Pricing Risks across Currency Denominations
Online Appendix

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A Stationarity

In our model we assume (A4) sufficient stationarity in the exchange rate processes. In particular, the assumption is that the composition of the PCs and the market prices of risks are constant through time. We validate this assumption using bootstrapping. We resample (with replacements) our monthly exchange rate growth data and construct the first two PCs and estimate the market price of risks. We repeat this 10,000 times and construct the distributions of the loadings of the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ on our 11 currencies and the FX market implied market prices $\hat{\gamma}_1^J$ and $\hat{\gamma}_2^J$ associated with the two PCs.

We first analyze the stationarity of the decomposition of the first two PCs. Figure A.1 reports the averages and the intervals spanned by the 5 and 95 percentiles of the loadings of the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ on our 10 exchange rates J/USD and (denoted by US) 1 minus the sum of the loadings on the 10 exchange rates. The 90% confidence interval is small indicating that the composition of the first two PCs does not vary much across the bootstrap samples.

Next, we analyze the distribution of the market prices of risks of the first two PCs. Figure A.2 reports the averages and the intervals spanned by the 5 and 95 percentiles of $\hat{\gamma}_1^J$ and $\hat{\gamma}_2^J$. Although the confidence intervals are wider than in the case of the PC decompositions, the variation of estimated market prices across bootstrap samples is relatively small. The standard deviations across bootstrap samples are more than one order of magnitude smaller than the averages of $\hat{\gamma}_1^J$

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Decomposition of the First Two Principal Components

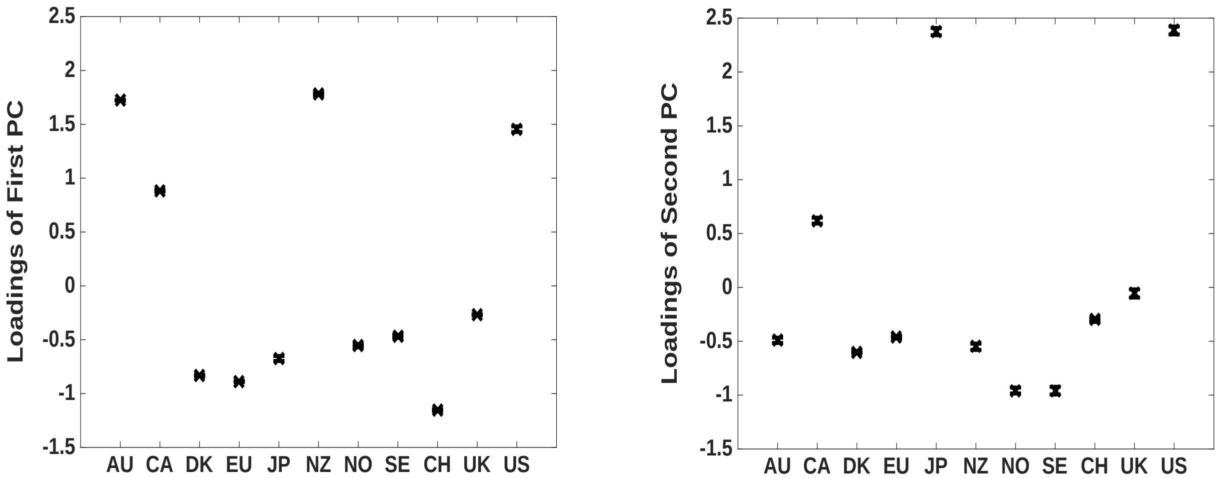

Figure A.1: Averages (indicated by x) and intervals spanned by the 5 and 95 percentiles of the loadings of the first PC $\overline{\Pi}_{t,1}$ (left) and second PC $\overline{\Pi}_{t,2}$ (right) on the 10 exchange rates J/USD and (denoted by US) 1 minus the sum of the loadings on the 10 exchange rates. Averages and percentiles are constructed from 10,000 bootstrap samples.

and $\hat{\gamma}_2^J$. This indicates that the estimated market prices are relatively stable across bootstrap samples. Moreover, Figure A.3 further characterizes the distribution of $\hat{\gamma}_1^{US}$ and $\hat{\gamma}_2^{US}$ from a US perspective in more detail. Figure A.3 confirms that the estimates across bootstrap samples are very much concentrated around the mean.

To sum up, the decomposition of our two PCs and the estimated market prices do not vary much across the bootstrap samples. We interpret this in favor of our assumption (A4) which requires sufficient stationarity to estimate our proposed model.

