)$ that maximizes her expected total profit given by (EC.33):

$$\mathbb{E}\left[\pi^{NE^*}(w_1,q_1,\alpha)\right] = \mathbb{E}\left[\pi_1^{NE}(q_1,\alpha)\right] - w_1 \cdot q_1. \tag{EC.33}$$

As is the case in Lemma 2, the retailer's expected total profit lacks concavity at $\underline{q_1}^{NE}$. There exists a threshold price $W^{NE}(\alpha) = \frac{a(3-\alpha)}{(5-\alpha)} - h$ such that

$$q_1^{NE^*}(w_1,\alpha) = \begin{cases} \frac{1}{8}\left(a\left(8-\alpha(5-\alpha)\right) + \alpha(6-\alpha)(w_1+h) - 5h - 9w_1\right) & \text{if } w_1 \in [0, W^{NE}(\alpha)), \\ \frac{a-w_1}{2} & \text{if } w_1 \in [W^{NE}(\alpha), a], \\ 0 & \text{if } w_1 \in (a,\infty). \end{cases} \tag{EC.34}$$

The manufacturer sets his first-period wholesale price to maximize his expected total profit given by (EC.35):

$$\mathbb{E}\left[\Pi^{NE^*}(w_1,\alpha)\right] = w_1 \cdot q_1^{NE^*}(w_1,\alpha) + \mathbb{E}\left[\Pi_2^{NE^*}(w_1,\alpha)\right], \tag{EC.35}$$

where

$$\mathbb{E}\left[\Pi_2^{NE^*}(w_1,\alpha)\right] = \mathbb{E}\left[\Pi_2^{NE^*}(w_2^{NE^*}(q_1^{NE^*}(w_1,\alpha),\alpha), I^{NE^*}(q_1^{NE^*}(w_1,\alpha),\alpha),\alpha)\right]. \tag{EC.36}$$

The lack of concavity of $\mathbb{E}\left[\pi^{NE^*}(w_1,q_1,\alpha)\right]$ at $\underline{q_1}^{NE}$ leads to a result similar to that in Lemma 3. We define two thresholds on the holding cost,

$$\begin{aligned} \underline{h}(\alpha) &= \frac{a(1-\alpha)^2\left(1-\alpha(4-\alpha)\right)}{2(5-\alpha)\left(13-\alpha(6-\alpha)\right)}, \text{ and} \\ \overline{h}(\alpha) &= \frac{a}{8(5-\alpha)}\left(\sqrt{(1-\alpha)\left(149-\alpha\left(31-5\alpha(3-\alpha)\right)\right)} - (1-\alpha)^2\right). \end{aligned}$$

Note that $\underline{h}(0) = \underline{h}$ and $\overline{h}(0) = \overline{h}$; and $\underline{h}(\alpha) \leq 0$ when $\alpha \geq 2 - \sqrt{3} \approx 0.268$. The manufacturer offers the first-period wholesale price $w_1^{NE^*}(\alpha)$ as given by (EC.37):

$$w_1^{NE^*}(\alpha) = \begin{cases} \frac{a(16-\alpha(1+\alpha(13-\alpha(7-\alpha))))-h(1-\alpha)(1+\alpha(13-\alpha(7-\alpha)))}{(3-\alpha)^3(1+\alpha)} & \text{if } h \in [0, \underline{h}(\alpha)), \\ W^{NE}(\alpha) = \frac{a(3-\alpha)}{(5-\alpha)} - h & \text{if } h \in [\underline{h}(\alpha), \overline{h}(\alpha)), \\ \frac{a}{2} & \text{if } h \in [\overline{h}(\alpha), \infty). \end{cases} \tag{EC.37}$$

