Online Appendix for "Consumer Search and Retail Market Structure"

Andrew Rhodes
Toulouse School of Economics
University of Toulouse Capitole

Jidong Zhou
School of Management
Yale University

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In this supplementary document, we characterize the asymmetric equilibrium in the symmetric market with two multiproduct firms A and B. Suppose all consumers visit firm A first in equilibrium. Let p^i_{mk} be firm k's price for its product i. Let $\mathbf{\Delta} = (\Delta_1, \Delta_2)$, where $\Delta_i \equiv p^i_{mB} - p^i_{mA}$ is the price difference of product i across firms. Denote by $S(\mathbf{\Delta})$ the stopping region in firm A's match utility space, and let $NS(\mathbf{\Delta})$ be the complement. The stopping region is characterized by a reservation frontier $\phi_{\mathbf{\Delta}}(u_1) \equiv \phi(u_1 + \Delta_1) - \Delta_2$, where $\phi(\cdot)$ is the reservation frontier in the symmetric case with $\mathbf{\Delta} = \mathbf{0}$ and it solves

$$\int_{u_1}^{\overline{u}} [1 - G(x)]dx + \int_{\phi(u_1)}^{\overline{u}} [1 - G(x)]dx = s \ .$$

A consumer will stop searching and buy both products immediately at firm A if the match utilities discovered there are such that $u_2 > \phi_{\mathbf{\Delta}}(u_1)$.

Since each firm's two products are symmetric, we look for an equilibrium where $p^1_{mk} = p^2_{mk} = p_{mk}$ and $\Delta_1 = \Delta_2 = \Delta = p_{mB} - p_{mA}$. Suppose an equilibrium with $\Delta \in (0, \overline{u} - \underline{u})$ exists for any $s \in (0, \overline{s})$ and in equilibrium all consumers buy (i.e., the market is fully covered).[1] Let π_{mk} be firm k's equilibrium profit from each product. Then the asymmetric market structure arises in equilibrium if the second pair of single-product firms become the non-prominent firm after merger and $\pi_{mB} < \pi_B$, or if the second pair become the prominent one after merger and $\pi_{mA} < \pi_B$. While if these inequalities are violated the equilibrium market structure has two multiproduct firms. The following two graphs depict the reservation frontier in equilibrium:

[1] If $\Delta \geq \overline{u} - \underline{u}$, no consumers will search beyond the first firm even if search is almost costless.