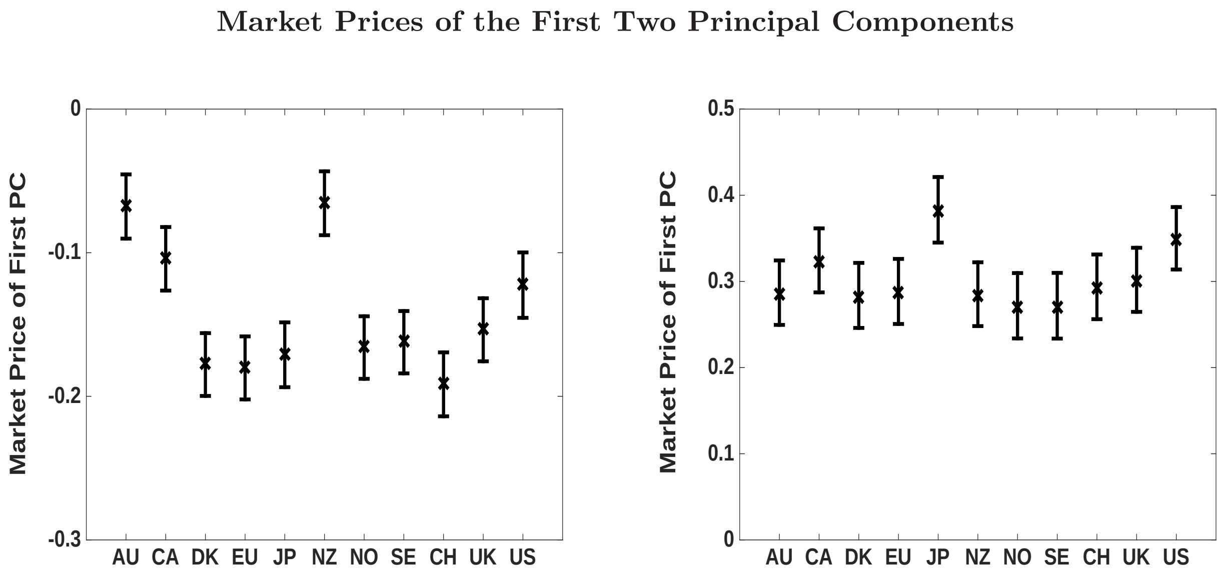

Figure A.2: Averages (indicated by x) and intervals spanned by the 5 and 95 percentiles of the market prices $\hat{\gamma}_1^J$ (left) and $\hat{\gamma}_2^J$ (right) for 11 developed countries J. Averages and percentiles are constructed from 10,000 bootstrap samples.

