Online Appendix to: (Non-)Precautionary Cash Hoarding and the Evolution of Growth Firms

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A Supplementary Material

A.1 Precautionary Hoarding

Assume that the time until the arrival of an investment opportunity follows an exponential distribution with parameter λ_a. It holds:

Proposition A.1 *The attractiveness of the firm's investment opportunity has opposite implications for hoarding depending on whether hoarding occurs in anticipation of an investment opportunity or whether it leads to the delay of an investment opportunity that is already present. In the former case, the manager hoards cash (until the investment opportunity arrives or she has sufficient funds at hand) only if the profitability of the investment opportunity is sufficiently high. In the latter case, the manager follows the hoarding and investment policies set out in Proposition 1.*

Proof of Proposition A.1. We only offer a sketch of the argument. To determine whether the manager should start hoarding, we have to compare the expected payoff from hoarding with paying out w_0. Since the investment opportunity's expected payoff is increasing in θ, there is a threshold $\widetilde{\theta}$, such that starting to hoard is optimal if $\theta > \widetilde{\theta}$.

For completeness, we briefly discuss the optimal hoarding strategy before the investment opportunity's arrival. Let w_0 denote the cash level that the manager has hoarded at the time of the arrival of the firm's investment opportunity. The expected value of the investment opportunity upon its arrival, $U(w_0, w_\theta^*)$, is given by (5), and it is strictly increasing and convex in w_0 for $w_0 < K$. This implies that the optimal hoarding level before arrival is $w_0^* \geq K$. To see this, suppose to a contradiction that the manager stops hoarding at $w_0^* < K$ and pays out $w_t - w_0^*$. Doing so cannot be optimal if hoarding until w_0^* is optimal. First, the probability of arrival is the same at every instant. Second, given that U is convex in w_0, the marginal increase in U is higher for any additionally hoarded unit of cash. In contrast, paying out a unit of cash has the same value to the manager regardless of the previously hoarded amount. Hence, if hoarding dominates paying out

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for $w_t < w_0^* < K$, it is even more beneficial for $w_t = w_0^*$, giving a contradiction. Hence, we must have that $w_0^* \geq K$.[1] **Q.E.D.**

A.2 Cash Hoarding when Delay Reduces Uncertainty

One of the results from Section *3.2* is that firms that choose private financing delay investment less than firms choosing public financing. We now show that this result is true even if delaying investment helps alleviate the uncertainty and disagreement about the project's fundamentals. As a simple modification to our baseline model, suppose that after receiving her initial signal, the manager believes that the project's value is $X(\theta)$ with probability p_0 (rather than one) and zero otherwise. The financier disagrees, believing that the value is $X(\theta)$ with probability $\rho < p_0$ and zero otherwise. Suppose further that before investing, the firm has a chance of observing a second signal that reveals whether the investment opportunity's value is $X(\theta)$ or zero is correct with certainty and is verifiable to all. The time until such an event follows an exponential distribution with parameter λ_e. If uncertainty disappears, the manager invests immediately if the project's value is $X(\theta)$, and her expected payoff is $X(\theta) - K$. If the project's value is zero, she abandons it and pays out w_t. If there is no signal, the manager can choose between investing and cash hoarding as in the baseline model, but also to continue waiting to observe a signal. For this extension, we assume in analogy to Section *3* that if the manager invests before the second signal arrives, the financier's monitoring and interference choice is $E \in \{0, 1\}$ and the manager's cost of interference are $\kappa(1)$ with $\phi \to \infty$. We assume that there is no monitoring and interference in case of the second fully-revealing signal.

The existence of such second reason to delay does not change that delay is less attractive when the firm's investment opportunity is better. If the manager uses delay also for hoarding, we obtain again that the manager delays and hoards less before investment, when choosing private financing. Hence, she is more likely to risk investing under uncertainty.

Proposition A.2 *Suppose that delaying investment could alleviate uncertainty about whether the investment opportunity's value is $X(\theta)$ or zero. When θ is higher, the manager is less willing to delay and is more likely to risk investing under uncertainty.*

Proof of Proposition A.2. We focus on the case in which the manager hoards cash while waiting. The alternative would be to pay out w_0, but this would have no implications for hoarding and the public versus private choice.

