An Online Appendix to
"Why Markdown as a Pricing Modality"

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Appendix A: Extensions and Discussions

Distribution of consumer valuations. We model possible consumer valuations with a uniform distribution on $[0, \bar{v}]$. Uniform distribution is widely used in the literature because it captures fickle consumers' tastes and also leads to the well-known linear demand function. The market segmentation obtained in Proposition 1 does not depend on this distribution. The form of the distribution plays a role only in the derivation of the aggregate demand at each retailer (Proposition 2). The form of the distribution also affects the profits, and therefore the best response decisions. Let $f(\cdot)$ denote the probability distribution over the possible consumer valuations $[0, \bar{v}]$. Proposition 1 continues to hold as is. The markdown retailer's Period 2 demand is then given by $N \int_0^{\bar{v}} I^M(v) f(v) dv$, where $I^M(v)$ equals to 1 if a consumer with valuation v attempts to purchase from the markdown retailer, and zero otherwise. This function can be obtained from Proposition 1; for instance when p_E is in the range matching case (ii), $I^M(v) = 0$ when $v < \delta p_1$, and $I^M(v) = 1$ otherwise. Similarly, we determine the spillover demand at the EDLP retailer as $(1-q)N \int_0^{\bar{v}} I^E(v) f(v) dv$, where $I^E(v)$ equals 1 if a consumer with valuation v finding a stockout would decide to spill over to the EDLP retailer, and zero otherwise. For example, when p_E is in the range matching case (ii), $I^E(v) = 0$ when $v < p_E$, and $I^E(v) = 1$ otherwise. Hence, the distribution of consumer valuation affects the demands shown in Proposition 2, and as a result it affects the profits. We can obtain closed-form expressions for demand and corresponding profit at each retailer when, for example, the distribution of consumer valuation is uniform, triangular or exponential. In Proposition 2, the non-zero demands depend on the distribution $f(\cdot)$, but the zero demands remain at zero regardless of this distribution. Also, the finding that the markdown retailer makes a positive profit in equilibrium and that the EDLP incumbent makes zero profit in equilibrium rely not on the actual demand and profit values, but on the cases leading to positive or zero demand. Therefore, these results would stand under different distributional assumptions. The closed-form expressions in Lemma 1 part (i) would change.

Symmetric procurement costs. Here we investigate the scenario in which the EDLP retailer has a lower cost c_E than the markdown retailer's cost c_M. Our results on the market segmentation (Proposition 1) and the aggregate demands (Proposition 2) remain unchanged as they are independent of the procurement costs. However the resulting profit expressions would need updating:

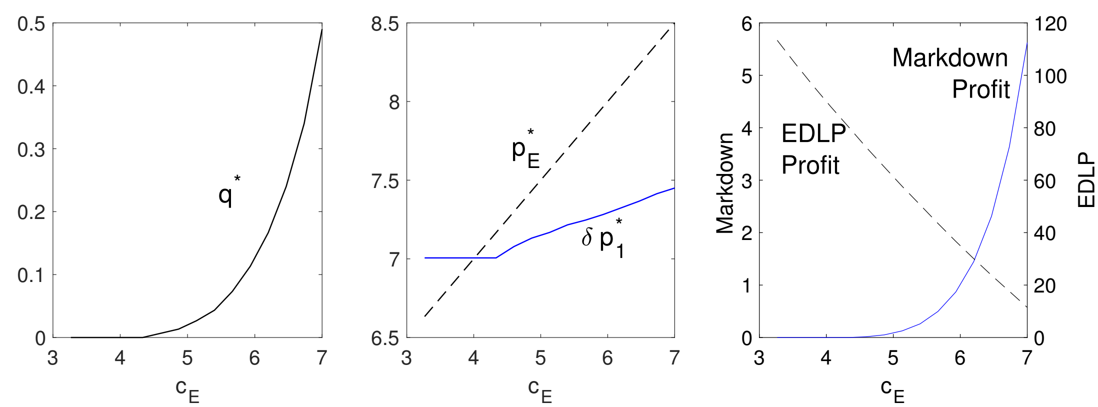
$$\Pi_M = (p_1 - c_M) D_1 + (\delta p_1 - c_M) q D_2,$$
$$\Pi_E = (p_E - c_E)(D_E^P + D_E^S).$$

The cost asymmetry affects the profitability of the markdown retailer, and in some cases, the EDLP retailer can exploit the cost differential to price lower and prevent the markdown retailer from earning any profit. Specifically, we obtain the following results.

Proposition A1. *Suppose $c_E < c_M$. When the EDLP retailer is the incumbent and the markdown retailer is the entrant, in equilibrium the markdown retailer chooses not to enter. The EDLP price is higher than the EDLP cost but lower than the markdown cost (i.e., $p_E^* = \min\{c_M, (c_E + \bar{v})/2\} \in (c_E, c_M]$).*

We study numerically the scenario in which the markdown retailer is the incumbent. We find that if the EDLP retailer's cost advantage becomes large, the markdown retailer may no longer be profitable in equilibrium. The markdown retailer must set its markdown price above its cost c_M to make a profit. This may end up being higher than the EDLP price if the EDLP retailer enjoys a large cost advantage. Even if the markdown retailer allows consumers to spill over to the EDLP retailer (by setting a low availability probability) and lowers its markdown price near its cost, the EDLP retailer may still find it profitable to undercut the markdown price and attract a large demand (scenario (i)), rather than price above the markdown price and obtain only spillover demand (scenarios (ii) or (iii)), when the EDLP cost is low. In an extreme case, when the EDLP cost is sufficiently low, the entrant EDLP retailer prefers to undercut the markdown price (by pricing below the markdown retailer's cost), pushing the incumbent markdown retailer out of the market. Such a strategy ensures a large demand and a sufficient profit margin, considering the low EDLP cost. We illustrate this finding by a numerical example, where $\alpha = 0, \beta = 1, \delta = 0.7, c_M = 7$ and c_E varies from 3 to 7. Note from Figure A.1 that when the EDLP cost drops below 4.4, the markdown incumbent achieves no profit.

Figure A.1 Effect of entrant EDLP retailer's cost on availability, prices and profits



Overall, when the EDLP retailer enjoys a cost advantage over the markdown retailer, the EDLP retailer may be able to dominate the markdown retailer (either by deterring entry, or by pushing the incumbent out of the market) as long as the EDLP retailer can set its price below the markdown retailer's cost. Perhaps Walmart enjoys a scale matched by no other retailer, allowing it to benefit from economies of scale, advanced supply chain management technology, high bargaining power with suppliers (all of which are supply side advantages and not due to the choice of pricing modality) to keep its costs very low. This cost advantage may be the reason why Walmart can use the EDLP pricing modality and deter a markdown retailer with higher cost from entering the market.