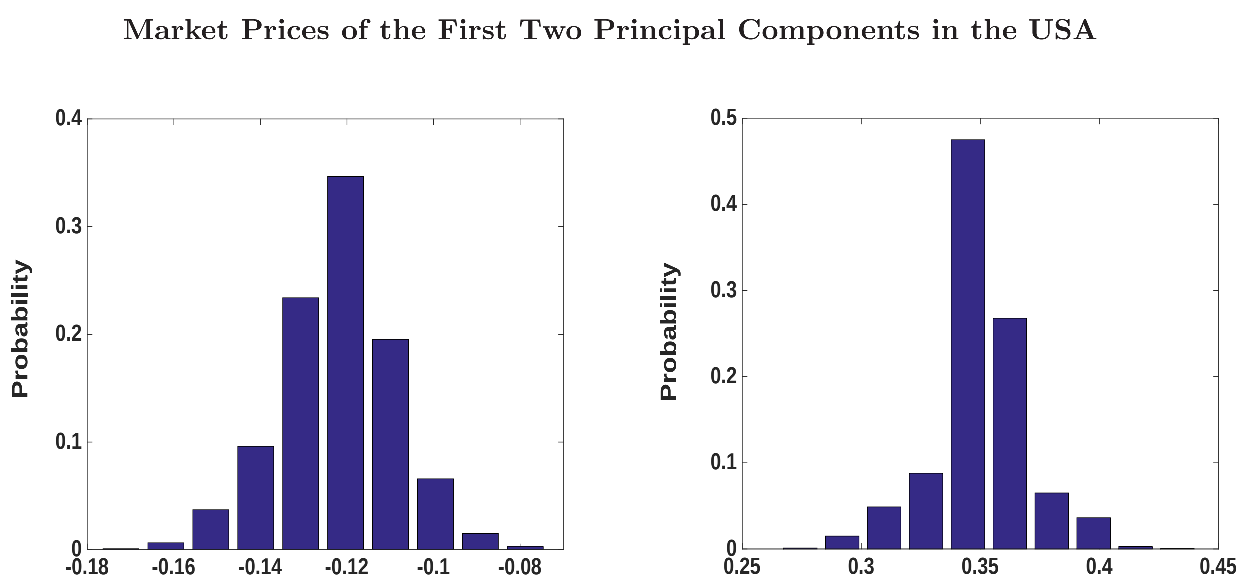

Figure A.3: Distribution of US market prices $\hat{\gamma}_1^{US}$ (left) and $\hat{\gamma}_2^{US}$ (right) constructed from 10,000 bootstrap samples.

B Eigenvalue and Growth Ratios

Table B.1 provides values of the Eigenvalue Ratio $ER(k)$ and the Growth Ratio $GR(k)$ for $k \in \{1, \ldots, k_{max}\}$ (with $k_{max} = 6$) in our PCA on 55 bilateral exchange rate growths. $ER(k)$ and $GR(k)$ both reach the maximum at $k = 2$.

Table B.1: Eigenvalue and Growth Ratios

k	$ER(k) = \frac{\lambda_k}{\lambda_{k+1}}$	$GR(k) = \frac{\ln(1+\lambda_k/V(k))}{\ln(1+\lambda_{k+1}/V(k+1))}$
1	1.587	1.050
2	1.713	1.151
3	1.430	1.040
4	1.189	0.896
5	1.205	0.871
6	1.588	1.040

Notes: The Table provides the value of the Eigenvalue Ratio $ER(k) = \frac{\lambda_k}{\lambda_{k+1}}$ and the Growth Ratio $GR(k) = \frac{\ln(1+\lambda_k/V(k))}{\ln(1+\lambda_{k+1}/V(k+1))}$ for the first $k_{max} = 6$ PCs of the $P = 55$ bilateral exchange rate growths of 11 developed currencies, where $V(j) = \sum_{i=j+1}^{P} \lambda_i$ and λ_j is the eigenvalue associated with the jth PCs.

C Time-Series of SDFs

Table C.2 provides augmented Dickey-Fuller test statistics for the time series $\ln\left(\widehat{M}_{t,J}\right)$, $\ln\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)$ and the errors $e_{t,J}$ from the Engle and Granger (1987) regression $\ln\left(\widehat{M}_{t,J}\right) = a + b\ln\left(\widehat{M}_{t,US}\right) + e_{t,J}$.[1] All tests include a constant and linear time trend.

[1] The table reports critical values for a Dickey-Fuller test including a constant and a linear trend. The critical values are very similar to the values from surface response functions provided by MacKinnon (1996) and our conclusions are unaffected, whether we use the standard Dicky-Fuller or the critical values of MacKinnon (1996). The critical values provided by MacKinnon (1996) are -3.9638, -3.4126 and -3.1279 for significance on the 1%, 5% and 10% levels.

Table C.2: Dickey-Fuller Tests

Country	$\ln\left(\widehat{M}_{t,J}\right)$	(p-value)	$\ln\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)$	(p-value)	$e_{t,J}$	(p-value)
Australia	-2.024	(0.581)	-86.406***	(0.000)	-4.200***	(0.005)
Canada	-2.171	(0.508)	-86.476***	(0.000)	-2.618	(0.287)
Denmark	-2.543	(0.324)	-86.551***	(0.000)	-2.297	(0.446)
Euro	-2.474	(0.358)	-86.552***	(0.000)	-4.499***	(0.002)
Japan	-2.104	(0.542)	-86.537***	(0.000)	-2.704	(0.244)
New Zealand	-2.052	(0.567)	-86.398***	(0.000)	-5.859***	(0.001)
Norway	-2.427	(0.381)	-86.553***	(0.000)	-1.914	(0.636)
Sweden	-2.521	(0.335)	-86.554***	(0.000)	-1.900	(0.642)
Switzerland	-2.443	(0.373)	-86.547***	(0.000)	-4.840***	(0.001)
UK	-2.364	(0.412)	-86.553***	(0.000)	-1.687	(0.748)
USA	-2.142	(0.523)	-86.494***	(0.000)		

Notes: provides augmented Dickey-Fuller test statistics for the time series $\ln\left(\widehat{M}_{t,J}\right)$, $\ln\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)$ and the errors $e_{t,J}$ from the Engle and Granger (1987) regression $\ln\left(\widehat{M}_{t,J}\right) = a + b\ln\left(\widehat{M}_{t,US}\right) + e_{t,J}$. All tests include a constant and linear time trend. Significance of the slope coefficients at the 1%, 5% and 10% level are indicated by ***, ** and *.