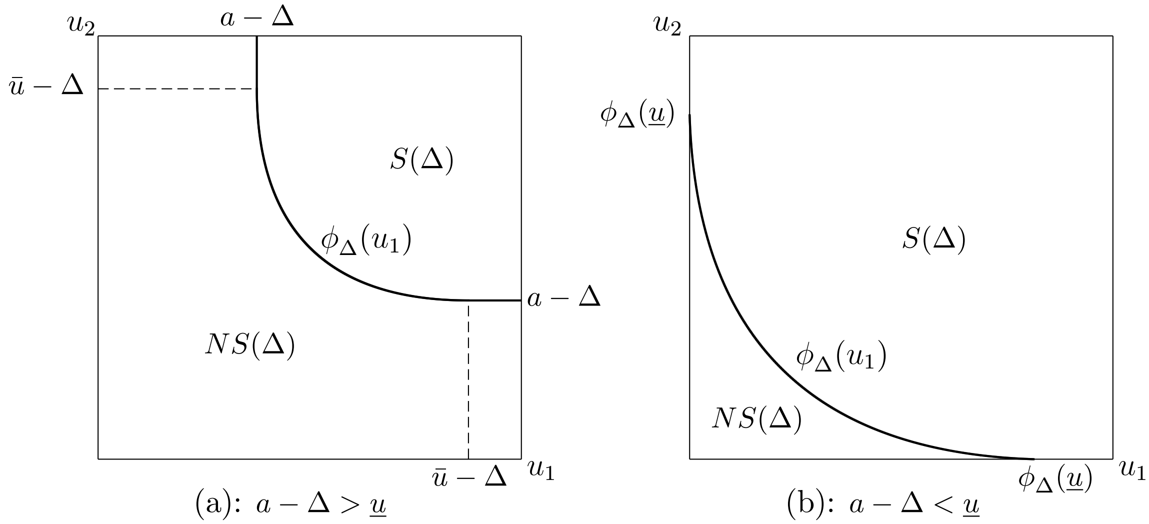


Figure 3: The reservation frontier in asymmetric equilibrium

Figure 3(a) is the case for $a-\Delta > \underline{u}$, where $\phi_{\mathbf{\Delta}}(u_1)$ has a vertical segment with $\phi_{\mathbf{\Delta}}(u_1) \in [\overline{u}-\Delta, \overline{u}]$ at $u_1 = a-\Delta$, and a horizontal segment with $\phi_{\mathbf{\Delta}}(u_1) = a-\Delta$ for $u_1 \in [\overline{u}-\Delta, \overline{u}]$; Figure 3(b) is the case for $a-\Delta < \underline{u}$, where $\phi_{\mathbf{\Delta}}(u_1)$ hits the vertical axis at $u_2 = \phi_{\mathbf{\Delta}}(\underline{u})$ and hits the horizontal axis at $u_1 = \phi_{\mathbf{\Delta}}(\underline{u})$ (where we have used the fact $\phi^{-1}(\cdot) = \phi(\cdot)$ since the two products are symmetric). Notice that $a = \overline{u}$ at $s = 0$, so the first case applies when s is small; while $a = \underline{u}$ at $s = \overline{s}$, so the second case applies when s is large.

We assume that the equilibrium prices p_{mA} and p_{mB} are determined by the first-order conditions (up to some possible corner solution adjustment when $a-\Delta = \underline{u}$ as we will discuss later). We first consider the case with $a-\Delta > \underline{u}$ (so that Figure 3(a) applies). Suppose firm A unilaterally deviates and charges $p_{mA}-\varepsilon$ for its product 2 so that $\Delta_2 = \Delta+\varepsilon$. This shifts the reservation frontier downward by ε everywhere. Let $\mathbf{\Delta}(\varepsilon) = (\Delta, \Delta+\varepsilon)$. Then firm A's deviation profit is

$$(2p_{mA}-\varepsilon)\int_{S(\mathbf{\Delta}(\varepsilon))} dG(\mathbf{u}) + \int_{NS(\mathbf{\Delta}(\varepsilon))} [p_{mA}G(u_1+\Delta) + (p_{mA}-\varepsilon)G(u_2+\Delta+\varepsilon)]dG(\mathbf{u}) \ .$$

Here the first term is the profit from consumers who buy immediately at firm A, and the second term is the profit from consumers who choose to search on and visit firm B but eventually come back to buy something from firm A (where $G(u_1+\Delta)$ is the chance that firm A's product 1 is better than firm B's product 1 and $G(u_2+\Delta+\varepsilon)$ is the chance that firm A's product 2 is better than firm B's product 2). Noticing that the price deviation affects both $S(\mathbf{\Delta}(\varepsilon))$ and $NS(\mathbf{\Delta}(\varepsilon))$, one can check that the first-order condition implies

$$p_{mA}=\frac{\int_{S(\mathbf{\Delta})}dG(\mathbf{u})+\int_{NS(\mathbf{\Delta})}G(u_2+\Delta)dG(\mathbf{u})}{\int_{NS(\mathbf{\Delta})}g(u_2+\Delta)dG(\mathbf{u})+\int_{a-\Delta}^{\overline{u}}[2-G(u_1+\Delta)-G(\phi(u_1+\Delta))]g(\phi_{\mathbf{\Delta}}(u_1))dG(u_1)}\ . \tag{1}$$

Here the numerator is the equilibrium demand for firm A's product 2.