EDLP inventory rationing. Because of its fixed price policy, an EDLP retailer typically enjoys a stable demand, which allows the retailer to avoid stockout. In practice, the EDLP retailer often stocks enough to satisfy all demand, as its strategy is to appeal to consumers by guaranteeing availability. In theory, the EDLP retailer, like the markdown retailer, could intentionally create scarcity (even though it makes little practical sense because there is little reason to induce customers to purchase in an earlier period under fixed price policy). Inventory rationing at the EDLP retailer might have two opposite effects: (i) high-valuation consumers could prefer to purchase from the markdown retailer in the first period to avoid the risk of finding a stockout; (ii) in the second period, some consumers may choose to attempt purchasing from the EDLP retailer rather than the markdown retailer if the EDLP price is low enough and/or the EDLP availability probability is high enough in comparison to the markdown retailer. We conjecture that when the EDLP retailer is the incumbent, an entrant markdown retailer could still capture the entire market demand by setting an availability probability slightly higher, and a markdown price slightly lower than the EDLP retailer, to attract both the low- and high-valuation consumers. The analysis of this scenario requires one to capture in the analysis the two-way demand spillover and two endogenous and inter-related availability probabilities.

Basket shoppers. Our model applies to a single-product case. In some other cases, consumers may select a retail venue for a basket of multiple products. This may provide an advantage to the EDLP retailer who has all the products in the basket in stock (Bell and Lattin 1998), while the markdown retailer may have stocked out for at least some of the products in the basket at the time of the consumer's visit. Our study may be viewed as a good approximation of the case of basket shoppers when the competing retailer offers a substitutable basket. To capture this situation, we may consider the basket as one product and assume that the consumer does not make a partial basket purchase and may spill over if at least one product in the basket is out of stock. The valuation would then be the valuation of the entire basket; the availability probability would be the probability that all the products in the basket are in stock; the discount rate at the markdown retailer would be the rate of discount on the overall basket in the future markdown period (even if not all products individually are discounted at that time).

Product differentiation. A retailer may successfully implement an EDLP pricing modality by differentiating its product from those of its competitor. Such differentiation could protect the retailer against the entry of a competitor, making co-existence possible. For example, Trader Joe's offers products, including a large collection of organic products, that are unavailable at other grocery stores, and it enjoys high consumer loyalty.

Simultaneous move. Here we determine whether a Nash equilibrium exists in pure strategies.

Proposition A2. When the two retailers make decisions simultaneously, the game has no equilibrium.

Without demand spillover. Here, we investigate the restrictive scenario in which spilling over is not an option, i.e., consumers never consider spilling over to the EDLP retailer in case of a stockout at the markdown retailer. A consumer with reservation price v would then have the following expected utilities when she purchases from the markdown retailer in Period 1, when she attempts to purchase from the markdown retailer in Period 2, and when she purchases from the EDLP retailer, respectively:

$$U_1 = (v - p_1) - q\alpha(p_1 - \delta p_1),$$
$$U_2 = q(v - \delta p_1) - (1-q)\beta \max\{v - p_1, 0\},$$
$$U_E = v - p_E.$$

The following proposition characterizes consumers' purchase decisions.

PROPOSITION A3. *We define thresholds Q_2, Q_3, Q_4 and cutoff values v_2, v_3, v_4 as follows:*

$$Q_2 = \frac{\bar{v} - p_E}{\bar{v} - \delta p_1}, \qquad v_2 = \frac{p_E - q\delta p_1}{1-q},$$
$$Q_3 = \frac{(1+\beta)\bar{v} - p_E - \beta p_1}{(1+\beta)\bar{v} - (\delta+\beta)p_1}, \qquad v_3 = \frac{p_E + p_1(\beta(1-q) - q\delta)}{(1+\beta)(1-q)},$$
$$Q_4 = \frac{(1+\beta)(\bar{v} - p_1)}{(1+\beta)\bar{v} + (\alpha - \beta - (1+\alpha)\delta)p_1}, \quad v_4 = \left(\frac{(1+\alpha)(1-\delta)q}{(1+\beta)(1-q)} + 1\right)p_1.$$

Then:

(i) When $p_E \in [0, \delta p_1]$, a consumer does not buy if her reservation price $v \in [0, p_E)$, and buys from the EDLP retailer if $v \in [p_E, \bar{v}]$.

(ii) When $p_E \in (\delta p_1, (1 - q(1-\delta))p_1]$,

(a) For any $q \in [0, Q_2]$, a consumer does not buy if her reservation price $v \in [0, \delta p_1)$, buys from the markdown retailer in Period 2 if $v \in [\delta p_1, v_2)$, and buys from the EDLP retailer if $v \in [v_2, \bar{v}]$.

(b) For any $q \in (Q_2, 1]$, a consumer does not buy if $v \in [0, \delta p_1)$ and buys from the markdown retailer in Period 2 if $v \in [\delta p_1, \bar{v}]$.

(iii) When $p_E \in ((1 - q(1-\delta))p_1, (1 + \alpha q(1-\delta))p_1]$,

(a) For any $q \in [0, Q_3]$, a consumer does not buy if her reservation price $v \in [0, \delta p_1)$, buys from the markdown retailer in Period 2 if $v \in [\delta p_1, v_3)$, and buys from the EDLP retailer if $v \in [v_3, \bar{v}]$.

(b) For any $q \in (Q_3, 1]$, a consumer does not buy if $v \in [0, \delta p_1)$ and buys from the markdown retailer in Period 2 if $v \in [\delta p_1, \bar{v}]$.

(iv) When $p_E \in ((1 + \alpha q(1-\delta))p_1, \infty)$,

(a) For any $q \in [0, Q_4]$, a consumer does not buy if her reservation price $v \in [0, \delta p_1)$, buys from the markdown retailer in Period 2 if $v \in [\delta p_1, v_4]$, and buys from the markdown retailer in Period 1 if $v \in (v_4, \bar{v}]$.

(b) For any $q \in (Q_4, 1]$, a consumer does not buy if $v \in [0, \delta p_1)$ and buys from the markdown retailer in Period 2 if $v \in [\delta p_1, \bar{v}]$.

Given this market segmentation, we characterize the resulting demand for the markdown retailer in each period (D_1, D_2) as well as for the EDLP retailer (D_E) as follows.

The aforementioned demand characterization at each retailer yields the following expected profits at the markdown and the EDLP retailer, respectively.

$$\Pi_M = (p_1 - c)D_1 + q(\delta p_1 - c)D_2,$$
$$\Pi_E = (p_E - c)D_E.$$

Table A.1 Demand at each retailer

$p_E \in$	(i) $[0, \delta p_1)$	(ii) $(\delta p_1, (1-q(1-\delta))p_1]$		(iii) $((1-q(1-\delta))p_1, (1+\alpha q(1-\delta))p_1]$		(iv) $((1+\alpha q(1-\delta))p_1, \infty)$	
		$q \le Q_2$	$q > Q_2$	$q \le Q_3$	$q > Q_3$	$q \le Q_4$	$q > Q_4$
D_1	0	0	0	0	0	$\frac{N(\bar{v}-v_4)}{\bar{v}}$	0
D_2	0	$\frac{N(v_2-\delta p_1)}{\bar{v}}$	$\frac{N(\bar{v}-\delta p_1)}{\bar{v}}$	$\frac{N(v_3-\delta p_1)}{\bar{v}}$	$\frac{N(\bar{v}-\delta p_1)}{\bar{v}}$	$\frac{N(v_4-\delta p_1)}{\bar{v}}$	$\frac{N(\bar{v}-\delta p_1)}{\bar{v}}$
D_E	$\frac{N(\bar{v}-p_E)}{\bar{v}}$	$\frac{N(\bar{v}-v_2)}{\bar{v}}$	0	$\frac{N(\bar{v}-v_3)}{\bar{v}}$	0	0	0

We obtain the following results. The proofs are similar to those in the main body of the manuscript and are not repeated here for brevity.