The retailer responds by purchasing $q_1^{NE^*}(\alpha)$ as given by (EC.38):

$$q_1^{NE^*}(\alpha) = \begin{cases} \frac{a(4+\alpha(1-\alpha))-h(1-\alpha)(7-3\alpha)}{4(3-\alpha)(1+\alpha)} & \text{if } h \in [0, \underline{h}(\alpha)), \\ \frac{1}{8}\left(a\left((1-\alpha) + \frac{8}{(5-\alpha)}\right) + 4h\right) & \text{if } h \in [\underline{h}(\alpha), \overline{h}(\alpha)), \\ \frac{a}{4} & \text{if } h \in [\overline{h}(\alpha), \infty). \end{cases} \tag{EC.38}$$

The inventory held by the retailer in the event she does not commit a mistake, is given by (EC.39):

$$I^{NE^*}(\alpha) = \begin{cases} \frac{a(1-\alpha)-h(13-\alpha(6-\alpha))}{2(3-\alpha)^2(1+\alpha)} & \text{if } h \in [0, \underline{h}(\alpha)), \\ a(\frac{1}{4} - \frac{1}{5-\alpha}) & \text{if } h \in [\underline{h}(\alpha), \overline{h}(\alpha)), \\ 0 & \text{if } h \in [\overline{h}(\alpha), \infty). \end{cases} \tag{EC.39}$$

Observe that as in the case for full observability, the retailer's equilibrium inventory $I^{NE^*}(\alpha)$ as well as her expected inventory $\mathbb{E}\left[I^{NE^*}(\alpha)\right]$ are decreasing in α when $h \in [0, \overline{h}(\alpha))$. Substituting (EC.37) and (EC.38) in (EC.33) and (EC.35) gives us $\mathbb{E}\left[\pi^{NE^*}(\alpha)\right]$ and $\mathbb{E}\left[\Pi^{NE^*}(\alpha)\right]$, the expected total profits for the retailer and the manufacturer, respectively. The retailer's expected total profit is

$$\mathbb{E}\left[\pi^{NE^*}(\alpha)\right] = \begin{cases} \frac{X^{NE}}{4(3-\alpha)^4(\alpha+1)^2} & \text{if } h \in [0, \underline{h}(\alpha)), \\ \left(\frac{1}{(5-\alpha)^2} + \frac{1}{16}\right)a^2 + \frac{ah}{5-\alpha} + \frac{h^2}{4} & \text{if } h \in [\underline{h}(\alpha), \overline{h}(\alpha)), \\ \frac{a^2}{8} & \text{if } h \in [\overline{h}(\alpha), \infty), \end{cases} \tag{EC.40}$$

where

$$X^{NE} = +(\alpha-1)(\alpha(\alpha(\alpha((\alpha-9)\alpha+27)-29)+40)-94)h^2$$
$$-(\alpha(\alpha(\alpha((\alpha-7)\alpha+8)+23)-23)-34)a^2$$
$$-2(\alpha-1)^2(\alpha((\alpha-6)\alpha+2)+19)ah.$$

The manufacturer's expected total profit is

$$\mathbb{E}\left[\Pi^{NE^*}(\alpha)\right] = \begin{cases} \frac{Y^{NE}}{8(\alpha-3)^3(\alpha+1)} & \text{if } h \in [0, \underline{h}(\alpha)), \\ \frac{Z^{NE}}{32(\alpha-5)^2} & \text{if } h \in [\underline{h}(\alpha), \overline{h}(\alpha)), \\ \frac{a^2}{4} & \text{if } h \in [\overline{h}(\alpha), \infty), \end{cases} \tag{EC.41}$$

where

$$Y^{NE} = a^2(\alpha(\alpha(\alpha(3\alpha-22)+43)+8)-64)$$
$$+2a(\alpha-4)(\alpha-1)^3h-(\alpha(5\alpha-34)+61)(\alpha-1)^2h^2,$$

and

$$Z^{NE} = (\alpha-3)(\alpha((\alpha-1)\alpha+15)-79)a^2+4(\alpha-5)(\alpha-1)^2ah-16(\alpha-5)^2h^2.$$