D Non-Parametric Decomposition of the SDF

In this seciton we describe the non-parametric approach of Christensen (2017) to decompose a SDF into a permanent and transitory component. We keep the our description brief and refer to Christensen (2017) and his references for details.

Alvarez and Jermann (2005) introduce a decomposition of SDF M_t into a permanent component M_t^P and a transitory component M_t^T such that $M_t = M_t^P M_t^T$ or in terms of growths,

$$\frac{M_{t+\tau}}{M_t} = \frac{M_{t+\tau}^P}{M_t^P}\frac{M_{t+\tau}^T}{M_t^T}.$$

Let the n-dimensional state variable X be a time-homogeneous, strictly stationary, and ergodic Markov process. $\mathbb{M}$ is a one-period pricing operator such that the price at time t of an asset with payoff $\psi(X_{t+1})$ at time $t+1$ is

$$\mathbb{M}\psi(x) = E\left[\frac{M_{t+1}(X_{t+1})}{M_t(X_t)}\psi(X_{t+1})\middle| X_t = x\right].$$

Hansen and Scheinkman (2009) provide conditions such that solving the Perron-Frobenius eigenfunction problem $\mathbb{M}\phi(x) = \rho\phi(x)$, (where the eigenvalue ρ is a positive scalar and the eigenfunction ϕ is positive) yields the decomposition

$$\frac{M^P_{t+\tau}}{M^P_t} = \rho^{-\tau}\frac{M_{t+\tau}}{M_t}\frac{\phi(X_{t+\tau})}{\phi(X_t)}, \qquad \frac{M^T_{t+\tau}}{M^T_t} = \rho^{\tau}\frac{\phi(X_t)}{\phi(X_{t+\tau})}.$$

The permanent component $M^P_t = E_t\left[M^P_{t+\tau}\right]$ is a martingale and $-\ln(\rho)$ may be interpreted as the yield on a long term bond (with infinite maturity).

Christensen (2017) proposes a sieve approach to reduce the infinite-dimensional eigenfunction problem to a low-dimensional eigenvector problem. In particular, he defines the basis functions $b_{k1}, \ldots, b_{kk}$ to approximate the state space spanned by X. The projection of the eigenfunction problem onto the linear subspace spanned by $b_{k1}, \ldots, b_{kk}$ is

$$\begin{aligned} G_k^{-1}M_k c_k &= \rho_k c_k \\ G_k &= \mathbb{E}[b^k(X_t)b^k(X_t')] \\ M_k &= \mathbb{E}[b^k(X_t)\frac{M_{t+1}(X_{t+1})}{M_t(X_t)}b^k(X_{t+1})'], \end{aligned}$$

where ρ_k is the largest real eigenvalue and c_k the associated eigenvector of matrix $G_k^{-1}M_k$. ρ_k and $\phi_k(X) = b^k(X)'c_k$ with vector $b^k(x) = (b_{k1}(x), b_{k2}(x), \ldots, b_{kk}(x))'$ are the approximate solution of the eigenvalue ρ and eigenfunction ϕ solving the Perron-Frobenius problem.

In our application we use the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ as the 2-dimensional state variable X. We choose Hermite polynomials of degree five as basis functions for each PC, then construct a tensor product basis from the univariate bases and discard tensor product polynomials whose total degree is order six or higher. The resulting sparse basis $b_{k1}, \ldots, b_{kk}$ has dimension $k = 15$.

Following Christensen (2017), the sample estimators of matrices G_k and M_k are

$$\widehat{G} = \frac{1}{T}\sum_{t=0}^{T-1} b^k(X_t)b^k(X_t)',$$
$$\widehat{M} = \frac{1}{T}\sum_{t=0}^{T-1} b^k(X_t)\frac{\widehat{M}_{t+1}}{\widehat{M}_t}b^k(X_{t+1})'.$$