Suppose now firm B unilaterally deviates and charges $p_{mB}-\varepsilon$ for its product 2. Then firm B's deviation profit is

$$\int_{NS(\mathbf{\Delta})}\{p_{mB}[1-G(u_1+\Delta)]+(p_{mB}-\varepsilon)[1-G(u_2+\Delta-\varepsilon)]\}dG(\mathbf{u})\ .$$

When a consumer who has discovered match utilities (u_1,u_2) at firm A comes to visit firm B, she will buy firm B's product 1 with probability $1-G(u_1+\Delta)$ and buy firm B's product 2 with probability $1-G(u_2+\Delta-\varepsilon)$. Notice that here the price deviation does not appear in $NS(\Delta)$, since whether a consumer will come to visit firm B or not depends on the expected equilibrium prices of firm B (instead of the actual deviation price). This also implies that firm B's pricing problem is totally separable between the two products. The first-order condition is then

$$p_{mB}=\frac{\int_{NS(\mathbf{\Delta})}[1-G(u_2+\Delta)]dG(\mathbf{u})}{\int_{NS(\mathbf{\Delta})}g(u_2+\Delta)dG(\mathbf{u})}\ . \tag{2}$$

Here the numerator is the equilibrium demand for firm B's product 2. (Given full market coverage, the sum of the two numerators in (1) and (2) equals 1.)

When $a-\Delta<\underline{u}$ (so that Figure 3(b) applies), the first-order conditions are the same except that $\int_{a-\Delta}^{\overline{u}}$ in the denominator of (1) is replaced by $\int_{\underline{u}}^{\phi_{\mathbf{\Delta}}(\underline{u})}$. An analytical investigation of the system of the first-order conditions is harder than in the case of asymmetric market structure. However, an approximation analysis when s is close to 0 or equivalently when a is close to $\overline{u}$ (in which case Figure 3(a) applies) can be done. As a result, we can prove a result parallel to result (i) in Proposition 2 in the main paper.

Claim 1 *Suppose $g(\underline{u}),g(\overline{u})>0$ and that two multiproduct firms play an asymmetric equilibrium. There exist $0<\hat{s}_1<\hat{s}_2<\overline{s}$ such that the equilibrium market structure is asymmetric if $s<\hat{s}_1$ and symmetric with two multiproduct firms if $s>\hat{s}_2$.*

Proof. As in the proof of result (ii) in Proposition 1 of the main paper, we approximate prices when a is close to $\bar{u}$ (or equivalently, s is close to 0). Hence the relevant prices to consider are those in equations (1) and (2). Consider $a=\bar{u}-\epsilon$ where $\epsilon>0$ but very small. Suppose the (first-order) linear approximations of the equilibrium prices are $p_{mA} \approx p + k_{mA}\epsilon$ and $p_{mB} \approx p + k_{mB}\epsilon$, where p is the price that prevails under full information and solves $1/p = 2\int_{\underline{u}}^{\overline{u}} g(u)^2\,du$. We now solve for k_{mA} and k_{mB}, and let $\delta = k_{mB} - k_{mA}$.

First consider the expression (2) for p_{mB}. The numerator can be written more explicitly as

$$\int_{\underline{u}}^{a-\Delta}\left(\int_{\underline{u}}^{\overline{u}-\Delta}\left[1-G(u_2+\Delta)\right]dG(u_2)\right)dG(u_1) + \int_{a-\Delta}^{\overline{u}-\Delta}\left(\int_{\underline{u}}^{\phi(u_1+\Delta)-\Delta}\left[1-G(u_2+\Delta)\right]dG(u_2)\right)dG(u_1) + \int_{\overline{u}-\Delta}^{\overline{u}}\left(\int_{\underline{u}}^{a-\Delta}\left[1-G(u_2+\Delta)\right]dG(u_2)\right)dG(u_1)\ .$$