Proposition A4. *When the markdown retailer is the incumbent and the EDLP retailer is the entrant, in equilibrium,*
(i) we observe from Table A.1 either demand scenario (ii) with $q \le Q_2$ or (iii) with $q \le Q_3$; as a result, the markdown retailer sells only in Period 2;
(ii) the EDLP price is higher than the markdown price, i.e., $p_E^* > \delta p_1^*$;
(iii) the markdown retailer's optimal stocking decision K^* is such that the resulting availability probability q^* during the markdown period is strictly less than 1 (that is, the markdown retailer's stock level is lower than the number of consumers attempting to purchase in Period 2). Specifically, in scenario (ii) then $q^* \le Q_2$; in scenario (iii) then $q^* \le Q_3$;
(iv) the markdown retailer captures consumers with moderate reservation price (i.e., $v \in [\delta p_1, v_2)$ in scenario (ii), and $v \in [\delta p_1, v_3)$ in scenario (iii)). The EDLP retailer captures demand from consumers with high reservation prices (i.e., $v \in [v_2, \bar{v}]$ in scenario (ii), and $v \in [v_3, \bar{v}]$ in scenario (iii)). Consumers with low reservation price (i.e., $v \in [0, \delta p_1^*)$) do not attempt to purchase.

Proposition A5. *When the EDLP retailer is the incumbent and the markdown retailer is the entrant, in equilibrium either demand scenario (ii) (with $q^* = 1$), (iii) (with $q^* = 1$) or (iv) (with $q^* = 1$ or $q^* \le Q_4$) of Table A.1 is observed. The EDLP price is higher than the discounted markdown price (i.e., $p_E^* > \delta p_1^*$). All consumers purchase from the markdown retailer. The EDLP retailer makes zero profit.*

Both of these key results are similar to the ones we obtain in the presence of spillover. Numerical study for the scenario in which the markdown retailer is the incumbent shows the following:

1. When the discount rate is low, both the availability probability and the high-price regret intensity affect even the EDLP retailer's price; this dynamic holds despite the fact that no consumer actually experiences high-price regret.
2. When discount rate is low, the rationing effect becomes more intense as the discount rate increases.
3. When the discount rate is moderate or low, the markdown retailer's discount rate affects even the EDLP retailer's pricing decision.
4. When the discount rate is high, the actual extent of the discount does *not* matter.
5. When the discount rate is high, the actual values of the availability and the high-price regret intensity do not matter.

6. The markdown retailer's profits are unimodal across the discount range.
7. An increase in availability regret intensity weakens the markdown retailer's rationing effect, while an increase in high-price regret intensity strengthens it.
8. When regret is ignored, the markdown retailer's potential profit loss is on average 12.5%, and up to 46%. With an average profit gain of 0.2%, the EDLP retailer also stands to lose a significant portion – up to 20% – of its potential profit.

Observations 3, 4, 5, 6 and 7 are similar to the case with spillover. Observation 2 is the opposite effect. Observation 1 holds with spillover with the distinction that some consumers do experience high-price regret. The rates of profit loss mentioned in observation 8 are lower in the presence of spillover. Indeed, the possibility of spilling over is a means to reduce opportunities to experience regret. When there is spillover, fewer consumers are exposed to both high-price and availability regret, hence ignoring regret leads to less forgone profit for the retailers.

Appendix B: Proofs

Proof of Proposition 1 We first state the Proposition in detail and next prove the proposition.

Proposition 1 (Unabridged):

We define thresholds Q_0, Q_1 and cutoff values $v_0, v_1, \bar{p}$ as follows:

$$Q_0 = \frac{p_E - p_1}{p_E - \delta p_1}, \qquad Q_1 = \frac{(1+\beta)(\bar{v} - p_1)}{\bar{v} - \delta p_1 + \alpha p_1(1-\delta) + \beta(\bar{v} - p_1)},$$
$$v_0 = \frac{p_E + \alpha(p_E - p_1) + \beta p_1}{1+\beta}, \quad v_1 = p_1 \frac{1 + \alpha q(1-\delta) - q\delta + \beta(1-q)}{(1+\beta)(1-q)}, \quad \bar{p} = \frac{\bar{v}(1+\beta) + (\alpha - \beta)p_1}{1+\alpha}.$$

Then:

(i) When $p_E \in [0, \delta p_1]$, a consumer does not buy if her reservation price $v \in [0, p_E)$, and buys from the EDLP retailer if $v \in [p_E, \bar{v}]$.

(ii) When $p_E \in (\delta p_1, p_1]$, a consumer does not buy if her reservation price $v \in [0, \delta p_1)$ and attempts to buy from the markdown retailer in Period 2 if $v \in [\delta p_1, \bar{v}]$. In the latter case, in case of stockout, she does not spill over to the EDLP retailer if $v \in [\delta p_1, p_E)$ and spills over if $v \in [p_E, \bar{v}]$.

(iii) When $p_1 < \bar{p}$ and $p_E \in (p_1, \bar{p}]$,

(a) For any $q \in [0, Q_0]$, a consumer does not buy if her reservation price $v \in [0, \delta p_1)$, attempts to buy from the markdown retailer in Period 2 if $v \in [\delta p_1, v_1)$ and, in case of stockout, does not spill over to the EDLP retailer, and buys from the markdown retailer in Period 1 if $v \in [v_1, \bar{v}]$.

(b) For any $q \in (Q_0, 1]$, a consumer does not buy if $v \in [0, \delta p_1)$ and attempts to buy from the markdown retailer in Period 2 if $v \in [\delta p_1, \bar{v}]$. In the latter case, in case of stockout, she does not spill over to the EDLP retailer if $v \in [\delta p_1, v_0)$ and spills over if $v \in [v_0, \bar{v}]$.

(iv) When $p_1 < \bar{p}$ and $p_E \in (\bar{p}, \infty)$ or when $p_1 \geq \bar{p}$ and $p_E \in (p_1, \infty)$,

(a) For any $q \in [0, Q_1]$, a consumer does not buy if her reservation price $v \in [0, \delta p_1)$, attempts to buy from the markdown retailer in Period 2 if $v \in [\delta p_1, v_1]$ and, in case of stockout, does not spill over to the EDLP retailer, and buys from the markdown retailer in Period 1 if $v \in (v_1, \bar{v}]$.