E Data Sources

Table E.3: Spot and Forward Exchange Rate Datastream Tickers

Country	Abbr.	Source	Spot Rate	Forward Rate
Australia	AU	Barclays	BBAUDSP(ER)	BBAUD1F(ER)
Canada	CA	Barclays	BBCADSP(ER)	BBCAD1F(ER)
Denmark	DK	Barclays	BBDKKSP(ER)	BBDKK1F(ER)
Eurozone	EU	WMR	EUDOLLR(ER)	EUDOL1F(ER)
Germany	DE	WMR	DMARKE$(ER)	USDEM1F(ER)
Japan	JP	Barclays	BBJPYSP(ER)	BBJPY1F(ER)
New Zealand	NZ	Barclays	BBNZDSP(ER)	BBNZD1F(ER)
Norway	NO	Barclays	BBNOKSP(ER)	BBNOK1F(ER)
Sweden	SE	Barclays	BBSEKSP(ER)	BBSEK1F(ER)
Switzerland	CH	Barclays	BBCHFSP(ER)	BBCHF1F(ER)
United Kingdom	UK	WMR	UKDOLLR(ER)	UKUSD1F(ER)

Table E.4: MSCI Total Stock Market Return Index Datastream Tickers

Country	Total Return	Country	Total Return
Australia	MSAUSTL(MSRI)	New Zealand	MSNZEAL(MSRI)
Canada	MSCNDAL(MSRI)	Norway	MSNWAYL(MSRI)
Denmark	MSDNMKL(MSRI)	Sweden	MSSWDNL(MSRI)
Eurozone	MSEMUIE(MSRI)	Switzerland	MSSWITL(MSRI)
Germany	MSGERML(MSRI)	United Kingdom	MSUTDKL(MSRI)
Japan	MSJPANL(MSRI)	United States	MSUSAML(MSRI)

Table E.5: Global Financial Data Total Return Index Tickers

Country	Total Return	Yield	Country	Total Return	Yield
Australia	TRAUSGVM	IGAUS10D	New Zealand	TRNZLGVD	IGNZL10D
Canada	TRCANGVM	IGCAN10D	Norway	TRNORGVM	IGNOR10D
Denmark	TRDNKGVM	IGDNK10D	Sweden	_RXTBD	IGSWE10D
Eurozone	TREURGVM	IGEUR10D	Switzerland	_SDGTD	IGCHE10D
Germany	TRDEUGVM	IGDEU10D	United Kingdom	TRGBRGVM	IGGBR10D
Japan	IGJPN10D	TRJPNGVM	United States	TRUSG10M	IGUSA10D

F Technical Details and Proofs

F.1 Diffusion Risk Model of FX Markets

Exchange rates: Consider a random and traded payoff Y_{t+dt} to be realized at $t+dt$ ($Y_{t+dt} \in \mathcal{F}_{t+dt}$) in units of currency I. Let the exchange rate $EX_{t,J/I}$ be the number of units of currency J which buys one unit of currency I at time t. Then time-t value Y_t of payoff Y_{t+dt} can either be computed directly in currency I (using I's SDF M_I), or in currency J (using J's SDF M_J) and exchanged back to currency I. That is,

$$E_t\left[\frac{M_{t+dt,I}}{M_{t,I}}Y_{t+dt}\right] = Y_t = \frac{1}{EX_{t,J/I}}E_t\left[\frac{M_{t+dt,J}}{M_{t,J}}\left(EX_{t+dt,J/I}Y_{t+dt}\right)\right].$$

Assuming complete financial markets so that SDFs M are unique, and the above pricing equations hold for a complete set of traded (Arrow-Debreu) assets Y. As a result, the exchange rate unambiguously is the ratio of the two SDFs, $EX_{t,J/I} = \frac{M_{t,I}}{M_{t,J}}$, $\forall t$, which is equation (2) in the main paper

$$\frac{dEX_{t,J/I}}{EX_{t,J/I}} = \left[r_J - r_I + \eta_J^T \Delta\eta_{J/I}\right]dt + \Delta\eta_{J/I}^T dZ_t, \qquad \text{where} \qquad \Delta\eta_{J/I} \equiv \eta_J - \eta_I. \tag{F.1}$$