(Recall that $\phi_{\boldsymbol{\Delta}}(u_1) = \phi(u_1+\Delta) - \Delta$.) Substituting in $a = \overline{u} - \epsilon$ and $\Delta = \delta\epsilon$, we can then write the numerator of (2) in terms of ϵ:

$$\int_{\underline{u}}^{\overline{u}-\epsilon-\delta\epsilon}\left(\int_{\underline{u}}^{\overline{u}-\delta\epsilon}\left[1-G(u_2+\delta\epsilon)\right]dG(u_2)\right)dG(u_1) + \int_{\overline{u}-\epsilon-\delta\epsilon}^{\overline{u}-\delta\epsilon}\left(\int_{\underline{u}}^{\phi(u_1+\delta\epsilon)-\delta\epsilon}\left[1-G(u_2+\delta\epsilon)\right]dG(u_2)\right)dG(u_1) + \int_{\overline{u}-\delta\epsilon}^{\overline{u}}\left(\int_{\underline{u}}^{\overline{u}-\epsilon-\delta\epsilon}\left[1-G(u_2+\delta\epsilon)\right]dG(u_2)\right)dG(u_1)\ .$$

Using the first-order Taylor approximation around the point $\epsilon = 0$, the first term in this expression is approximated by

$$\frac{1}{2} - \left[\frac{(1+\delta)\,g(\overline{u})}{2} + \frac{\delta}{2p}\right]\epsilon\ ,$$

whilst the second term is approximately equal to $g(\overline{u})\,\epsilon/2$ and the third term is approximately equal to $\delta g(\overline{u})\,\epsilon/2$. Hence we conclude that

$$\int_{NS(\Delta)}\left[1-G(u_2+\Delta)\right]dG(u) \approx \frac{1}{2} - \frac{\delta}{2p}\epsilon\ . \tag{3}$$

Following the same procedure, it is also straightforward to derive that

$$\int_{NS(\Delta)} g(u_2+\Delta)\,dG(u) \approx \frac{1}{2p} - \delta\frac{g(\underline{u})^2 + g(\bar{u})^2}{2}\epsilon\ . \tag{4}$$

Consequently using equation (2) and dropping higher order terms, we obtain the following equation which determines k_{mA} and k_{mB}:

$$k_{mB} + \delta - \delta\left[g(\overline{u})^2 + g(\underline{u})^2\right]p^2 = 0\ . \tag{5}$$

Second consider the expression (1) for p_{mA}. Since the numerator is firm A's demand, which consists of all consumers who do not purchase from firm B, we can immediately use equation (3) to infer that

$$\int_{S(\Delta)} dG(u) + \int_{NS(\Delta)} G(u_2 + \Delta)\, dG(u) \approx \frac{1}{2} + \frac{\delta}{2p}\epsilon \ .$$

Moreover the first term in the denominator of equation (1) has already been approximated above in equation (4). In addition it is straightforward to see that the second term in the denominator is not first order. Combining this information with equation (1), and again dropping higher order terms, we obtain another equation which determines k_{mA} and k_{mB}:

$$k_{mA} - \delta - \delta\left[g(\overline{u})^2 + g(\underline{u})^2\right]p^2 = 0 \ . \tag{6}$$

Notice that equations (5) and (6) have a unique solution given by $k_{mA} = k_{mB} = 0$ (and so $\delta = 0$), and thus we conclude that $p_{mA} \approx p_{mB} \approx p$, that $\int_{NS(\Delta)} [1 - G(u_2 + \Delta)]\, dG(u) \approx 1/2$, and hence $\pi_{mA} \approx \pi_{mB} \approx p/2$. We have shown in the proof of result (i) in Proposition 2 in the main paper that π_B in the asymmetric market is greater than $p/2$ when $a = \overline{u} - \epsilon$. Therefore, we can conclude that $\pi_{mA}, \pi_{mB} < \pi_B$ when a is sufficiently close to $\overline{u}$, or equivalently when the search cost is sufficiently small. This implies an asymmetric market structure for small s.