(b) For any $q \in (Q_1, 1]$, a consumer does not buy if $v \in [0, \delta p_1)$ and buys from the markdown retailer in Period 2 if $v \in [\delta p_1, \bar{v}]$ and, in case of stockout, does not spill over to the EDLP retailer.

We first provide four results that are useful in proving Proposition 1.

Preliminary Lemma 1: Suppose $p_1 < p_E$. A consumer selecting the markdown retailer in Period 2 spills over to the EDLP retailer in case of stockout iff $v > v_0$. In addition, $v_0 > p_1$. Furthermore, $v_0 \leq \bar{v}$ iff $p_E \leq (\bar{v}(1+\beta) + (\alpha - \beta)p_1)/(1+\alpha)$.

Proof: There is spillover iff $v - p_E - \alpha(p_E - p_1) \geq -\beta(v - p_1)^+$. If $v \leq p_1$, then $v \leq p_1 < p_E < p_E + \alpha(p_E - p_1)$, thus the consumer does not spill over. If $v \geq p_E + \alpha(p_E - p_1)(> p_1)$, then $v - p_E - \alpha(p_E - p_1) \geq 0 \geq -\beta(v - p_1)^+$, thus the consumer spills over. Now suppose $p_1 < v < p_E + \alpha(p_E - p_1)$. There is spillover iff $v - p_E - \alpha(p_E - p_1) \geq -\beta(v - p_1)$, that is, iff $v \geq v_0$. It is straightforward to check that $p_1 < v_0 < p_E + \alpha(p_E - p_1)$, and that $v_0 \leq \bar{v}$ iff $p_E \leq (\bar{v}(1+\beta) + (\alpha - \beta)p_1)/(1+\alpha)$. The result follows.

Preliminary Lemma 2: Suppose $p_1 < p_E$. Then $v - p_1 - q\alpha p_1(1-\delta) \geq q(v - \delta p_1) - (1-q)\beta(v - p_1)$ iff $v > v_1$. Furthermore, $v_1 \geq p_1 + q\alpha p_1(1-\delta)$. In addition, $v_1 \leq v_0$ iff $q \leq Q_0$. Finally, $v_1 \leq \bar{v}$ iff $q \leq Q_1$.
Proof: follows from straightforward algebra.

Preliminary Lemma 3: Suppose $p_1 < p_E$ and $p_1 \leq v < \min\{v_0, v_1\}$. Then $U_2 \geq 0$.
Proof: we have $U_2 \geq 0$ iff $v[q - \beta(1-q)] \geq p_1[q\delta - (1-q)\beta]$. If $q - \beta(1-q) > 0$, $v > p_1$ and $\delta < 1$ imply $v[q-\beta(1-q)] > p_1[q-\beta(1-q)] > p_1[\delta q - \beta(1-q)]$, and thus $U_2 \geq 0$. If $q - \beta(1-q) \leq 0$, that is, if $q < \beta/(1+\beta)$, then $U_2 \geq 0$ iff $v \leq p_1[(1-q)\beta - q\delta]/[(1-q)\beta - q]$. After substituting the definition of v_1 into the inequality and simplifying, we find that $p_1[(1-q)\beta - q\delta]/[(1-q)\beta - q] > v_1$ iff $\alpha(\beta(1-q) - q) < 1$. The latter inequality holds because $\alpha, \beta < 1$. Hence, when $q - \beta(1-q) \leq 0$, we have $v < v_1 < p_1[(1-q)\beta - q\delta]$, and thus $U_2 \geq 0$.

Preliminary Lemma 4: Suppose $p_1 < p_E$. Then $v - p_1 - q\alpha p_1(1-\delta) \geq q(v - \delta p_1)$ iff $v \geq p_1[1 + \alpha q(1-\delta) - q\delta]/(1-q)$. In addition, $p_1[1 + \alpha q(1-\delta) - q\delta]/(1-q) > p_1$.
Proof: follows from straightforward algebra.

Next, we consider four cases to prove Proposition 1.

case (i): $p_E \leq \delta p_1$: then $U_1 = v - p_1 - q\alpha(p_1 - p_E) - (1-q)\alpha(p_1 - p_E) = v - p_1 - \alpha(p_1 - p_E) < v - p_1 < v - p_E = U_E$. Also, if $v \geq p_E$, then $v \geq p_E - \beta(v - p_1)^+$ so the consumer spills over in case of stockout, and we have $U_2 = q(v - \delta p_1) - q\alpha(\delta p_1 - p_E) + (1-q)(v - p_E) \leq q(v - \delta p_1) + (1-q)(v - p_E) \leq v - p_E = U_E$. If $v < p_E$, then $v < p_1$ and the consumer does not spill over in case of stockout, and since $v < p_E \leq \delta p_1$, we have $U_2 = q(v - \delta p_1) - q\alpha(\delta p_1 - p_E) \leq 0$. Thus the markdown retailer is never the preferred option. Consumer chooses the EDLP retailer iff $U_E = v - p_E \geq 0$.

case (ii): $\delta p_1 < p_E \leq p_1$: then $U_1 = v - p_1 - q\alpha p_1(1-\delta) - (1-q)\alpha(p_1 - p_E) \leq v - p_1 - q\alpha(p_1 - \delta p_1) \leq v - p_E - q\alpha(p_E - \delta p_1) = U_E$ so the markdown retailer in Period 1 is not the preferred option. Also, if $v \geq p_E$, then $v \geq p_E - \beta(v - p_1)^+$ so the consumer spills over in case of stockout, and we have $U_2 = q(v - \delta p_1) + (1-q)(v - p_E) \geq v - p_E \geq U_E$. If $v < p_E$, then $U_E \leq v - p_E < 0$. Hence, the consumer prefers either the markdown retailer in Period 2 or no purchase. Thus a consumer purchases from the markdown retailer in Period 2 iff $U_2 \geq 0$. When $v \geq p_E (> \delta p_1)$, then $U_2 = q(v - \delta p_1) + (1-q)(v - p_E) = v - p_E + q(p_E - \delta p_1) > 0$. When $v < p_E$, then $v < p_1$ so $U_2 = q(v - \delta p_1)$ so $U_2 \geq 0$ iff $v \geq \delta p_1$ (or $q = 0$, in which case the amount sold is 0). Hence the consumer chooses the markdown retailer in Period 2 iff $v \geq \delta p_1$, and only consumers with $v \geq p_E$ spill over to the EDLP retailer in case of stockout.