Realized carry trade excess returns: Consider the following net-zero strategy denominated in currency I: (i) at time t, borrow $\frac{M_{t,I}}{M_{t,B}}$ units of currency B (worth one unit of currency I) paying interest rate r_B, and simultaneously lend $\frac{M_{t,I}}{M_{t,L}}$ units of currency L (also worth one unit of currency I) earning interest rate r_L, (ii) at time $t+dt$, close all positions and convert the proceeds to denomination currency I. The realized excess return of the strategy is, after using differential representation (1) of the main paper ($\frac{dM_{t,I}}{M_{t,I}} = -r_I dt - \eta_I^T dZ_t, \forall I, t$) and applying Ito's lemma,

$$\begin{aligned}
CT^I_{t+dt,-B/+L} &= \frac{M_{t,I}}{M_{t,L}}(1+r_L dt)\frac{M_{t+dt,L}}{M_{t+dt,I}} - \frac{M_{t,I}}{M_{t,B}}(1+r_B dt)\frac{M_{t+dt,B}}{M_{t+dt,I}} \\
&= \frac{M_{t,I}}{M_{t+dt,I}} \times \left[\frac{M_{t+dt,L}}{M_{t,L}}(1+r_L dt) - \frac{M_{t+dt,B}}{M_{t,B}}(1+r_B dt)\right] \\
&= \frac{1}{1 - r_I dt - \eta_I^T dZ_t} \times \left[(1 - r_L dt - \eta_L^T dZ_t)(1+r_L dt) - (1 - r_B dt - \eta_B^T dZ_t)(1+r_B dt)\right] \\
&= \left[1 + (r_I + ||\eta_I||^2)dt + \eta_I^T dZ_t\right] \times \left[\eta_B^T dZ_t - \eta_L^T dZ_t\right] = \eta_I^T\left(\eta_B - \eta_L\right)dt + \left(\eta_B^T - \eta_L^T\right)dZ_t,
\end{aligned}$$

which yields equation (3) of the main paper, ie.

$$\begin{aligned}
CT^I_{t+dt,-B/+L} &= \eta_I^T \Delta\eta_{B/L} dt + \Delta\eta^T_{B/L} dZ_t, \\
ECT^I_{-B/+L} &= \eta_I^T \Delta\eta_{B/L} dt, \qquad \Delta\eta_{B/L} \equiv \eta_B - \eta_L.
\end{aligned} \tag{F.2}$$

F.2 Details on PCA and the FX-Based SDF Estimates

We begin with the mean-zero innovations $X_{t,J/I} \equiv \left(\eta_J^T - \eta_I^T\right) dZ_t$ of exchange rate growths, (F.1),

$$X_{t,J/I} = \sum_i^n dZ_{t,i}(\eta_{J,i} - \eta_{I,i}) = \sum_i^n dZ_{t,i}\eta_{J/I,i}; \quad t \in [0,s]; \quad \forall J/I \in \mathcal{P}, \tag{F.3}$$

$$\text{where:} \qquad \eta_{J/I} \equiv (\eta_J - \eta_I) \in \mathbf{R}^n; \quad \forall J/I \in \mathcal{P}.$$

Above, $dZ_{t,i}$ is a normally distributed random variable with mean zero and variance dt, index $i \in \{1, \ldots, n\}$ denotes n such independent risks in the setting, and $\mathcal{P}$ denotes the set of $P \equiv dim\mathcal{P}$ bilateral exchange rates (i.e., currency pair $\{J/I\}$) in the data (see (F.4) below). For each $J/I \in \mathcal{P}$, let $s \times 1$ column vector $X_{J/I}$ denote the *demeaned* exchange rate growth time series (F.3), where s is the number of observations in each time series. We arrange these P time series into P columns of $s \times P$ matrix $X = [X_1; \ldots; X_P]$ (F.4). Hence, $\frac{1}{dt}$ is the number of observations for each exchange rate time series per year, and $s \times dt$ is the length of each time series in years. Similarly, for country each currency pair $J/I \in \mathcal{P}$, let $n \times 1$ column vector $\Delta\eta_{J/I}$ denote the differential prices of prices of n risks across the two respective currencies (F.2), $\Delta\eta_{k,J/I} \equiv \eta_{k,J} - \eta_{k,I}$, $k \in \{1, \ldots, n\}$. Finally, for each $t \in \{1, \ldots, s\}$, let $1 \times n$ row vector $dZ_t \equiv [dZ_{t,1}, \ldots, dZ_{t,n}]$ denote the n contemporaneous innovations. Therefore, we have explicitly,

$$X \equiv \begin{bmatrix} X_{1,1} & & X_{1,J/I} & & X_{1,P} \\ \vdots & & \vdots & & \vdots \\ X_{t,1} & \ldots & X_{t,J/I} & \ldots & X_{t,P} \\ \vdots & & \vdots & & \vdots \\ X_{s,1} & & X_{s,J/I} & & X_{s,P} \end{bmatrix} =$$