(ii) Suppose now that s is close to $\overline{s}$. Clearly since firm A is searched first it must earn a strictly positive profit (i.e. $\pi_{mA} > 0$). We now argue that firm B must also earn strictly positive profit (i.e. $\pi_{mB} > 0$). Suppose to the contrary that it does not (i.e. all consumers buy immediately at firm A). (i) Suppose that the consumer who finds $(\underline{u}, \underline{u})$ at firm A strictly prefers to buy immediately without searching. Then firm A could slightly increase both prices without losing any demand, which would contradict the assumption that its price is determined by the first-order condition. (ii) Suppose instead that the consumer who finds $(\underline{u}, \underline{u})$ at firm A is just indifferent between searching and not. Then if firm A slightly increases both prices, consumers around the corner of $(\underline{u}, \underline{u})$ in the match utility space will start searching firm B. In other words, the non-stopping region of NS will appear around that corner. But this only has a second-order effect on firm A's demand, so firm A's deviation must be profitable. This again yields a contradiction.

In the proof of result (i) of Proposition 2 in the main paper, we have shown that $\pi_B \to 0$ as $s \to \overline{s}$. Therefore, we have $\pi_{mA}, \pi_{mB} > \pi_B$ when $s \to \overline{s}$. This implies that the second pair of single-product firms will choose to merge, and so a symmetric market structure with two multiproduct firms arises in equilibrium for large s. ■

We now proceed to study the uniform-distribution example. When $a > \Delta$, one can check that the first-order conditions simplify to

$$p_{mA} = \frac{Q(\Delta)}{1+s-\Delta} \tag{7}$$

and

$$p_{mB} = \frac{1-Q(\Delta)}{1-\frac{1}{2}\pi s-(1+\sqrt{2s})\Delta}\ , \tag{8}$$

where $Q(\Delta) = \frac{1}{2}+\frac{2}{3}s\sqrt{2s}+(1+s)\Delta-\frac{1}{2}\Delta^2$ is the demand for a product of firm A. When $a < \Delta$, the first-order conditions simplify to

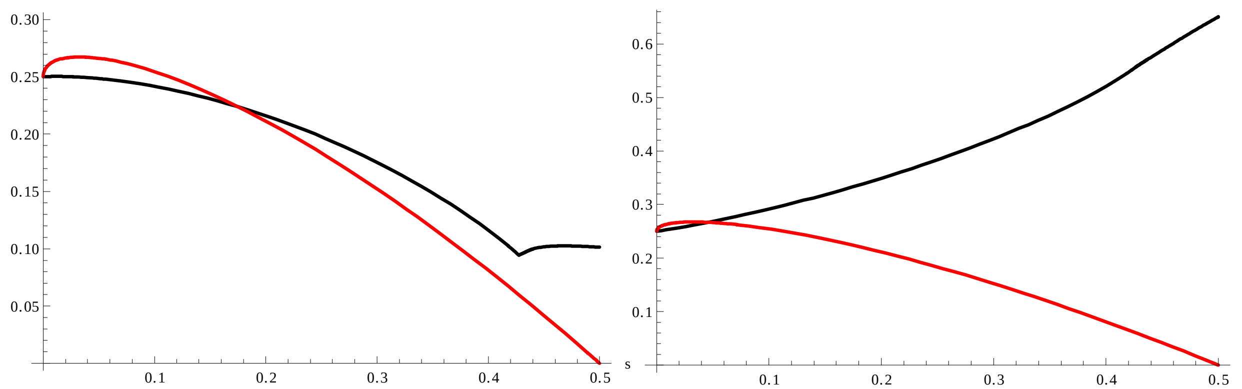
$$p_{mA} = \frac{\hat{Q}(\Delta)}{3s-2A(\Delta)-(1-\Delta)\sqrt{A(\Delta)}} \tag{9}$$