case (iii): $p_1 < p_E \leq (\bar{v}(1+\beta) + (\alpha - \beta)p_1)/(1+\alpha)$. (The last inequality implies $p_1 < \bar{v}$.) Then we have $p_1 < v_0 \leq \bar{v}$, where v_0 is such that a consumer finding a stockout at the markdown retailer spills over to the EDLP retailer iff $v > v_0$ (Preliminary Lemma 1). In this case, $U_1 = v - p_1 - q\alpha p_1(1-\delta)$,

$$U_2 = \begin{cases} q(v - \delta p_1) + (1-q)(v - p_E - \alpha(p_E - p_1)) & \text{if } v \geq v_0, \\ q(v - \delta p_1) - (1-q)\beta(v - p_1) & \text{if } p_1 \leq v < v_0, \\ q(v - \delta p_1) & \text{if } v < p_1, \end{cases}$$

and $U_E = v - p_E - q\alpha(p_E - \delta p_1) - (1-q)\alpha(p_E - p_1)$. We observe that $U_E \leq v - p_E - q\alpha(p_E - \delta p_1) < v - p_1 - q\alpha(p_1 - \delta p_1) = U_1$, thus the EDLP retailer is never the preferred option. It is straightforward to check that $U_1 \geq q(v - \delta p_1) + (1-q)(v - p_E - \alpha(p_E - p_1))$ iff $q \leq Q_0$.

case (iii)a: $q \le Q_0$. If $v_0 \le v \le \bar{v}$, we have $U_1 \ge U_2$, thus the consumer chooses either the markdown retailer in Period 1 or no purchase. We have $U_1 \ge 0$ iff $v \ge p_1 + q\alpha p_1(1-\delta)$. We now show that $p_1 + q\alpha p_1(1-\delta) \le v_0$. We proceed by contradiction. Suppose $p_1 + q\alpha p_1(1-\delta) > v_0$. This inequality simplifies to $q > (p_E - p_1)(1+\alpha)/(\alpha p_1(1-\delta)(1+\beta))$. When $q \le Q_0$, this implies $(p_E - p_1)(1+\alpha)/(\alpha p_1(1-\delta)(1+\beta)) < Q_0$, which simplifies to $p_E < p_1[\alpha(1+\beta)(1-\delta) + \delta(1+\alpha)]/(1+\alpha)$. After rearranging the terms, we find that $[\alpha(1+\beta)(1-\delta) + \delta(1+\alpha)]/(1+\alpha) < 1$ iff $(1-\delta)(1-\alpha\beta) > 0$, which holds since $\alpha, \beta, \delta < 1$. Hence, the previous inequality implies $p_E < p_1$, yielding a contradiction. It follows that $v_0 < v < \bar{v}$, $U_1 \ge 0$ and so the consumer chooses the markdown retailer in Period 1.

If $p_1 \le v < v_0$, by Preliminary Lemma 2, $U_1 \ge U_2$ iff $v > v_1$, with $p_1 < v_1 \le v_0$. Thus we have $U_1 \ge U_2$ for $v_1 < v \le v_0$ and $U_2 \ge U_1$ for $p_1 < v \le v_1$. It remains to show that $U_1 \ge 0$ when $v_1 < v \le v_0$, and $U_2 \ge 0$ for $p_1 < v \le v_1$. Since $U_1 \ge 0$ iff $v \ge p_1 + q\alpha p_1(1-\delta)$, with $v_1 \ge p_1 + q\alpha p_1(1-\delta)$ (Preliminary Lemma 2), we have $U_1 \ge 0$ when $v_1 < v \le v_0$, so the consumer chooses the markdown retailer in Period 1. When $p_1 < v \le v_1$, by Preliminary Lemma 3, we have $U_2 \ge 0$, that is, the consumer selects the markdown retailer in Period 2 and, because $v \le v_1 \le v_0$, the consumer does not spill over to the EDLP retailer in case of stockout (Preliminary Lemma 1).

If $\delta p_1 \le v < p_1$, by Preliminary Lemma 4 we have $v < p_1 < p_1[1 + \alpha q(1-\delta) - q\delta]/(1-q)$, so $U_2 > U_1$. Since $v \ge \delta p_1$, we have $U_2 \ge 0$. Thus the consumer chooses the markdown retailer in Period 2. Also, $v < p_1 < v_0$ ensures that no consumer spills over to the EDLP retailer in case of stockout (Preliminary Lemma 1).

If $v \le \delta p_1$, by Preliminary Lemma 4 we have $v < p_1 < p_1[1 + \alpha q(1-\delta) - q\delta]/(1-q)$, so $U_2 > U_1$. Furthermore, $U_2 \le 0$, so the consumer does not purchase.

case (iii)b: $q > Q_0$. If $v_0 \le v \le \bar{v}$, we have $U_2 \ge U_1$, thus the consumer chooses either the markdown retailer in Period 2 or no purchase. We have $U_2 \ge 0$ iff $v \ge q\delta p_1 + (1-q)(p_E + \alpha(p_E - p_1))$. We now show that $q\delta p_1 + (1-q)(p_E + \alpha(p_E - p_1)) \le v_0$. We proceed by contradiction. Suppose $q\delta p_1 + (1-q)(p_E + \alpha(p_E - p_1)) > v_0$. This inequality simplifies to $q < (\beta/(1+\beta)) \cdot (p_E - p_1 + \alpha(p_E - p_1))/(p_E - \delta p_1 + \alpha(p_E - p_1))$. When $q > Q_0$, this implies $(\beta/(1+\beta)) \cdot (p_E - p_1 + \alpha(p_E - p_1))/(p_E - \delta p_1 + \alpha(p_E - p_1)) > Q_0$, which simplifies to $(p_E - \delta p_1)[1 - (1+\alpha)\beta/(1+\beta)] + \alpha(p_E - p_1) < 0$. Observing that $\alpha\beta < 1$ implies $(1+\alpha)\beta/(1+\beta) < 1$ yields a contradiction. It follows that for $v_0 < v < \bar{v}$, $U_2 \ge 0$ and so the consumer chooses the markdown retailer in Period 2. In addition, because $v \ge v_0$, in case of stockout the consumer would spill over to the EDLP retailer (Preliminary Lemma 1).

If $p_1 \le v < v_0$, by Preliminary Lemma 2, $U_1 \ge U_2$ iff $v > v_1$, with $v_1 > v_0$. Thus for $p_1 \le v < v_0$, we have $v < v_1$ and thus $U_1 < U_2$. By Preliminary Lemma 3, we have $U_2 \ge 0$, that is, the consumer chooses the markdown retailer in Period 2. Furthermore, since $v < v_0$, the consumer does not spill over to the EDLP retailer in case of stockout (Preliminary Lemma 1).

If $\delta p_1 \le v < p_1$ or if $v \le \delta p_1$, the reasoning from case (iii)a applies.

case (iv): $(\delta p_1 <) p_1 < p_E$ and $p_E > (\bar{v}(1+\beta) + (\alpha - \beta)p_1)/(1+\alpha)$. Then we have $v_0 > \bar{v}$ (Preliminary Lemma 1), that is, we have $v < v_0$ for all $v \in [0, \bar{v}]$, thus no consumer would spill over to the EDLP retailer if selecting the markdown retailer in Period 2 and finding a stockout.

case (iv)a: $q \leq Q_1$. If $p_1 \leq v \leq \bar{v}$, the reasoning from case (iii)a applies after substituting $\bar{v}$ for v_0.