$$\begin{bmatrix} dZ_{1,1} & & dZ_{1,k} & & dZ_{1,n} \\ \vdots & & \vdots & & \vdots \\ dZ_{t,1} & \ldots & dZ_{t,k} & \ldots & dZ_{t,n} \\ \vdots & & \vdots & & \vdots \\ dZ_{s,1} & & dZ_{s,k} & & dZ_{s,n} \end{bmatrix} \times \begin{bmatrix} \Delta\eta_{1,1} & & \Delta\eta_{1,J/I} & & \Delta\eta_{1,P} \\ \vdots & & \vdots & & \vdots \\ \Delta\eta_{k,1} & \ldots & \Delta\eta_{k,J/I} & \ldots & \Delta\eta_{k,P} \\ \vdots & & \vdots & & \vdots \\ \Delta\eta_{n,1} & & \Delta\eta_{n,J/I} & & \Delta\eta_{n,P} \end{bmatrix} \equiv dZ \times \Delta\eta. \quad \text{(F.4)}$$

Since the sum of each column of X is zero (the law of large numbers), the symmetric matrix $X^T X$ is proportional to the empirical (i.e., sample) covariance matrix of the exchange rate fluctuations.

In the PCA, we solve for the eigenvalues and eigenstates of this $P \times P$ empirical covariance matrix $X^T X$ to identify and sort out the most important risks in FX markets. Because $X^T X$ is symmetric, it can be diagonalized by an $P \times P$ orthogonal matrix W (that is $W^T W = W W^T = \mathbf{1}_{P\times P}$): $W^T \left[X^T X\right] W = \text{Diag}[\lambda_1; \ldots; \lambda_P]$. Note that because $dZ^T dZ = s \times dt \times \mathbf{1}_{n\times n}$, then by virtue of (F.4), the same orthogonal matrix W also diagonalizes $\Delta\eta^T \Delta\eta$ (that is, $W^T \Delta\eta^T \Delta\eta W = s \times dt \times \text{Diag}[\lambda_1; \ldots; \lambda_P]$). We can also define $n \times P$ score matrix $\Pi \equiv XW$, which then satisfies, $\Pi^T \Pi = \text{Diag}[\lambda_1; \ldots; \lambda_P]$, or columns of matrix Π are pairwise orthogonal.

In PCA literature, elements and columns of orthogonal matrix W are respectively referred to as *loadings* and loading vectors, and columns of Π are P principal components when (without loss of generality) eigenvalues are arrange in descending order of magnitudes $\lambda_1 \geq \ldots \geq \lambda_P$ (which we also do here). For the notational convenience in our asset pricing tests, however, we work with rescaled $P \times P$ loading matrix $\overline{W} \equiv W\text{Diag}\left[\sqrt{\lambda_1}; \ldots; \sqrt{\lambda_P}\right]$ (so that $\overline{W}^T\overline{W} = \text{Diag}[\lambda_1; \ldots; \lambda_P]$), rescaled $n \times P$ score matrix $\overline{\Pi} \equiv \Pi\,\text{Diag}\left[\frac{1}{\sqrt{\lambda_1}}; \ldots; \frac{1}{\sqrt{\lambda_P}}\right]$ (so that $\overline{\Pi}^T\overline{\Pi} = \mathbf{1}_{P\times P}$), and rescaled differential price of risk matrix $\Delta\overline{\eta} \equiv \Delta\eta W\text{Diag}\left[\frac{1}{\sqrt{\lambda_1}}; \ldots; \frac{1}{\sqrt{\lambda_P}}\right]$ (so that $\Delta\overline{\eta}^T\Delta\overline{\eta} = \mathbf{1}_{P\times P}$), which explain the definitions in equation (4) of the main paper. In the paper, we refer to K-th column of matrix $\overline{\Pi}$ as the K-th (rescaled) principal component.