Given that the true value of the investment opportunity is revealed with probability λ_e, we can use (24) to derive the manager's expected payoff as

$$\frac{\lambda_e p_0 (X - K)}{(r + \lambda_e)} + \left(\left(1 - \frac{K - \widehat{w}}{\rho X} \right) p_0 X - \kappa(1) - \widehat{w} - \frac{\lambda_e p_0 (X - K)}{(r + \lambda_e)} \right) \left(\frac{w_t}{\widehat{w}} \right)^{\gamma_e}$$

[1] Since the maximum value of α is one, the term $(1 - \alpha) X$ in (5) remains constant for $w_0 \geq K$. Thus, the benefit of hoarding more than K decreases in w_0 for $w_0 \geq K$ and there is a certain hoarding level $w_0^* \geq K$, beyond which the manager pays out all additionally generated cash above w_0^*.

where γ_e is the positive root to $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r - \lambda_e = 0$. This gives the following first-order condition with respect to $\widehat{w}$

$$0 = \left(-\frac{\gamma_e}{\widehat{w}}\left(\left(1 - \frac{K - \widehat{w}}{\rho X}\right)p_0 X - \kappa(1) - \widehat{w} - \frac{\lambda_e p_0 (X - K)}{(r + \lambda_e)}\right) + \left(\frac{p_0}{\rho} - 1\right)\right)\left(\frac{w}{\widehat{w}}\right)^{\gamma_e}$$

implying that the optimal hoarding level is given by

$$w^* = \frac{\gamma_e}{(\gamma_e - 1)}\left(\frac{(\rho\kappa(1) + K - \rho X)p_0}{(p_0 - \rho)} + \frac{\lambda_e p_0 \rho (X - K)}{(r + \lambda_e)(p_0 - \rho)}\right).$$

Differentiating with respect to θ, we have

$$\frac{\partial w^*}{\partial \theta} = \frac{\gamma_e \rho p_0}{(\gamma_e - 1)(p_0 - \rho)}\frac{\partial X}{\partial \theta}\left(\frac{\lambda_e}{r + \lambda_e} - 1\right) < 0.$$

Q.E.D.

A.3 Market Timing with Time Varying Profitability

We now let the NPV of the investment opportunity change over time. This is a standard assumption in the related real options literature (Bolton et al., 2013). Define $\widetilde{K}_t := \rho\kappa(1) + K_t - \rho X_t$ and let

$$\frac{d\widetilde{K}}{\widetilde{K}} = \mu_{\widetilde{K}} dt + \sigma_{\widetilde{K}} dZ_{\widetilde{K}}$$

where $Z_{\widetilde{K}}$ is standard Brownian motion and $\sigma_{\widetilde{K}} > 0$ with a correlation ψ to Z (we assume that the change in $\widetilde{K}$ comes from K, but analogous arguments apply if it would come from X). We assume that $\mu_{\widetilde{K}} < 0$ implying that the NPV from the financier's perspective (i.e., $\rho X - K$) increases on average over time. Delay in investment could, then, occur for two reasons: delaying not only to hoard cash, but also to wait for the value of the investment opportunity to increase. Our results remain robust also in such a setting.

Proposition A.3 *Along the optimal investment barrier, the optimal cash level decreases when the firm's investment opportunity is better.*

Proof of Proposition A.3. Following similar steps to Proposition 1, the manager's expected payoff is the solution to the following partial differential equation

$$\begin{aligned} rU &= \mu w U_w + \frac{1}{2}\sigma^2 w^2 U_{ww} + \mu_{\widetilde{K}} \widetilde{K} U_{\widetilde{K}} \\ &\quad + \frac{1}{2}\sigma_{\widetilde{K}}^2 \widetilde{K}^2 U_{\widetilde{K}\widetilde{K}} + \psi\sigma\sigma_{\widetilde{K}} w \widetilde{K} U_{w\widetilde{K}} \end{aligned} \quad (A.1)$$

where the subscripts w and $\widetilde{K}$ denote the partials of U with respect to w and $\widetilde{K}$, respectively. Define $\chi = \frac{w}{\widetilde{K}}$ so that $U(w, \widetilde{K}) = \widetilde{K} U(\chi)$, where we use that U is homogenous of degree one in

$(w, \widetilde{K})$ (Doubling $\widetilde{K}$ and doubling w would merely double the manager's expected payoff). We have

$$\begin{aligned} U_w &= U_\chi;\ U_{ww} = \frac{1}{\widetilde{K}} U_{\chi\chi};\ U_{w\widetilde{K}} = -\frac{w}{\widetilde{K}^2} U_{\chi\chi} \\ U_{\widetilde{K}} &= U - \frac{w}{\widetilde{K}} U_\chi;\ U_{\widetilde{K}\widetilde{K}} = \frac{w^2}{\widetilde{K}^3} U_{\chi\chi}. \end{aligned}$$