If $\delta p_1 \leq v < p_1$ or if $v \leq \delta p_1$, the reasoning from case (iii)a applies.

case (iv)b: $q > Q_1$. If $p_1 \leq v \leq \bar{v}$, by Preliminary Lemma 2, $U_1 \geq U_2$ iff $v > v_1$, with $v_1 > \bar{v}$. This implies $v < v_1$ and thus $U_1 < U_2$. By Preliminary Lemma 3, we have $U_2 \geq 0$, that is, the consumer selects the markdown retailer in Period 2 and does not spill over to the EDLP retailer in case of stockout.

If $\delta p_1 \leq v < p_1$ or if $v \leq \delta p_1$, the reasoning from case (iii)a applies. □

Proof of Proposition 2 This result follows from the market segmentation described in Proposition 1 and the assumption that consumers' reservation prices are uniformly distributed on $[0, \bar{v}]$. □

Proof of Lemma 1 We first state the first part of the Lemma in detail and next prove the lemma.

Lemma 1 (Unabridged):
Over the region of EDLP price where scenario (i) of Proposition 2 holds, the EDLP retailer's optimal price response and profit are as follows:

$$\begin{array}{lll} p_E = \delta p_1, & \Pi_E = N(\delta p_1 - c)(\bar{v} - \delta p_1)/\bar{v}, & \textit{if } p_1 \in [0, (\bar{v}+c)/(2\delta)); \\ p_E = (\bar{v}+c)/2, & \Pi_E = N(\bar{v}-c)^2/(4\bar{v}), & \textit{if } p_1 \in [(\bar{v}+c)/(2\delta), \infty). \end{array}$$

Over the region of EDLP price where scenario (ii) of Proposition 2 holds, the EDLP retailer's optimal price response and profit are as follows: if $q = 1$, then $\Pi_E = 0$ for all p_E; otherwise, that is, $q < 1$, then:

$$\begin{array}{lll} p_E = p_1, & \Pi_E = (1-q)N(p_1 - c)(\bar{v} - p_1)/\bar{v}, & \textit{if } p_1 \in [0, (\bar{v}+c)/2); \\ p_E = (\bar{v}+c)/2, & \Pi_E = (1-q)N(\bar{v}-c)^2/(4\bar{v}), & \textit{if } p_1 \in [(\bar{v}+c)/2, (\bar{v}+c)/(2\delta)); \\ p_E = \delta p_1 + \epsilon, & \Pi_E = (1-q)N(\delta p_1 - c)(\bar{v} - \delta p_1)/\bar{v}, & \textit{if } p_1 \in [(\bar{v}+c)/(2\delta), \infty), \end{array}$$

where $\epsilon > 0$ is a small increment[1] (such as \$0.01). Over the region of EDLP price where scenario (iii) of Proposition 2 holds, the markdown retailer's price $p_1 \in [0, \bar{v})$ and the EDLP retailer's optimal price response and profit are as follows: if $q = 1$, then $\Pi_E = 0$ for all p_E; otherwise, that is, $q < 1$, then:

$p_E =$	$\Pi_E = (1-q)\,\frac{N}{\bar{v}} \times \dots$	*if* ...
$p_1 \frac{1-\delta q}{1-q} - \epsilon$	$(p_1 \frac{1-\delta q}{1-q} - c)\left(\bar{v} - p_1 \frac{1-\delta q + \alpha q(1-\delta) + \beta(1-q)}{(1+\beta)(1-q)}\right)$	$p_1 \in [0, \min\{\bar{v},\ l_1\}) \cap [0, l_3]$
$\frac{\bar{v}(1+\beta) + c(1+\alpha) - p_1(\beta-\alpha)}{2(1+\alpha)}$	$\frac{[\bar{v}(1+\beta) - c(1+\alpha) - p_1(\beta-\alpha)]^2}{4(1+\alpha)(1+\beta)}$	$p_1 \in [0, \min\{\bar{v},\ l_2\}) \cap (l_3, l_4]$[2]
$\frac{\bar{v}(1+\beta) + (\alpha-\beta)p_1}{1+\alpha}$	0	$p_1 \in [l_1, \bar{v}) \cap (l_4, \infty)$[3]
$p_1 + \epsilon$	$(p_1 - c)(\bar{v} - p_1)$	$p_1 \in [l_2, \bar{v})$

where $l_1 = \bar{v} \frac{(1+\beta)(1-q)}{1-\delta q + \alpha q(1-\delta) + \beta(1-q)}$, $l_2 = \frac{\bar{v}(1+\beta) + c(1+\alpha)}{2+\alpha+\beta}$, $l_3 = (1-q)\frac{\bar{v}(1+\beta) + c(1+\alpha)}{2(1+\alpha)(1-\delta q) + (1-q)(\beta-\alpha)}$, $l_4 = \frac{c(1+\alpha) - \bar{v}(1+\beta)}{\alpha-\beta}$, *and* $\epsilon > 0$ *is a small increment (such as \$0.01).*

[1] Technically, when the solution of a constrained continuous optimization problem is on the boundary of the feasible region but the boundary is excluded from the feasible set, the problem has no solution mathematically. We assume here that when such a situation occurs, we can set the optimal solution at the value located a small increment ϵ away from the boundary.

Part (i): Under case (i), the EDLP retailer solves the following maximization problem:

$$\max_{0 \le p_E \le \delta p_1} \quad \frac{N}{\bar{v}}(p_E - c)(\bar{v} - p_E).$$

The optimal solution follows from the concavity of the objective and using the first-order conditions.

Under case (ii), the EDLP retailer solves the following maximization problem:

$$\max_{\delta p_1 < p_E \le p_1} \quad (1-q)\frac{N}{\bar{v}}(p_E - c)(\bar{v} - p_E).$$

The optimal solution follows from the concavity of the objective and using the first-order conditions.

Under case (iii), the EDLP retailer makes a non-zero profit only when the availability probability exceeds Q_0. Hence, the EDLP retailer solves the following maximization problem (which is feasible only when $p_1 < \bar{v}$):

$$\begin{aligned} \max_{p_E} \quad & (1-q)\frac{N}{\bar{v}}(p_E - c)(\bar{v} - v_0) \\ \text{s.t.} \quad & p_1 < p_E \le \bar{p} \\ & q > Q_0 \equiv \frac{p_E - p_1}{p_E - \delta p_1} \\ & v_0 = \frac{p_E + \alpha(p_E - p_1) + \beta p_1}{1+\beta}. \end{aligned}$$

Note that when $q = 1$, the profit is zero for any p_E. When $q < 1$, the problem can be rewritten as follows:

$$\begin{aligned} \max_{p_E} \quad & (1-q)\,\frac{N}{\bar{v}}(p_E - c)\left(\bar{v} - \frac{p_E + \alpha(p_E - p_1) + \beta p_1}{1+\beta}\right) \\ \text{s.t.} \quad & p_1 < p_E \le \bar{p} \\ & \frac{p_E - p_1}{p_E - \delta p_1} < q. \end{aligned}$$