and

$$p_{mB} = \frac{1-\hat{Q}(\Delta)}{\int_0^{\phi(\Delta)-\Delta}\int_0^{\phi(u_1+\Delta)-\Delta} du_2 du_1} \tag{10}$$

where $A(\Delta) = 2s-(1-\Delta)^2$, $\phi(\Delta) = 1-\sqrt{A(\Delta)}$, and $\hat{Q}(\Delta) = 1-\frac{1-\Delta}{3}(s-2A(\Delta))-\frac{A(\Delta)}{3}\sqrt{A(\Delta)}$ is the demand for a product of firm A. (The denominator in the p_{mB} equation does not have a simple elementary expression.) Unfortunately, neither of the two systems has a simple analytical solution. But numerical calculation is easy to do. In the following, we report the details.

The exact nature of the equilibrium depends on how s compares with two thresholds s' and s'', where $s' \approx 0.427$ and $s'' \approx 0.436$. When $s < s'$ the equilibrium prices satisfy $a > \Delta$ and jointly solve equations (7) and (8). On the other hand, when $s > s''$ the equilibrium prices satisfy $a < \Delta$ and jointly solve equations (9) and (10). Interestingly we find that when $s \in (s', s'')$, the equilibrium prices satisfy $a = \Delta$ and firm B's price is pinned down by equation (8). In other words, in this case firm A's problem has a corner solution. To understand why, notice that for s relatively small the reservation frontier is as depicted in Figure 3(a). Therefore when firm A reduces its price for, say, product 2 the whole frontier shifts down, and demand is relatively price elastic. However as s increases, the reservation frontier moves south-west, and touches the axes when $s = s'$. At this point firm A faces a kinked demand curve and wants to price in such a way that $a = \Delta$. To see why, notice that if firm A increases price, the reservation frontier moves north-east and demand is relatively sensitive as before. However if firm A slightly decreases price, the situation resembles that depicted in Figure 3(b), where suddenly the horizontal segment on the reservation frontier with $u_1 \in [\overline{u}-\Delta, \overline{u}]$ disappears and so the length of the reservation frontier is decreased by a discrete amount; equivalently, demand is much less price sensitive. Finally though, as s increases from s' to s'', firm A becomes relatively more expensive because its price satisfies $a = \Delta$. Consequently at $s = s''$ it becomes worthwhile for firm A to reduce price in relative terms to attract more marginal consumers, and so the equilibrium solution again satisfies the interior first-order conditions.



(a): Non-prominent case (b) Prominent case

Figure 4: Profit comparison for second merger

Figures 4(a) and 4(b) above plot equilibrium profits in this uniform example. The red lines depict π_B, the profit earned by a non-prominent firm in the asymmetric market structure. The black lines depict (per-product) profit earned by the second pair of single-product firms if they proceed to merge: Figure 4(a) assumes they remain non-prominent after merger and so the black line is π_{mB}, whilst Figure 4(b) assumes that they become prominent after merger and so the black line is π_{mA}. In either case, the post-merger profit is less than π_B for a sufficiently small s. As reported in Section 5 in the main paper, the asymmetric market structure arises for s below approximately 0.17 and 0.045 respectively, and otherwise the equilibrium market structure consists of two multiproduct firms.[2]

[2]One interesting observation is that whilst π_{mA} monotonically increases in s, π_{mB} is non-monotonic. Intuitively the latter arises because as discussed above, once s reaches s' the prominent multiproduct firm has less incentive to reduce price. Consequently the prominent firm becomes a weaker competitor, which by strategic complementarity benefits the non-prominent multiproduct firm as well. However once s is sufficiently above s'', the prominent firm starts to become more aggressive again, and steals demand away from the non-prominent firm causing its profit to fall again.