Plugging into (A.1), we obtain the simple ordinary differential equation

$$\underbrace{\left(r - \mu_{\widetilde{K}}\right)}_{r'} U = \underbrace{\left(\mu - \mu_{\widetilde{K}}\right)}_{\mu'} \chi U_\chi + \underbrace{\left(\frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_{\widetilde{K}}^2 - \psi\sigma\sigma_{\widetilde{K}}\right)}_{\sigma'} \chi^2 U_{\chi\chi} \tag{A.2}$$

with a value matching condition $U(\chi^*) = \left(\frac{\chi^* - 1}{\rho}\right) - \chi^*$. Defining φ as the positive root to $\frac{1}{2}\sigma'^2 y(y-1) + \mu' y = r'$ (where r', μ' and σ' are defined in (A.2)), and following the same steps as in Section *3*, we obtain

$$\chi^* = \frac{w^*}{\rho\kappa(1) + K^* - \rho X} = \frac{\varphi}{\varphi - 1}\left(\frac{1}{1-\rho}\right).$$

We see, thus, that the optimal co-investment w and the NPV from the financier's point of view are in a constant proportion at the optimal investment barrier. Along this barrier, the optimal cash level w^* increases with the investment cost K^*, and this level is lower when the investment opportunities are better (high θ). **Q.E.D.**

A.4 Dilution of Effort Incentives and Hoarding

We have derived our results assuming that the manager and the financier have different visions. However, hoarding could also be caused by other financing frictions. We have already discussed information asymmetry. Another friction for growth firms is that external financing dilutes ownership and, hence, could reduce the manager's incentives to exert effort, increasing the cost of external financing (Holmstrom and Tirole, 1997).[2]

Specifically, suppose that conditional on being undertaken, the investment succeeds with probability e, in which case it yields X. With probability $1-e$, it fails and yields zero. Let the success probability e reflect the effort exerted by the manager at cost $\frac{e^2}{2\nu}$ after the investment is undertaken. To illustrate the main idea, assume that the firm raises equity from a passive financier. Hence, the financier needs to obtain a stake $\alpha = \frac{K-w}{e^* X}$ to break even, where e^* is the manager's equilibrium effort choice. Conditional on investing, the manager's problem is to choose the optimal level of effort $\widehat{e}$ that maximizes her expected payoff

$$\max_{\widehat{e}} \left(1 - \frac{K-w}{e^* X}\right) \widehat{e} X - w - \frac{\widehat{e}^2}{2\nu}. \tag{A.3}$$

[2] We thank Andrey Malenko for suggesting this discussion.

Hence, her effort choice is

$$\widehat{e} = \nu \left(1 - \frac{K - w}{e^* X}\right) X. \tag{A.4}$$

Since in equilibrium, we must have $e^* = \widehat{e}$, we obtain

$$e^* = \frac{1}{2}\nu X + \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu (K - w)}. \tag{A.5}$$

To avoid corner solutions, assume that ν is sufficiently small so that $e^* \leq 1$. We see immediately that the manager's effort e^* is increasing in her co-investment w. Furthermore, plugging (A.4) into (A.3), we obtain that, prior to investing, the manager chooses her optimal hoarding level w^* to solve

$$U = \max_{w^*} \left(\frac{(e^*)^2}{2\nu} - w^*\right) \left(\frac{w_t}{w^*}\right)^\beta. \tag{A.6}$$

It is now straightforward to show that this alternative setting leads to the same qualitative results as our baseline model. In particular, it continues to be true that firms with better investment opportunities hoard less, as delay is more costly for them.

Proposition A.4 *Consider a reformulation of our model as a moral hazard problem, in which the manager's effort determines the probability of the new investment's success. In this setting, hoarding reduces the dilution of the manager's effort incentives, and it continues to be true that firms with better investment opportunities hoard less.*

Proof of Proposition A.4. The optimal level of effort solving (A.5) is

$$e^* = \frac{1}{2}\nu X \pm \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu (K - w)} \tag{A.7}$$

Since, for any given w, the manager's payoff is increasing in e, the global maximum is at $e^* = \frac{1}{2}\nu X + \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu (K - w)}$. This proves (A.6). As usual, we obtain the solution for w^* from the manager's first-order condition

$$\left(-\frac{\beta}{\widehat{w}}\left(\frac{\left(\frac{1}{2}\nu X + \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu (K - \widehat{w})}\right)^2}{2\nu} - \widehat{w}\right) + \left(\frac{\nu X - \sqrt{(\nu X)^2 - 4\nu (K - \widehat{w})}}{2\sqrt{(\nu X)^2 - 4\nu (K - \widehat{w})}}\right)\right)\left(\frac{w}{\widehat{w}}\right)^\beta = 0.$$

By standard monotone comparative statics arguments, observe that $\frac{\partial^2 U}{\partial \widehat{w} \partial \theta} < 0$ implies that $\widehat{w}$ is decreasing in θ. Hence, as in our baseline model, firms with better investment opportunities hoard less cash. **Q.E.D.**