We can rewrite the last inequality constraint as $p_E < p_1 \frac{1-\delta q}{1-q}$, where the right-hand side is larger than p_1. Hence, the inequality is compatible with the bounds on p_E. After plugging in the definition of $\bar{p}$ and simplifying th terms, we find that

$$p_1 \frac{1-\delta q}{1-q} < \bar{p} \Leftrightarrow p_1 < \bar{v}\frac{(1+\beta)(1-q)}{1-\delta q + \alpha q(1-\delta) + \beta(1-q)} = l_1,$$

where l_1 can be shown to be lower than $\bar{v}$. Hence, when $q < 1$ and $p_1 < l_1$, the problem can be rewritten as follows:

$$\begin{aligned} \max_{p_E} \quad & (1-q)\,\frac{N}{\bar{v}}(p_E - c)\left(\bar{v} - \frac{p_E + \alpha(p_E - p_1) + \beta p_1}{1+\beta}\right) \\ \text{s.t.} \quad & p_1 < p_E < p_1 \frac{1-\delta q}{1-q}, \end{aligned}$$

[2] When $\beta < \alpha$ the condition becomes $p_1 \in [0, \min\{\bar{v},\ l_2\}) \cap ((l_3, l_1) \cup [\max\{l_1, l_4\}, \infty))$. When $\beta = \alpha$, the condition becomes $p_1 \in (l_3, l_2)$.

[3] When $\beta < \alpha$ the condition becomes $p_1 \in [l_1, \min\{\bar{v}, l_4\})$. When $\beta = \alpha$, this case does not occur.

while when $q<1$ and $l_1 \le p_1 < \bar{v}$, the problem can be rewritten as follows:

$$\max_{p_E} \quad (1-q)\,\frac{N}{\bar{v}}(p_E - c)\left(\bar{v} - \frac{p_E + \alpha(p_E - p_1) + \beta p_1}{1+\beta}\right)$$
$$\text{s.t.} \quad p_1 < p_E \le \bar{p}.$$

In either case, this problem can thus be seen as a concave maximization problem over an interval. Using the first-order conditions, it is straightforward that, when $q<1$ and $p_1 < l_1$, the optimal solution is

$$p_E = \begin{cases} \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} & \text{if } p_1 < \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} < p_1\frac{1-\delta q}{1-q} \\ p_1 + \epsilon & \text{if } \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} \le p_1 \\ p_1\frac{1-\delta q}{1-q} - \epsilon & \text{if } \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} \ge p_1\frac{1-\delta q}{1-q}, \end{cases}$$

that is,

$$p_E = \begin{cases} \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} & \text{if } l_3 = (1-q)\frac{\bar{v}(1+\beta)+c(1+\alpha)}{2(1+\alpha)(1-\delta q)+(1-q)(\beta-\alpha)} < p_1 < \frac{\bar{v}(1+\beta)+c(1+\alpha)}{2+\alpha+\beta} = l_2 \\ p_1 + \epsilon & \text{if } p_1 \ge l_2 \\ p_1\frac{1-\delta q}{1-q} - \epsilon & \text{if } p_1 \le l_3. \end{cases}$$

The EDLP profit is then

$$\Pi_E = \begin{cases} (1-q)\,\frac{N}{\bar{v}}\,\frac{[\bar{v}(1+\beta)-c(1+\alpha)-p_1(\beta-\alpha)]^2}{4(1+\alpha)(1+\beta)} & \text{if } l_3 < p_1 < l_2 \\ (1-q)\,\frac{N}{\bar{v}}(p_1-c)(\bar{v}-p_1) & \text{if } p_1 \ge l_2 \\ (1-q)\,\frac{N}{\bar{v}}\left(p_1\frac{1-\delta q}{1-q} - c\right)\left(\bar{v} - p_1\frac{1-\delta q+\alpha q(1-\delta)+\beta(1-q)}{(1+\beta)(1-q)}\right) & \text{if } p_1 \le l_3, \end{cases}$$

When $q<1$ and $l_1 \le p_1 < \bar{v}$, the optimal solution is

$$p_E = \begin{cases} \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} & \text{if } p_1 < \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} \le \bar{p} \\ p_1 + \epsilon & \text{if } \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} \le p_1 \\ \bar{p} & \text{if } \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} > \bar{p}, \end{cases}$$

that is,

$$p_E = \begin{cases} \frac{\bar{v}(1+\beta)+c(1+\alpha)-p_1(\beta-\alpha)}{2(1+\alpha)} & \text{if } p_1(\alpha-\beta) \ge c(1+\alpha) - \bar{v}(1+\beta) \equiv l_4' \text{ and } p_1 < l_2 \\ p_1 + \epsilon & \text{if } p_1 \ge l_2 \\ \bar{p} & \text{if } p_1(\alpha-\beta) < l_4'. \end{cases}$$

The EDLP profit is then

$$\Pi_E = \begin{cases} (1-q)\,\frac{N}{\bar{v}}\,\frac{[\bar{v}(1+\beta)-c(1+\alpha)-p_1(\beta-\alpha)]^2}{4(1+\alpha)(1+\beta)} & \text{if } p_1(\alpha-\beta) \ge l_4' \text{ and } p_1 < l_2 \\ (1-q)\,\frac{N}{\bar{v}}(p_1-c)(\bar{v}-p_1) & \text{if } p_1 \ge l_2 \\ 0 & \text{if } p_1(\alpha-\beta) < l_4'. \end{cases}$$

Part (ii): In case (i), it follows from the expressions in Lemma 1 that the best response decision is independent of the regret factors. In case (ii), it follows from the expressions in Lemma 1 that the best response decision is independent of the regret factors. In case (iii), it follows from the expressions in Lemma 1 that the best response decision is independent of the regret factors when $p_1 \in [0, \min\{\bar{v},\ l_1\}) \cap [0, l_3]$ or when $p_1 \in [l_2, \bar{v})$. When $p_1 \in [0, \min\{\bar{v},\ l_2\}) \cap (l_3, l_4]$, we have $\partial p_E/\partial\beta = (\bar{v} - p_1)/(2+2\alpha) > 0$, and $\partial p_E/\partial\alpha =$

$(p_1 - \bar{v})(1+\beta)/[2(1+\alpha)^2] < 0$. When $p_1 \in [l_1, \bar{v}) \cap (l_4, \infty)$, we have $\partial p_E/\partial\beta = (\bar{v} - p_1)/(1+\alpha) > 0$, and $\partial p_E/\partial\alpha = (p_1 - \bar{v})(1+\beta)/(1+\alpha)^2 < 0$. In case (iv), the EDLP retailer achieves zero profit regardless of its decision. □

Proof of Proposition 3 Parts (i) and (ii) follow from Proposition 2 when cases (ii) or (iii) hold. Part (iii): $q < 1$ is necessary to guarantee that $\Pi_E > 0$ so that the EDLP retailer responds by selecting case (ii) or case (iii) with $q > Q_0$. Finally, part (iv) follows from Proposition 1 when cases (ii) or (iii) with $q > Q_0$ hold. □