From (EC.26) and (EC.40), $\mathbb{E}\left[\pi^{FE^*}(\alpha)\right]$ and $\mathbb{E}\left[\pi^{NE^*}(\alpha)\right]$ intersect at

$$h_N(\alpha) = a \cdot \frac{(32\alpha+17)\cdot U+V}{2(\alpha-5)(2\alpha+5)(4\alpha+3)(50\alpha-1)},$$

where

$$U = \sqrt{(\alpha(\alpha(\alpha(\alpha(4\alpha(16\alpha-45)-2275)-2197)+10610)+22035)+10359)},$$

and

$$V = (2\alpha(\alpha((235-28\alpha)\alpha+1565)+2157)+1746).$$

It is easy to confirm that when $\alpha \in [0,1]$, $h_N(\alpha)$ is decreasing in α, and $h_N(\alpha) > 0$ for $\alpha < 1$, and

$h_N(\alpha) = 0$ for $\alpha = 1$.

EC.7 Proof of Proposition 2

From Sections (FE) and (NE) respectively, we have

$$\begin{aligned}
\widehat{h}(\alpha) &= \frac{a}{4}, \text{ and} \\
\overline{h}(\alpha) &= \frac{a}{8(5-\alpha)}\left(\sqrt{(1-\alpha)\left(149-\alpha\left(31-5\alpha(3-\alpha)\right)\right)}-(1-\alpha)^2\right).
\end{aligned}$$

(a) Recall from the baseline in Section 3.3 that $\widehat{h} = \frac{a}{4}$. Therefore, $\widehat{h}(\alpha) = \widehat{h}$. It is easy to confirm that $\overline{h}(\alpha)$ is (concave) decreasing in α for all $\alpha \in [0,1]$.

(b) Observe that $\overline{h}(0) = \overline{h} > \widehat{h} = \widehat{h}(0)$ and $\overline{h}(1) = 0 < \widehat{h} = \widehat{h}(0)$. Thus, there exists an $\overline{\alpha} \in [0,1]$ such that for all $\alpha > \overline{\alpha}$, $\overline{h}(\alpha) < \widehat{h}(\alpha)$.

(c) Note that the dynamic contracts under the FE and the NE scenarios converge to the commitment contract when $h \geq \widehat{h}(\alpha)$ and $h \geq \overline{h}(\alpha)$, respectively. Since the manufacturer benefits from strategic inventory when in either scenario, h is lower than the corresponding thresholds $\widehat{h}(\alpha)$ and $\overline{h}(\alpha)$, it is easy to confirm from (EC.27) that $\mathbb{E}\left[\Pi^{FE^*}(\alpha)\right] > \Pi^{C^*} = \frac{a^2}{4}$ when $h < \widehat{h}(\alpha)$, and $\mathbb{E}\left[\Pi^{FE^*}(\alpha)\right] = \Pi^{C^*}$ when $h \geq \widehat{h}(\alpha)$. Similarly, from (EC.41), $\mathbb{E}\left[\Pi^{NE^*}(\alpha)\right] > \Pi^{C^*}$ when $h < \overline{h}(\alpha)$, and $\mathbb{E}\left[\Pi^{NE^*}(\alpha)\right] = \Pi^{C^*}$ when $h \geq \overline{h}(\alpha)$.

From (b), for all $\alpha > \overline{\alpha}$, $\overline{h}(\alpha) < \widehat{h}(\alpha)$. Therefore, when $\alpha > \overline{\alpha}$, for $h \geq \overline{h}(\alpha)$, $\mathbb{E}\left[\Pi^{NE^*}(\alpha)\right] = \Pi^{C^*} \leq \mathbb{E}\left[\Pi^{FE^*}(\alpha)\right]$. Further, when $h \in (\overline{h}(\alpha), \widehat{h}(\alpha))$, $\mathbb{E}\left[\Pi^{NE^*}(\alpha)\right] = \Pi^{C^*}$ and $\mathbb{E}\left[\Pi^{FE^*}(\alpha)\right]$, thus $\mathbb{E}\left[\Pi^{NE^*}(\alpha)\right] < \mathbb{E}\left[\Pi^{FE^*}(\alpha)\right]$.