Employing matrix (stacking) notation, the linear system determining principal factor prices γ's,

$\frac{1}{dt}ECT^I_{-B/+L} = \sum_{C/D\in\mathcal{P}} \gamma^I_{C/D}\overline{W}_{B/L,C/D}, \forall B/L \in \mathcal{P}$, can be written as,

$$\frac{1}{dt}\begin{bmatrix} ECT^I_1 \\ \vdots \\ ECT^I_{-B/+L} \\ \vdots \\ ECT^I_P \end{bmatrix} = \begin{bmatrix} \overline{W}_{1,1} & \dots & \overline{W}_{1,C/D} & \dots & \overline{W}_{1,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{W}_{B/L,1} & \dots & \overline{W}_{B/L,C/D} & \dots & \overline{W}_{B/L,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{W}_{P,1} & \dots & \overline{W}_{P,C/D} & \dots & \overline{W}_{P,P} \end{bmatrix} \begin{bmatrix} \gamma^I_1 \\ \vdots \\ \gamma^I_{C/D} \\ \vdots \\ \gamma^I_P \end{bmatrix} \tag{F.5}$$

From this follow the OLS estimates

$$\widehat{\gamma}^I = \frac{1}{dt}\left(\overline{W}^T\overline{W}\right)^{-1}\overline{W}^T ECT^I. \tag{F.6}$$

Similar to (F.5), we can also stack the (time series) regression equations in the Fama-MacBeth first stage into,

$$\begin{bmatrix} CT^I_{1,1} & \dots & CT^I_{1,-B/+L} & \dots & CT^I_{1,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ CT^I_{t,1} & \dots & CT^I_{t,-B/+L} & \dots & CT^I_{t,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ CT^I_{s,1} & \dots & CT^I_{s,-B/+L} & \dots & CT^I_{s,P} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \overline{\Pi}_{1,1} & \dots & \overline{\Pi}_{1,K} & \dots & \overline{\Pi}_{1,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{\Pi}_{t,1} & \dots & \overline{\Pi}_{t,K} & \dots & \overline{\Pi}_{t,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{\Pi}_{s,1} & \dots & \overline{\Pi}_{s,K} & \dots & \overline{\Pi}_{s,P} \end{bmatrix}}_{\equiv\overline{\Pi}} \times \underbrace{\begin{bmatrix} b^I_{1,1} & \dots & b^I_{1,B/L} & \dots & b^I_{1,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ b^I_{K,1} & \dots & b^I_{K,B/L} & \dots & b^I_{K,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ b^I_{P,1} & \dots & b^I_{P,B/L} & \dots & b^I_{P,P} \end{bmatrix}}_{\equiv b^I} \tag{F.7}$$

Clearly, B/L-th column of matrix b^I denotes the P loadings of the carry trade strategy $CT^I_{t,-B/+L}$ (borrowing B, lending L, denominated in I) on P respective principal factors (i.e., P columns of the score matrix $\overline{\Pi}$). Therefore, if γ^I_K, $K \in \mathcal{P}$ denote the factor prices of the respective principal factors (nominated in currency I), and $\widehat{b}^I$'s denote the estimates of factor loadings b^I's, the expected carry trade return reads,

$$ECT^I_{-B/+L} = \sum_{K\in\mathcal{P}} \widehat{b}^I_{B/L,K}\gamma^I_K.$$

We can stack these strategies for all currency pairs in the set $\mathcal{P}$ and obtain in the matrix form,

$$\begin{bmatrix} ECT^I_1 \\ \vdots \\ ECT^I_{-B/+L} \\ \vdots \\ ECT^I_P \end{bmatrix} = \underbrace{\begin{bmatrix} \widehat{b}^I_{1,1} & \dots & \widehat{b}^I_{1,K} & \dots & \widehat{b}^I_{P,1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \widehat{b}^I_{B/L,1} & \dots & \widehat{b}^I_{B/L,K} & \dots & \widehat{b}^I_{B/L,K} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \widehat{b}^I_{1,P} & \dots & \widehat{b}^I_{K,P} & \dots & \widehat{b}^I_{P,P} \end{bmatrix}}_{\widehat{b}^{I,T}} \times \underbrace{\begin{bmatrix} \gamma^I_1 \\ \vdots \\ \gamma^I_K \\ \vdots \\ \gamma^I_P \end{bmatrix}}_{\gamma^I} = \left(\widehat{b}^I\right)^T \gamma^I. \tag{F.8}$$

This is the basis for the cross-sectional regressions in the Fama-MacBeth second stage on expected carry trade returns ECT on factor loading estimates $\widehat{b}^I$'s. From this follow the OLS principal factor price estimates, $\widehat{\gamma}^I = (\widehat{b}^I \, \widehat{b}^{I,T})^{-1} \, \widehat{b}^I \, ECT^I$, which is identical to our PCA-based estimates $(\overline{W}^T \, \overline{W})^{-1} \, \overline{W}^T \, ECT^I$ (F.6) (because of the identity $\widehat{b}^I = \overline{W}^T$).

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