Proof of Proposition 4 We proceed by backward induction, considering that p_E is fixed, and we solve for the markdown retailer's best response price p_1 and stock level K. These decisions determine which of scenarios (i), (ii), (iii) and (iv) from Proposition 2 holds, and hence what the demands and profits at each retailer are (according to Proposition 2). At its best response, the markdown retailer would not select a price leading to scenario (i), as in scenario (i) the markdown retailer receives no demand ($D_1 = D_2 = 0$) and hence makes no profit ($\Pi_M = 0$). Thus, we have $\delta p_1 < p_E$. If the markdown retailer selects its price and stock level to be in scenario (iv) or scenario (iii) with $q \le Q_0$, the EDLP retailer receives no demand ($D_E = 0$) and hence makes zero profit ($\Pi_E = 0$). If the markdown retailer selects scenario (ii) or scenario (iii) with $q > Q_0$, it makes all its sales in Period 2 ($D_1 = 0$). Therefore, the markdown retailer's profit is $\Pi_M = (\delta p_1 - c)qD_2 = (\delta p_1 - c)q\frac{N}{\bar{v}}(\bar{v} - \delta p_1)$, a quantity that is increasing in q. Hence, it is optimal for the markdown retailer to select a stock level high enough to satisfy all Period 2 demand, i.e., $q = 1$ (that is, $K = (N/\bar{v})(\bar{v} - \delta p_1)$). It thus follows that the EDLP retailer receives no demand, and thus makes zero profit.□

Proof of Proposition A1 If the EDLP sets its price above c_M, according to Proposition 4, the markdown chooses to enter and sets its price in a away that leaves no profit to the EDLP retailer. This choice of EDLP price is thus not optimal for the EDLP retailer. However, if the EDLP retailer sets its price in $(c_E, c_M]$, the markdown retailer cannot set its discount price below the EDLP price (to ensure that scenario (i) does not occur) without eliminating any profit margin. Meanwhile, because of its cost advantage, the EDLP retailer still maintains a profit margin even at that low price point. Hence, the markdown retailer does not enter, and the EDLP retailer captures all demand and achieves a positive profit. The optimal price is the price that maximizes $(p_E - c_E)(\bar{v} - p_E)$ on $(c_E, c_M]$. □

Proof of Proposition A2 When the EDLP retailer moves first, according to Proposition 4, the markdown retailer's best response pricing and stocking decisions are such that either (a) scenario (ii) with $q = 1$ or scenario (iii) with $q = 1$ of Proposition 2 occur, or (b) scenario (iii) with $q \le Q_0$ or scenario (iv) of Proposition 2 occur. When the markdown retailer moves first, according to Section 5.1, the EDLP retailer's best response price is such that either (c) scenario (i) of Proposition 2 occurs, or, if $q < 1$, (d) scenario (ii) of Proposition 2 occurs, or, if $q < 1$, (d) scenario (iii) with $q > Q_0$ of Proposition 2 occurs. We observe that these best responses have no overlap. As a result, there is no Nash equilibrium. □

Proof of Proposition A3 Case (i): $p_E \le \delta p_1$; i.e., $U_1 \le U_E$, $p_1 \ge v_3$, $p_1 \ge v_2$ and $v_2 \le p_E$. In addition, $v \ge p_1$ implies $v \ge p_E$ on this domain for p_E, hence $U_E \ge 0$. The consumer chooses to purchase from the

EDLP retailer. If $v < p_1$, we distinguish two cases: $p_E \leq q\delta p_1$ and $p_E > q\delta p_1$. If $p_E \leq q\delta p_1$, we have $v_2 \leq 0$, so $v \geq v_2$, resulting in $U_E \geq U_2$. The consumer purchases from the EDLP retailer if $U_E \geq 0$ (i.e., $v \geq p_E$) and does not purchase otherwise. If $p_E > q\delta p_1$, $v_2 > 0$ and the inequality $p_E \leq \delta p_1$ implies $v_2 \leq \delta p_1$. Hence, when $v < v_2$, we have $U_E \leq U_2 < 0$ and the consumer chooses not to purchase. When $v_2 \leq v < p_E$, $0 > U_E \geq U_2$, the consumer does not purchase. When $v \geq p_E (\geq v_2)$, then $U_E \geq U_2$ and $U_E \geq 0$, and the consumer purchases from the EDLP retailer.

Case (ii): $\delta p_1 < p_E \leq (1 - q(1-\delta))p_1$; i.e., $U_1 \leq U_E$, $p_1 \geq v_3$, $p_1 \geq v_2$ and $v_2 > p_E$. If $v \geq p_1$, then $v \geq v_3$ so $U_2 \leq U_E$. By the same reasoning as Case (i), if $v \geq p_1$ (which implies $v \geq p_E$), the consumer chooses to purchase from the EDLP retailer. If $v < p_1$, $U_2 > U_E$ when $v < v_2 (\leq p_1)$, and $U_2 \leq U_E$ when $v_2 \leq v \leq \bar{v}$. In addition, if $v < p_1$, $U_2 \geq 0$ iff $v \geq \delta p_1$. Furthermore, because $\delta p_1 < p_E$, we have $v_2 > p_E$, hence $U_E \geq 0$ when $v \geq v_2$. Hence the consumer chooses the markdown retailer in Period 2 if $\delta p_1 \leq v < v_2$ and chooses the EDLP retailer if $v_2 \leq v \leq \bar{v}$. The latter requires $v_2 \leq \bar{v}$, that is, $q \leq Q_2$.

Case (iii): $(1 - q(1-\delta))p_1 < p_E \leq (1 + \alpha q(1-\delta))p_1$; i.e., $U_1 \leq U_E$, $p_1 < v_3$ and $p_1 < v_2$. The consumer decides between purchasing from the markdown retailer in Period 2, purchasing from the EDLP retailer, or not purchasing at all. If $v < p_1$, we have $v < v_2$ so $U_2 = q(v - \delta p_1) > U_E$. Thus the consumer chooses the markdown retailer in Period 2 if $\delta p_1 \leq v < p_1$, and does not purchase if $v < \delta p_1$. If $v \geq p_1$, $U_2 = q(v - \delta p_1) - (1-q)\beta(v - p_1) > U_E$ when $v < v_3$, and $U_2 \leq U_E$ otherwise. In addition, when $v \geq p_1$, from Özer and Zheng (2016, Proposition 1), $U_2 \geq 0$. Hence the consumer chooses the markdown retailer in Period 2 if $p_1 \leq v < v_3$ and chooses the EDLP retailer if $v_3 \leq v \leq \bar{v}$. The latter requires $v_3 \leq \bar{v}$, that is, $q \leq Q_3$.

Case (iv): $p_E > (1 + \alpha q(1-\delta))p_1$; i.e., $U_1 > U_E$. The consumer decides between purchasing from the markdown retailer in Period 1 or in Period 2 or not purchasing at all. The result follows from Özer and Zheng (2016, Proposition 1). □

References

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